Defining Chiral Gauge Theories Beyond Perturbation Theory

Lattice Regulating Chiral Gauge Theories

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Work done with David B. Kaplan:
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arXiv:1610.02151 (accepted to PRD)

Parity Violation in Standard Model

First conclusively seen in decay of ^{60}Co

$$^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + e^{-} + \bar{\nu}_e + 2\gamma$$

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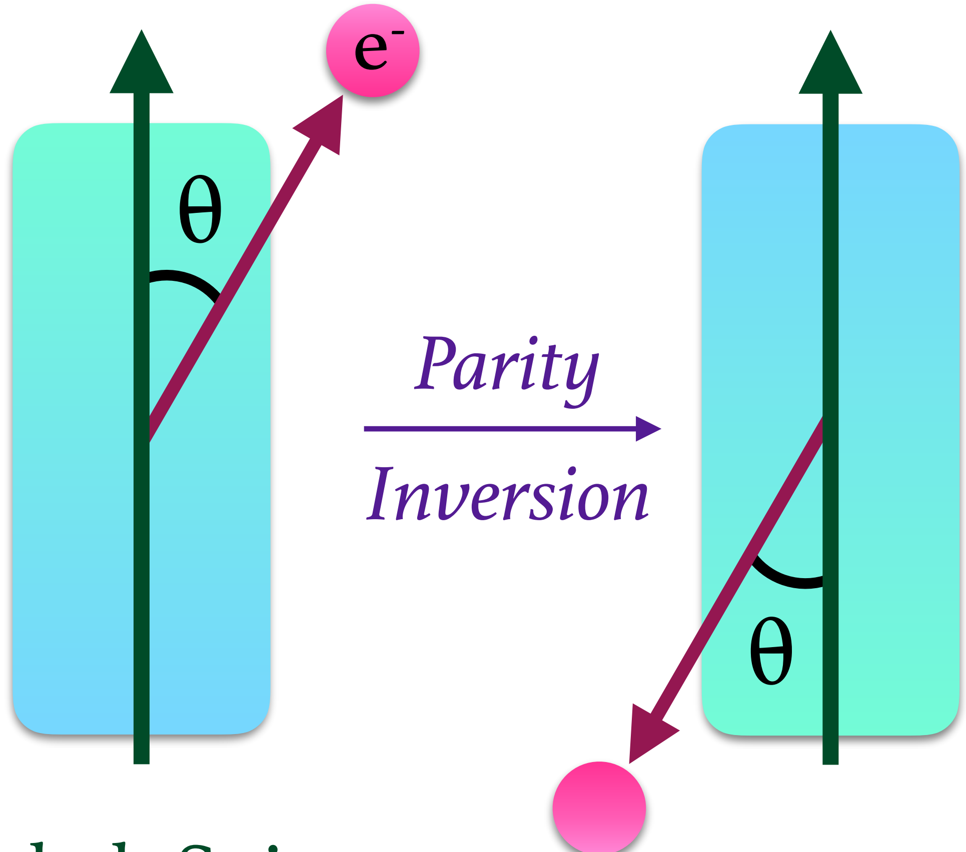
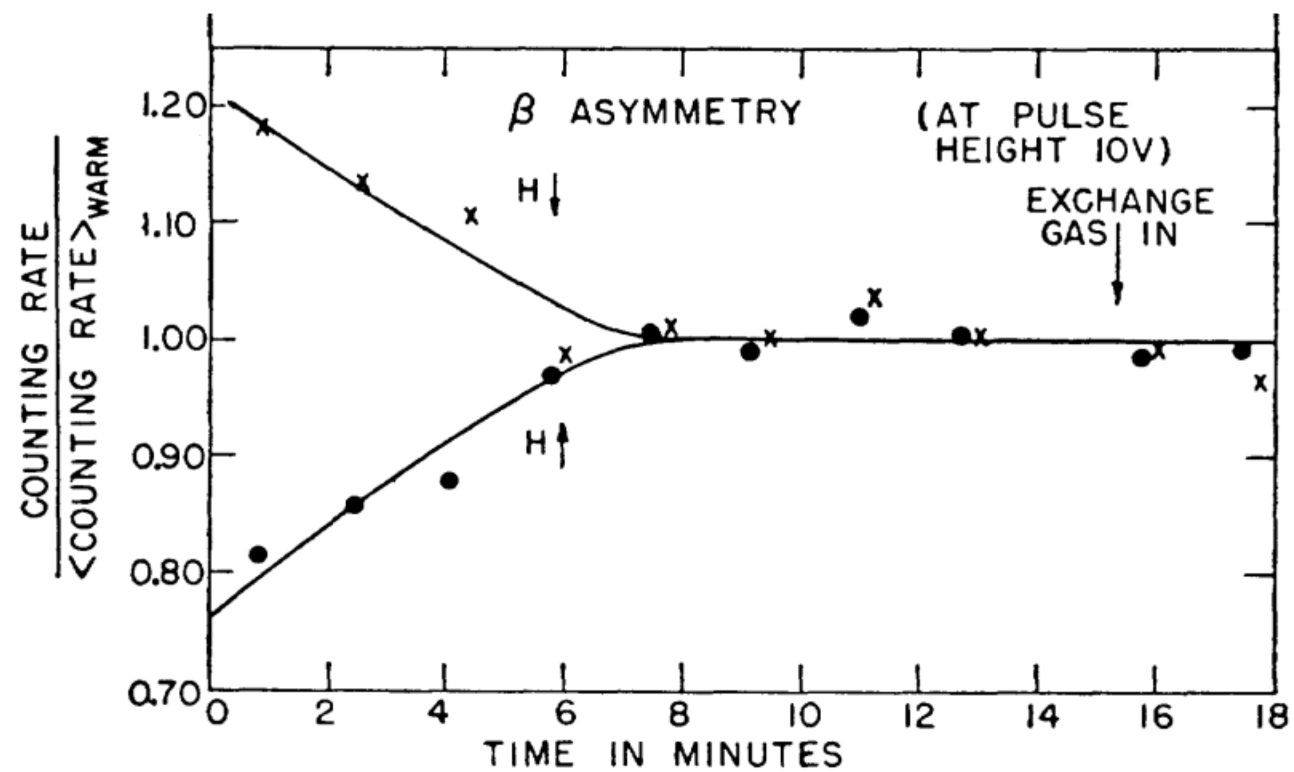


Cobalt Spin Orientation



Phys. Rev. 105, 1413

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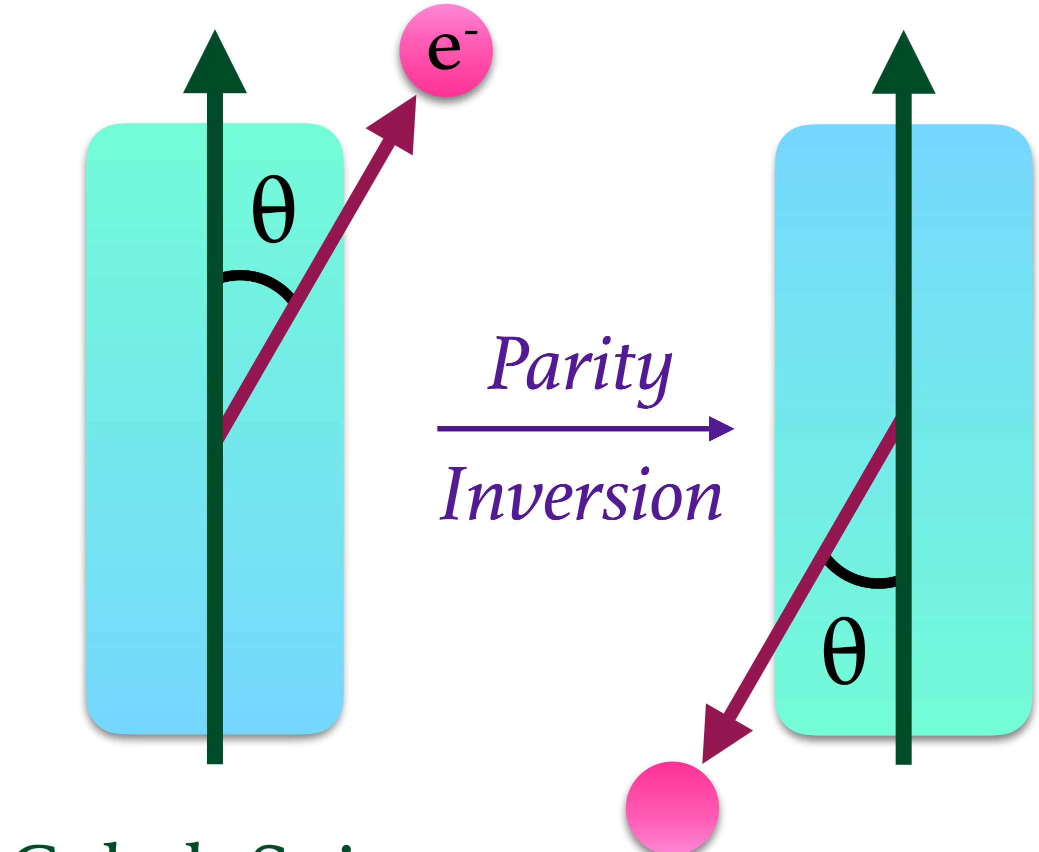

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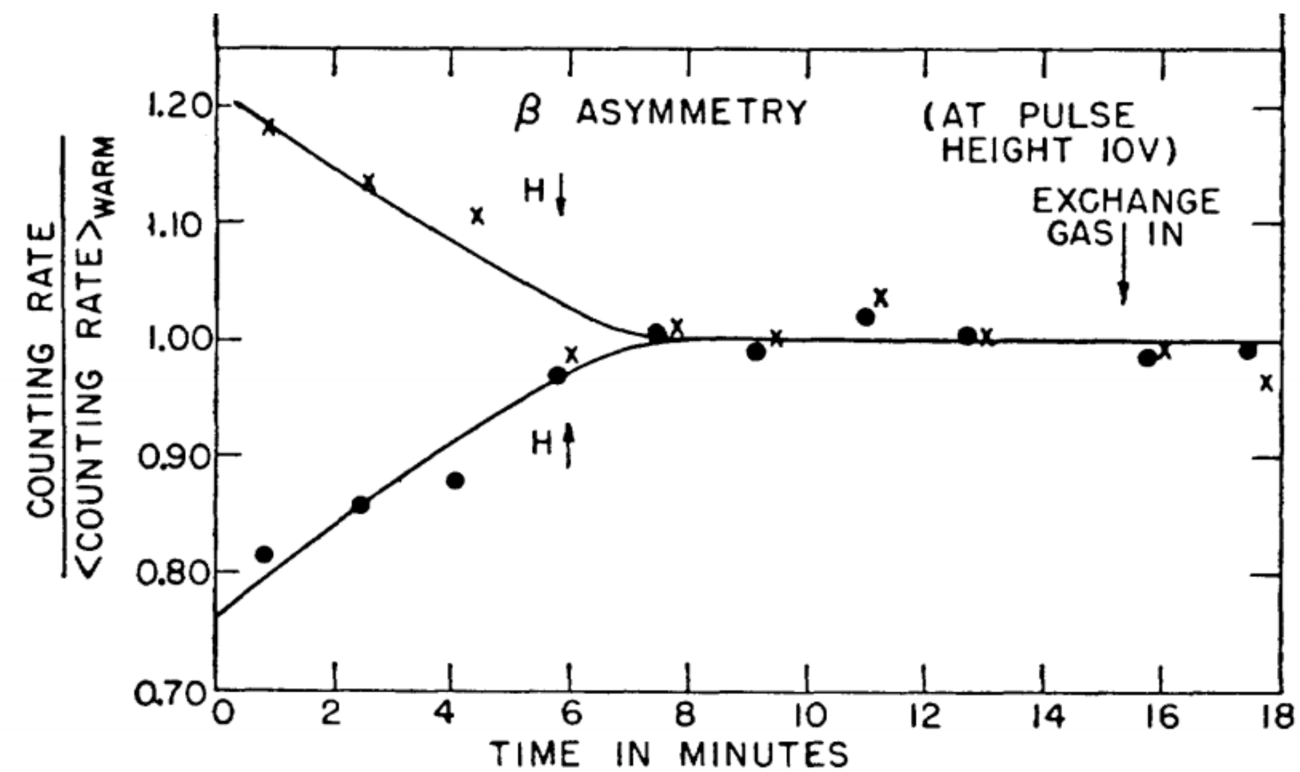

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Electroweak interactions differentiate between left-handed and right-handed fermions

Motivation

Big Question: What are the necessary ingredients of a well-defined chiral gauge theory

- Experimental tests of Standard Model only probe weakly coupled chiral gauge theories
- No experimental access to nonperturbative regime of chiral gauge theories
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To answer these, must first find a nonperturbative regulator

Vector vs Chiral Gauge Theories

Vector Theory (QED, QCD)

- **Real** fermion representation
- Gauge symmetries **allow** fermion mass term
- Gauge-invariant massive regulator (Pauli-Villars) **can** be used
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Need to control UV divergences in gauge-invariant manner

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Anomalies

Quantum violations of classical symmetry

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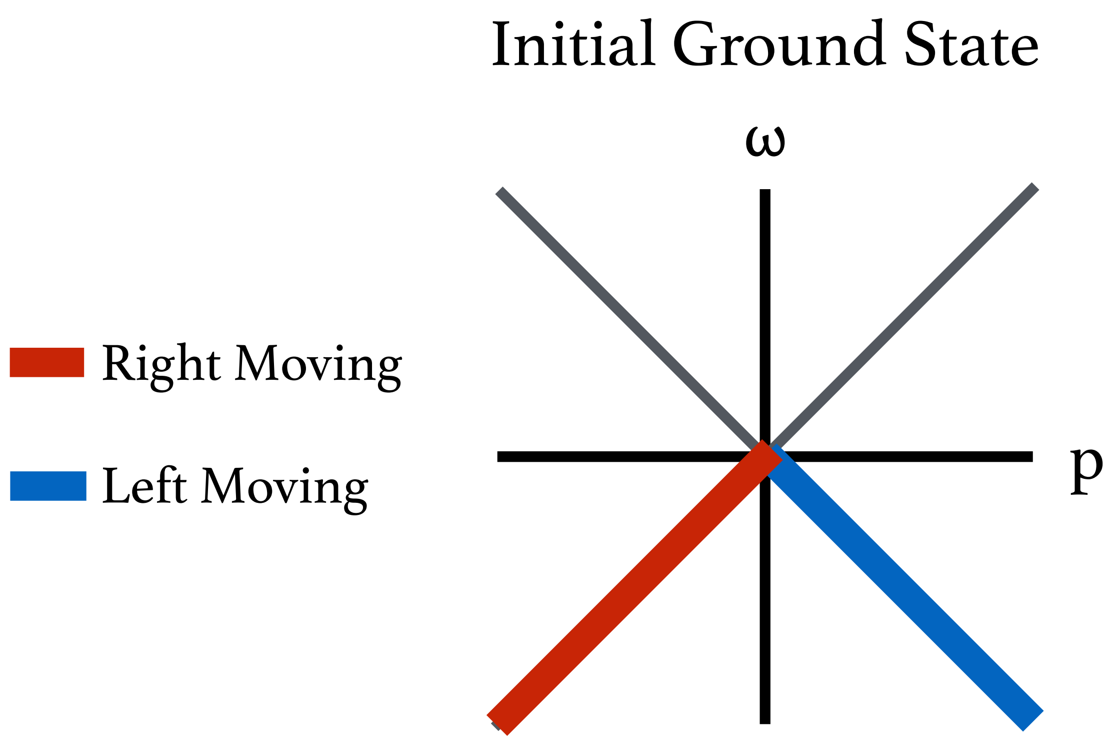
Example: Massless electrons in a box (2d QED)



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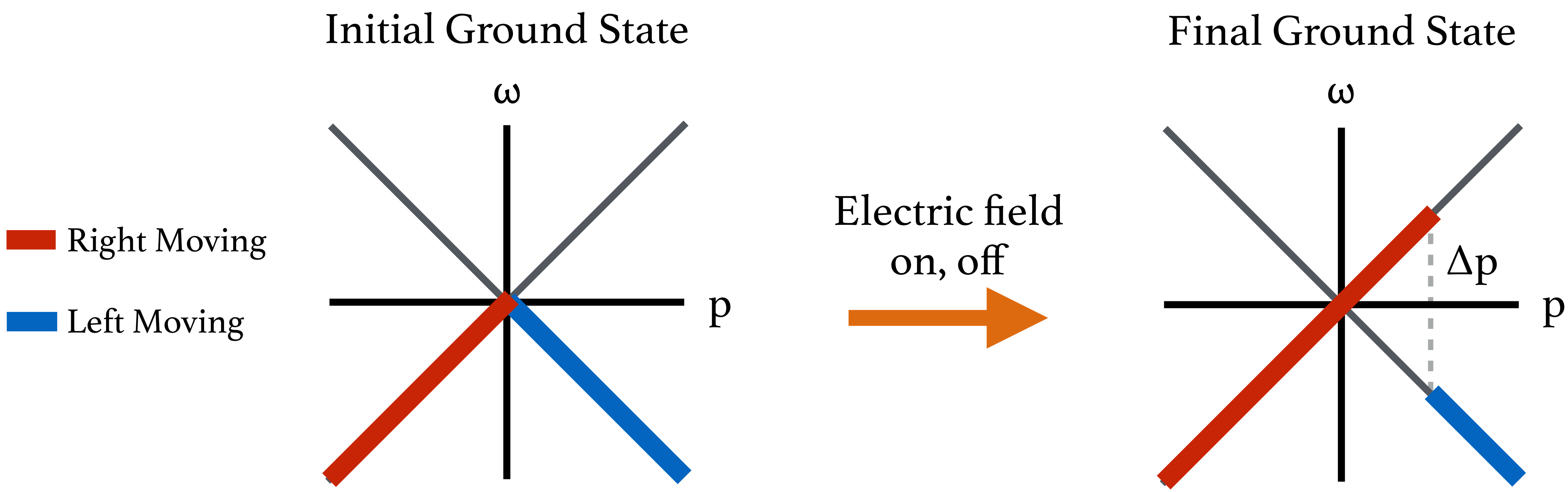

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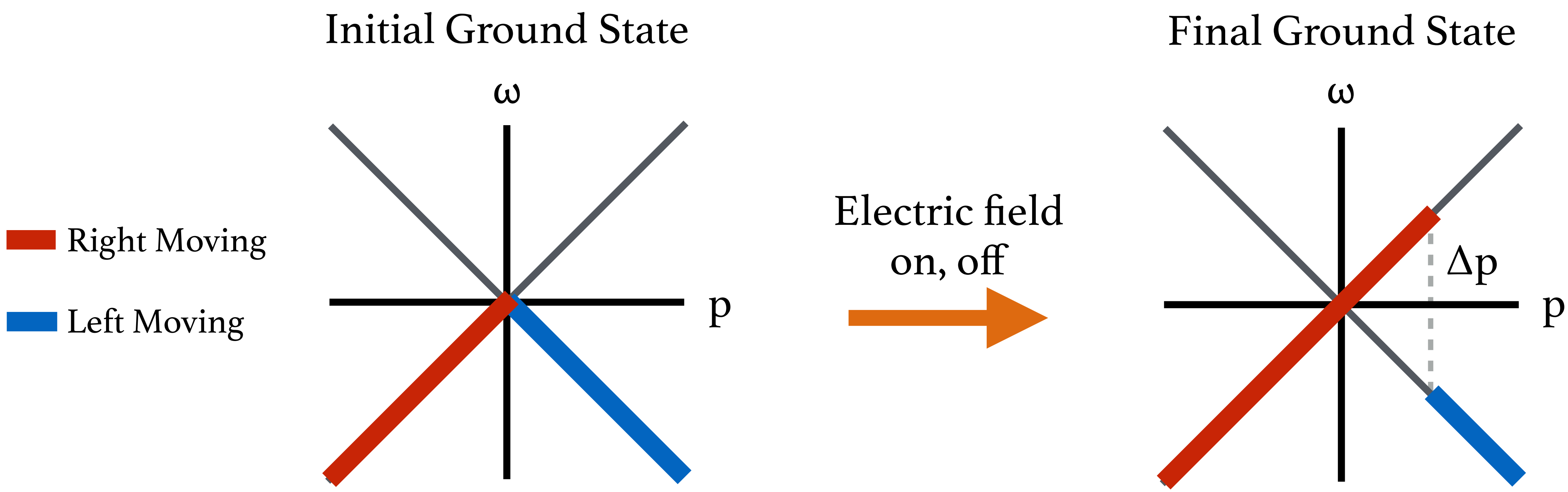

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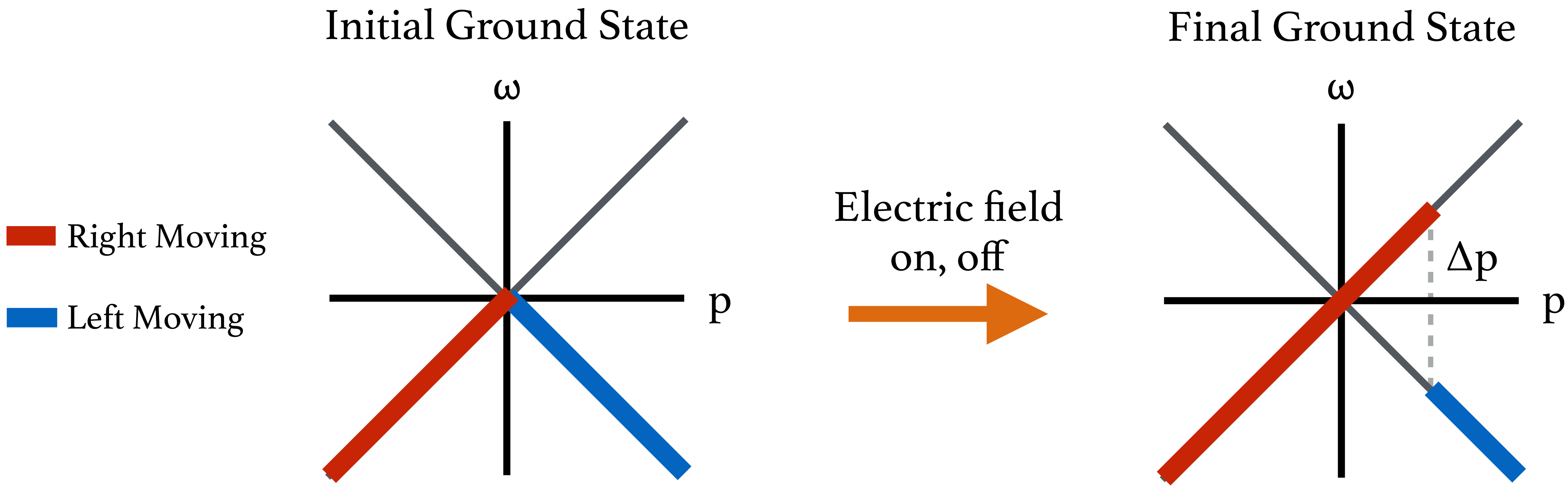

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Chiral charge changes only if Dirac sea is infinitely deep

Perturbative vs Nonperturbative Construction

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- Global symmetry violated explicitly
- Need “road to failure” for anomalous fermion reps.
- No-go for massless fermions with full chiral symmetry*

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Is there new physics 'hidden' in the mismatch between what we expect a nonperturbative regulator to do and what lattice can seem to do?

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Regulating Fermions

Fermion contribution to path integral encoded in Δ(A)

$$\Delta(A) \equiv \int [D\Psi]\,[D\bar{\Psi}]\, e^{-S_F(A)}$$

- Dirac operator maps $V_L \oplus V_R$ to $V_L \oplus V_R$ and so Δ(A) can be determined unambiguously

$$\mathcal{D}_V = \begin{pmatrix} 0 & D_\mu(A)\sigma^\mu \\ D_\mu(A)\bar{\sigma}^\mu & 0 \end{pmatrix} \longrightarrow \Delta_V = \det \mathcal{D}_V$$

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Additional information is necessary to define δ(A)

Nielson-Ninomiya Theorem

No-Go Theorem: No lattice fermion operator can satisfy all four conditions simultaneously:

1. Periodic and analytic in momentum space
2. Reduces to Dirac operator in continuum limit
3. Invertible everywhere except at zero momentum
4. Anti-commutes with γ_5

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Ex: Discretized Dirac Operator

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Lattice regulated chiral fermions violate at least one condition

Ginsparg-Wilson Equation*

Block spin-averaged continuum theory results in operator that obeys

$$\gamma_5 D + D\gamma_5 = aD\gamma_5 D$$

Lattice spacing

- $U(1)_A$ explicitly violated only at finite lattice spacing
- Operator obeys index theorem correctly relating zero modes to winding number*
- New symmetry protects fermion mass from additive renormalization

*Ginsparg & Wilson, '85
*Luscher, '98

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- Operator obeys index theorem correctly relating zero modes to winding number*
- New symmetry protects fermion mass from additive renormalization
- Solution has the form

$$\mathcal{D}^{-1} = \begin{pmatrix} 0 & S_1 \\ S_2 & 0 \end{pmatrix} + \frac{a}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Requires LH and RH fermions have same gauge transformation

*Ginsparg & Wilson, '85
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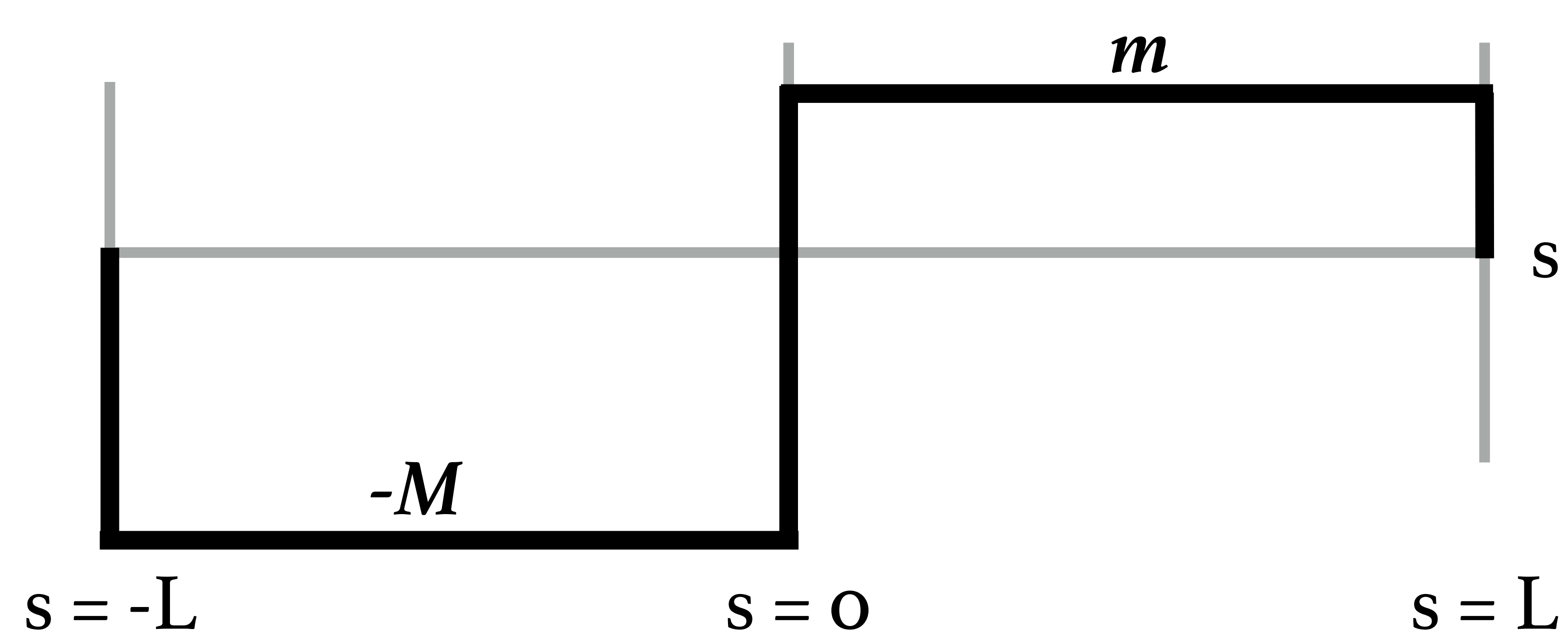
Domain Wall Fermions*

Idea: New mechanism/symmetry to allow 'naturally' light fermions

- Introduce extra compact dimensions, s = [-L, L]
- 5d fermion mass term depends on s



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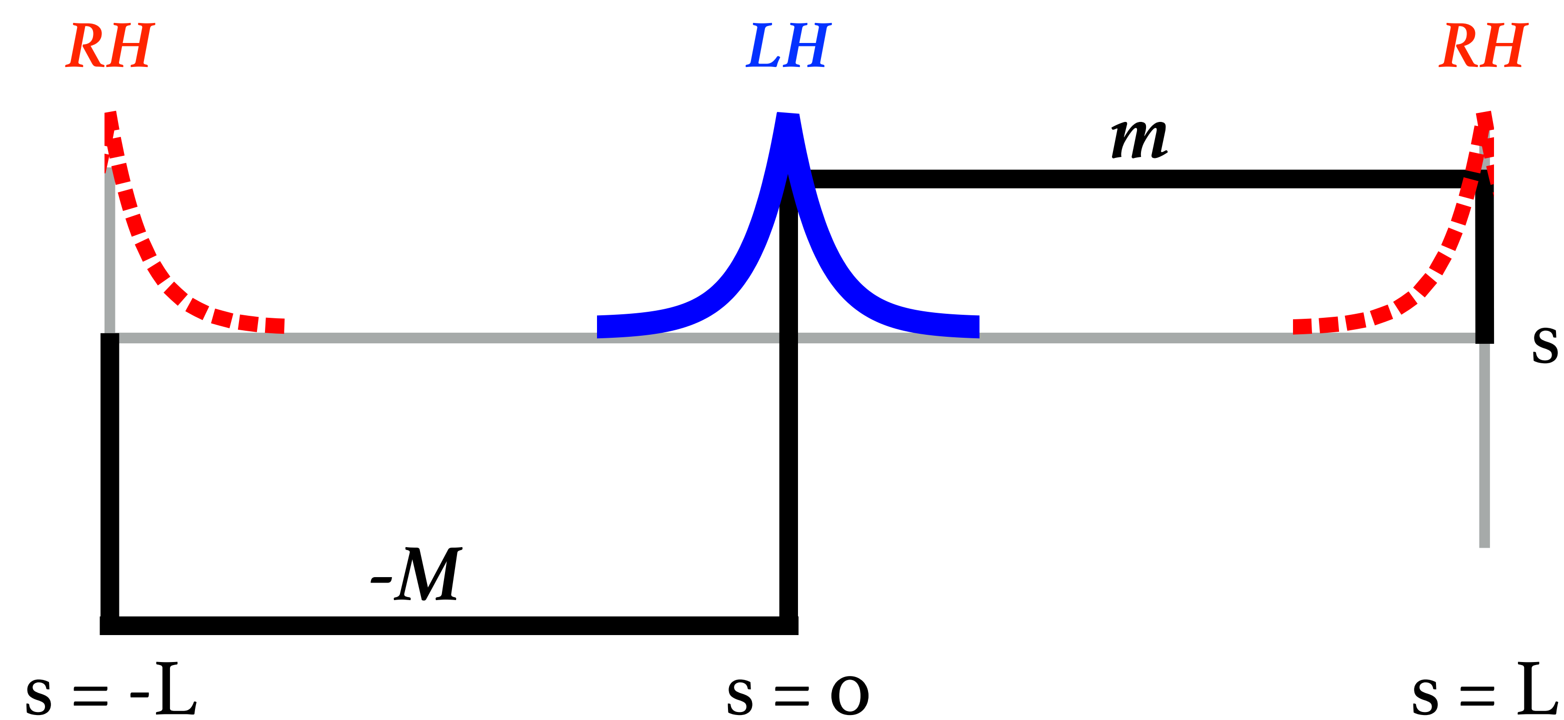
- 5d fermion mass term depends on s
- Spectrum contains both light and heavy fermions modes
- Light modes exponentially localized onto the boundaries



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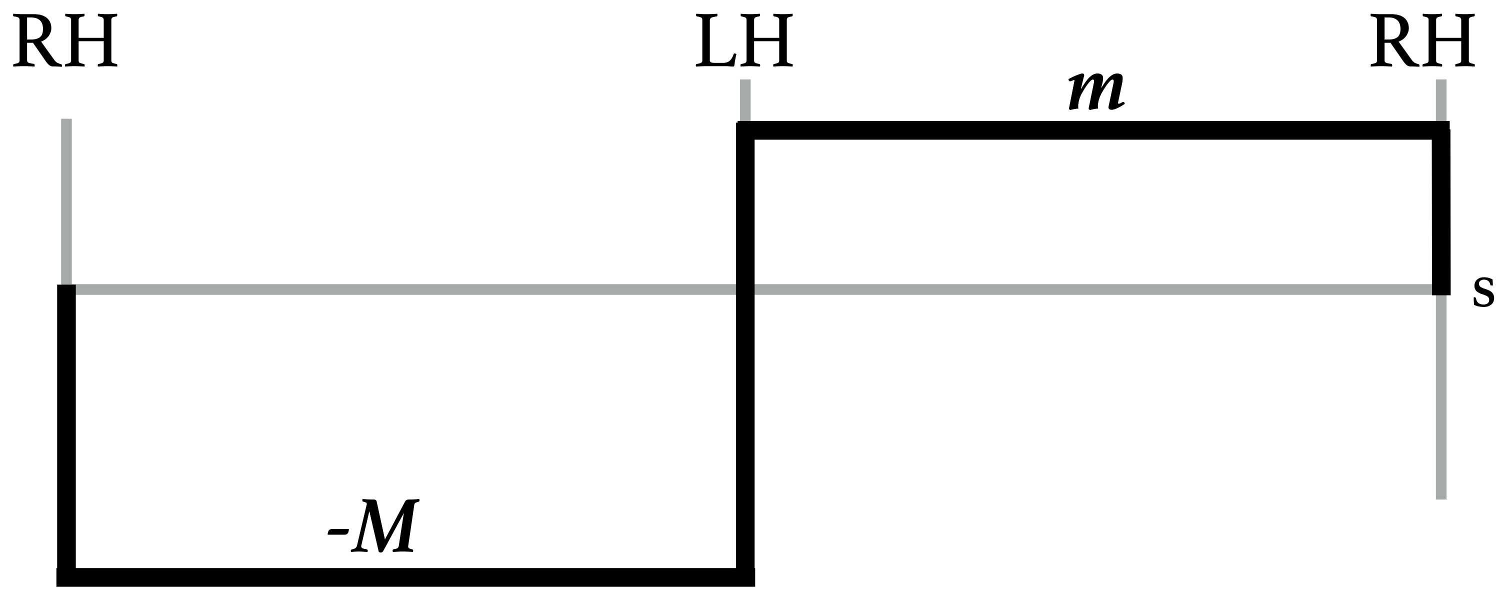
U(1)$_A$ Anomaly on the Lattice



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- Introduce ***s-independent*** 4d gauge field

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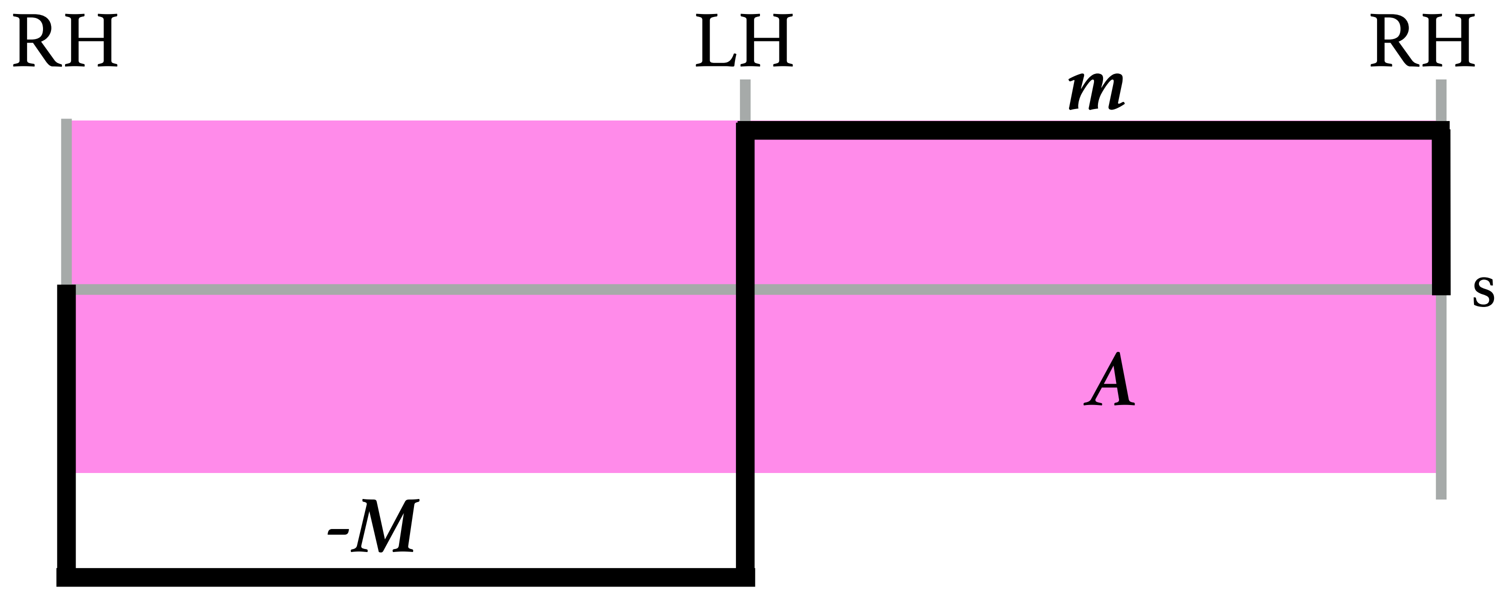

Idea: U(1)$_A$ is explicitly broken at finite lattice spacing

- Introduce ***s-independent*** 4d gauge field
- Heavy modes decouple apart from Chern-Simons operator (Callan-Harvey Mechanism*)
- Light fermions on boundary see Chern-Simons operator as explicit U(1)$_A$ symmetry breaking

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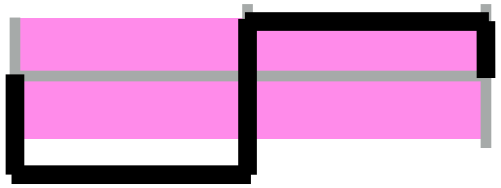
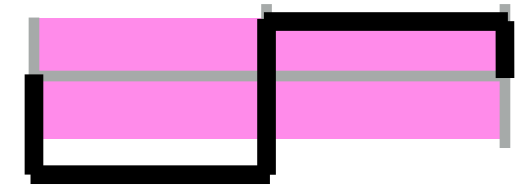
Effective Fermion Operator



Construction describes one massless Dirac fermion in limit of infinite extra dimension

- Overlap of LH and RH modes goes to zero

*Narayan & Neuberger, '95



Effective Fermion Operator

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$$a\mathcal{D}_V = 1 + \gamma_5 \epsilon \qquad \epsilon = \mathrm{sgn}\left[\gamma_5 \left(aD_w - 1\right)\right]$$

m = 1
M infinite

Lattice spacing *Wilson operator*

- Operator obeys Ginsparg-Wilson equation

$$\gamma_5 \mathcal{D}_V + \mathcal{D}_V \gamma_5 = a\,\mathcal{D}_V \gamma_5 \mathcal{D}_V$$

important: $\epsilon^2 = 1$

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Can we find similar operator for chiral gauge theories?

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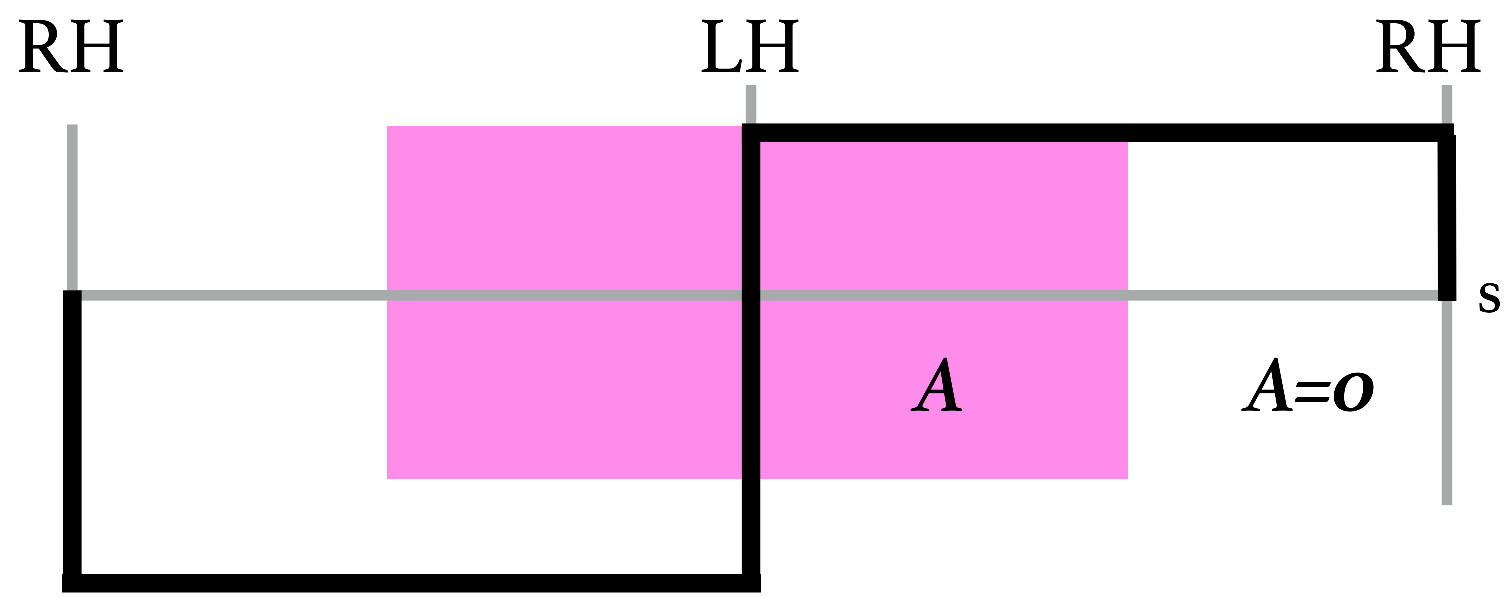
Attempt One: Chiral Gauge Theories



Idea: Localize gauge field around LH brane

- Theory is not gauge-invariant

discretize extra dimension

$$\bar{\Psi}\gamma_5\partial_s\Psi \longrightarrow \bar{\psi}_n\gamma_5\left(\psi_{n+1} - \psi_n\right)$$

'flavor' index

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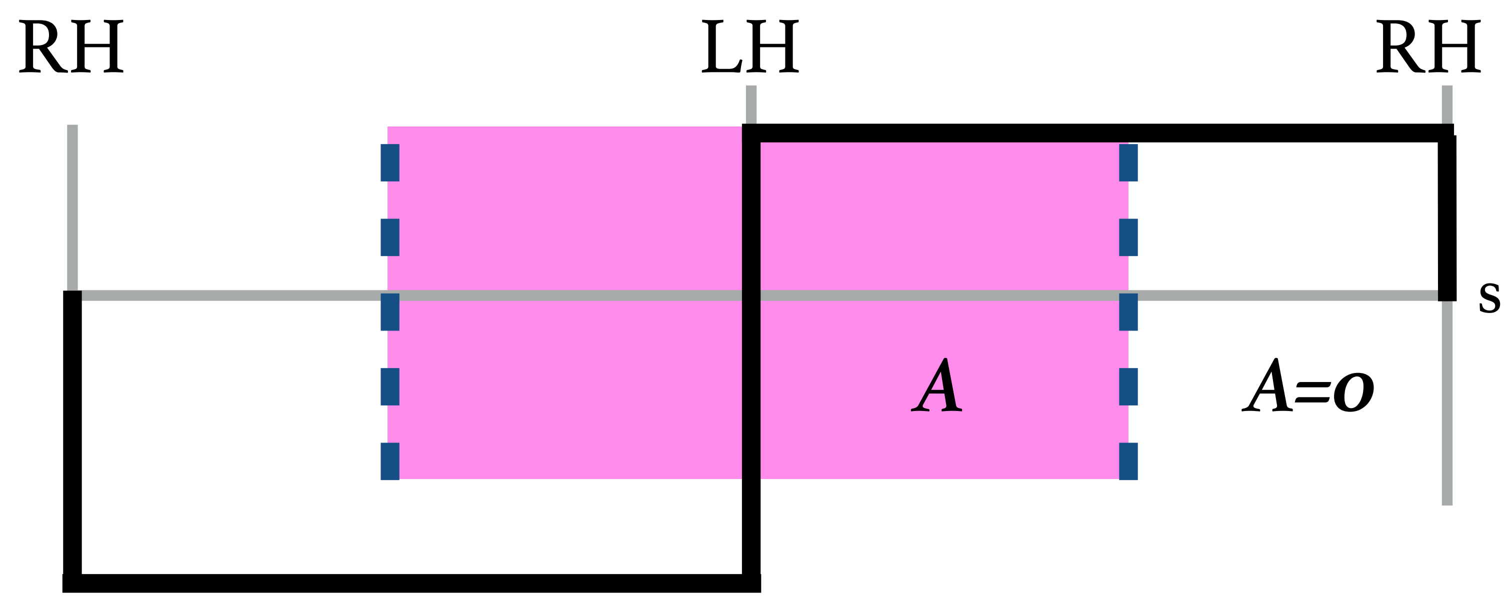

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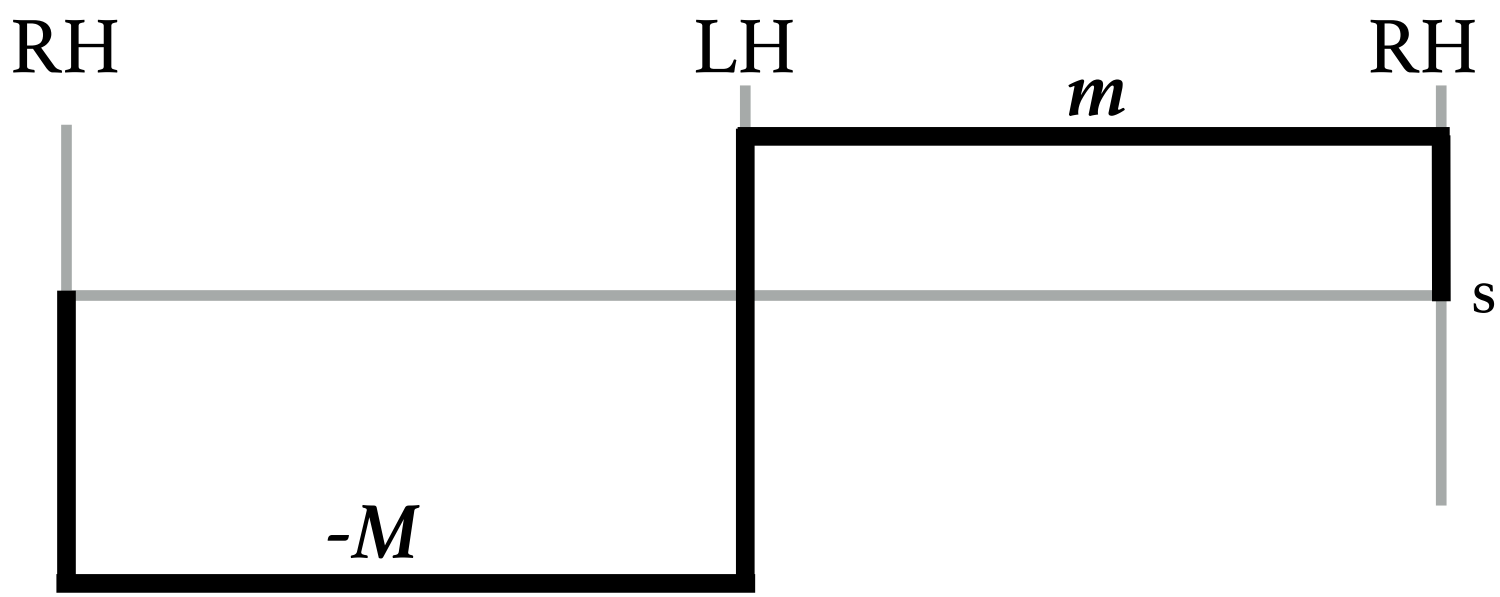
- Add Yukawa-like coupling at discontinuity
- Spectrum is vector-like as new light modes become localized at discontinuity*

*Golterman, Jansen & Vink, '93

Attempt Two: Chiral Gauge Theories



Idea: Localize gauge field using gauge-covariant flow equation*

Flow Eq: $\partial_s \mathcal{A}_\mu = \frac{\text{sgn}(s)}{\Lambda} \mathcal{D}_\nu \mathcal{F}_{\nu\mu}$ ***BC:*** $\mathcal{A}_\mu(x,0) = A_\mu(x)$

Integration variable in path integral

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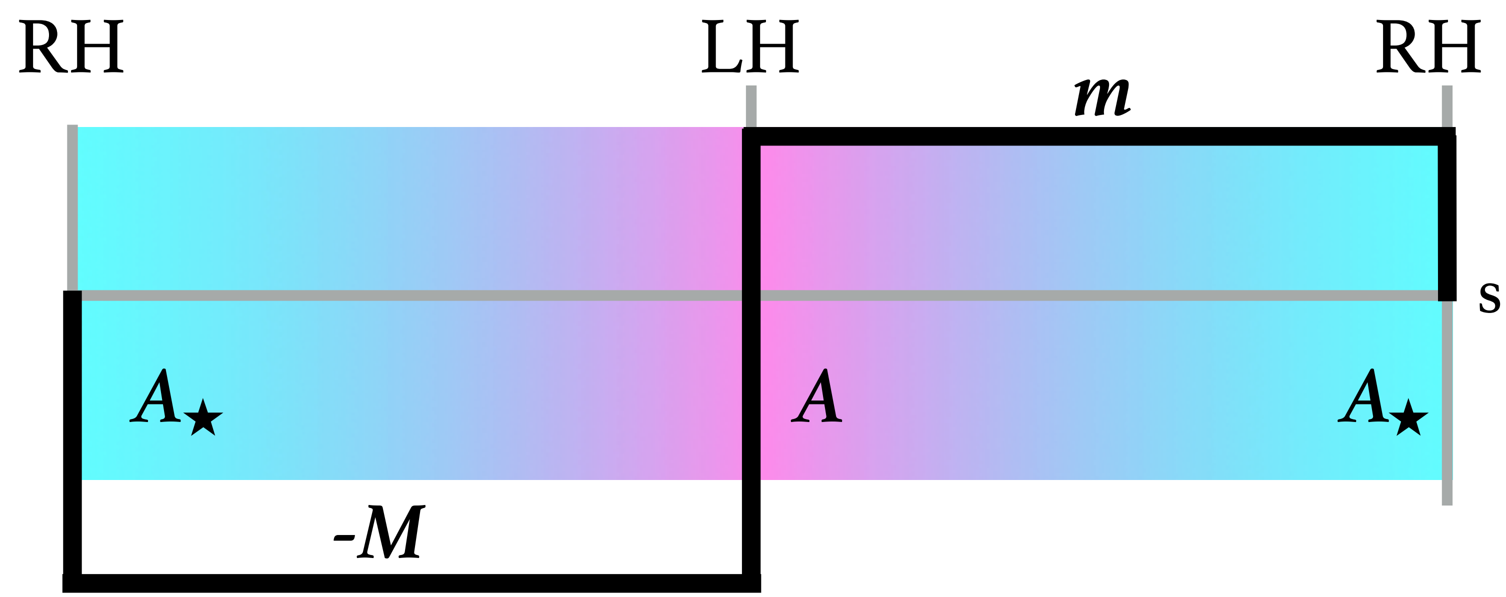

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Integration variable in path integral

- $A_\star$ completely determined by A

$$A_\star^\mu(x) \equiv \mathcal{A}^\mu(x, \pm L)$$

- Gradient flow damps out high momentum gauge fields

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Gradient Flow*

Behaves like a heat equation

Ex: 2d QED

- Gauge field decomposes into gauge and physical degrees of freedom

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*Previously used in LQCD (Luscher '10 etc)

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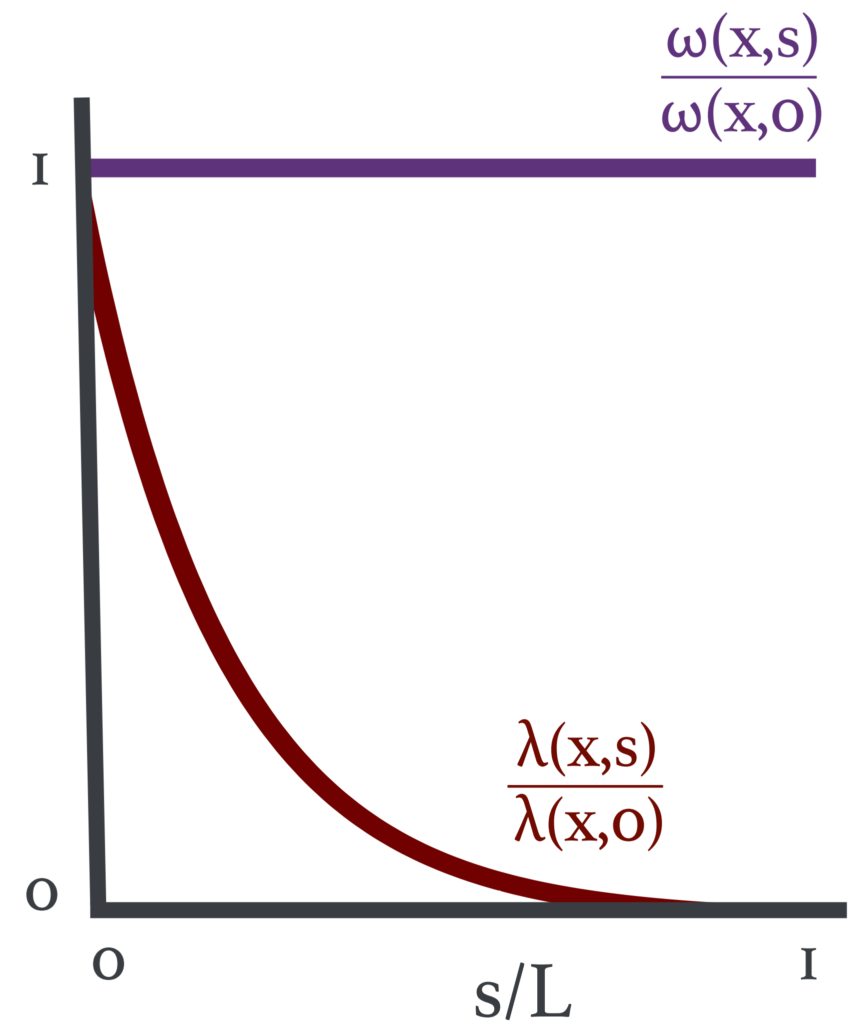
High momentum modes damped out — *Gauge DoF unaffected*



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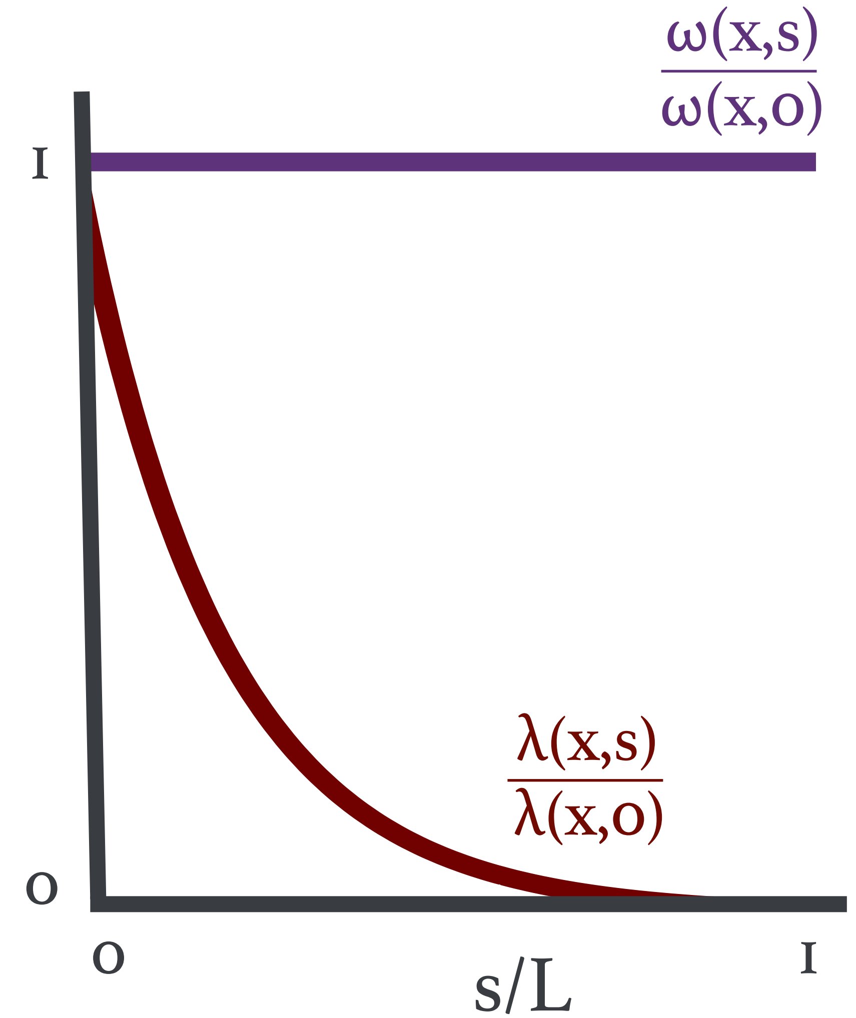

RH fermions have exponentially soft form factor

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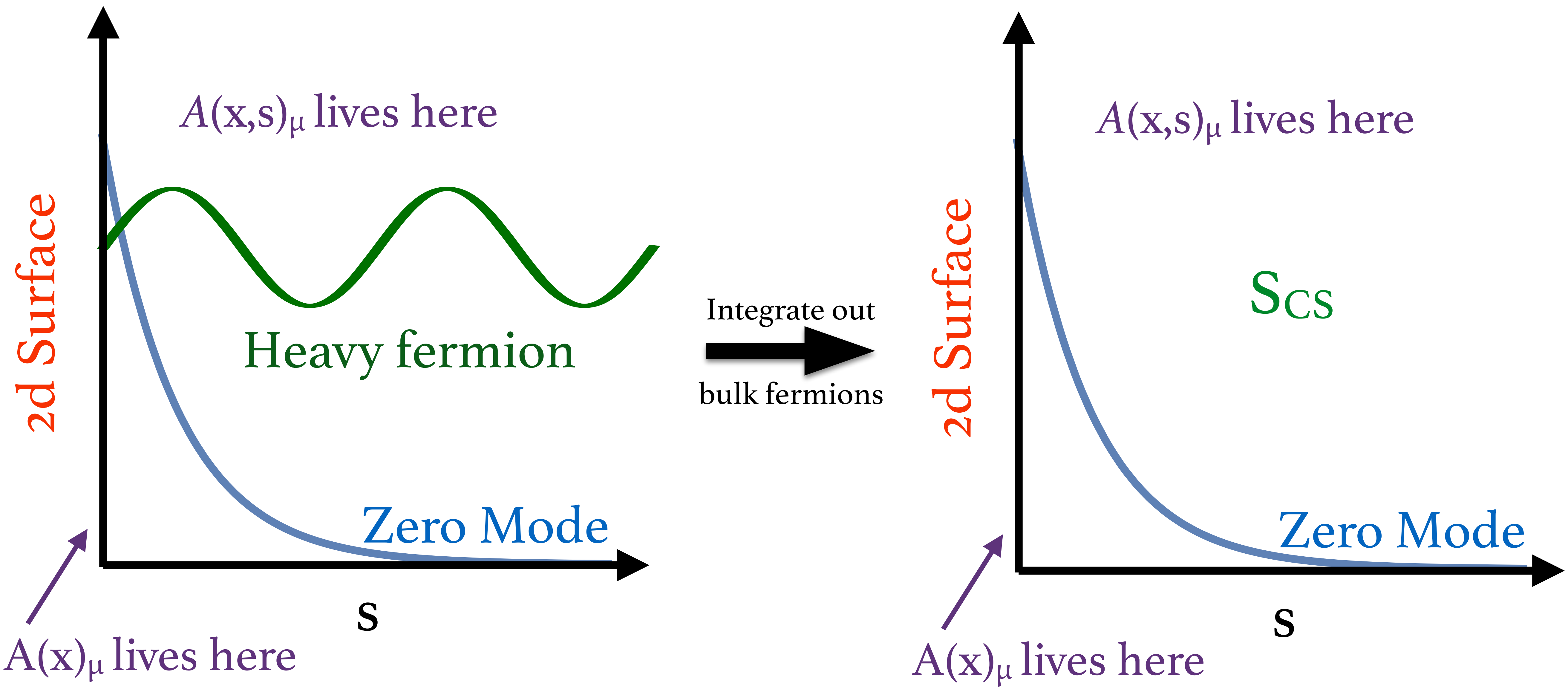
Anomalies and Callan-Harvey Mechanism*

Integrating out bulk fermions generates a Chern-Simons term



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Anomalies and Callan-Harvey Mechanism

Bulk fermions generate Chern Simons action

- In 3 dimensions, the Chern Simons action is

$$S_3^{\text{bulk}} = c_3 \frac{\Lambda}{|\Lambda|} \int (\epsilon(s) - 1) \, \text{Tr} \left(\mathcal{F} \mathcal{A} - \tfrac{1}{3} \mathcal{A}^3 \right)$$

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Fermion Contribution

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- This approximation is only valid far away from domain wall

Gauge Anomalies

Chern-Simons Term for 3d QED with flowed gauge fields

$$S_3^{\rm bulk} = 2e^2 c_3 \frac{\Lambda}{|\Lambda|} \int dx^2 dy^2 \left(\frac{\partial_\mu \partial_\alpha}{\Box} A_\alpha(x) \right) \Gamma(x-y) \left(\frac{\partial_\mu \partial_\beta}{\Box} \epsilon_{\beta\gamma} A_\gamma(y) \right)$$

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- Effective two point function is nonlocal

$$\Gamma(r) = \left(\delta^2(r) - \frac{\mu^2}{4\pi} e^{-\mu^2 r^2/4} \right) \qquad \mu \equiv \sqrt{\frac{\Lambda}{\xi L}}$$

- When flow is turned off ($\mu \to \infty$), Γ vanishes

Determines speed of flow

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Effective 2d theory is nonlocal due to Chern-Simons operator

Anomaly Cancellation

DWF with flowed gauge fields give rise to nonlocal 2d theory

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Fermion Chirality

- Theory is local if prefactor vanishes

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- Chern-Simons has prefactor if have multiple fermion fields

$$\sum_i e_i^2 \boxed{\frac{\Lambda_i}{|\Lambda_i|}}$$

Fermion Chirality

- Theory is local if prefactor vanishes

Chiral fermion representations that satisfy this criteria are gauge anomaly free representations in continuum

Summary (so far)

Proposal: Couple DWF to gradient-flowed gauge fields to lattice regulate chiral gauge theories

- Massless fermions localized on boundaries in limit of infinite extra dimension
- Fermions on far boundary couple with exponentially soft form factor

$$e^{-\xi p^2 L/\Lambda}$$

- Gauge-anomaly cancellation criteria in continuum is analogous to locality criteria on the lattice

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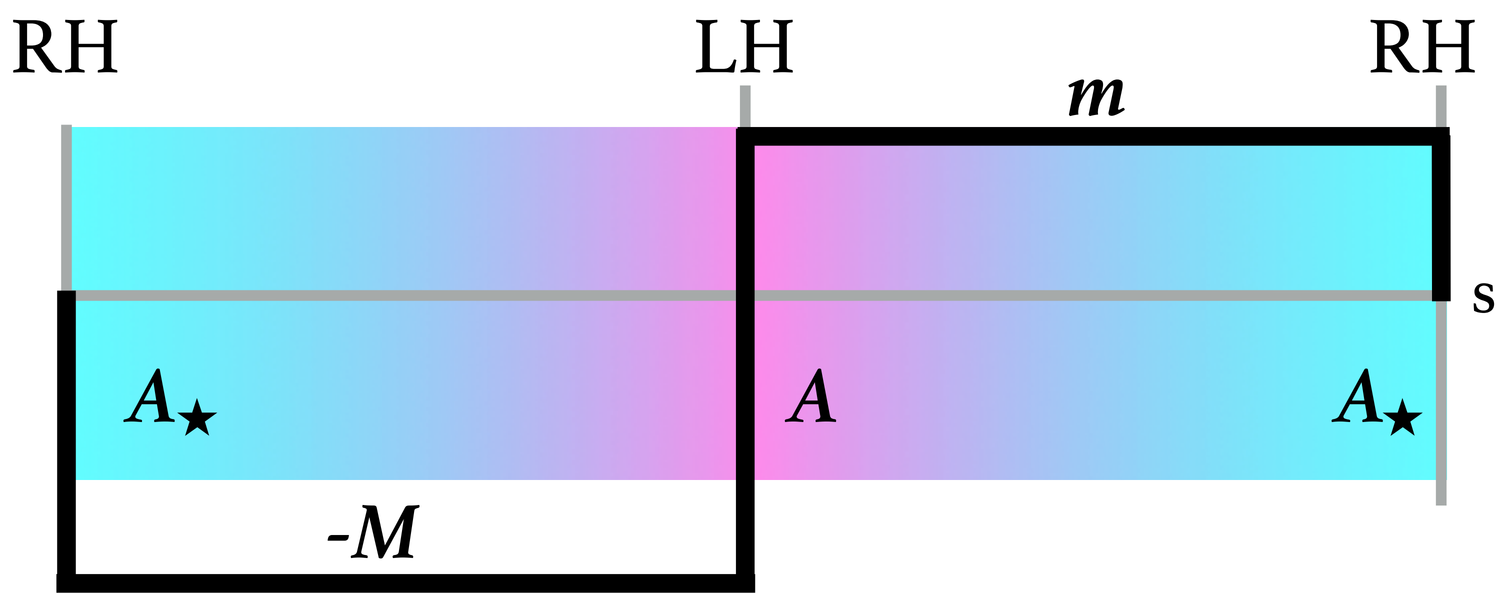
Question 1: Do mirror partners decouple completely and if not, what are the physical implications?

Question 2: What is the effective fermion operator for the massless fermions in this construction?

Gradient Flow Fixed Points



Idea: Continuum gradient flow equation has multiple attractive fixed points*

Flow Eq: $\partial_s \mathcal{A}_\mu = \frac{\text{sgn}(s)}{\Lambda} \mathcal{D}_\nu \mathcal{F}_{\nu\mu}$ ***BC:*** $\mathcal{A}_\mu(x,0) = A_\mu(x)$

- Flow does not affect topological gauge configurations (ex: instantons)
- RH fermions couple to these configurations

*(Probably) not true for discretized flow equation

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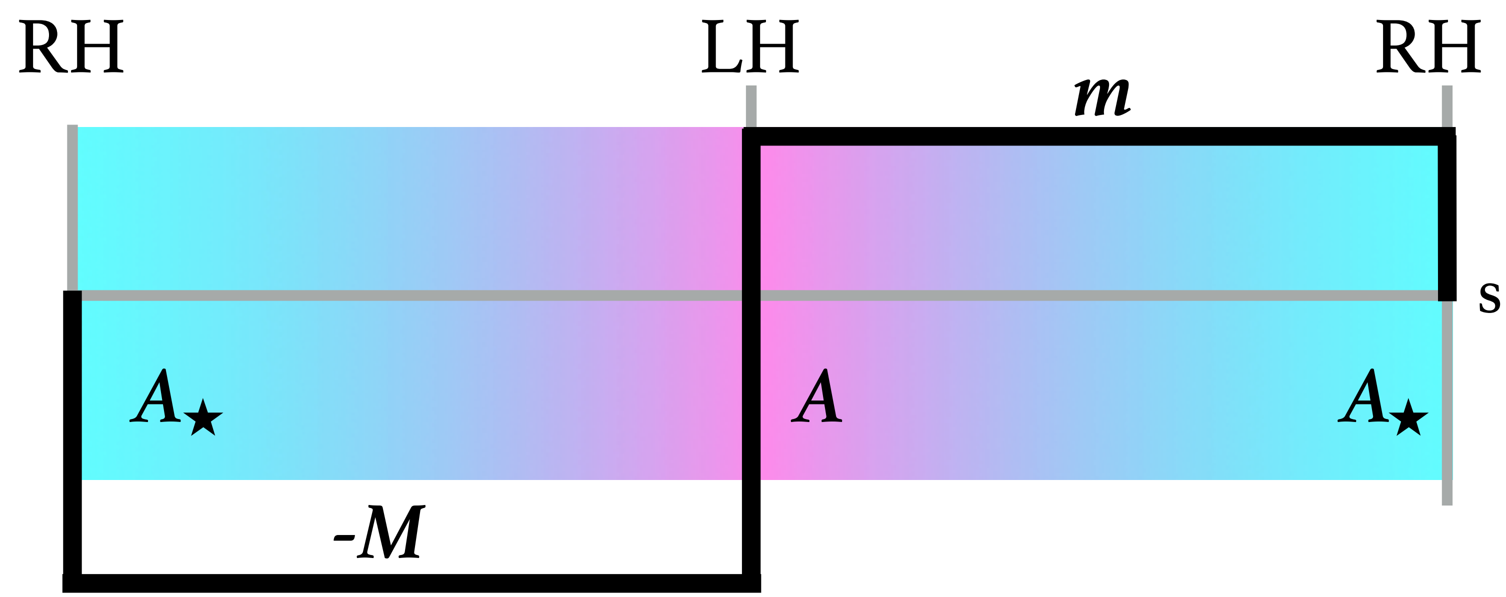

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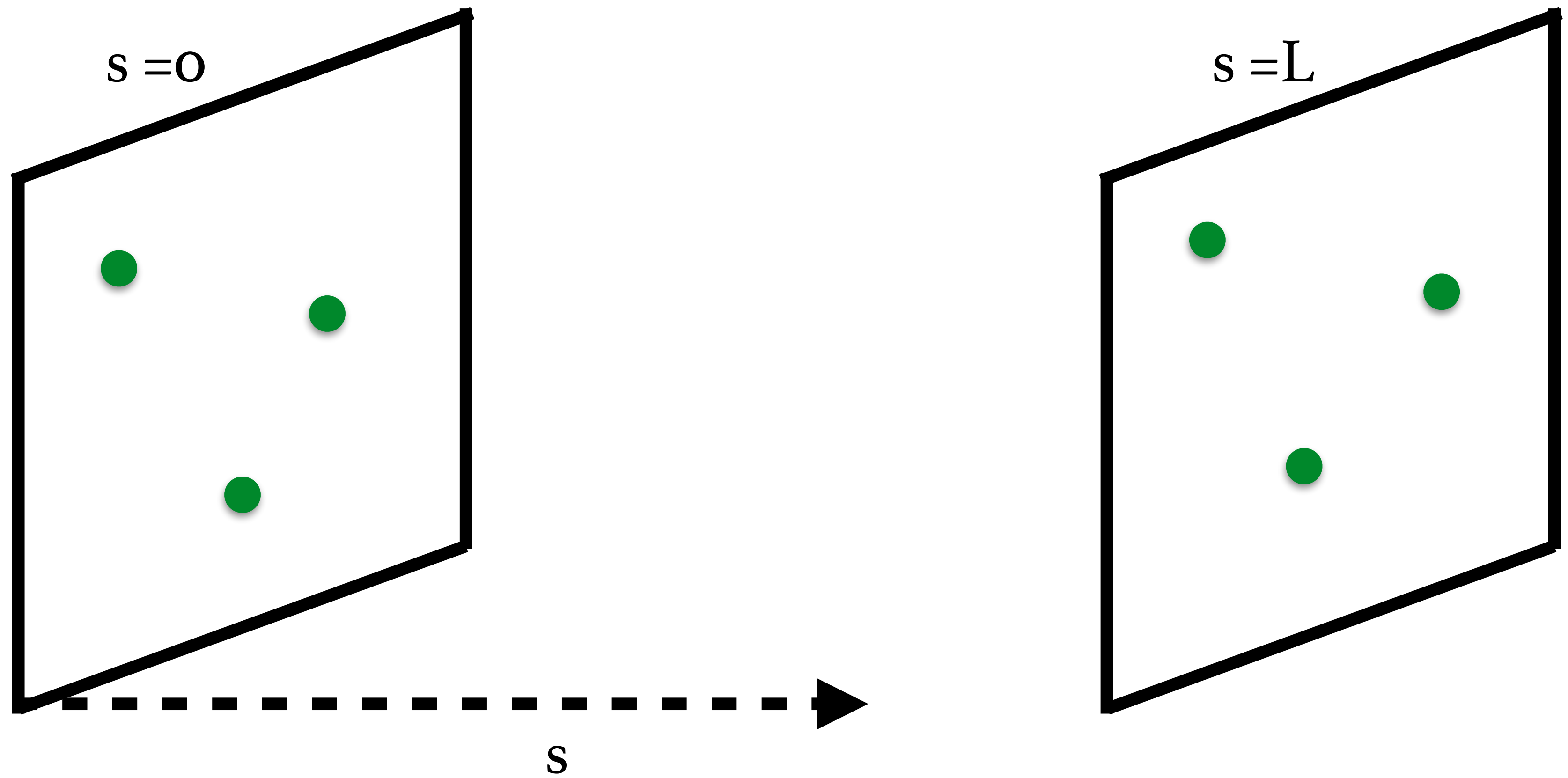
Weak Coupling

Winding number = 3



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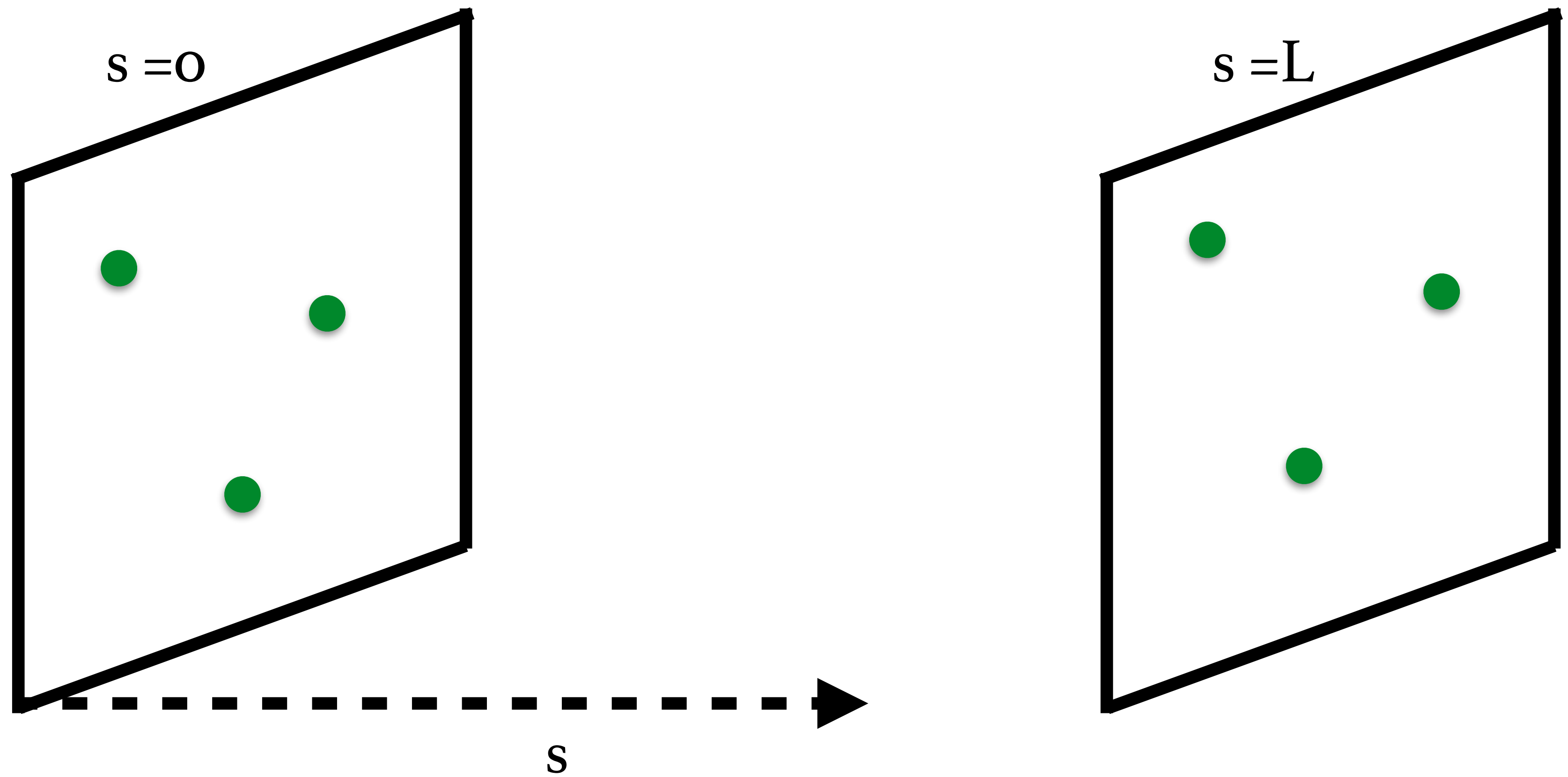
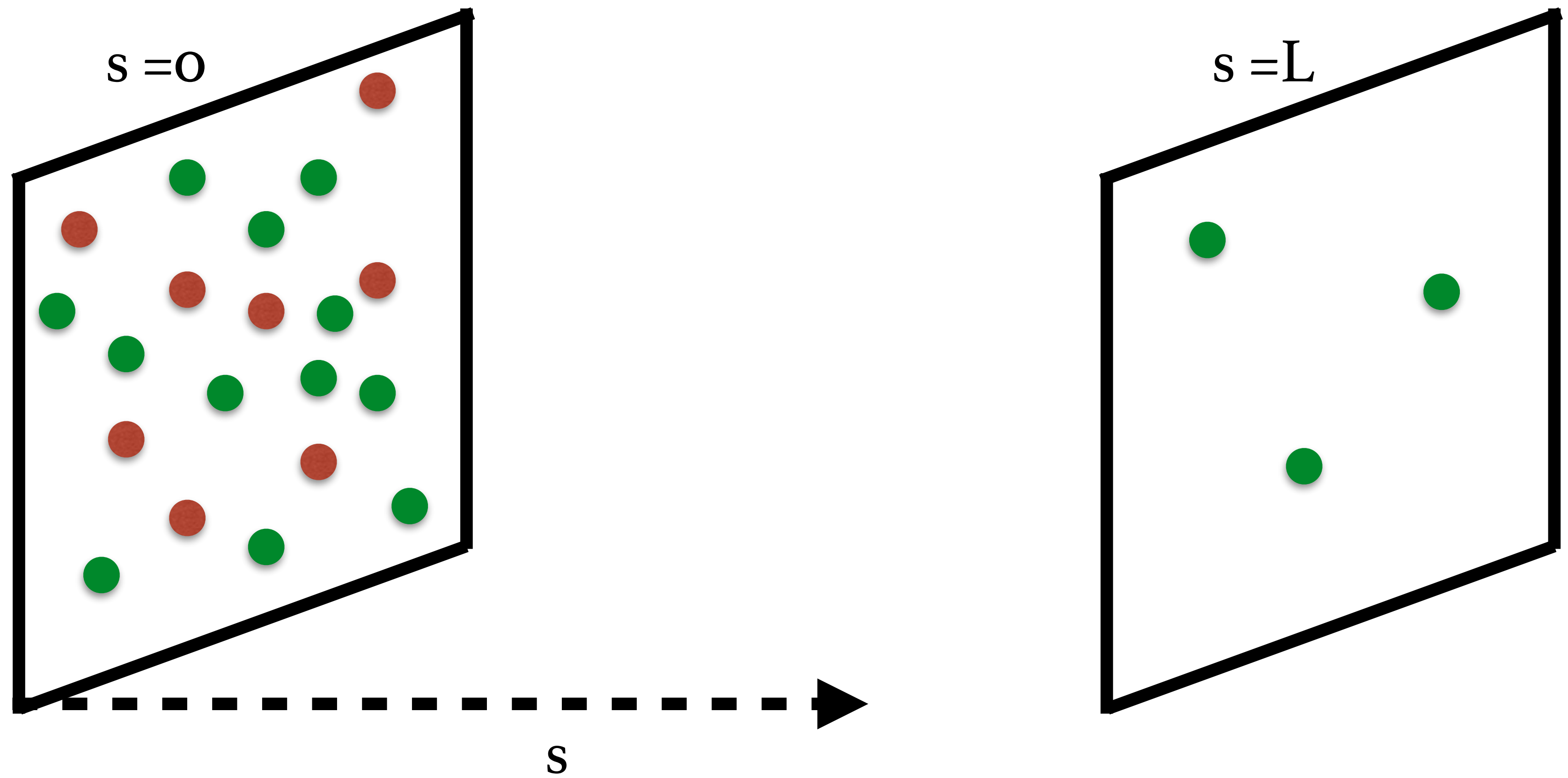


At weak coupling, instanton contribution is most important

- Flow does not affect location of instantons
- Correlation between location of instantons on the two boundaries allows for exchange of energy/momentum
- Highly suppressed process, so difficult to observe

Strong Coupling

Winding number = 3



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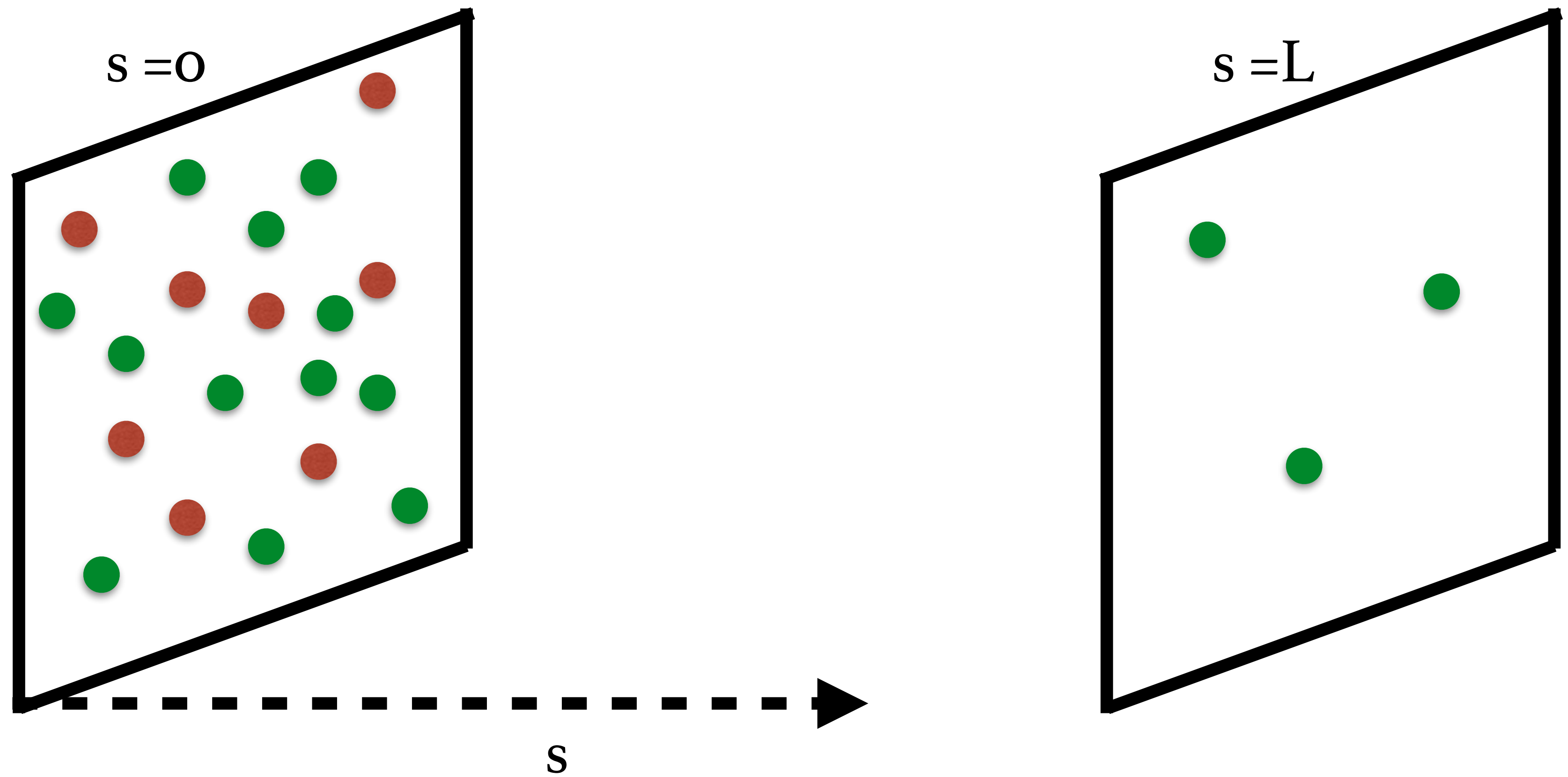
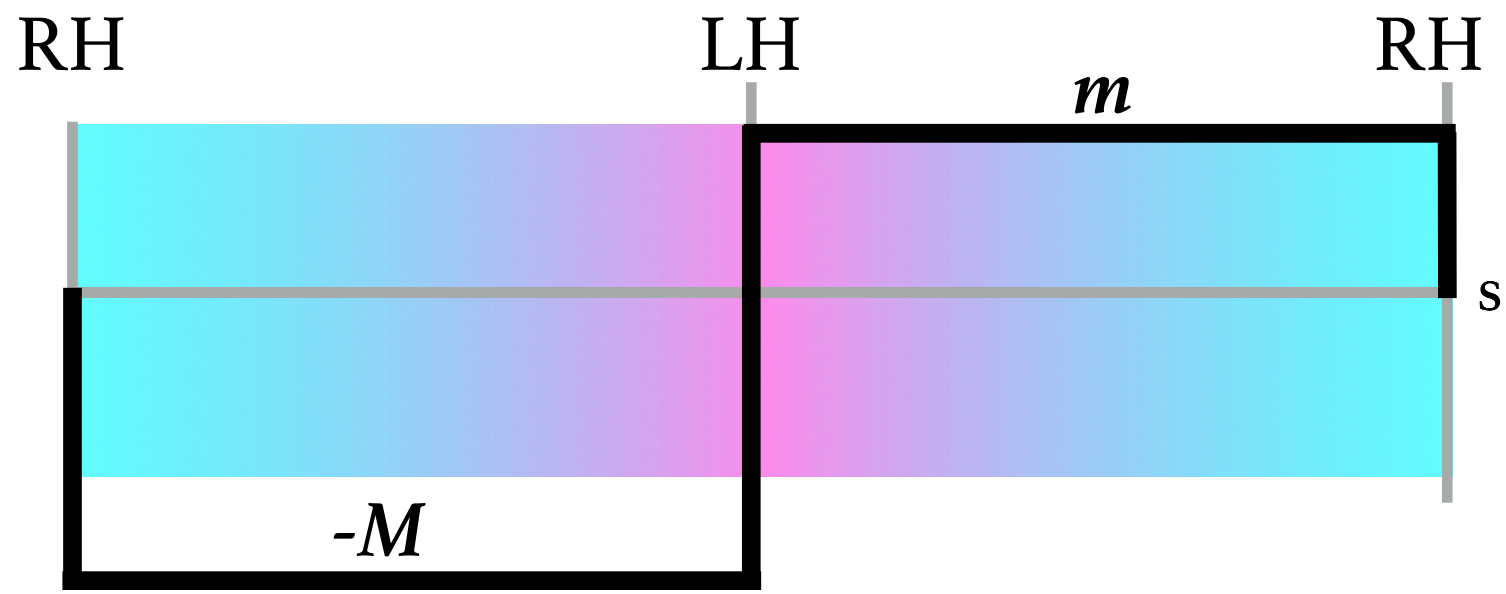


At strong coupling, need to include instanton-anti instanton pairs

- I-A pairs DO NOT satisfy equations of motion
- If flow for sufficiently long time, all pairs will annihilate
- If no correlation between location of instantons on the two boundary, standard fermions and Fluff do not exchange energy/momentum

Effective Fermion Operator



Idea: Derive effective fermion operator in limit of infinite extra dimension*

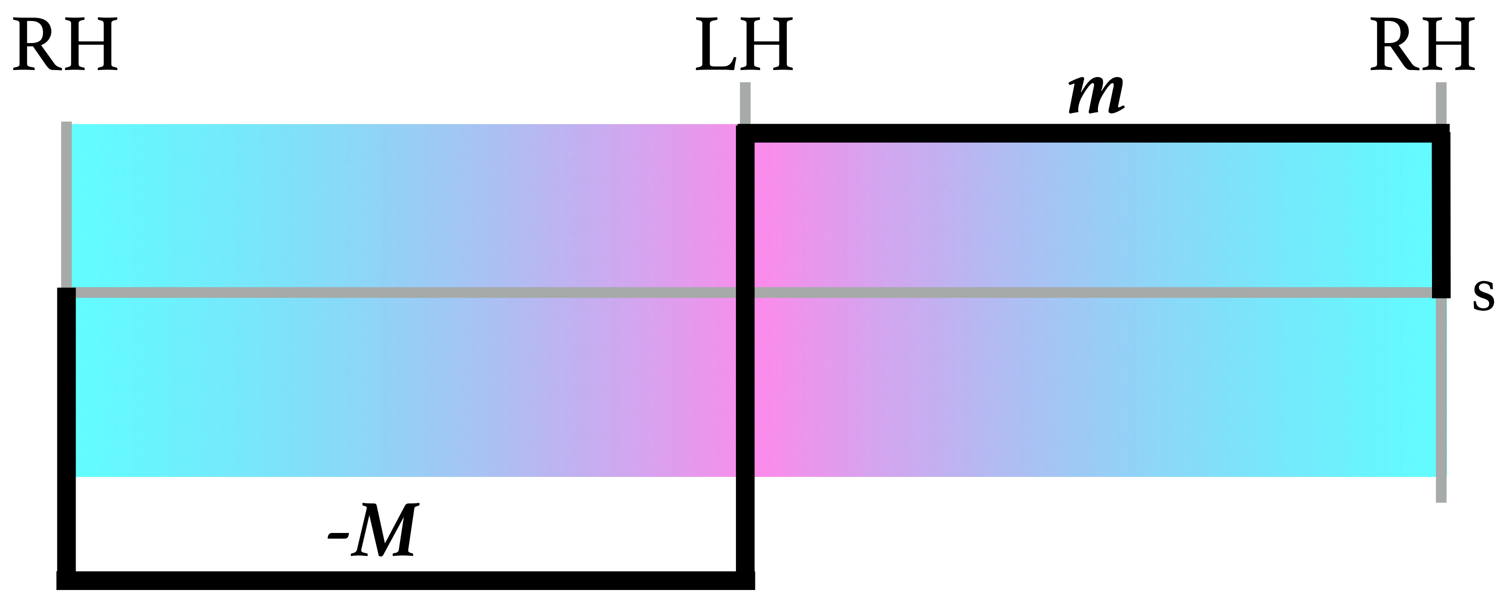
$$\mathcal{D}_\chi = \lim_{L\to\infty}\left(1+\gamma_5\frac{1-\prod_s e^{-\mathcal{H}(s)}}{1+\prod_s e^{-\mathcal{H}(s)}}\right)$$

Hamiltonian for translations in s direction

$$\mathcal{H}(s) = \gamma_5\left[D_w(\mathcal{A})-1\right]$$

Flowed gauge field

Effective Fermion Operator



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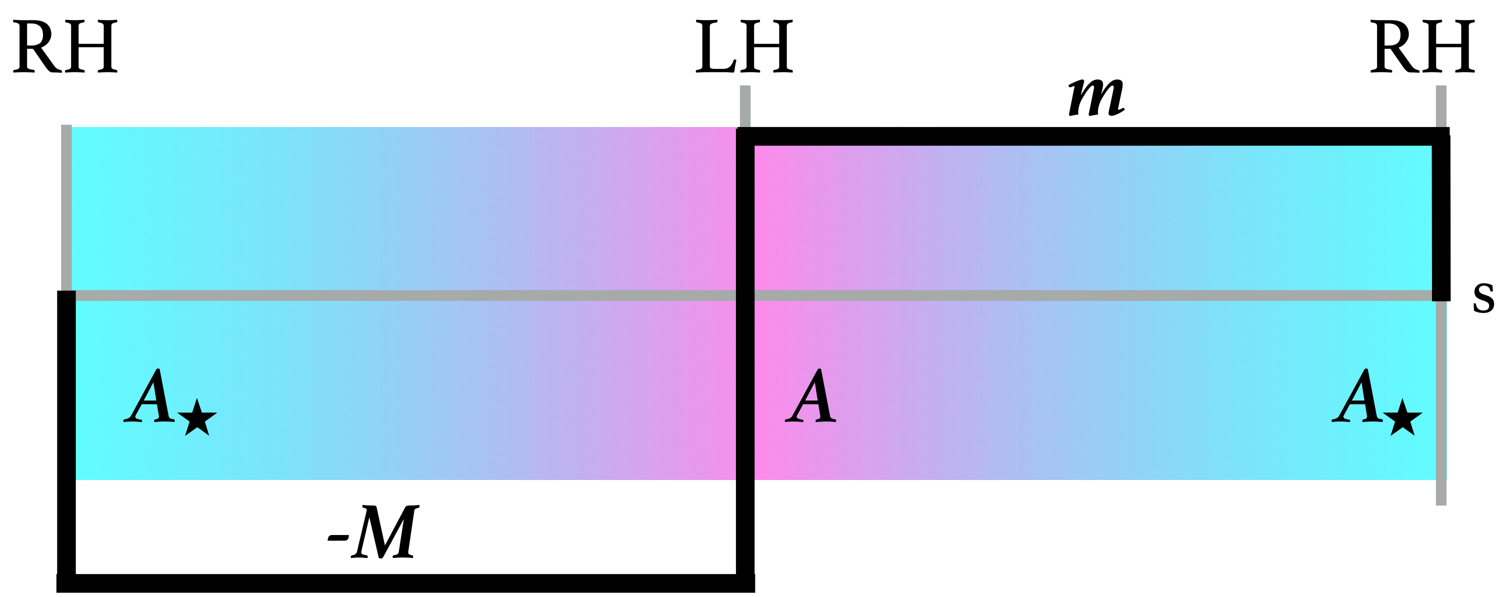
Hamiltonian for translations in s direction

Flowed gauge field

- Satisfies Ginsparg-Wilson as eigenvalues of *H* are real
- Analytic form for Dχ can be explicitly derived for special case

*DMG & Kaplan '16

“Abrupt Flow” operator



Hard to take large L limit analytically, due to s-dependent flow

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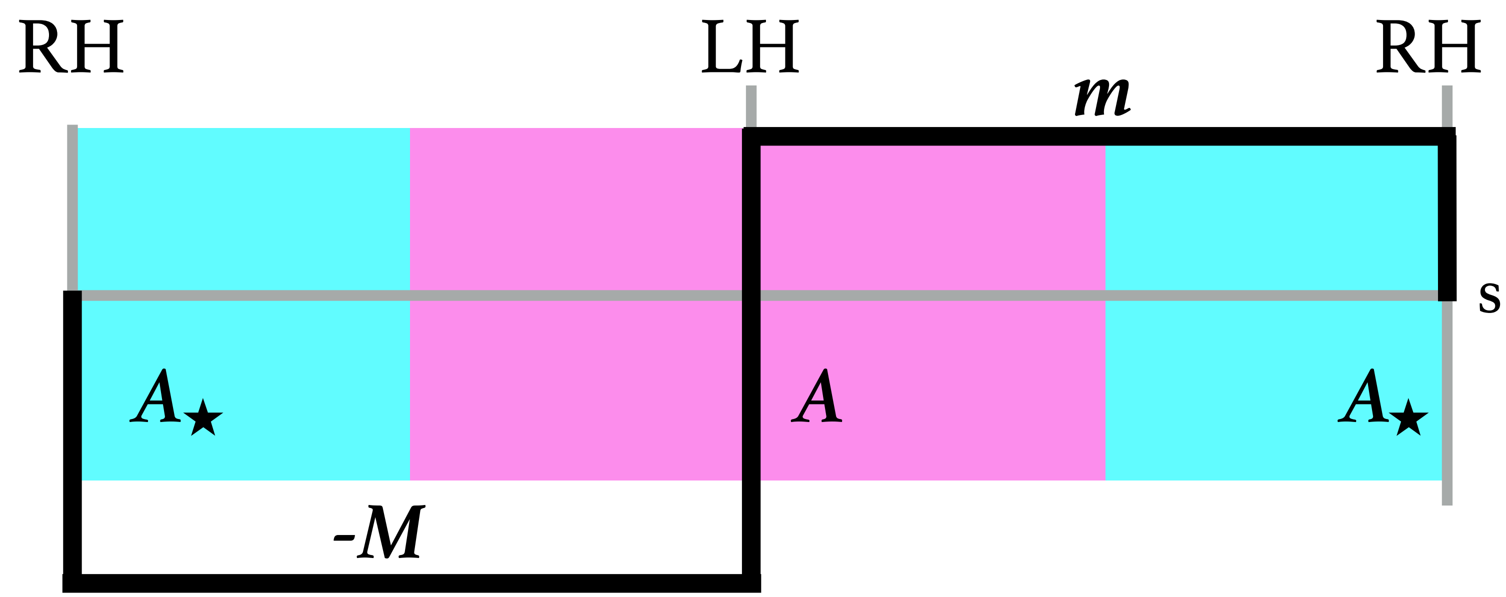

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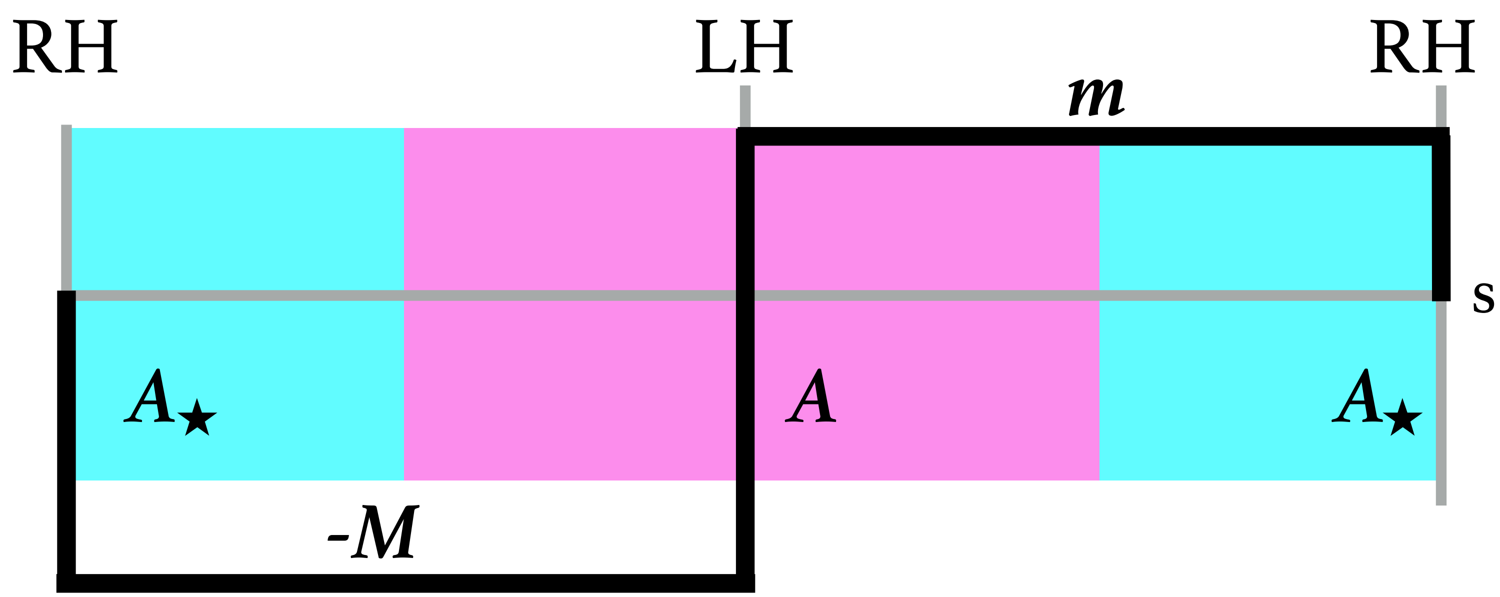

Hard to take large L limit analytically, due to s-dependent flow

- Use ‘abrupt flow’ approximation*
- Assume A and $A_\star$ have same topology

$$\mathcal{D}_\chi = 1 + \gamma_5 \left\{ 1 - (1 - \epsilon_\star) \frac{1}{1 + \epsilon\epsilon_\star} (1 - \epsilon) \right\}$$

$$\epsilon = \mathrm{sgn}\left[\gamma_5 \left(D_w(A) - 1\right)\right]$$
$$\epsilon_\star = \mathrm{sgn}\left[\gamma_5 \left(D_w(A_\star) - 1\right)\right]$$

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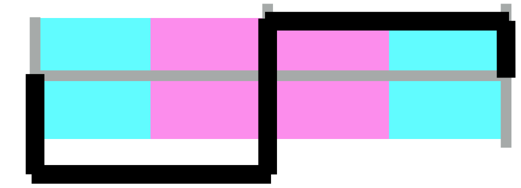
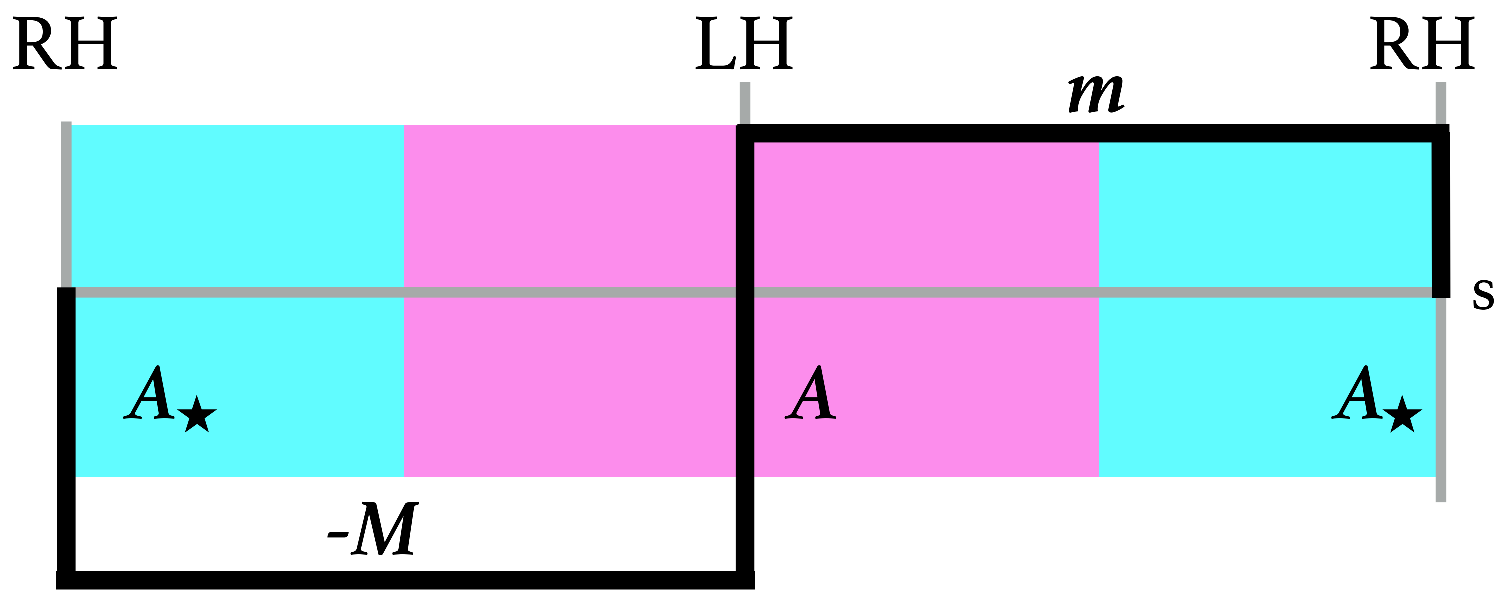


‘Abrupt Flow’ Operator



Key property: Continuum Limit

$$\mathcal{D}_\chi = \begin{pmatrix} 0 & D_\mu(A)\sigma^\mu \\ D_\mu(A_\star)\bar{\sigma}^\mu & 0 \end{pmatrix}$$

- Gauge invariance preserved as A and $A_\star$ transform identically
- Mirror partners should* decouple completely if $A_\star$ is pure gauge

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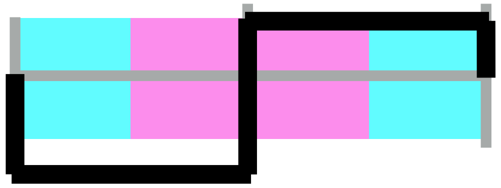

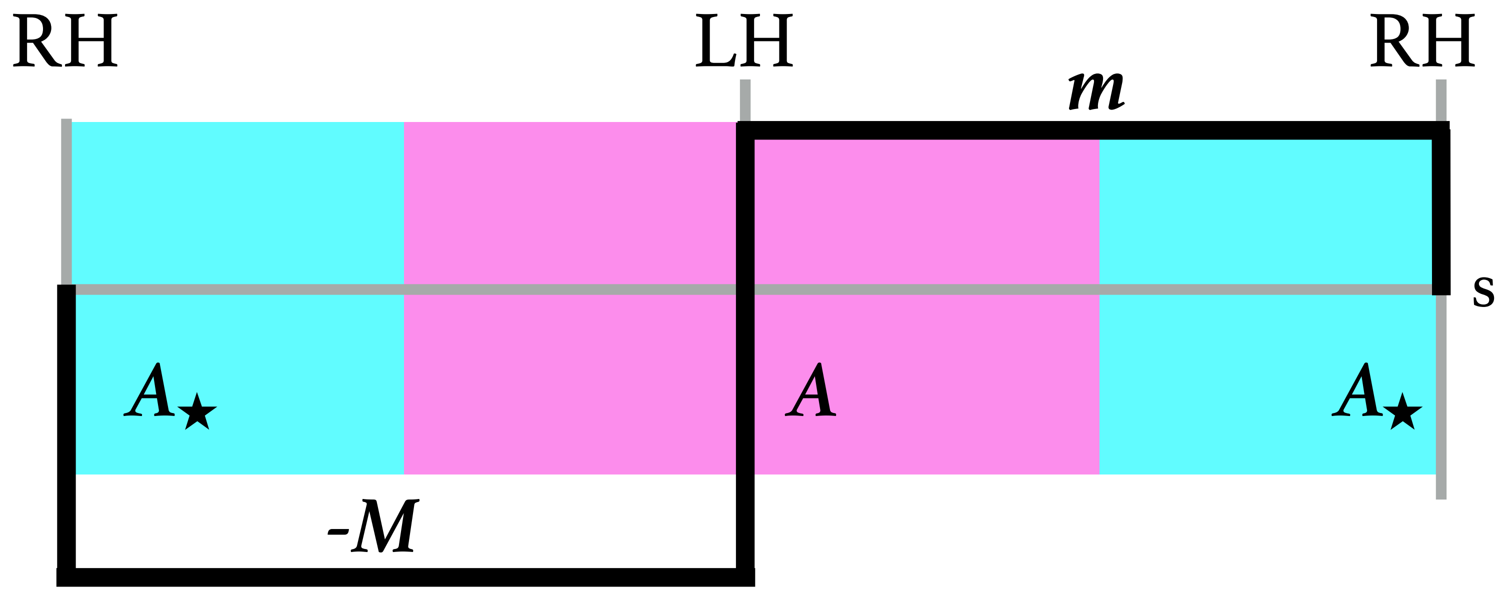

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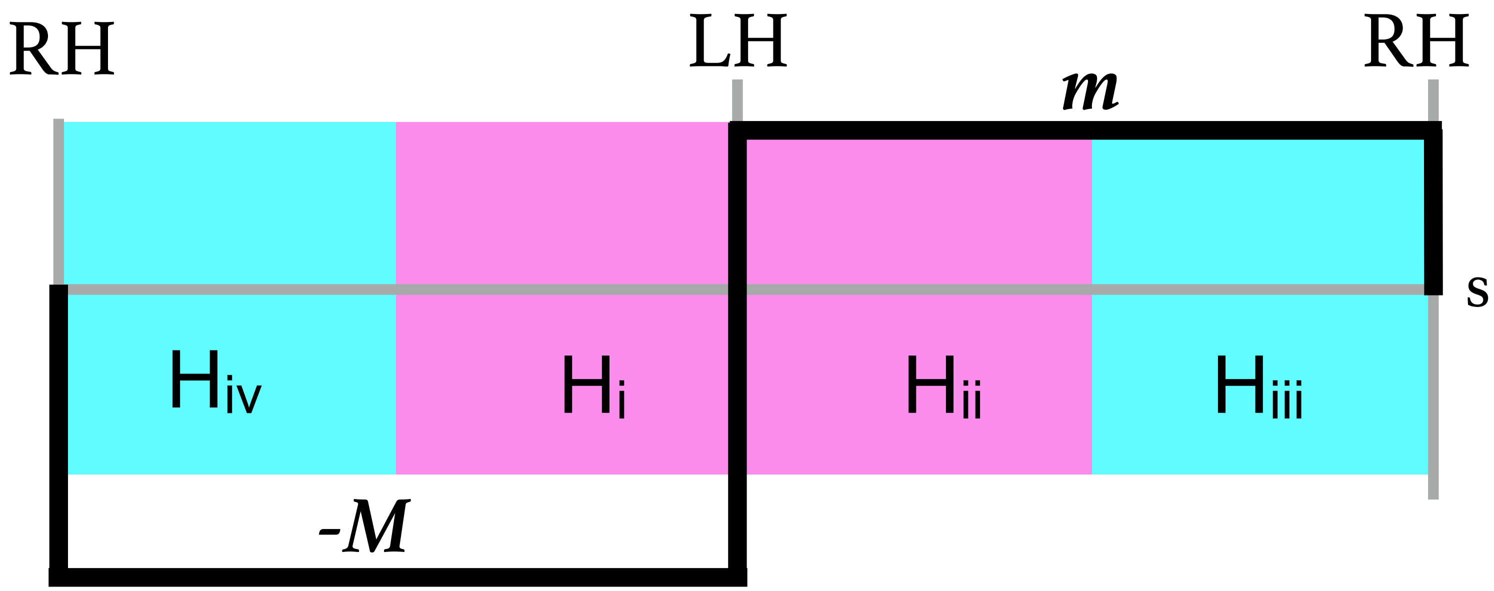
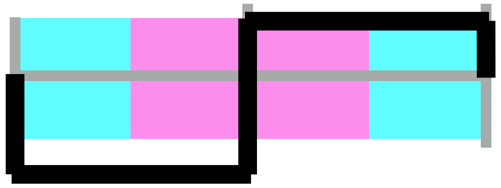
Open Question: How discretized gradient flow affect ‘topology’

*Abrupt flow might disrupt decoupling

'Abrupt Flow' Operator

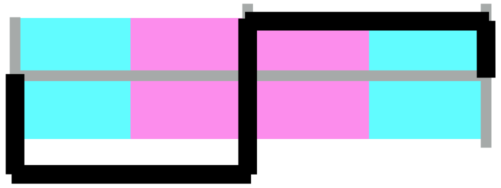




Key property: Phase ambiguity

- $\Delta(A)$ determined by treating extra dimension as time component
- Infinite extra dimension corresponds to projecting onto multi-particle ground states of Hamiltonians, $H_{i\text{-}iv}$

*Narayan & Neuberger, '95



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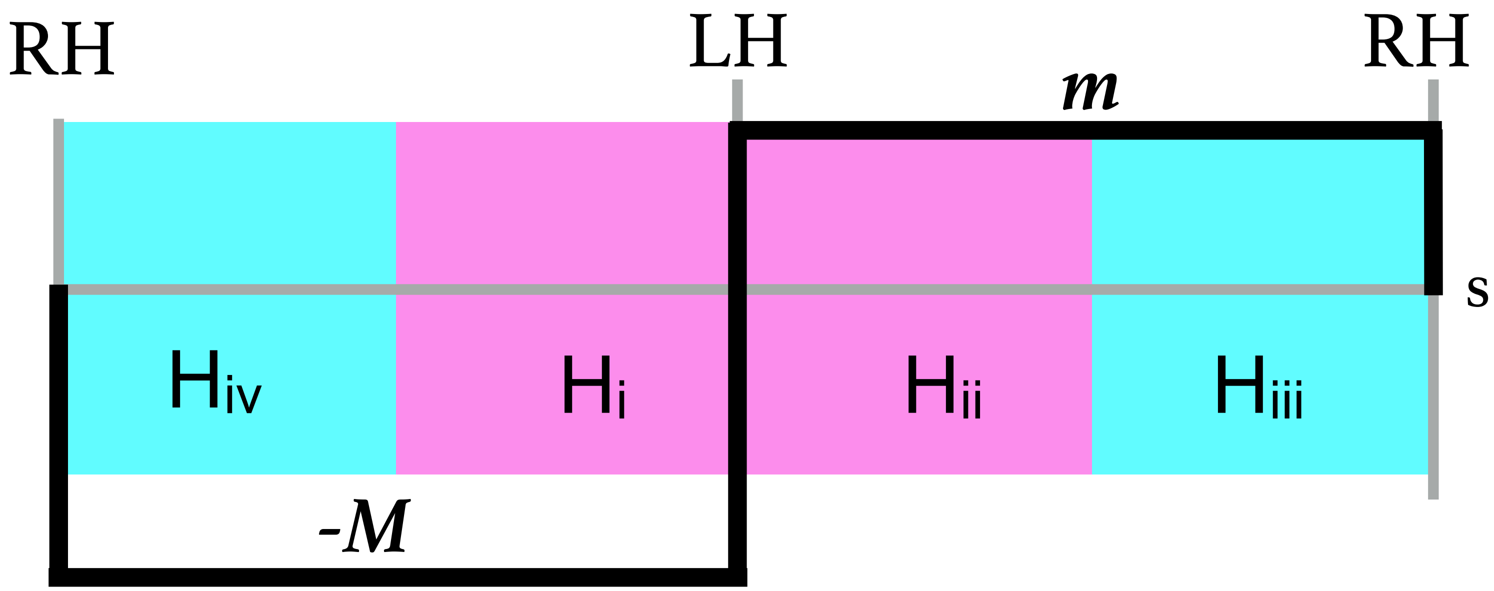

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Ground state

$$\Delta = \frac{\langle \Omega, H_i | \Omega, H_{ii} \rangle \langle \Omega, H_{ii} | \Omega, H_{iii} \rangle \langle \Omega, H_{iii} | \Omega, H_{iv} \rangle \langle \Omega, H_{iv} | \Omega, H_i \rangle}{|\langle \Omega, H_{iv} | \Omega, H_i \rangle|^2 \langle |\Omega, H_{ii} | \Omega, H_{iii} \rangle|^2}$$

- Phase of $\Delta(A)$ unaffected by arbitrary phase rotations on eigenbases of $H_{i\text{-}iv}$

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New physics may be hidden in Standard Model due to nonperturbative chiral dynamics