

UC Davis: The Physics and Mathematics of the Universe  
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# BLACK HOLES, HOLOGRAPHY & ENTANGLEMENT

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# OUTLINE

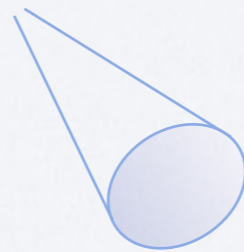
- Black holes in many guises



- AdS/CFT correspondence



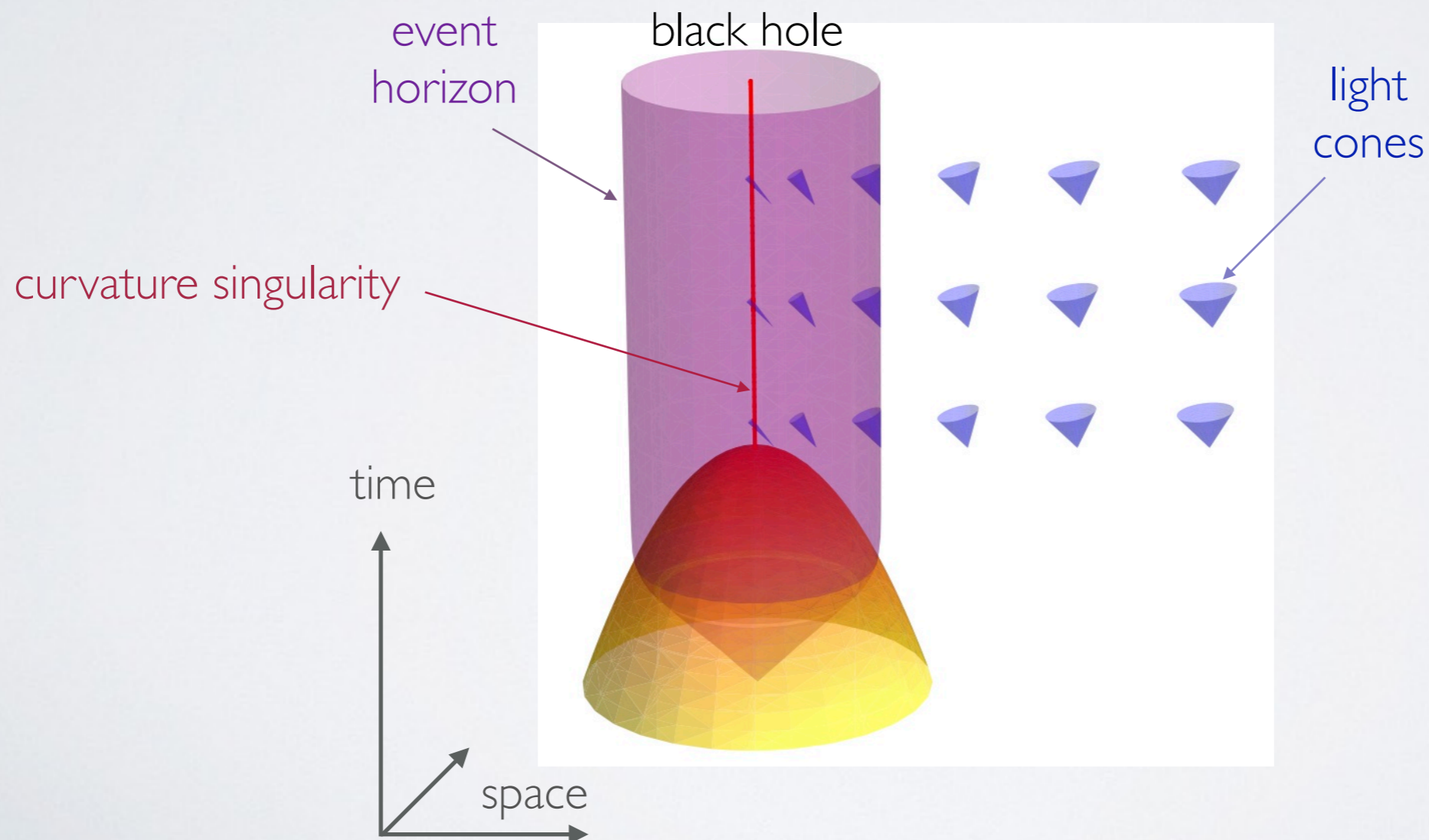
- Holographic entanglement entropy



- Emergent spacetime

# Black holes

- **Black hole** = region of spacetime which cannot communicate with the external Universe
- In Nature, results as endpoint of gravitational collapse



# Many guises of black holes

- Astronomical objects, powering some of the most energetic processes in the Universe
- Mathematically beautiful: “*the most perfect macroscopic objects there are*” [Chandrasekhar]
- Lie at heart of profound dualities (e.g. AdS/CFT)
- Remarkably related to ‘ordinary’ systems (e.g. fluids)
- May hold a key to quantum gravity...

# Black hole thermodynamics

- Stationary black hole characterized by 3 parameters:
  - mass  $M$ , angular momentum  $J$ , and charge  $Q$
- Important properties: horizon area  $A$  and surface gravity  $\kappa$



Laws of BH mechanics mimic laws of thermodynamics:

0.  $\kappa$  is constant over horizon  
for stationary BH
1.  $dM = (1/8\pi G)\kappa dA + \Omega_H dJ$
2.  $\delta A \geq 0$  in any process
3. Impossible to achieve  $\kappa = 0$   
by a physical process

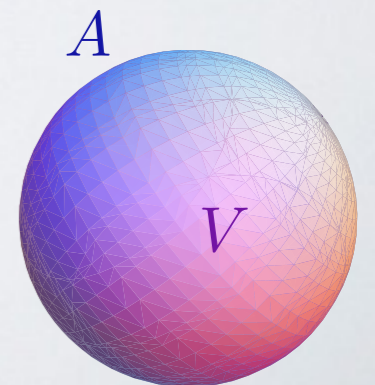
0.  $T$  is constant over system  
in thermal equilibrium
1.  $dE = T dS + \text{work terms}$
2.  $\delta S \geq 0$  in any process
3. Impossible to achieve  $T = 0$   
by a physical process

Hence natural to identify  $M \sim E$ ,  $\kappa \sim T$ , and  $A \sim S$

# Black holes as thermodynamic objects

- $S$  motivated by gedanken-experiments of matter falling into BH [Bekenstein]
- $T$  substantiated by semi-classical calculations [Hawking]: black holes radiate
- Specifically identify  $S_{BH} = \frac{k_B c^3}{G \hbar} \frac{A}{4}$  and  $T_{BH} = \frac{\hbar c}{k_B} \frac{\kappa}{2\pi}$
- Natural question: statistical mechanics origin of BH entropy?
- Generalized Second Law: combined matter+BH entropy increases
  - ⇒ Bekenstein bound (weakly gravitating systems):  $S_{\text{matter}} \leq 2\pi E R$
  - ⇒ Spherical entropy bound (slowly evolving systems): [‘t Hooft, Susskind]

$$S_{\text{matter}} \leq \frac{A}{4} \quad \Rightarrow \quad \text{entropy } S \text{ is not extensive:}$$
$$S \not\propto V$$

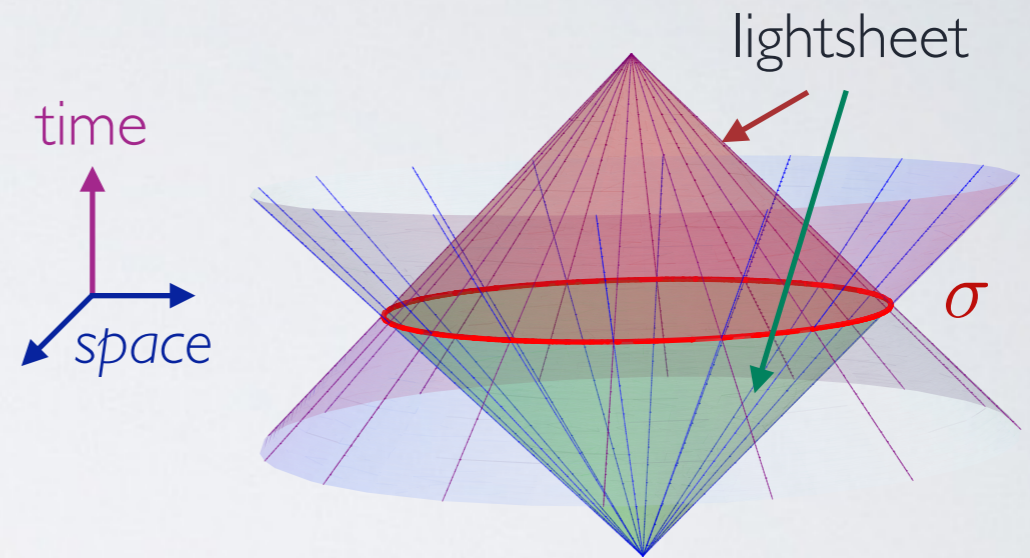


# Holographic Principle

- Covariant entropy bound: full spacetime construct [Bousso]

Entropy on any lightsheet  $L$  of a surface  $\sigma$  cannot exceed the area of  $\sigma$ :

$$S(L) \leq \frac{A(\sigma)}{4}$$



- Holographic Principle: in a theory of gravity, the number of degrees of freedom describing the physics on lightsheet  $L(\sigma)$  cannot exceed  $A(\sigma)/4$   
 $\Rightarrow$  physical equivalence between 2 theories living in different # of dimensions!
- Concrete realization: AdS/CFT correspondence

# AdS/CFT correspondence

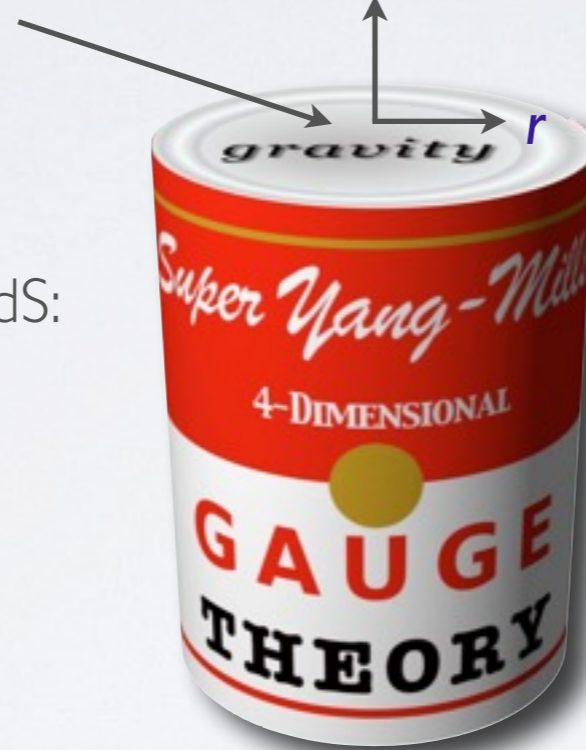
String theory ( $\ni$  gravity)  $\iff$  gauge theory (CFT)

“in bulk” asymp.  $\text{AdS} \times S$

“on boundary”

[Maldacena, '97]

‘soup can’ diagram of AdS:



here label is everything...

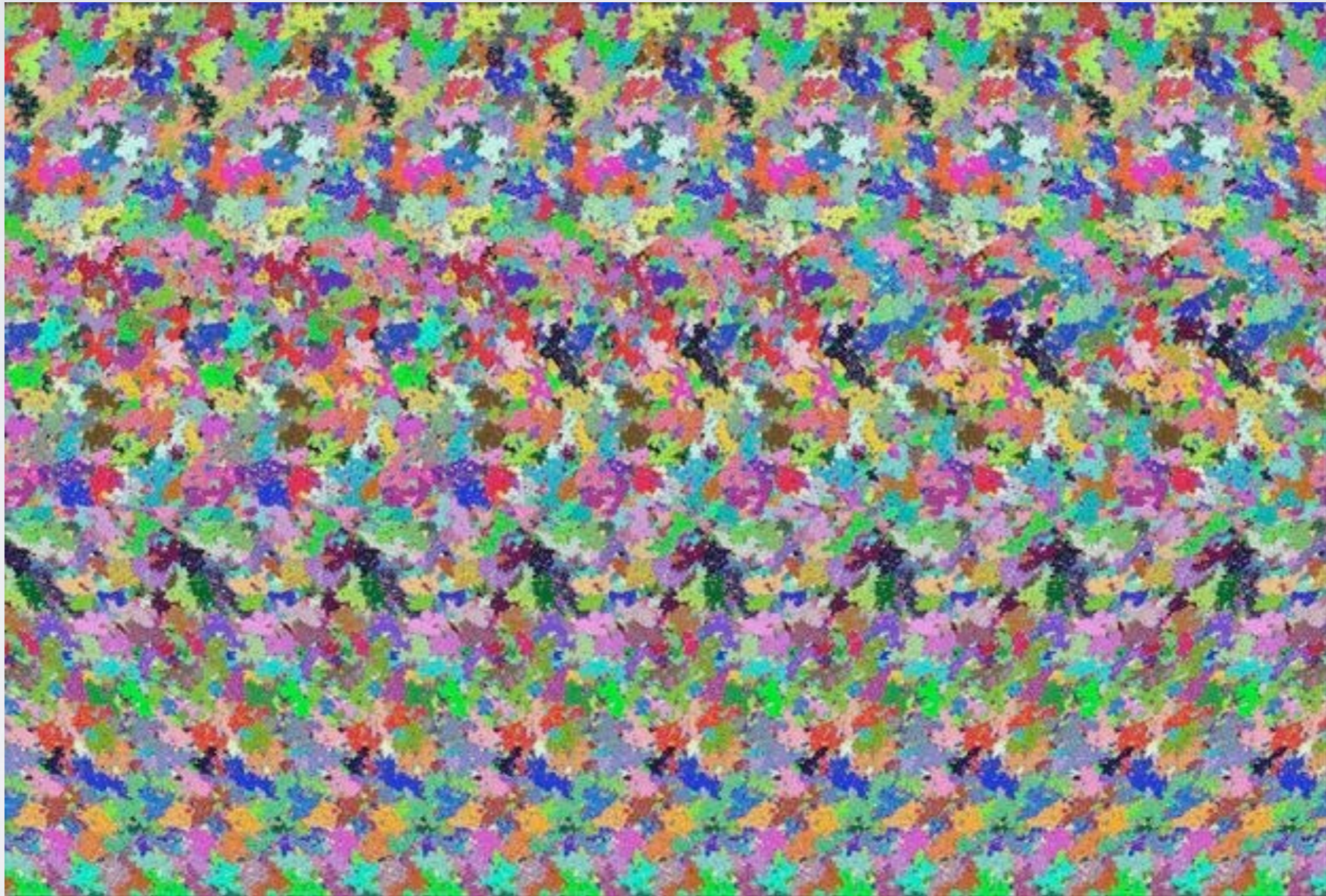
## Key aspects:

- \* Gravitational theory maps to non-gravitational one!
- \* *Holographic*: gauge theory lives in fewer dimensions.



# AdS/CFT correspondence

\* better analogy: stereogram...



...but infinitely more complicated

# AdS/CFT correspondence

String theory ( $\ni$  gravity)  $\iff$  gauge theory (CFT)

*“in bulk”* asymp. AdS  $\times$  K

*“on boundary”*

## Key aspects:

- \* Gravitational theory maps to non-gravitational one!
- \* *Holographic*: gauge theory lives in fewer dimensions.
- \* Strong/weak coupling duality.

## Invaluable tool to:

- ~ Use gravity on AdS to learn about strongly coupled field theory  
(as successfully implemented in e.g. AdS/QCD & AdS/CMT programs)
- ~ Use the gauge theory to define & study quantum gravity in AdS

**Pre-requisite:** Understand the AdS/CFT ‘dictionary’...

# Scale/radius duality

What CFT quantity encodes the extra bulk direction?

- Scale/radius (or UV/IR) duality:

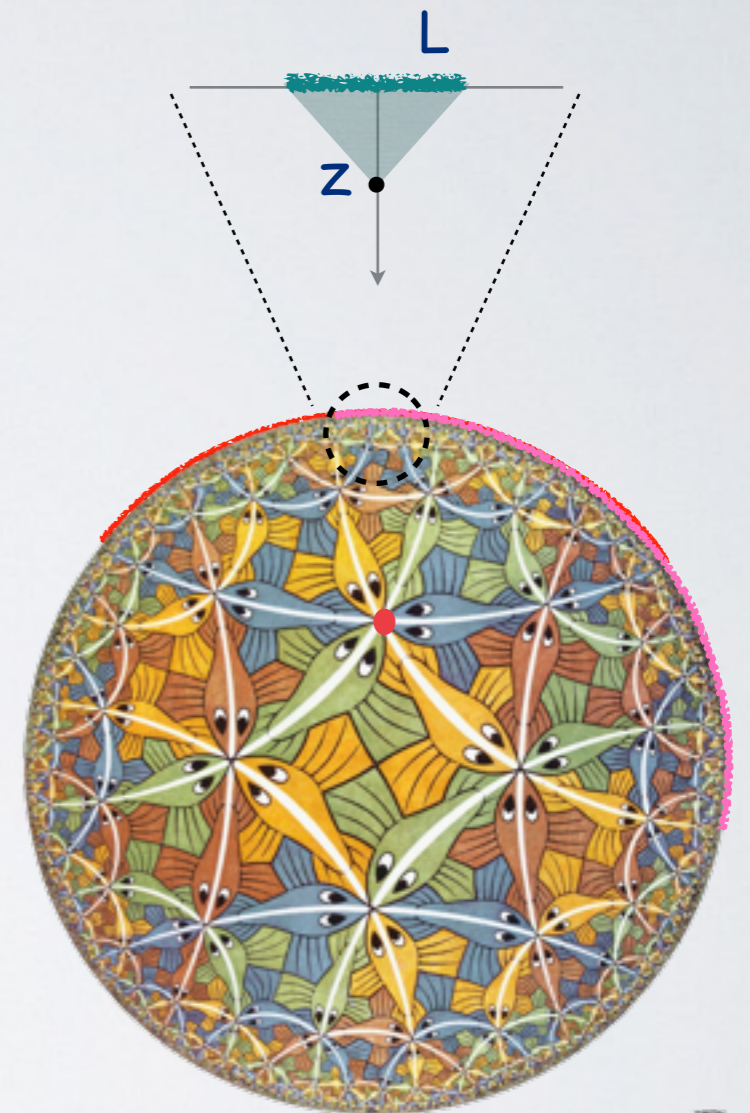
- UV (small scale) in CFT  $\leftrightarrow$  IR (large radius) in AdS
- Local bulk excitation at radial position  $z$  in AdS is manifested by CFT excitation at scale  $L \sim z$ .

[Susskind & Witten]

- Follows from AdS geometry...

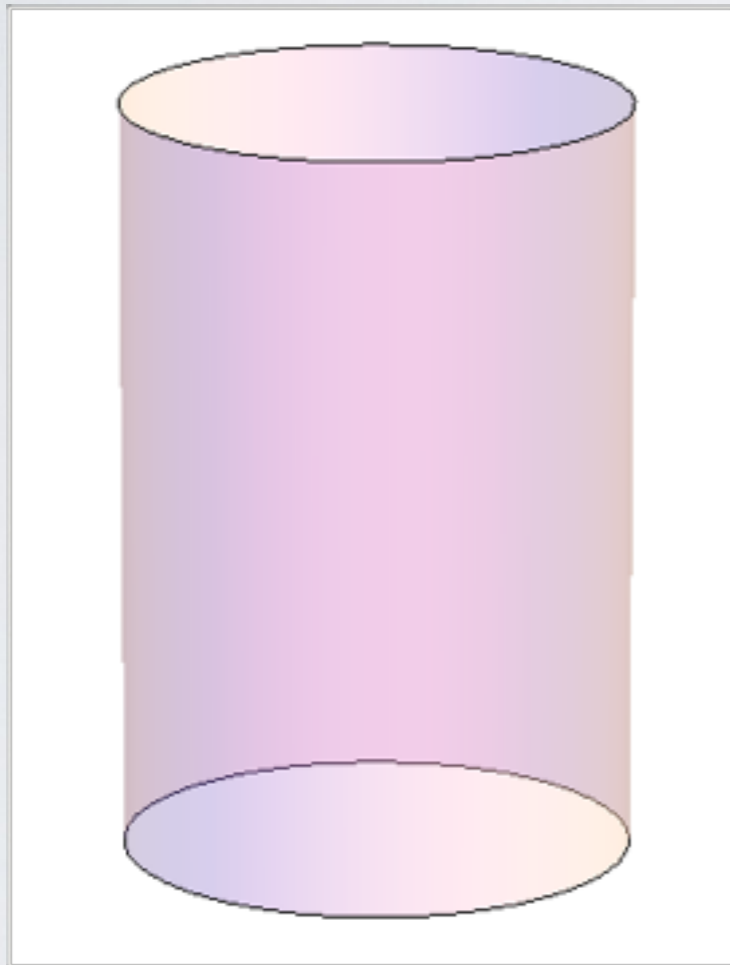
- Provides useful intuition: e.g. object falling into a black hole  $\leftrightarrow$  CFT excitation spreads & thermalizes [Banks, Douglas, Horowitz, Martinec]

- Asymptotic fall-off of bulk fields  $\leftrightarrow$  Expectation values of local gauge-invariant operators in CFT



# Bulk geometries and CFT states

different bulk geometries  $\leftrightarrow$  different states in CFT  
(asymptotically AdS)

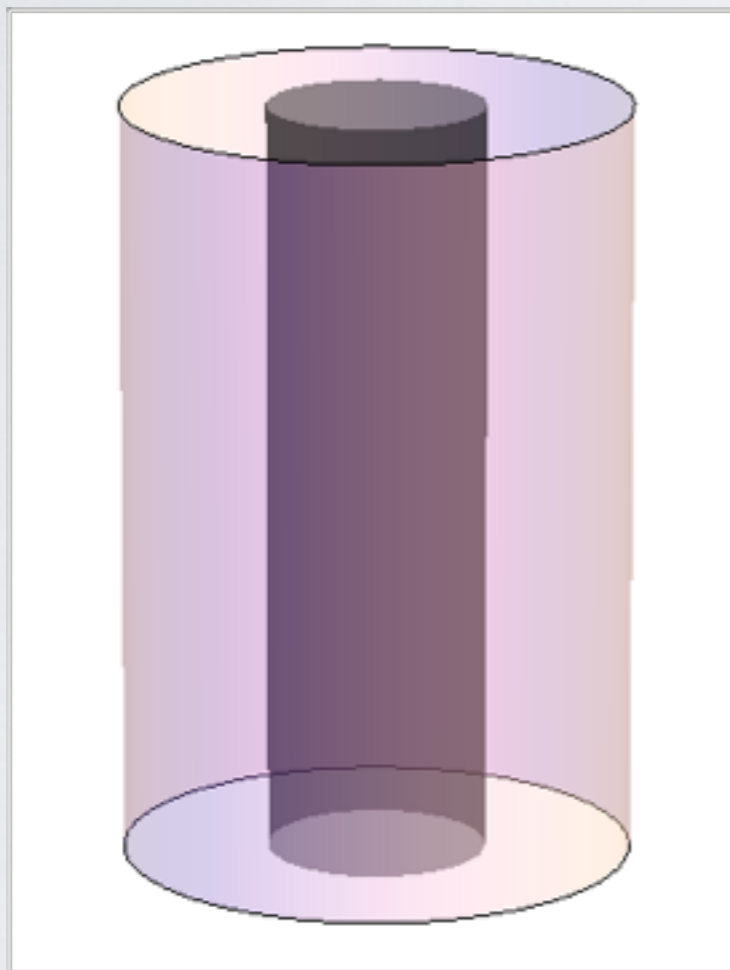


- Pure AdS  $\leftrightarrow$  vacuum state in CFT

Finite-mass deformations of the bulk geometry result in non-zero boundary stress-energy-momentum tensor

# Black holes in equilibrium

different bulk geometries  $\leftrightarrow$  different states in CFT



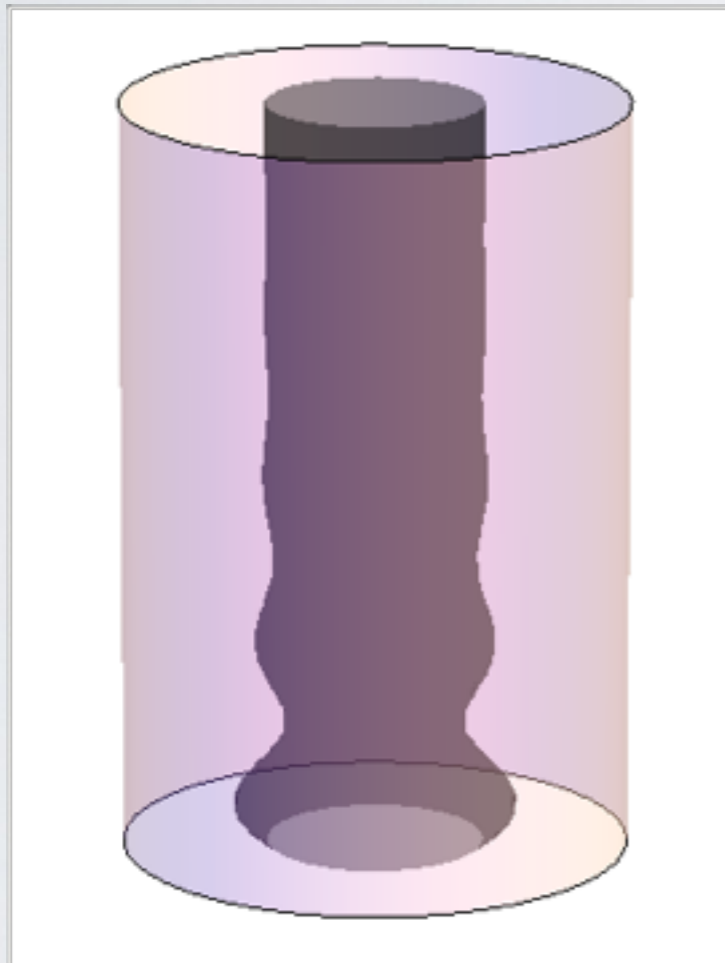
- Pure AdS  $\leftrightarrow$  vacuum state in CFT
- Black hole  $\leftrightarrow$  thermal state in CFT

But need more refined understanding:

- (How) Does the CFT describe physics behind an event horizon?
  - What is the nature of the BH singularity?
  - What is the CFT description of causal structure?
- 
- What about time-evolving geometries?

# Small deviations from equilibrium

evolving bulk geometries  $\leftrightarrow$  corresponding dynamics

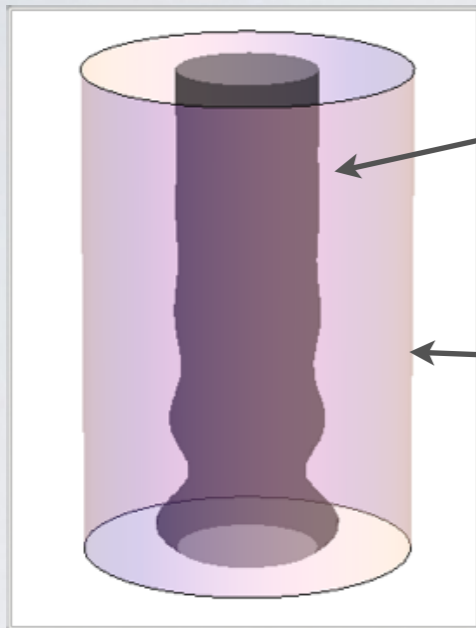


- Pure AdS  $\leftrightarrow$  vacuum state in CFT
- Black hole  $\leftrightarrow$  thermal state in CFT
- Quasinormal modes of perturbed black hole  $\leftrightarrow$  approach to thermal equilibrium

[Horowitz & Hubeny]

\* Horizon response properties  $\leftrightarrow$  transport coefficients in CFT  
[Kovtun, Son, Starinets]

# Description of dynamics



bulk geometry specified by  $g_{ab}(r, x^\mu)$   
(asymptotic falloff of  $g_{ab}$  induces  $T_{\mu\nu}$ )

boundary state characterized by  $T_{\mu\nu}(x^\mu)$   
(for slow variations, describes a fluid)

- \* Bulk dynamics is specified by Einstein's equations.

$$E_{ab} \equiv R_{ab} - \frac{1}{2}R g_{ab} + \Lambda g_{ab} = 0$$

- \* Boundary dynamics satisfies stress tensor conservation.

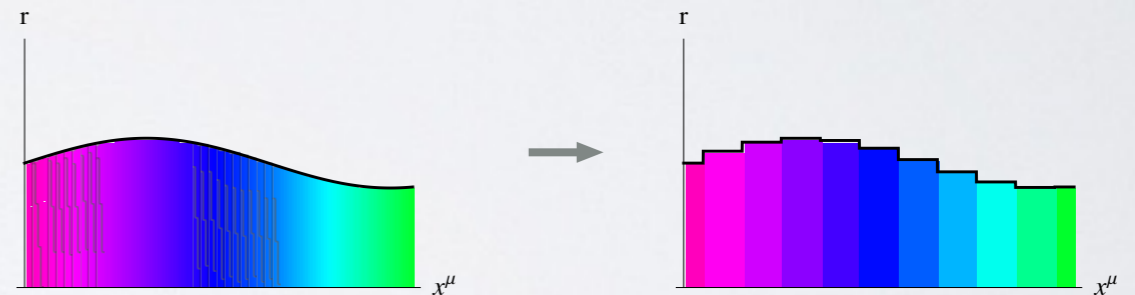
$$\nabla_\mu T^{\mu\nu} = 0$$

# Fluid/Gravity correspondence

[Bhattacharyya, Hubeny, Minwalla, Rangamani, 2008]

- Dynamics of **bulk** black hole  $\Leftrightarrow$  fluid dynamics on **boundary**  
5-d Einstein's equations  $\supset$  4-d Navier-Stokes equations  
w/ negative cosmological const. (describing relativistic, conformal fluid)
- For any given fluid flow, we iteratively construct a solution of a dynamical black hole in AdS whose horizon mimics the fluid.

- Technically: expand in boundary derivatives and solve order by order
- The radial equation is fully nonlinear, and gives patched 'tubes' of different black holes



- We calculate 2nd order transport coefficients for the conformal fluid.
- The pull-back of the horizon area form gives a natural entropy current on the boundary, with automatically non-negative divergence.

[Bhattacharyya, Hubeny, Loganayagam, Mandal, Minwalla, Morita, Rangamani, Reall]



# Fluid/Gravity correspondence

[Bhattacharyya, Hubeny, Minwalla, Rangamani, 2008]

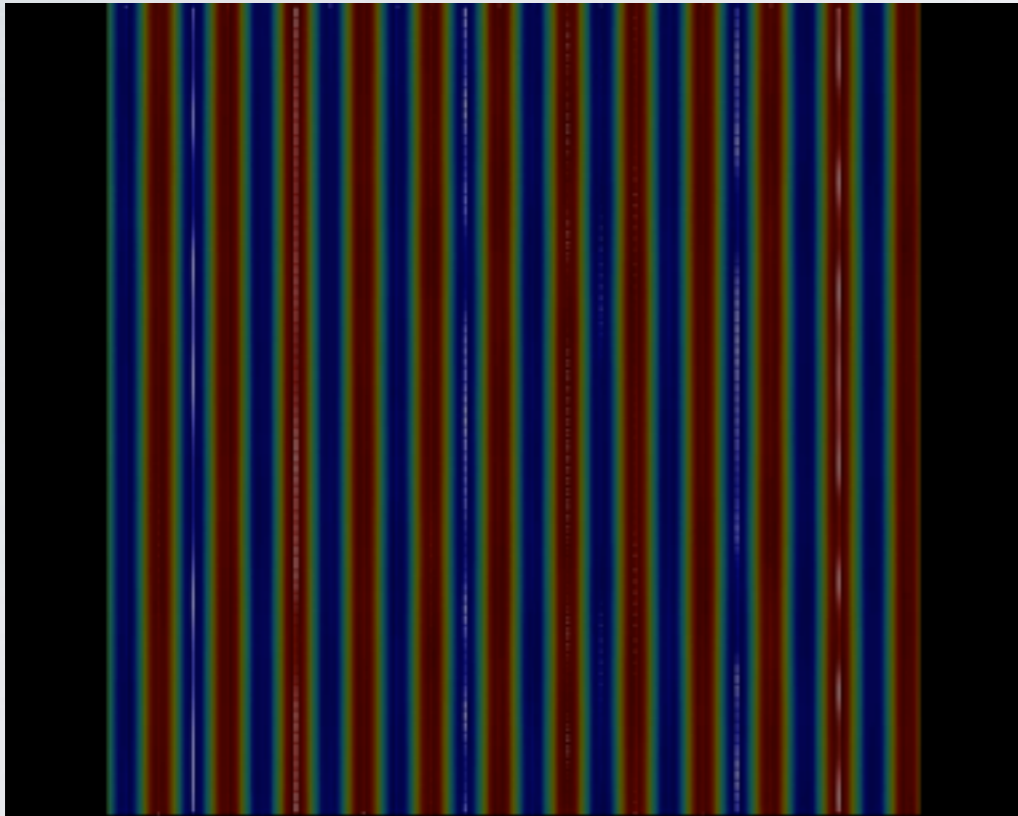
- Dynamics of **bulk** black hole  $\Leftrightarrow$  fluid dynamics on **boundary**
  - 5-d Einstein's equations w/ negative cosmological const.  $\supset$  4-d Navier-Stokes equations (describing relativistic, conformal fluid)
- For any given fluid flow, we iteratively construct a solution of a dynamical black hole in AdS whose horizon mimics the fluid.
  - Generalizations:
    - charged fluids,
    - superfluids,
    - non-conformal fluids,
    - non-relativistic fluids,
    - fluids with boundary,
    - forced fluids,
    - other dimensions, ...
  - Applications:
    - black hole physics,
    - fluid dynamics,
    - nuclear physics,
    - condensed matter physics,
    - solid state physics, ...
  - But not yet everyday liquids, e.g. non-Newtonian fluids.

# Holographic Turbulence

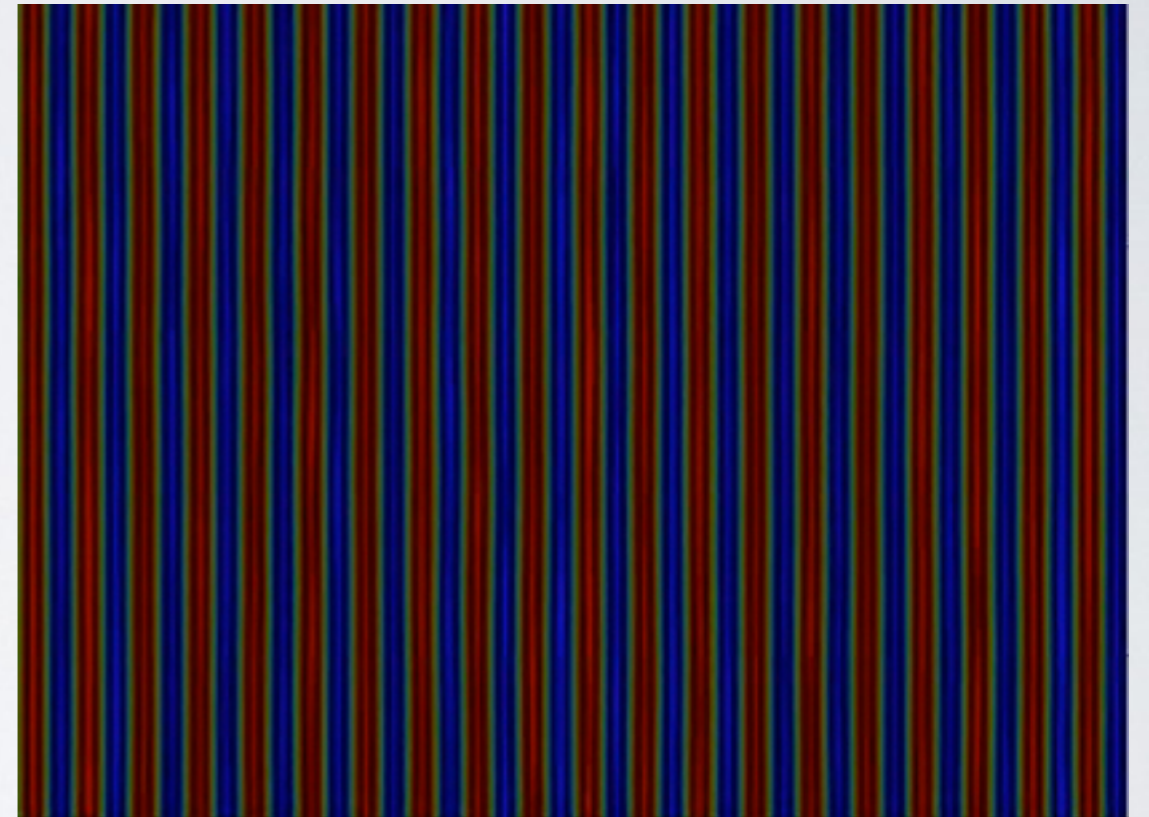
## Beyond fluid/gravity:

- Recently, [Adams, Chesler, Liu] constructed turbulent black holes in asymptotically AdS<sub>4</sub> spacetime by numerically solving Einstein equations
- Resulting bulk solution is well-approximated by the metric derived from fluid/gravity expansion
- Both dual holographic fluid and bulk geometry display signatures of an inverse cascade (see hints of Kolmogorov scaling for driven steady-state turbulence: the power spectrum  $P$  of the fluid velocity  $\sim k^{-5/3}$ )
- Surprise for GR: statistically steady-state black holes dual to  $d$  dimensional turbulent flows have horizons which are approximately fractal with fractal dimension  $D = d + 4/3$

# Holographic Turbulence



<http://turbulent.lns.mit.edu/Turbulence/1307.7267/1307.7267.html>



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The vorticity field of the quantum liquid holographically derived from the turbulent numerical metric: instability of initial perturbation drives ordered state into turbulent evolution with inverse cascade (small vortices merge into larger ones)

# Onward from AdS/CFT

String theory ( $\ni$  gravity)  $\iff$  gauge theory (CFT)

*“in bulk”* asymp.  $\text{AdS} \times \text{K}$

*“on boundary”*

Applied AdS/CFT:

- study specific system via its dual
- e.g. AdS/QCD, AdS/CMT, ...

Fundamentals of AdS/CFT:

- why/how does the duality work
- map between the 2 sides

Holographic Entanglement Entropy

Quantum Gravity

# Entanglement

- Most non-classical manifestation of quantum mechanics
  - “Best possible knowledge of a whole does not include best possible knowledge of its parts — and this is what keeps coming back to haunt us” [Schrodinger '35]
- New quantum resource for tasks which cannot be performed using classical resources [Bennet '98]
- Plays a central role in wide-ranging fields
  - quantum information (e.g. cryptography, teleportation, ...)
  - quantum many body systems
  - quantum field theory
- Hints at profound connections to geometry...

# Entanglement in 2 qubit system

Consider a system of 2 spins, labeled  $A$  and  $B$



- Simple product state:  $|\psi\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B \equiv |\uparrow\downarrow\rangle$

- More complicated product state:

$$|\psi\rangle = \frac{|\downarrow\rangle_A + |\uparrow\rangle_A}{\sqrt{2}} \otimes \frac{|\downarrow\rangle_B + |\uparrow\rangle_B}{\sqrt{2}} = \frac{1}{2} (|\downarrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\uparrow\uparrow\rangle)$$

- Generic state (with arbitrary  $c_{ij}$  s.t.  $\sum c_{ij}^2 = 1$ )

$$|\psi\rangle = c_{00} |\downarrow\downarrow\rangle + c_{01} |\downarrow\uparrow\rangle + c_{10} |\uparrow\downarrow\rangle + c_{11} |\uparrow\uparrow\rangle$$

is **entangled** when it is not a product state.

- A Bell (EPR) pair, such as  $|\psi\rangle = \frac{1}{\sqrt{2}} (|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle)$

is **maximally entangled**.

# Entanglement Entropy (EE)

The amount of entanglement is characterized by **Entanglement Entropy**  $S_A$ . Since we can only measure **A**, integrate out **B**:

- reduced density matrix  $\rho_A = \text{Tr}_B |\psi\rangle\langle\psi|$   
(more generally, for a mixed total state,  $\rho_A = \text{Tr}_B \rho$ )

- **EE** = von Neumann entropy  $S_A = -\text{Tr} \rho_A \log \rho_A$

- For the maximally entangled state  $|\psi\rangle = \frac{1}{\sqrt{2}} (|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle)$

$$\rho_A = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow S_A = \log 2$$

- For the non-entangled state  $|\psi\rangle = \frac{1}{2} (|\downarrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\uparrow\uparrow\rangle)$

$$\rho_A = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \Rightarrow S_A = 0$$

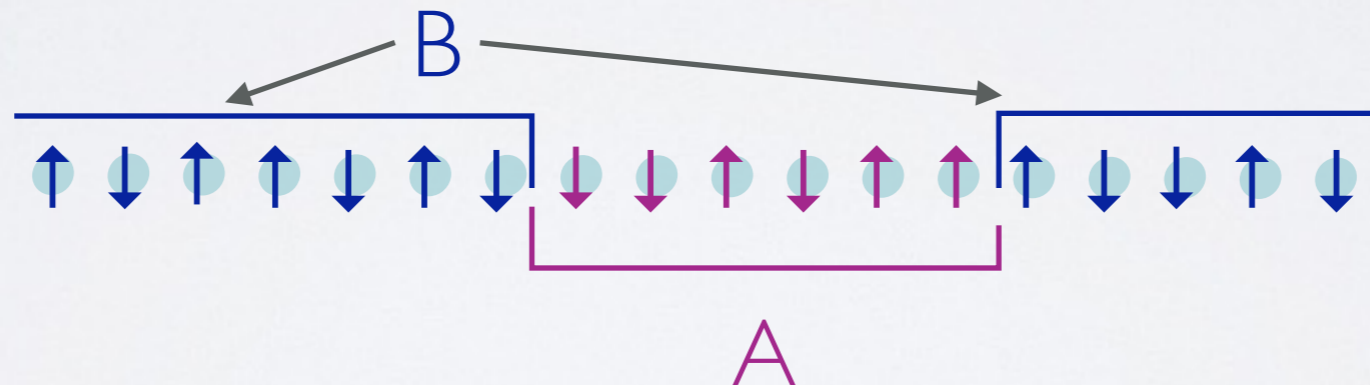
# EE more generally

More generally: divide a quantum system into a subsystem  $A$  and its complement  $B$ , such that the Hilbert space decomposes:

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

e.g.:

- spin chain





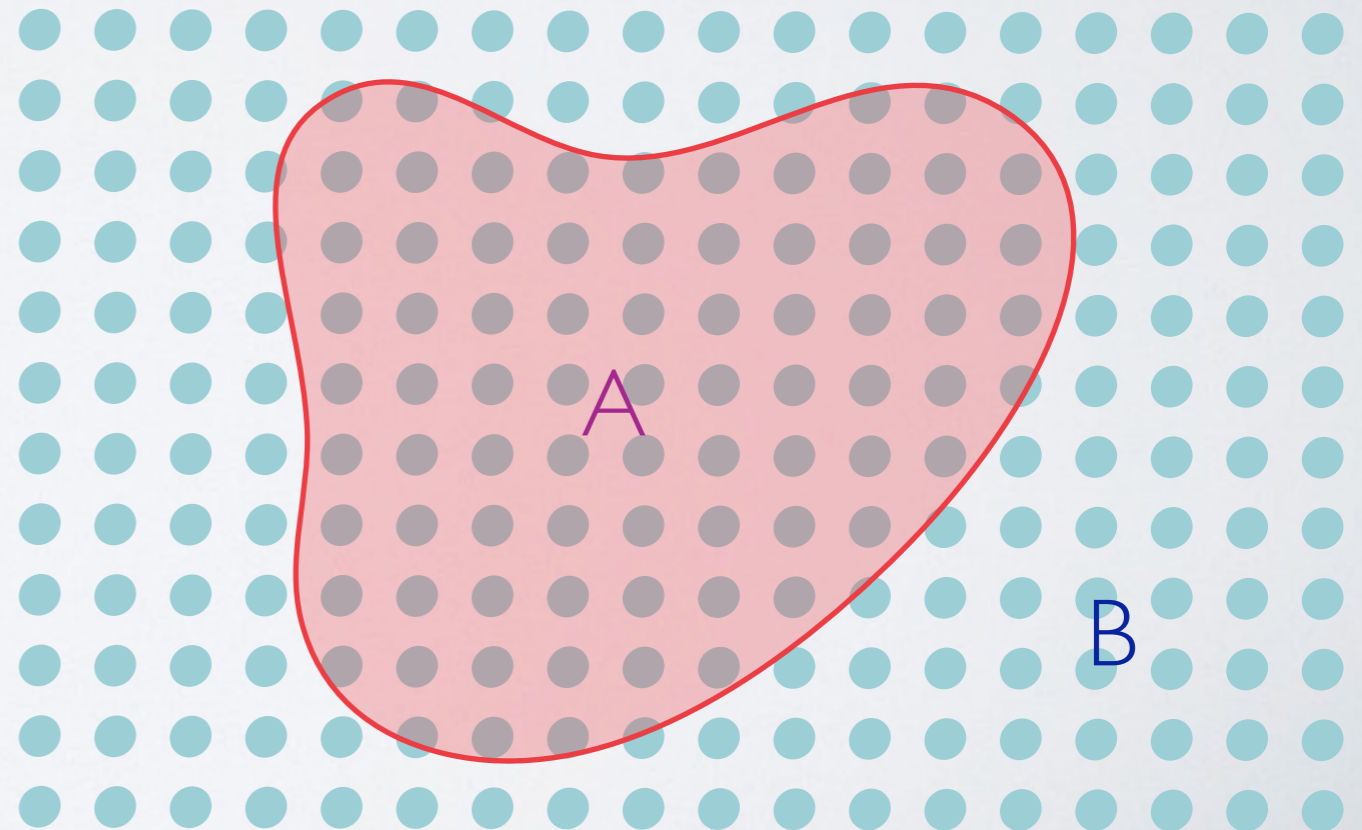
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- spin chain
- many-body quantum system, e.g. on a lattice



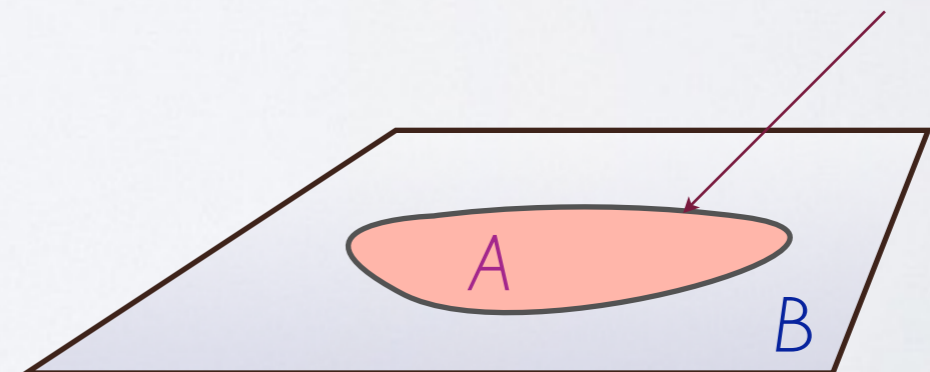
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e.g.:

- spin chain
- many-body quantum system, e.g. on a lattice
- QFT:  $A$  and  $B$  can be spatial regions, separated by a smooth entangling surface

In all cases,  $S_A = -\text{Tr} \rho_A \log \rho_A$ , where  $\rho_A = \text{Tr}_B \rho$ .

# Applications of EE

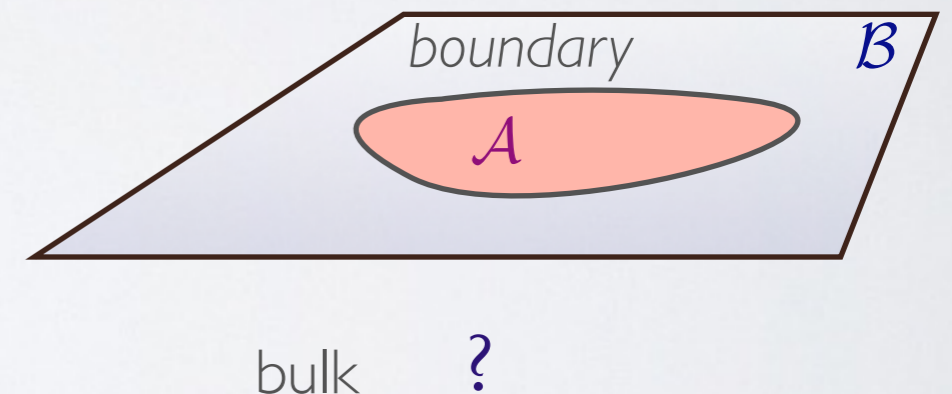
- Quantum Information theory: new quantum resource [Bennett '98 & Masanes '05]
  - quantum cryptography [Ekert, '91]
  - quantum dense coding [Bennett and Wiesner, '92]
  - quantum teleportation [Bennett et al., '93]
- Condensed Matter theory: diagnostic
  - quantum critical points
  - topological phases
  - computational difficulty, e.g. MERA [Vidal '09]
- Quantum Gravity:
  - suggested as origin of black hole entropy [Bombelli, Koul, Lee & Sorkin, '86 Srednicki, Frolov & Novikov, Callan & Wilczek, Susskind ...]
  - origin of macroscopic spacetime [Van Raamsdonk et al., Maldacena & Susskind]

# The good news & the bad news

- **But** EE is hard to deal with...
  - non-local quantity, intricate & sensitive to environment
  - difficult to measure
  - difficult to calculate... especially in strongly-coupled quantum systems

- **AdS/CFT to the rescue?**

- ~ Is there a natural bulk dual of EE?  
(= “Holographic EE”)



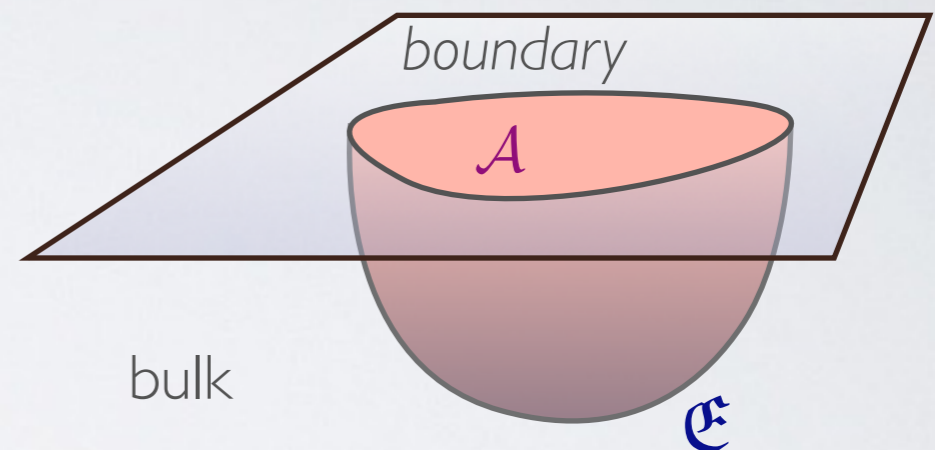
Yes! - described geometrically...

# Holographic Entanglement Entropy

Proposal [Ryu & Takayanagi, '06] for *static* configurations:

In the bulk, EE  $S_{\mathcal{A}}$  is captured by the area of minimal co-dimension 2 bulk surface  $\mathcal{E}$  (at constant  $t$ ) anchored on  $\partial\mathcal{A}$ .

$$S_{\mathcal{A}} = \min_{\partial\mathcal{E}=\partial\mathcal{A}} \frac{\text{Area}(\mathcal{E})}{4G_N}$$



Remarks:

- cf. black hole entropy...
- Minimal surface “hangs” into the bulk due to large distances near bdy.
- Note that both LHS and RHS are in fact infinite...

# Evidence for HEE

- ✓ Leading contribution correctly reproduces the area law
- ✓ Recover known results of EE for intervals in 2-d CFT [Calabrese&Cardy] both in vacuum and in thermal state
- ✓ Derivation of holographic EE for spherical entangling surfaces [Cassini,Huerta,&Myers]
- ✓ Attempted proof by [Fursaev] elaborated & refined by [Headrick, Faulkner, Hartman, Maldacena&Lewkowycz]

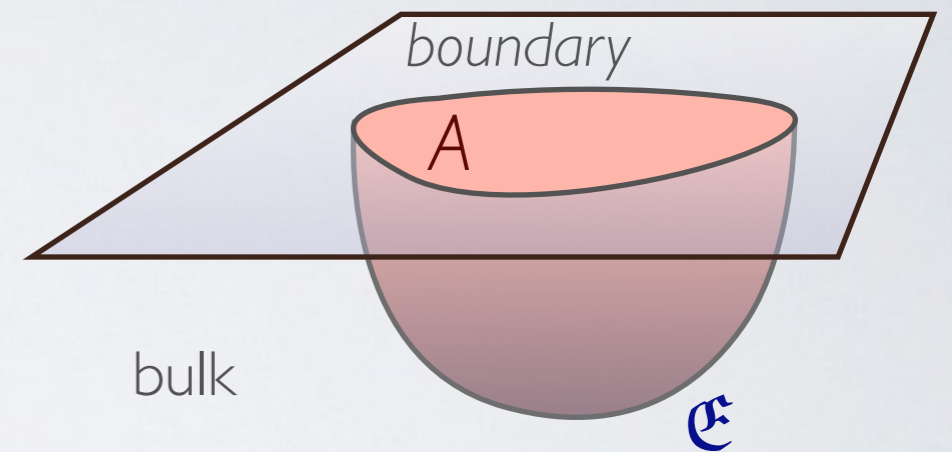
Further suggestive evidence:

- ✓ Automatically satisfies  $S_{\mathcal{A}} = S_{\mathcal{A}^c}$  for pure states
- ✓ Automatically satisfies (strong) subadditivity [Lieb&Ruskai] & Araki-Lieb inequality -- easy to prove on the gravity side, far harder within field theory

# Covariant Holographic EE

But the RT prescription is not well-defined outside the context of static configurations:

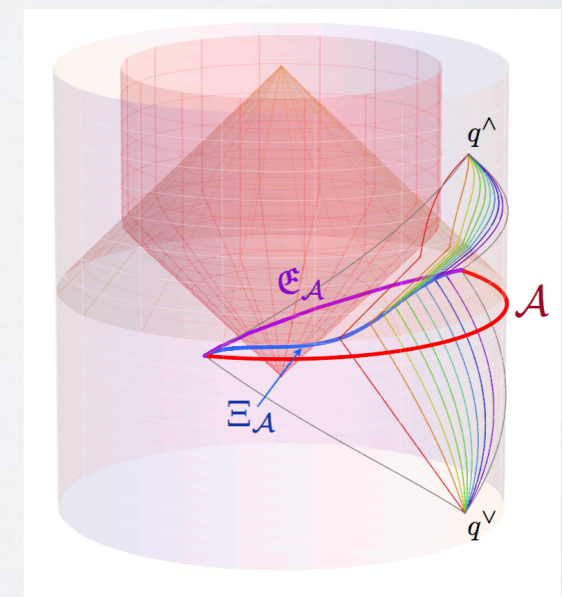
- In Lorentzian geometry, we can decrease the area arbitrarily by timelike deformations
- In time-dependent context, no natural notion of “const. t” slice...



In *time-dependent* situations, RT prescription must be covariantized:

4 natural candidates: [Hubeny, Rangamani, Takayanagi '07]

- $\mathcal{E}$  = Extremal surface
- $\Psi$  = Minimal-area surface on maximal-volume slice
- $\Phi$  = Surface with zero null expansions
- $\Xi$  = Causal wedge rim

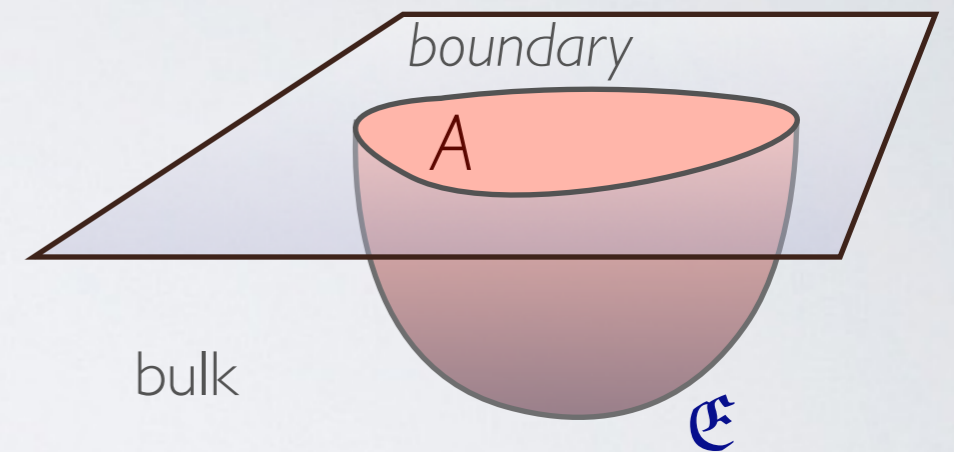




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  - $\Phi$  = Surface with zero null expansions
  - $\Xi$  = Causal wedge rim
- Two blue arrows point from the first three items to the text:  $\mathcal{E} = \Phi$  is correct = 'HRT prescription'

Later known as Causal Information Surface;  
w/ area = causal holographic information  $\chi$

[Hubeny, Rangamani '12]

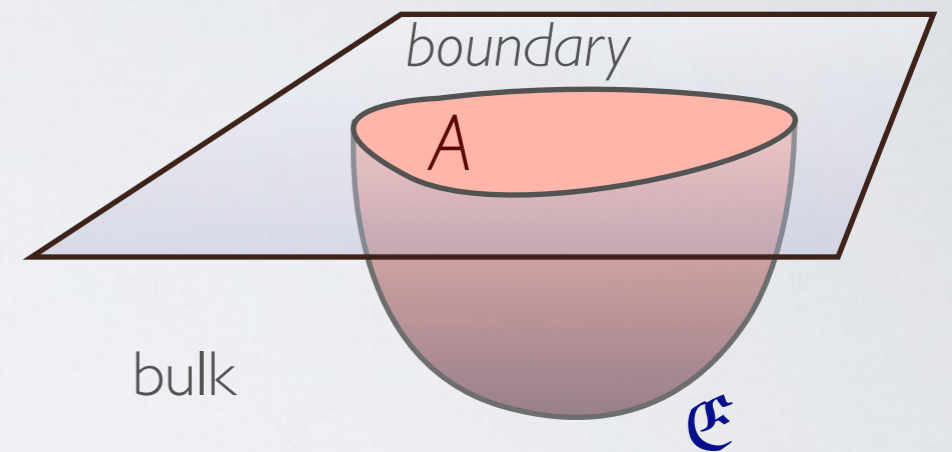
# Covariant Holographic EE

HRT Prescription:

[Hubeny, Rangamani, Takayanagi '07]

In the bulk EE  $S_{\mathcal{A}}$  is captured by the area of extremal co-dimension 2 bulk surface  $\mathcal{E}$  anchored on  $\partial\mathcal{A}$  & homologous to  $\mathcal{A}$

$$S_{\mathcal{A}} = \min_{\partial\mathcal{E}=\partial\mathcal{A}} \frac{\text{Area}(\mathcal{E})}{4G_N}$$



This gives a well-defined quantity in any (arbitrarily time-dependent asymptotically AdS) spacetime  $\Rightarrow$  equally robust as in CFT

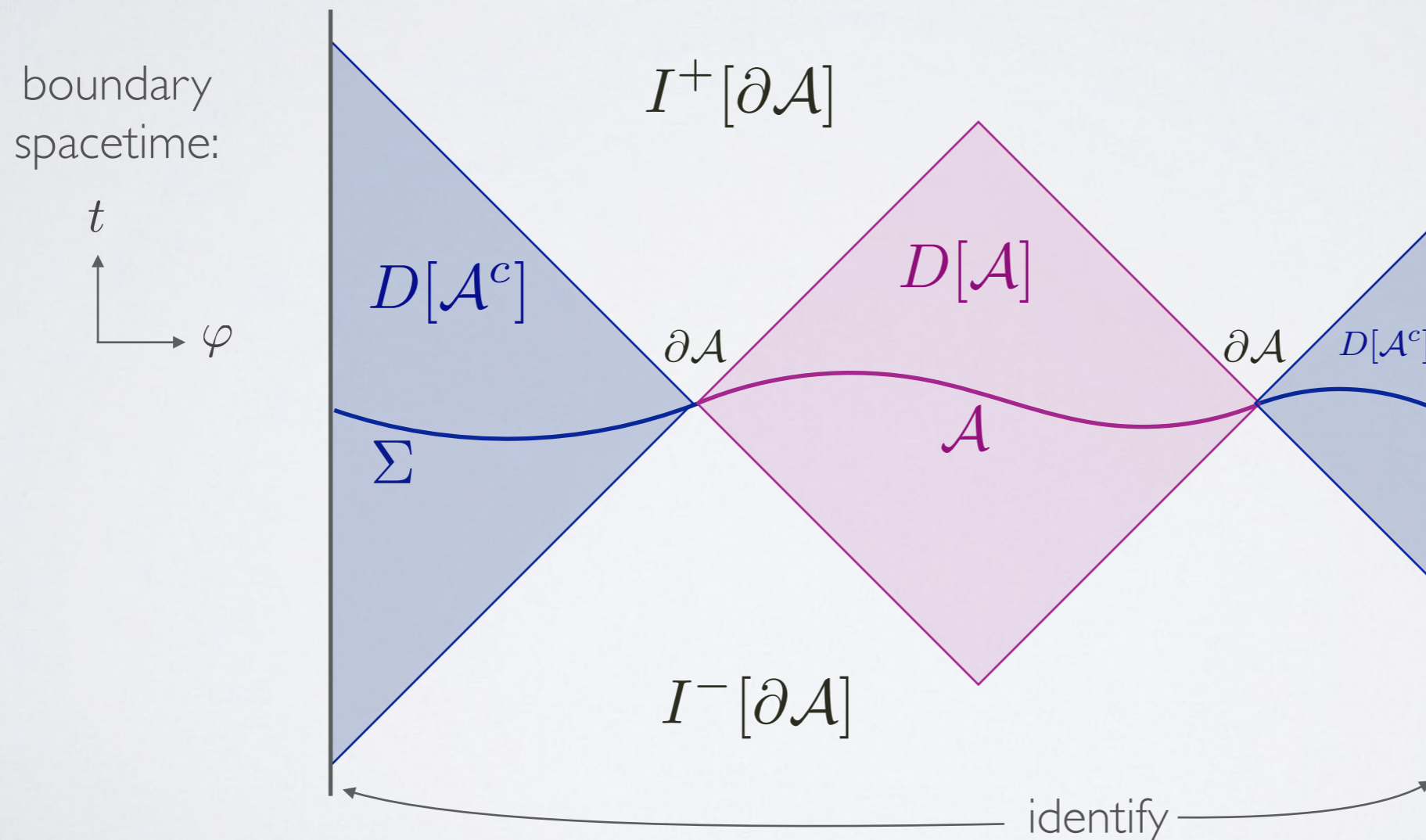
But we can't use Euclidean techniques for proof...

?: Is HRT prescription consistent with CFT constraints, e.g. causality?

# CFT causal restriction

- Entanglement entropy  $S_{\mathcal{A}}$  only depends on  $D[\mathcal{A}]$  and not on  $\Sigma$ .
- Natural separation of boundary spacetime into 4 regions:

$$\partial\mathcal{M} = D[\mathcal{A}] \cup D[\mathcal{A}^c] \cup I^-[\partial\mathcal{A}] \cup I^+[\partial\mathcal{A}]$$



- EE should not be influenced by any change to state within  $D[\mathcal{A}]$  or  $D[\mathcal{A}^c]$ .

# CFT causal requirement on bulk

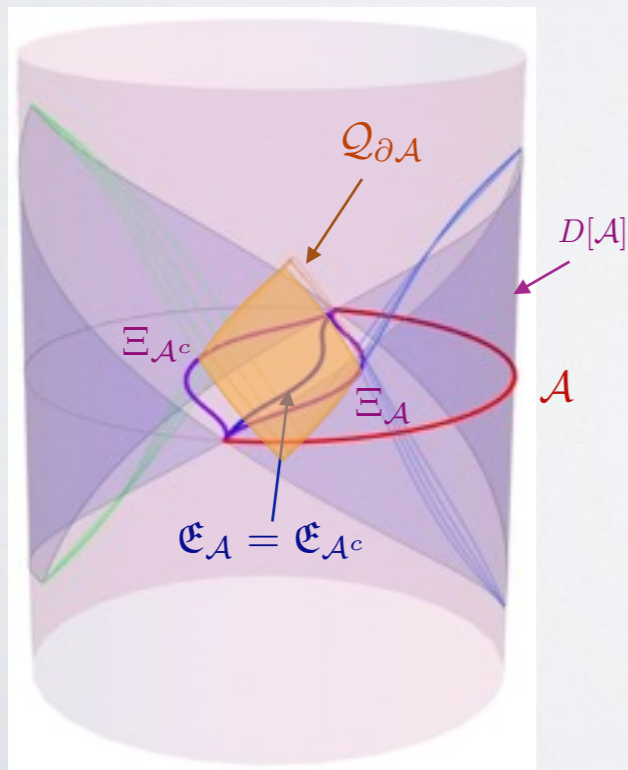
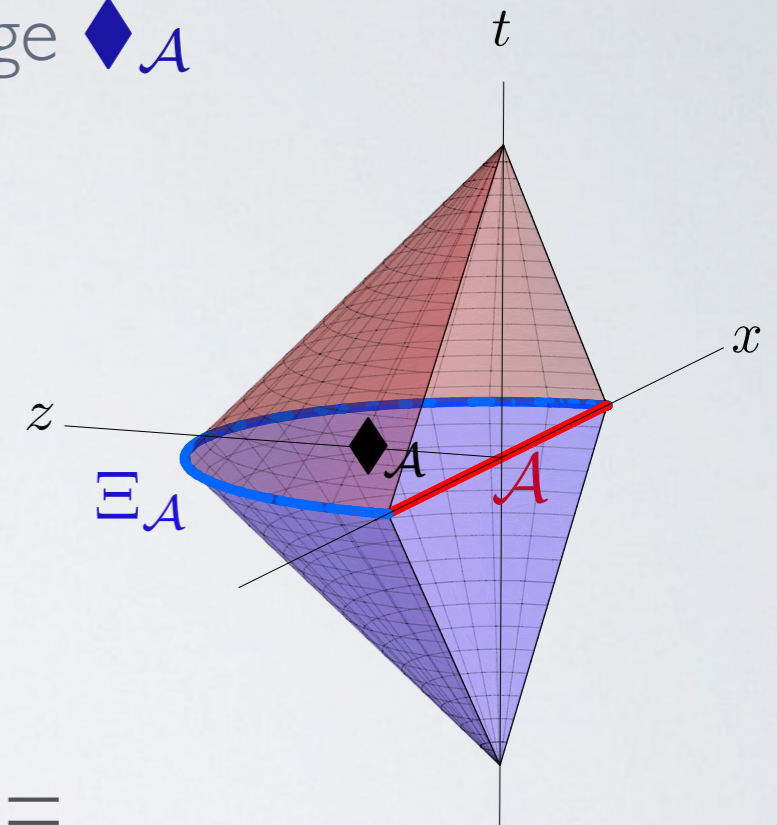
- Extremal surface cannot lie within the bulk causal wedge  $\blacklozenge_{\mathcal{A}}$

$$\blacklozenge_{\mathcal{A}} \equiv J^{-}[D[\mathcal{A}]] \cap J^{+}[D[\mathcal{A}]]$$

= { bulk causal curves which  
begin and end on  $D[\mathcal{A}]$  }

shown in [Hubeny, Rangamani '12]

- In fact it must lie in the causal shadow  $\mathcal{Q}_{\partial\mathcal{A}}$



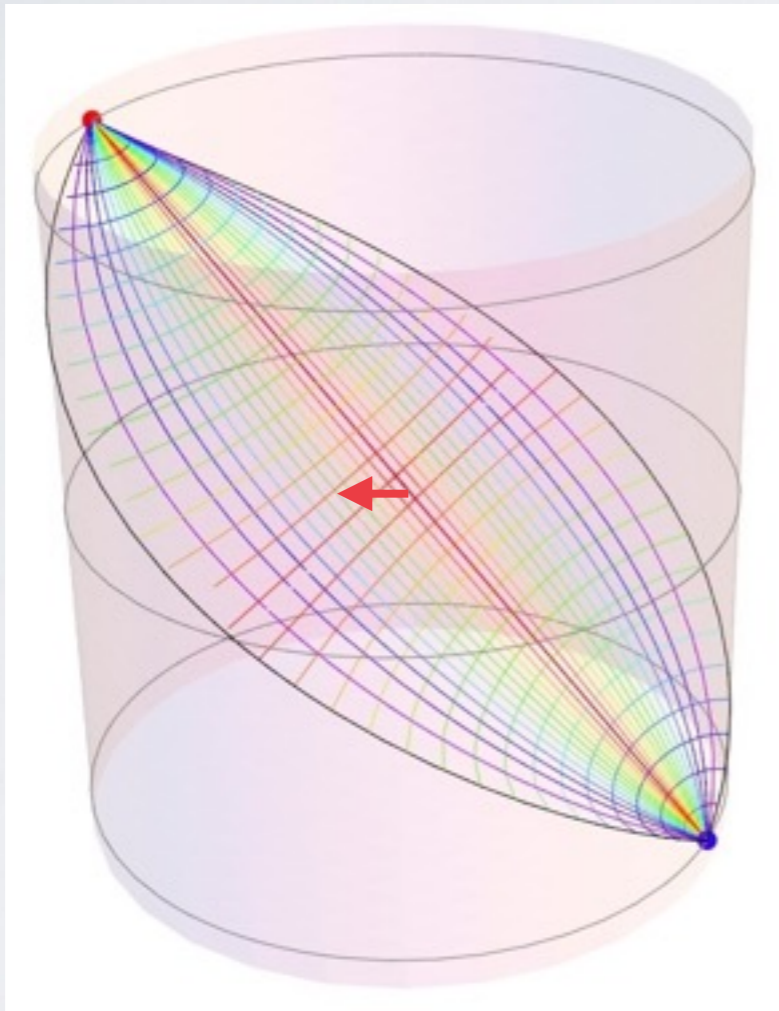
$\mathcal{Q}_{\partial\mathcal{A}}$  = causal shadow =  
bulk region which is  
causally disconnected  
from both  $\mathcal{A}$  and  $\mathcal{A}^c$

- Shown in [Headrick, Hubeny, Lawrence, Rangamani '14]
- Non-trivial condition on holographic EE

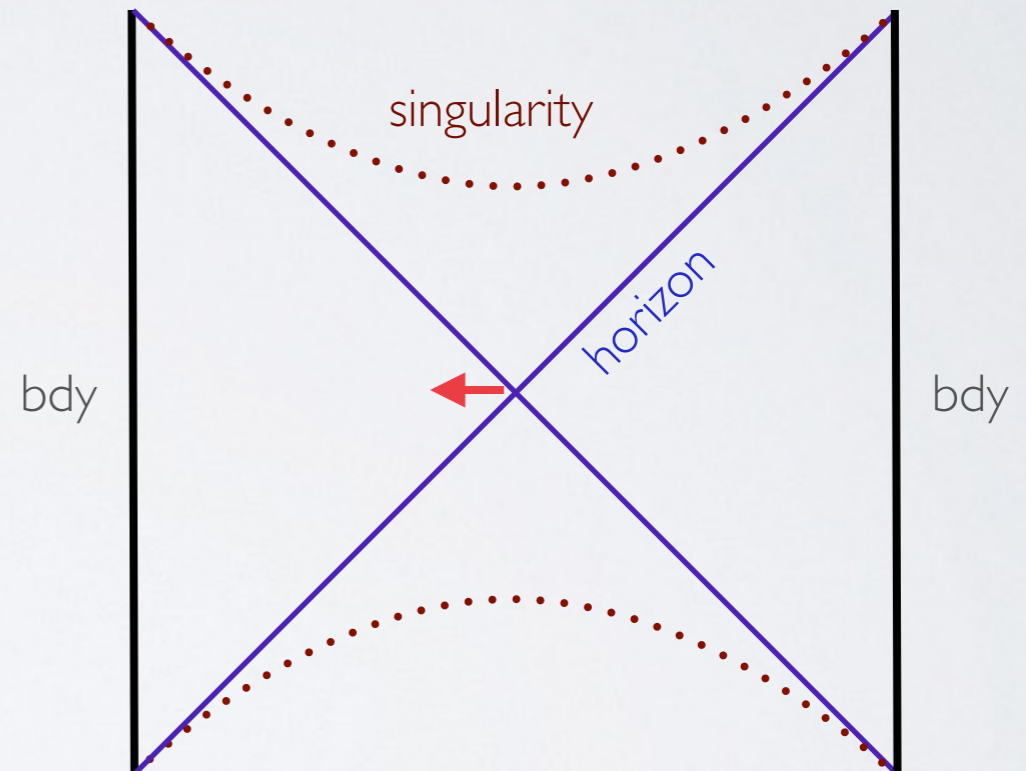
# Marginal for static case...

- In static situations where RT applies, causality is upheld just marginally

pure AdS:



Schwarzschild-AdS black hole



- ♦ **Danger**: arb. small deformation of extremal surface could violate causality!

# Entanglement wedge

- Boundary spacetime separation:

$$\partial\mathcal{M} = D[\mathcal{A}] \cup D[\mathcal{A}^c] \cup I^-[\partial\mathcal{A}] \cup I^+[\partial\mathcal{A}]$$

- This naturally induces a corresponding separation into 4 bulk regions:

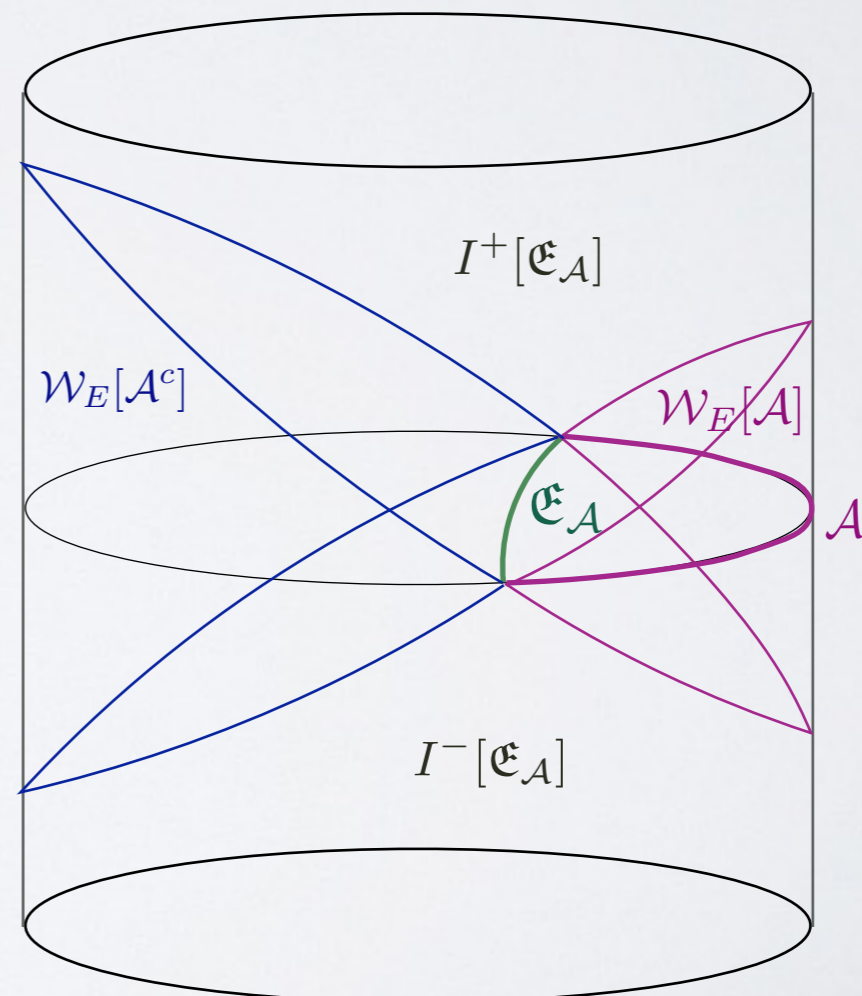
$$\mathcal{M} = \mathcal{W}_E[\mathcal{A}] \cup \mathcal{W}_E[\mathcal{A}^c] \cup I^-[\mathfrak{E}_\mathcal{A}] \cup I^+[\mathfrak{E}_\mathcal{A}]$$

↓ (for pure state)

entanglement wedge of  $\mathcal{A}$

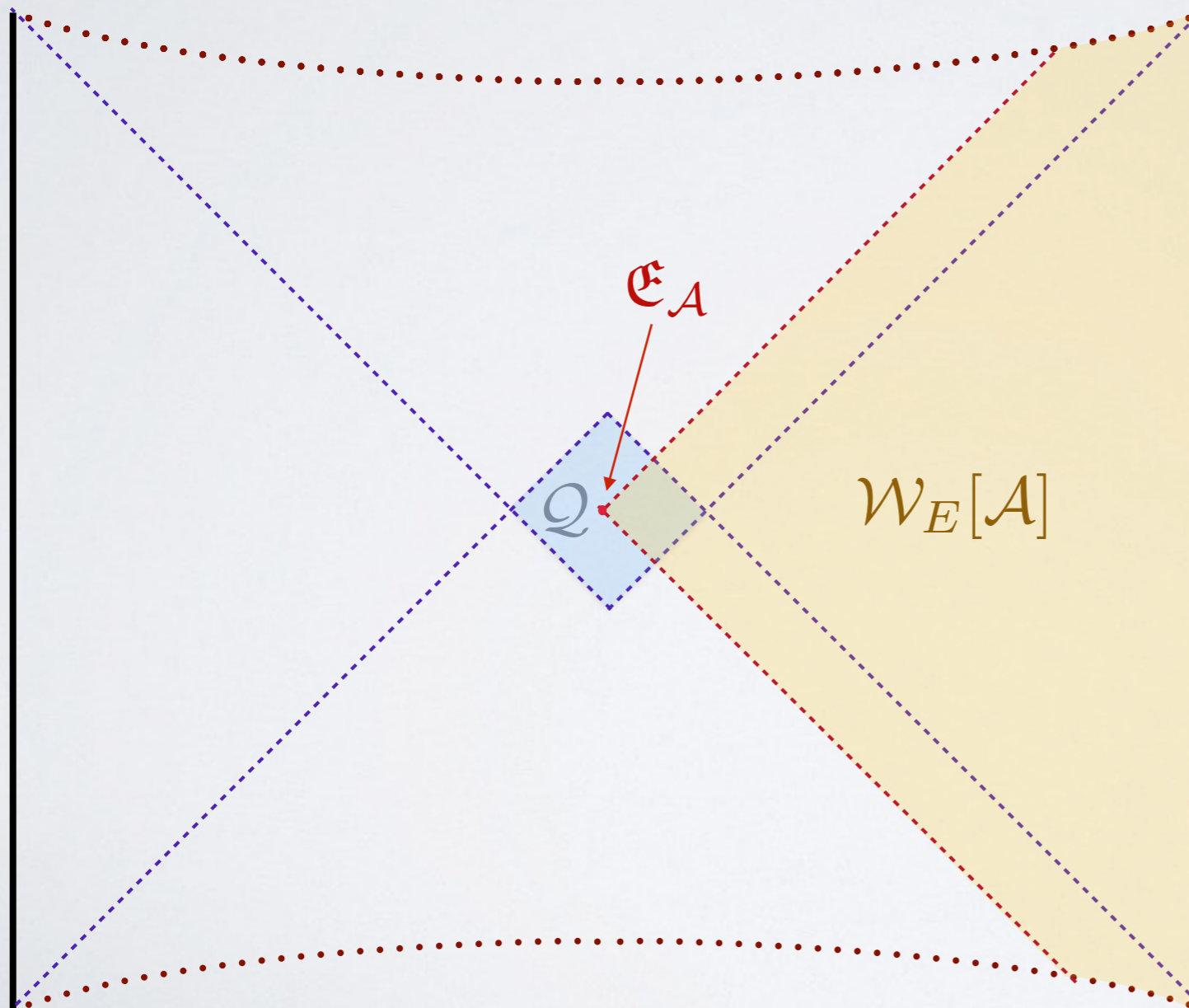
- $\mathcal{W}_E[\mathcal{A}]$  ends on  $D[\mathcal{A}]$
- contains the causal wedge  $\blacklozenge_{\mathcal{A}}$
- generated by null geodesics normal to  $\mathfrak{E}_\mathcal{A}$

⇒ natural ‘dual’ of  $\rho_{\mathcal{A}}$



# Entanglement wedge in deformed SAdS

In deformed eternal Schw-AdS, (compact) extremal surface corresponding to  $\mathcal{A} = \Sigma_L$  or  $\mathcal{A} = \Sigma_R$  must lie in the 'shadow region'  $\diamond Q$



i.e. causally disconnected from both boundaries...

(for static Schw-AdS, shadow region = bifurcation surface)

$\Rightarrow$  Entanglement wedge extends past event horizon

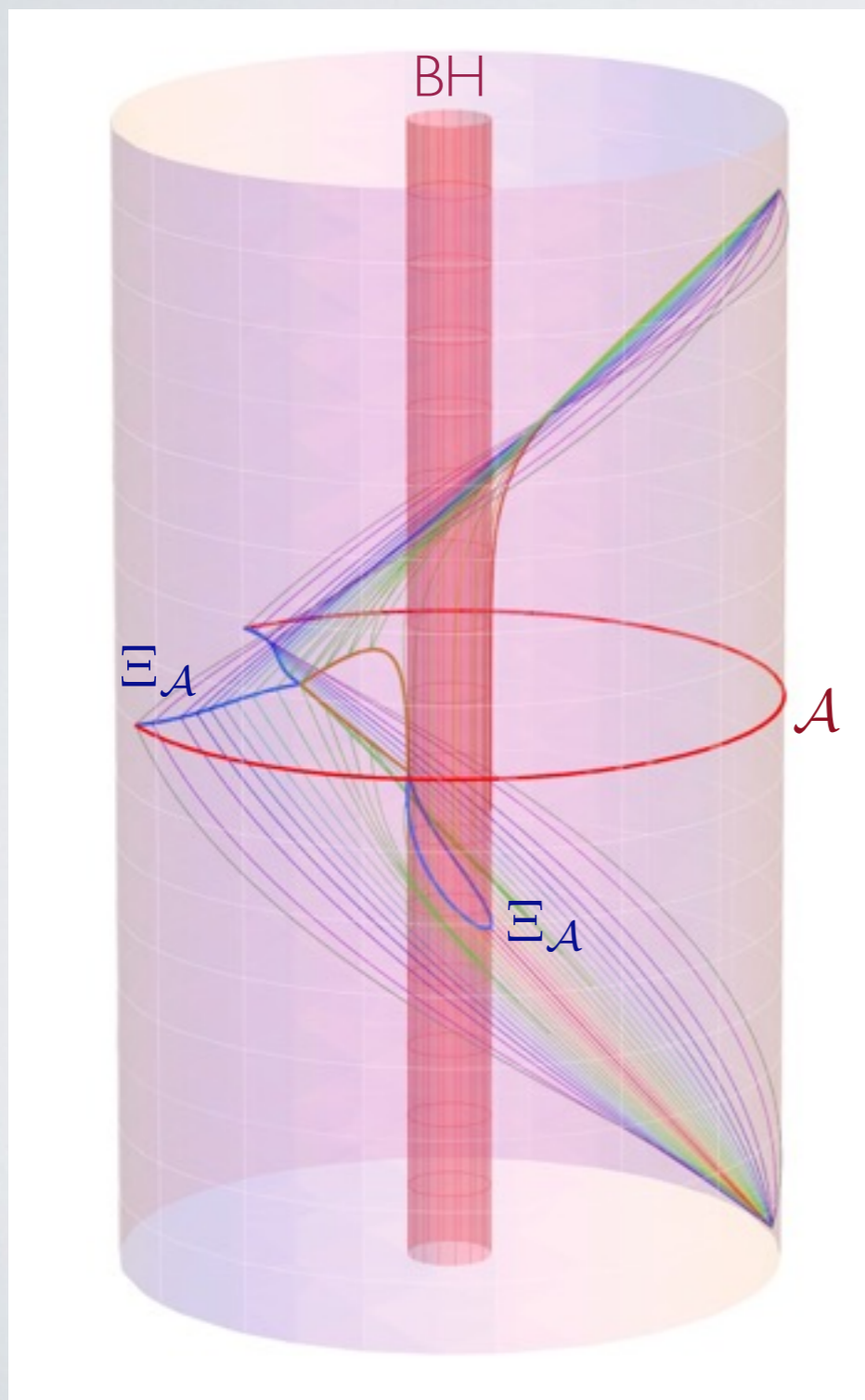
## Curious properties of EE:

- EE satisfies very nontrivial causality constraints
- Entanglement plateaux (  $\delta S_{\mathcal{A}}$  saturates to  $S_{\rho_{\Sigma}}$  for large enough  $\mathcal{A}$  )
- EE has two separate components
- EE is a 'fine-grained' observable

These are all easy to see from the holographic dual!



# Aside: one use of causal wedge

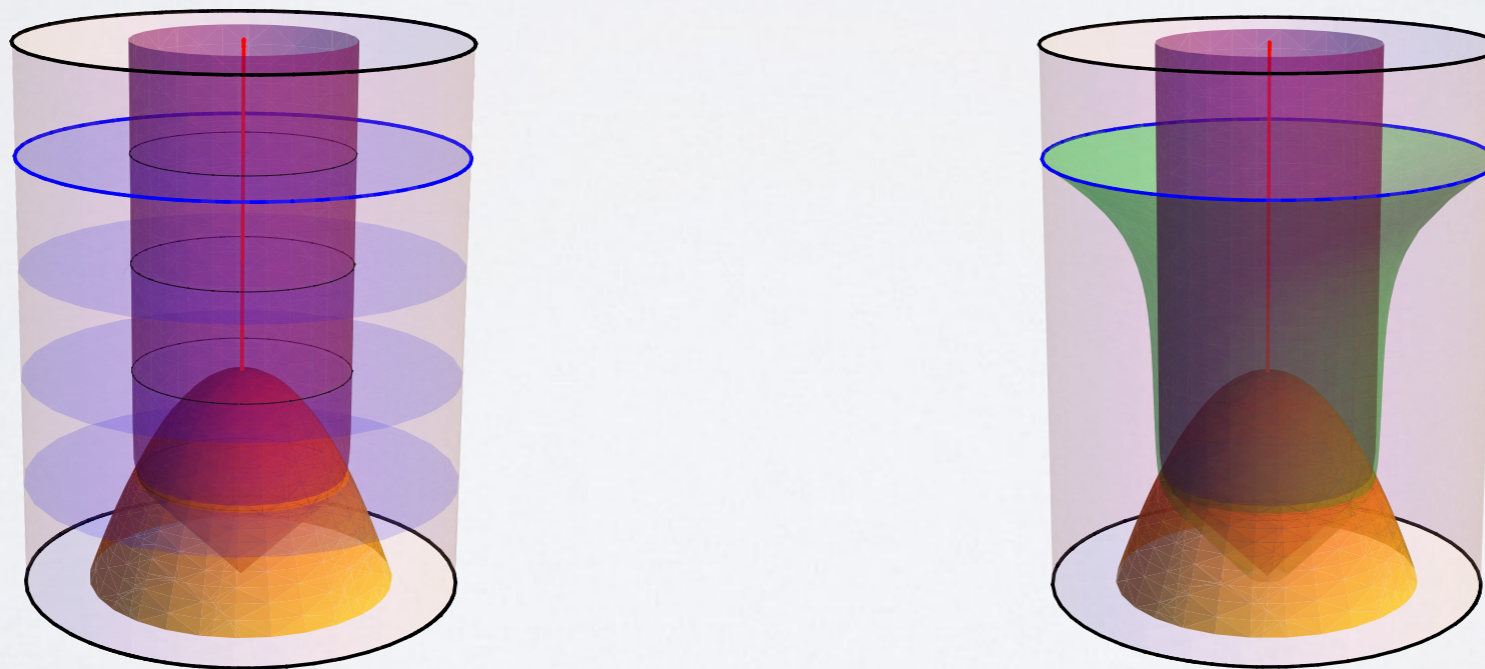


- Causal wedge can have holes...
- Important implication for entanglement:
  - whenever  $\mathcal{A}$  is large enough for  $\Xi_{\mathcal{A}}$  to have two disconnected pieces, there **cannot exist** a single connected extremal (minimal) surface  $\mathfrak{E}_{\mathcal{A}}$  homologous to  $\mathcal{A}$ !
  - in such cases,  $\Rightarrow S_{\mathcal{A}} = S_{\mathcal{A}^c} + S_{\text{BH}}$   
(saturates Araki-Lieb inequality)
    - *entanglement plateau*  
[VH, Maxfield, Rangamani, Tonni, '13]
    - two components to entanglement
- Causal wedge argument guarantees this even for generic time-dependent BHs.

# EE is fine-grained observable!

Example: black hole formed from a collapse

- In contrast to the static (i.e. eternal) black hole, for a collapsed black hole, there is no non-trivial homology constraint on extremal surfaces. [cf. Takayanagi & Ugajin]



- Hence we always have  $S_{\mathcal{A}} = S_{\mathcal{A}^c}$  as for a pure state.

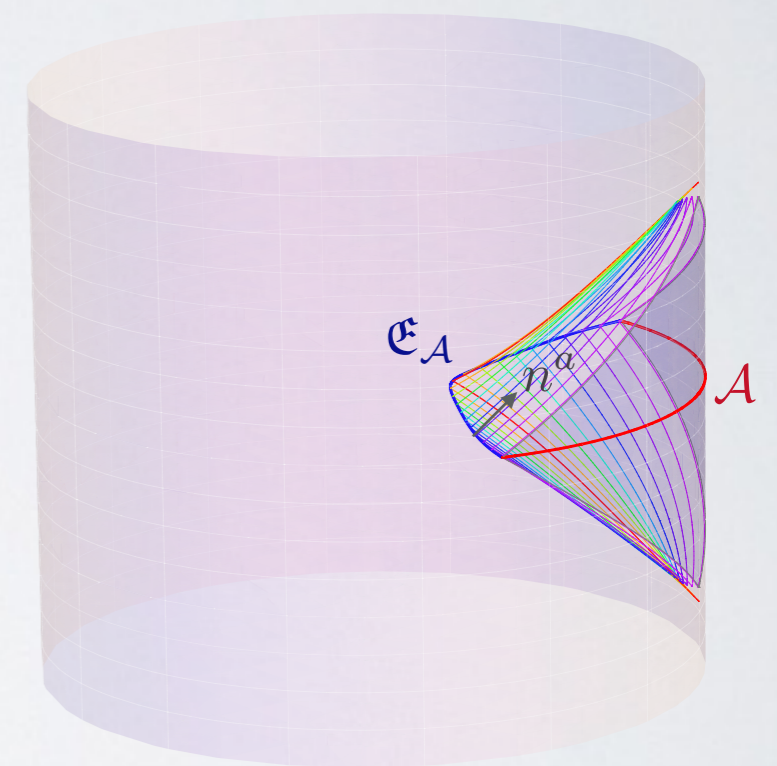
# Bulk dynamics from EE?

- We can in principle decode the bulk geometry from  $\{S_{\mathcal{A}}\}$  for a suitable set of  $\mathcal{A}$ 's.
- But can we extract bulk dynamics more directly?
  - Use the strong subadditivity property of EE:

$$\delta_{\mathcal{A}}^2 S_{\mathcal{A}} \sim \int_{\mathfrak{E}_{\mathcal{A}}} E_{ab} n^a n^b \geq 0$$

cf. Null Energy Condition

specific 2nd order variation of region



- proved at linearized level in 3-d, but conjectured to hold more generally...

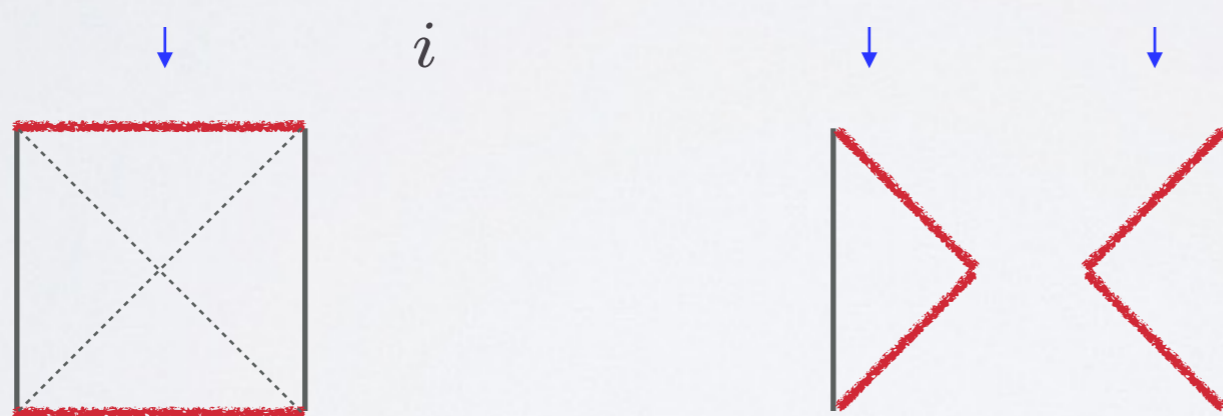
[Bhattacharya, Hubeny, Rangamani, Takayanagi, '14]  
cf. [Lashkari, Rabideau, Sabella-Garnier, Van Raamsdonk]

# Spacetime from entanglement?

How does bulk spacetime emerge in the first place?

- Some connected spacetimes emerge as superpositions of disconnected spacetimes [Van Raamsdonk; Swingle]

eg. eternal AdS black hole as thermofield double:

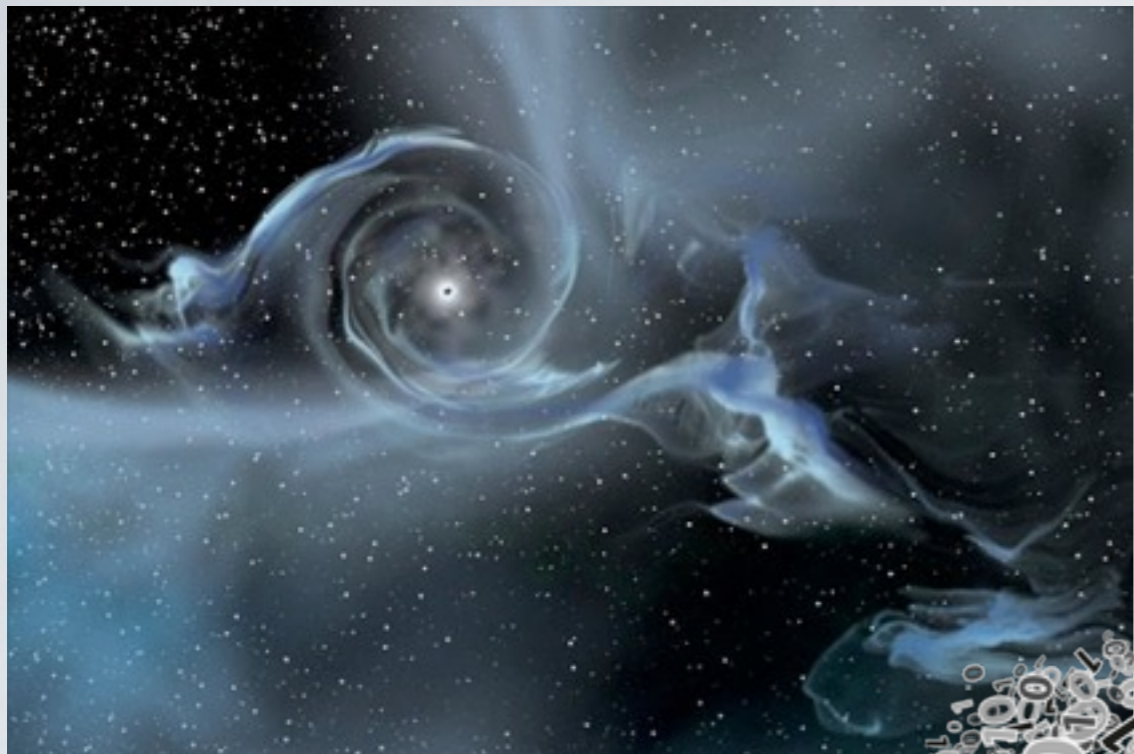
$$|\psi\rangle = \sum_i e^{-\frac{\beta E_i}{2}} |E_i\rangle \otimes |E_i\rangle$$


- Entanglement builds bridges: 'ER = EPR' [Maldacena, Susskind]

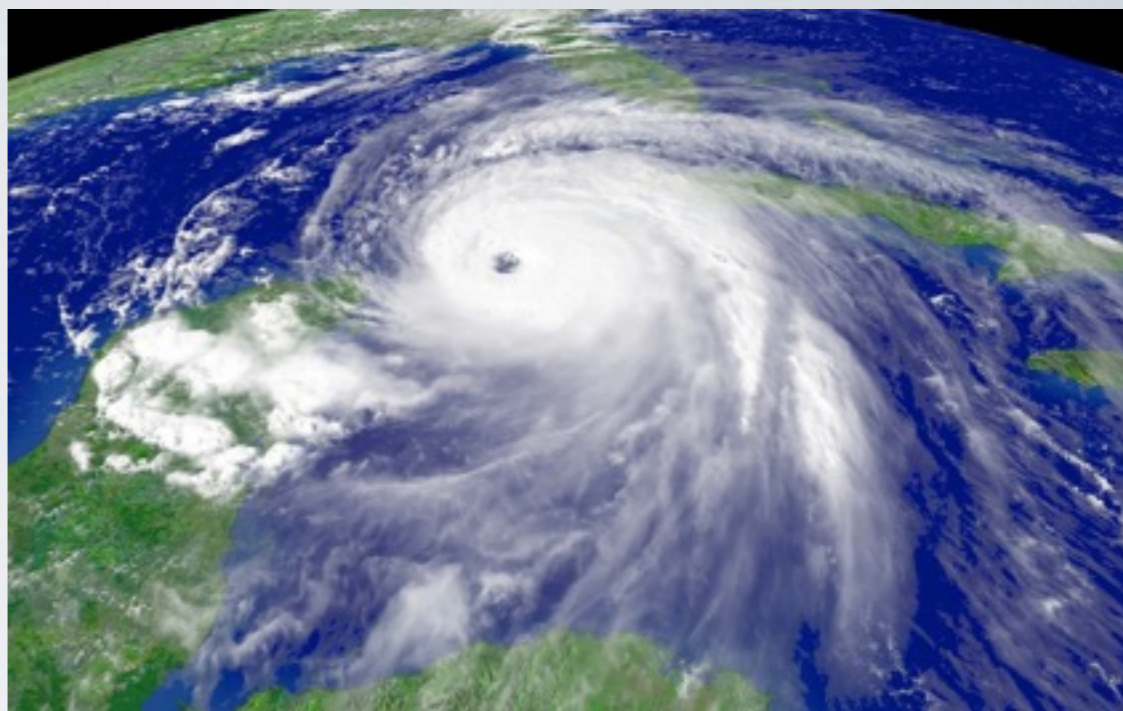


Einstein-Rosen bridge

Einstein-Podolsky-Rosen entanglement



Space Ref (Harvard-Smithsonian CfA)

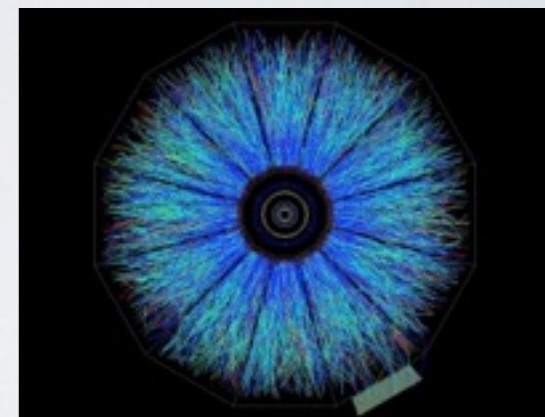


Hurricane Ivan, NOAA



soihub.org/itschool

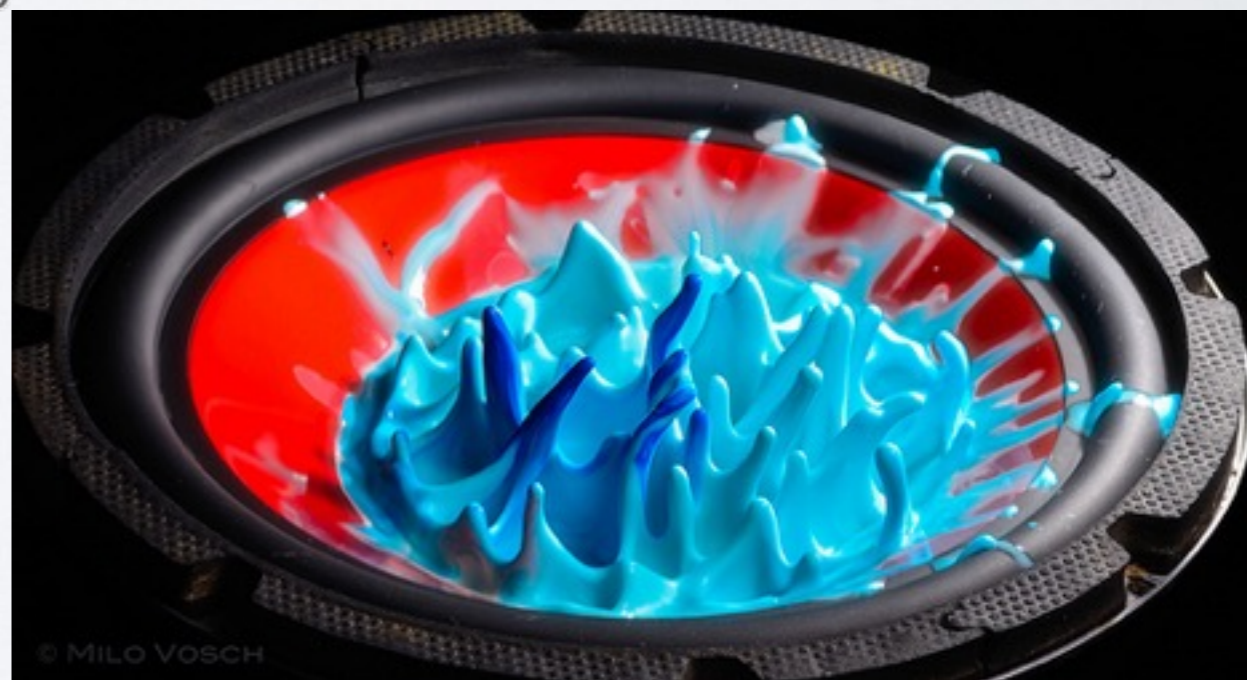
Thank you!



RHIC event, LBL



Roller Wave, by William Dalton



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Dancing Non-Newtonian fluid by Milo Vosch