Black Hole Microstate Counting using Pure D-brane Systems

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based on JHEP10(2014)186 [arXiv:1405.0412] and upcoming paper

with Abhishek Chowdhury, Richard Garavuso & Ashoke Sen

Plan of the talk

- Overview
- Our work
  - Motivation
  - Our system
  - Warm up : an easier toy
  - Actual system
  - Towards a conjecture
- Conclusion

Summary

- Long term goal:
  to perform exact microscopic counting in $\mathcal{N} = 2$ theories using pure D brane systems.

- In these works, we test our methods for an intersecting D brane system in type IIA string theory, compactified on T^6 and our computation yields the expected result.

- All microstates we find carry zero angular momentum.

conjecture : At a generic point in moduli space, microstates of a single centred black hole all carry zero angular momentum.

Our conjecture has interesting impact on fuzzball program.

Overview

Black Holes: General Introduction

- Special solutions of Einstein's equations, in which there is a region of space time, bounded by an "event horizon", from which nothing can escape!
- They have been observed in our universe.
  (Hence not just a fancy theoretical stuff !)
- Black holes solutions are completely specified by only a few parameters (like mass, charge, angular momentum)
  $\Rightarrow$ regarded as thermodynamical system, there seems to be no microstates, hence no entropy.

Black Holes should have entropy

What if black holes do not have entropy ?

- 2^{nd} law in danger : What happens to the entropy of a bucket of hot water, when thrown into a black hole ?
- postdiction in danger : If black holes do not have any microstates, how does it remeber the state of the star that colapsed into a black hole ?

Black Holes better have microstates and entropy.

Any candidate for entropy of a black hole?

- Laws of black hole mechanics $\Rightarrow$ area of event horizon never decreases.
- Black Hole entropy $\propto$ Black Hole area
  [proportionality constant] $=$ *length*$^{-2}$ (in units $k_B = 1$).
- But there is no natural length scales in classical physics!
- There is Planck length in quantum gravity.

> Lesson: black hole entropy is a window to quantum gravity !

Importance for a theory of quantum gravity

- What is the statistical understanding of entropy ?
  Since classical gravity deos not answer this, it is for a quantum theory of gravity to answer this question.
- A test for quantum gravity :
  There may be many phases of a theory of quantum gravity, many of them would have black hole solutions. For each of them this question can be posed and must be answered.
- An opportunity !
  An experimental test of any theory of quantum gravity is highly unlikely in near future.
  $\Rightarrow$ Theoretical tests such as this provide opportunities to check whether such a theory is consistent.

Score card of string theory

very high score !

* In string theory one can answer this question (with high accuracy) for phases with high enough supersymmetry.

Many fascinating features of string theory (SUSY, dualities, AdS/CFT, extra dimensions) appear together in this problem. Thus success in this direction validates the very structure of string theory.

* Considerable progress has been achieved for $\mathcal{N} = 8$, $\mathcal{N} = 4$ theories.

* Not much achievement for $\mathcal{N} = 2$ theories.

The general story

- In supersymmetric theories certain quantities (called index) do not change as one changes the coupling of the theory.
- For supersymmetric black holes, one can relate degeneracy to index. It is enough to be able to compute the index for any coupling.
- Black holes are good descriptions for small G, large GM.
- microscopic description : For smaller G, small GM gravity is decoupled and the system contains stringy objects (like D branes). Computing index is easier in this description.
- How to get to microscopic description?
  Track the charges carried by the black hole.

Our Work

Motivation

D brane systems are special

Only option for microscopic system in $\mathcal{N} = 2$ theories (CY3 compactifications).

$\Rightarrow$ Need to develop methods of microstate counting using pure D brane systems.

Steps . . .

- $\mathcal{N} = 8$ theory (T^6 compactification).

  (for smallest charges positive result in JHEP 10(2014)186 , arXiv:1405.0412 [hep-th] , recent progress for larger charges.)

- $\mathcal{N} = 4$ theory (K3 compactification)

- $\mathcal{N} = 2$ theory, (Calabi Yau compactificaion)

Our system

Our system

* 1/8 BPS black holes in $\mathcal{N} = 8$ theory.
* Relevant index is $B_{14} = \frac{1}{14!} \mathrm{Tr}\, (-1)^F (2J_3)^{14}$ (same as Witten index with Goldstinos removed).
* Has already been computed by Shih, Strominger, Yin. N_1 KK monopoles associated with x^5, N_2 units of momentum along the x^5, N_3 D1 branes along x^5, N_4 D5-branes along $x^5 \times T^4$ and N_5 units of momentum along the x^4.

Our system

* Using various dualities, this can be mapped to a pure D brane system.

Table : Brane configuration

brane	123	45	67	89
N_1 D2		$\surd$		
N_2 D2			$\surd$	
N_3 D2				$\surd$
N_4 D6		$\surd$	$\surd$	$\surd$

* First let us consider $(N_1, N_2, N_3, N_4) = (1, 1, 1, 1)$ case.

  The index is known to be 12 in this case.

What to do ?

- Calculate Witten Index for the given brane system (after throwing the Goldstinos and Godstones).
- Only minimum energy modes are relevant
  $\rightarrow$ concentrate on 0 modes.
- Witten Index in the SUSY QM (that lives on the intersection of the branes).
- But how to get that SUSY QM ?

What to do ?

- Calculate massless open string spectrum in this brane background.
- Arrange in SUSY multiplets.
- SUSY dictates their interactions (mostly).
- Witten Index = Euler characteristic of the vacuum manifold.
- Write down the potential, calculate the Euler number of the vacuum manifold.

Warm up: 2 intersecting branes

2 Intersecting D-branes

Table : Brane configuration

brane	123	45	67	89
1 D2		$\surd$		
1 D2			$\surd$	

SUSY multiplets

Preserved number of supercharges $= 32/(2 \times 2) = 8$
$\Rightarrow$ Arrange fields in $\mathcal{N} = 2$ multiplets .

Table : $\mathcal{N} = 2$ multiplets

Fields	$\mathcal{N} = 2$ multiplet
$V^{(i)}, \Phi_3^{(i)}$	$\mathcal{N} = 2$ vector multiplets
$\Phi_1^{(i)}, \Phi_2^{(i)}$	$\mathcal{N} = 2$ hypermultiplet
$Z^{(12)}, Z^{(21)}$	$\mathcal{N} = 2$ hypermultiplet

Physical interpretation of bosonic fields

Table : Interpretation of on brane fields

Fields	Physical Interpretation
$V^{(1)}$	1, 2, 3 coordinates of 1-st brane.
$\Phi_1^{(1)}$	Wilson lines of the 1-st brane along 4, 5 .
$\Phi_2^{(1)}$	6, 7 coordinates of 1-st brane.
$\Phi_3^{(1)}$	8, 9 coordinates of 1-st brane.

Interactions of the multiplets

Table : Interactions

Fields	Interactions
$V, \Phi_1, \Phi_2, \Phi_3$	$\mathcal{N} = 4$ SYM (free for U(1))
$V^{(1)} - V^{(2)}, \Phi_3^{(1)} - \Phi_3^{(2)}, Z^{(12)}, Z^{(21)}$	$\mathcal{N} = 2$ vector + $\mathcal{N} = 2$ hyper

Superpotentials

- $\mathcal{W}_{\mathcal{N}=4} \sim \mathrm{Tr}\left(\Phi_1[\Phi_2, \Phi_3]\right)$

  vanishes for Abelian case.

- $\mathcal{W}_{\mathcal{N}=2} \sim Z^{(12)}(\Phi_3^{(1)} - \Phi_3^{(2)})Z^{(21)}$

  Mixed strings sense separation of branes.

Goldstones

Table : Goldstones

Goldstone	Physical interpretation
$A_\mu^{(1)} + A_\mu^{(2)}$	c.o.m along flat directions
$\phi_1^{(1)}$	Wilson line
$\phi_2^{(2)}$	Wilson line
$\phi_2^{(1)}$	1st brane moving along 2nd brane
$\phi_1^{(2)}$	2nd brane moving along 1st brane
$\phi_3^{(1)} + \phi_3^{(2)}$	c.o.m along x^8, x^9

6 Goldstones $\rightarrow$ 6 Goldstinos $\rightarrow$ $4 \times 6 = 24$ broken SUSY

$\therefore 32 - 24 = 8$ remaining SUSY.

The actual problem

The actual problem

Table : Brane configuration

brane	123	45	67	89
1 D2		$\surd$		
1 D2			$\surd$	
1 D2				$\surd$
1 D6		$\surd$	$\surd$	$\surd$

- preserved SUSY : $\mathcal{N} = 1$
- The Lagrangian :

$$L = \sum_{i=1}^{4} (\mathcal{N} = 4\, SYM)_i + \sum_{(ij);i,j=1}^{4} (\mathcal{N} = 2)_{(ij)} + \mathcal{W}_{\mathcal{N}=1}$$

Various pieces of $\mathcal{W}_{\mathcal{N}=1}$

- $\mathcal{W}_{\mathcal{N}=1} = \mathcal{W}_1 + \mathcal{W}_2$
- $\mathcal{W}_1 = \sqrt{2} C \sum_{(ij);\ i,\ j=1}^{4} (sign) Z^{ij} Z^{jk} Z^{ki}$
  - ∗ has origin in 3 string interaction.
  - ∗ the constant C and the signs are in principle calculable from 3 string amplitudes.
- $\mathcal{W}_2 = c^{(12)}(\Phi_3^1 - \Phi_3^2) + \dots$
  - ∗ caused by metric and B field fluctuations.
  - ∗ as a side effect this introduces FI parameters.
  - ∗ both $\mathcal{W}_2$ and FI parameters have the effect of making the mixed strings non vanishing.

The vacuum manifold

$V = V_D + V_F$

- D term :
  D term eqn + gauge invariance = complexified gauge invariance
  $\therefore U(1)^3 \rightarrow (\mathcal{C}^*)^3$
  - $*$ 6 ϕ-s, all neutral.
  - $*$ 12 z-s, all charged $\rightarrow$ 12-3=9 dimensional toric variety.
- F term :
  - $*$ ϕ-s are uniquely fixed in terms of z-s $\rightarrow$ can be safely forgotten.
  - $*$ 9 equations involving only z-s. Thus,

    vacuum manifold $\rightarrow$ intersection of hypersurfaces in a toric variety.

.

The equations (in homogeneous coordinates)

- ϕ eqns :

  $z_{ij} z_{ji} = -c_{ij}$

  $\Rightarrow$ all z-s are non-zero

  $\Rightarrow$ a single patch of the toric variety suffices.

  $\Rightarrow$ can be treated as equations on $\mathbb{C}^9$.

- z eqns :
  * ϕ-s are fixed in terms of z-s
  * consistency conditions:

$$z_{23} z_{31} z_{12} + z_{23} z_{34} z_{42} = z_{32} z_{21} z_{13} + z_{32} z_{24} z_{43}$$
$$z_{24} z_{41} z_{12} + z_{24} z_{43} z_{32} = z_{42} z_{21} z_{14} + z_{42} z_{23} z_{34}$$
$$z_{34} z_{42} z_{23} - z_{34} z_{41} z_{13} = z_{43} z_{31} z_{14} + z_{43} z_{32} z_{24}$$

- 9 equations on 9 $\mathbb{C}$ variables $\Rightarrow$ vacuum manifold is 0 dimensional

Affine coordinates (on relevant patch)

$$u_1 \equiv z_{12}z_{21}$$

$$u_2 \equiv z_{23}z_{32}$$

$$u_3 \equiv z_{31}z_{13}$$

$$u_4 \equiv z_{14}z_{41}$$

$$u_5 \equiv z_{24}z_{42}$$

$$u_6 \equiv z_{34}z_{43}$$

$$u_7 \equiv z_{12}z_{24}z_{41}$$

$$u_8 \equiv z_{13}z_{34}z_{41}$$

$$u_9 \equiv z_{23}z_{34}z_{42}$$

The final result

Number of solutions $= 12$

exactly the expected result !

larger charges: difficulty

Natural attempt → formulate the problem in terms gauge invariant objects.

* variables are now vectors and matrices.
* Affine coordinates → generators of the ring of invariants.
* Generally such a ring contains more generators than naively expected and some compenstaing syzygies.
  We are unaware of any straightforward formula for the generators (and syzygies) of this ring.
  checking by hand is a hopeless task.

larger charges: possible methods

- nice method : Hilbert series + computer algebra
  - Hilbert series: knows about number of monomials for any given charge, for a graded polynomial ring.
  - If the vacuum variety is zero dimensional, the the number of points can be read from the Hilbert series (after some manipulations).
  - Given the variables and their charges, Macaulay2 can generate the Hilbert series.
  - did not work out due to computational limitations :(
- naive method : Gauge fix !

  It works !

(1,1,1,2) and (1,1,1,3)

- We are able to handle these cases by gauge fixing.
- Degeneracies are known to be 56 for (1,1,1,2) and 208 for (1,1,1,3).
- We are able to get the same result.

Towards a conjecture . . .

zero angular momenta microstates

* Matching index does not imply one to one matching of the microstates.
* In gravity side, (single centred) SUSY black holes define an ensemble of states with strictly 0 angular momenta, i.e. all bosonic.
* In our work we are able to capture the microstates themsleves and find they are all zero angular momentum as well !
* suspicion : Is this true at a generic point of moduli space ?

suspicion to conjecture

known results do not contradict the proposed conjecture.

* When blackhole description is valid, microstates of single centred black holes are all zero angular momentum.

* In existing computations, index usually takes contribution from both bosonic and fermionic states.
  But such computations usualy take various moduli to vanish, hence are not done in a generic point of moduli space.

* On the contrary, our computation requires turning on various moduli, hence is being done at a "generic point".

guideline for fuzzball program : In order to be trustable as black hole microstates, solutions must be constructed at a generic point of moduli space and have zero angular momentum there.

Conclusion

Summary and scorecard

* Initial motivation: to develop methods for microstate counting using pure D brane systems.
* We tested our methods for 1/8 BPS pure D brane configuration in type *IIA* theory for a few small charges.
  These at the least are some more non trivial checks of U duality.
* All our microstates are zero angular momentum and hence in one to one correspondence with black hole microstates.
* In the view of other known results, we are led to the conjecture that at a generic point of moduli space all microstates of a single centred black hole have zero angular momentum.

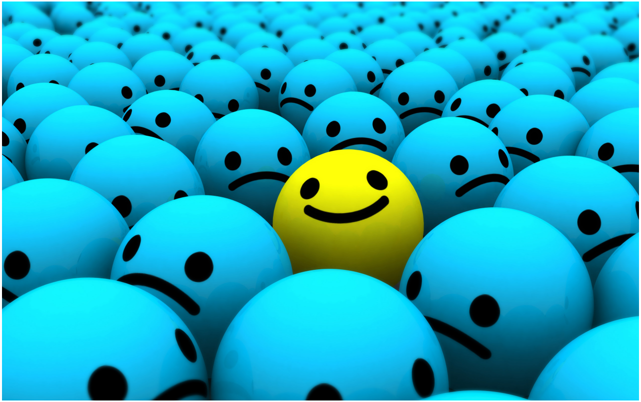
Future taks

* Counting the index for large charges.

* Apply similar techniques to $\mathcal{N} = 4$ theory.
  (We are thinking of starting with $T^4/\mathbb{Z}_2$ and then blowing up to $K3$.)

* Apply similar techniques to $\mathcal{N} = 2$ theory.

thank you !



The equations (in affine coordinates)

$$
\begin{aligned}
m_{13}u_7^2u_9^2 - m_{23}m_{34}m_{24}^2u_7u_8 + m_{24}u_7u_8u_9^2 - m_{24}m_{23}m_{12}u_8^2 &= 0 \\
u_7^2u_9 - u_7u_9^2 + m_{23}m_{24}m_{34}u_7 - m_{12}m_{14}m_{24}u_9 &= 0 \\
u_8^2u_9 + u_8u_9^2 - m_{23}m_{24}m_{34}u_8 - m_{13}m_{14}m_{34}u_9 &= 0\,.
\end{aligned}
$$

with $m_{ij} = -c_{ij}$

The system concerned

Original System IIB on T^6 , D1-D5 system (some results are known here)	D Dual IIA on T^6 , only R-R charges (computations $\Rightarrow$ check of U duality)
KK along 4	D2-branes along 45
momentum along 5	D2-branes along 67
D1-brane along 5	D2-branes along 89
D5-brane along 56789	D6-branes along 456789
momentum along 4	D4-branes along 4589

Dualities relating two systems

1. T duality along 4-5
2. T duality along 6-7
3. S duality
4. T duality along 5-8-9

Thumb Rules: S Duality

Initial configuration	Final configuration
momentum	momentum
F1	D1
D1	F1
KK monopole	KK monopole
NS5 brane	D5 brane
D3 brane	D3 brane

Table : S Duality

Thumb Rules: T Duality

Initial configuration	Final configuration
momentum (4)	F1 (4)
F1 (4)	momentum (4)
momentum (a), $a \neq 4$	momentum (a)
F1 (a), $a \neq 4$	F1 (a)
KK monopole (4)	NS5 (56789)
NS5 (5-6-7-8-9)	KK monopole (4)
KK monopole (a), $a \neq 4$	KK monopole (a), $a \neq 4$
NS 5 (4) $\times$ T^4	NS5 (4) $\times$ T^4

Table : T Duality (along X^4)

on the signs in $\mathcal{W}_{\mathcal{N}=1}$

- Look at exchange symmetries such as $(x^4 \leftrightarrow x^6, x^5 \leftrightarrow x^7)$, alongwith exchange of brane indices.
- various components of g_{ij}, b_{ij} gets exchanges and/or picks up signs $\rightarrow$ so do c^{ij}-s.
- Through $\mathcal{W}_2$ this affects Φ-s, that in turn affect Z-s through $ZZ\Phi$.
- Demanding invariance of $\mathcal{W}_1$ gives a set of possible choices of relative signs.
- All these choices are related through $Z^{ij} \rightarrow -Z^{ij}$ field redefinitions.
- We work with the choice where only $Z^{13}Z^{34}Z^{41}$ term appears with negative sign.