Effective Field Theories
From
String Compactification

Jonathan J. Heckman

UNC Chapel Hill

Question:

What is String Theory Good For?

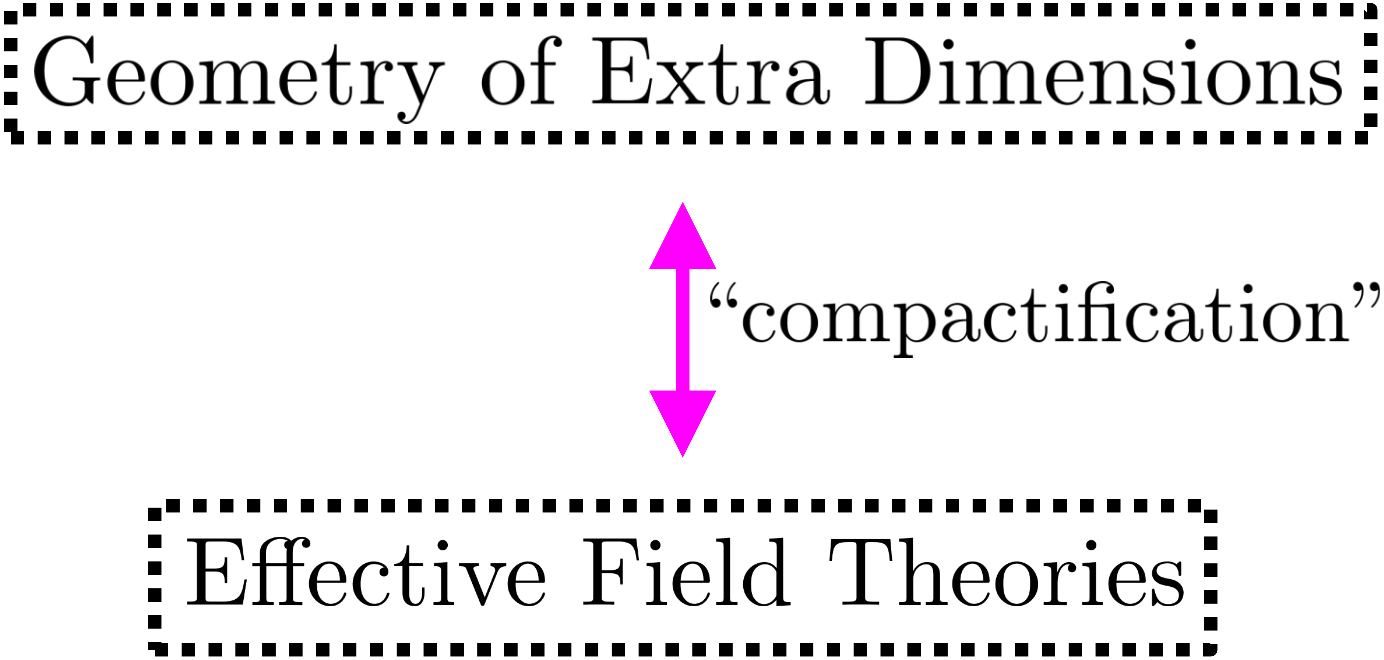
Potential Answer(s):

- As a model of quantum gravity
- Framework for high energy physics
- Tool for strongly coupled systems
- Tool for revealing maths

Focus Of This Talk:

- As a model of quantum gravity
- Framework for high energy physics
- Tool for strongly coupled systems
- Tool for revealing maths

What This Talk Is About



Geometry of Extra Dimensions

“compactification”

Effective Field Theories

Goals

Short-Term: Classical Geometry → EFT

- What are the rules of the game?
- Spectrum of physical states?
- Physical / Mathematical structures?

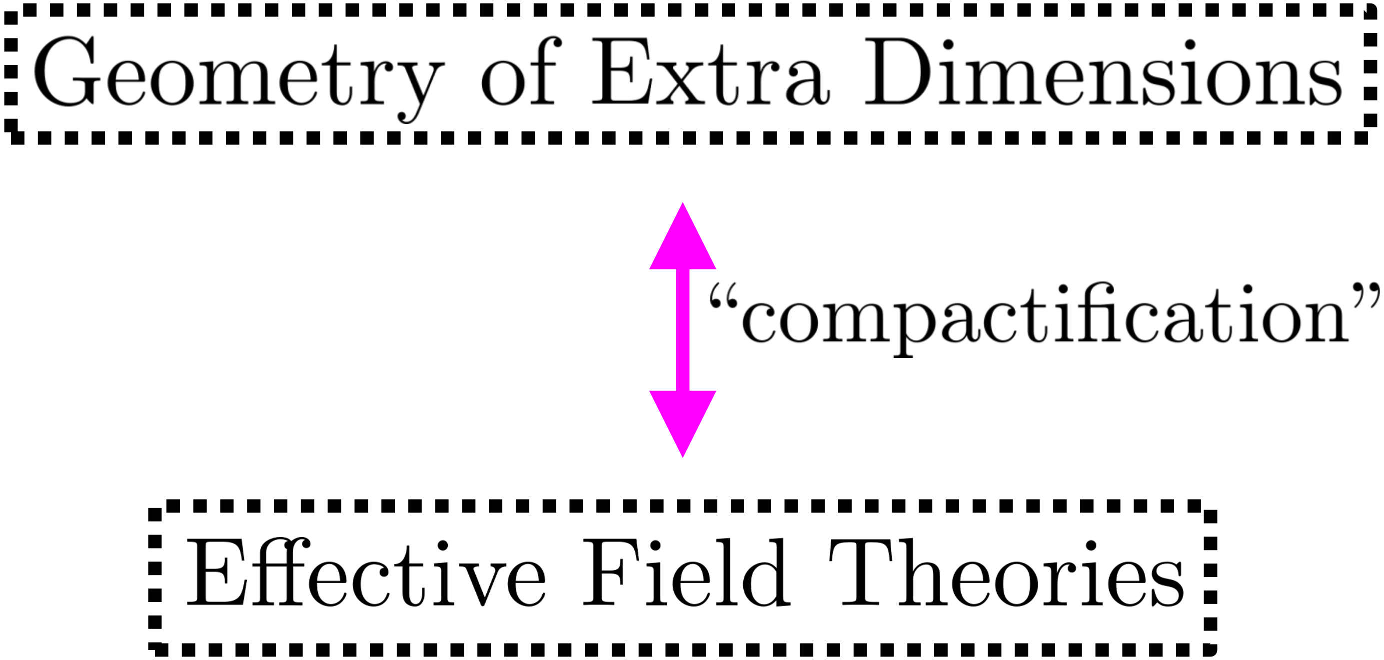
Goals

Short-Term: Classical Geometry $\rightarrow$ EFT

Longer Term: Quantum Geometry $\rightarrow$???

- Quantum Gravity + QFT?
- Emergent Spacetime?
- ...?

What This Talk Is About



Geometry of Extra Dimensions

“compactification”

Effective Field Theories

What's an EFT?

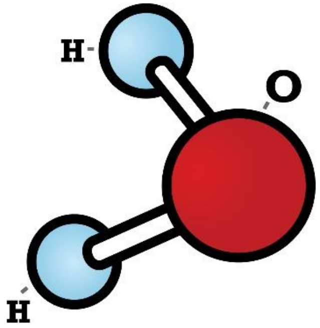
Effective (Quantum) Field Theory

Physics at Energies $\ll \Lambda_{max}$ (the cutoff)

- Remain agnostic about $E \sim \Lambda_{max}$
- Describes many systems!

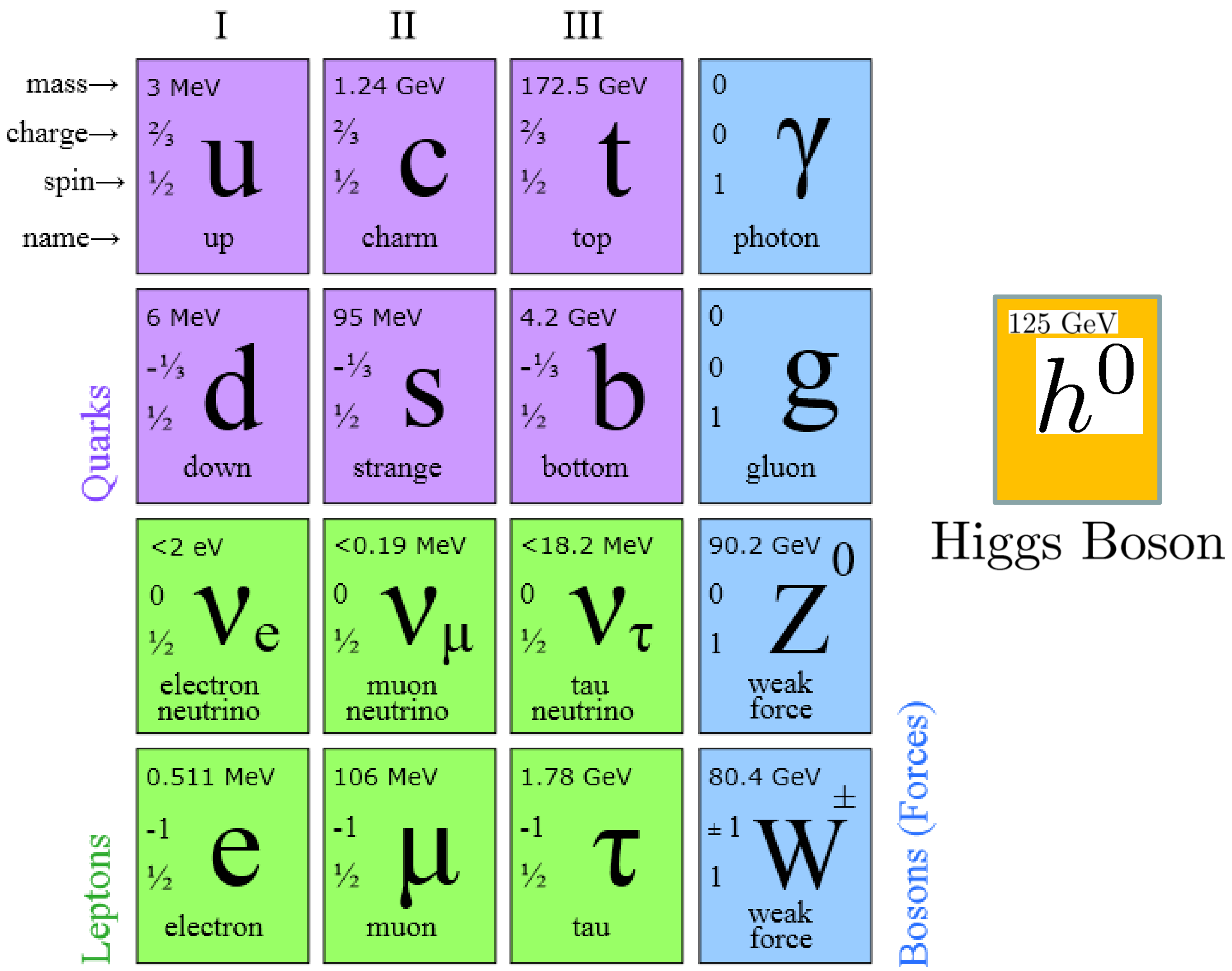
Everyday Example: Water

Short Distance Description:



Long Distance Description:

The Standard Model



	I	II	III		
mass→ charge→ spin→ name→	3 MeV ⅔ ½ u up	1.24 GeV ⅔ ½ c charm	172.5 GeV ⅔ ½ t top	0 0 1 γ photon	
Quarks	6 MeV -⅓ ½ d down	95 MeV -⅓ ½ s strange	4.2 GeV -⅓ ½ b bottom	0 0 1 g gluon	
	<2 eV 0 ½ ν_e electron neutrino	<0.19 MeV 0 ½ ν_μ muon neutrino	<18.2 MeV 0 ½ ν_τ tau neutrino	90.2 GeV 0 1 Z^0 weak force	Bosons (Forces)
Leptons	0.511 MeV -1 ½ e electron	106 MeV -1 ½ μ muon	1.78 GeV -1 ½ τ tau	80.4 GeV ±1 1 $W^\pm$ weak force	

125 GeV

h^0

Higgs Boson

The Standard Model

- Works great at energies $\ll 10^{19}$ GeV (Planck Scale)
- Old Idea: Near Planck Scale, complete via strings
- Still Active Area: Standard Model $\subset$ String Theory

Other Examples of EFTs

Conformal Field Theories (no scales, just angles)

- Phase Transitions
- AdS/CFT (quantum gravity)
- Conformal Geometry

What This Talk Is About

Geometry of Extra Dimensions

“compactification”

Effective Field Theories

Strings?

Even after 40 years: A Work in Progress

One Use: To Study Quantum Gravity

e.g. graviton

closed string

Strings?

Even after 40 years: A Work in Progress

New Use: To Study Novel EFTs

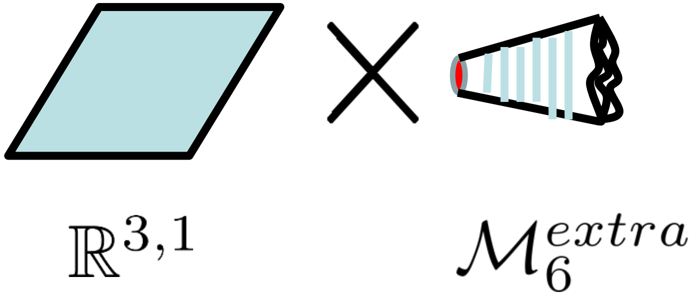
AdS/CFT: (QFT via classical gravity)

String Compactification: (Geometrize EFT)

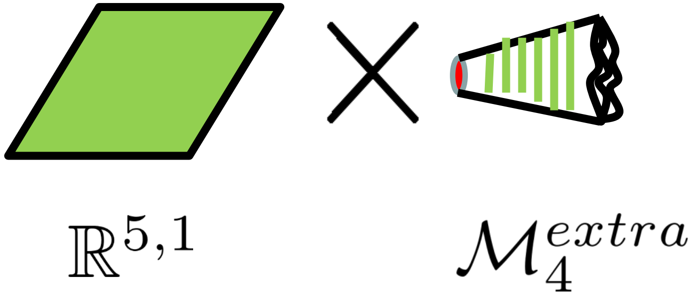
Ingredients: Extra Dim^{n}s

Self-Consistency $\Rightarrow$ 10 Spacetime Dimensions
(some may be small)

4D Effective Theory

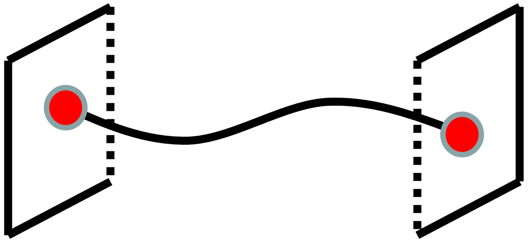


6D Effective Theory



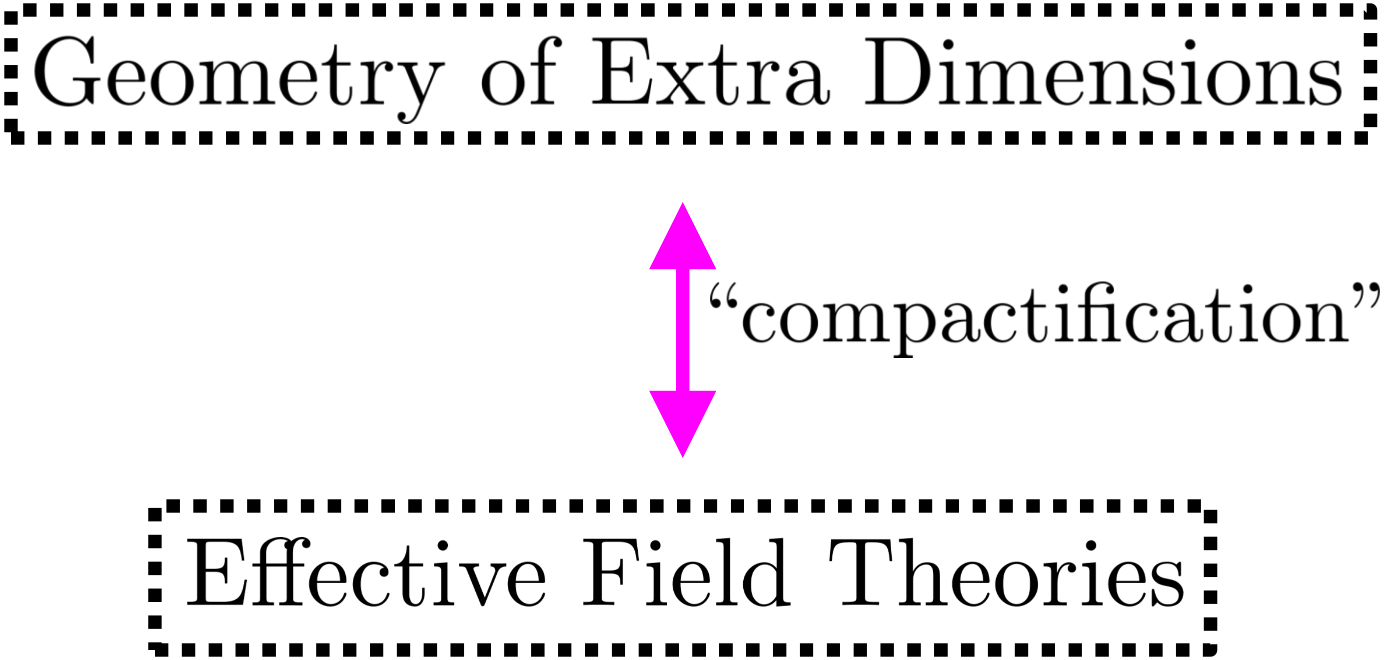
Ingredients: P-Branes

Self-Consistency ⇒
Strings Can End on "P-Branes"
(P spatial dimensions)



	$\mathbb{R}^{5,1}$						$\mathcal{M}_4^{extra}$			
	0	1	2	3	4	5	6	7	8	9
1-Brane	×	×								
3-Brane	×	×					×	×		
7-Brane	×	×	×	×	×	×	×	×		

What This Talk Is About

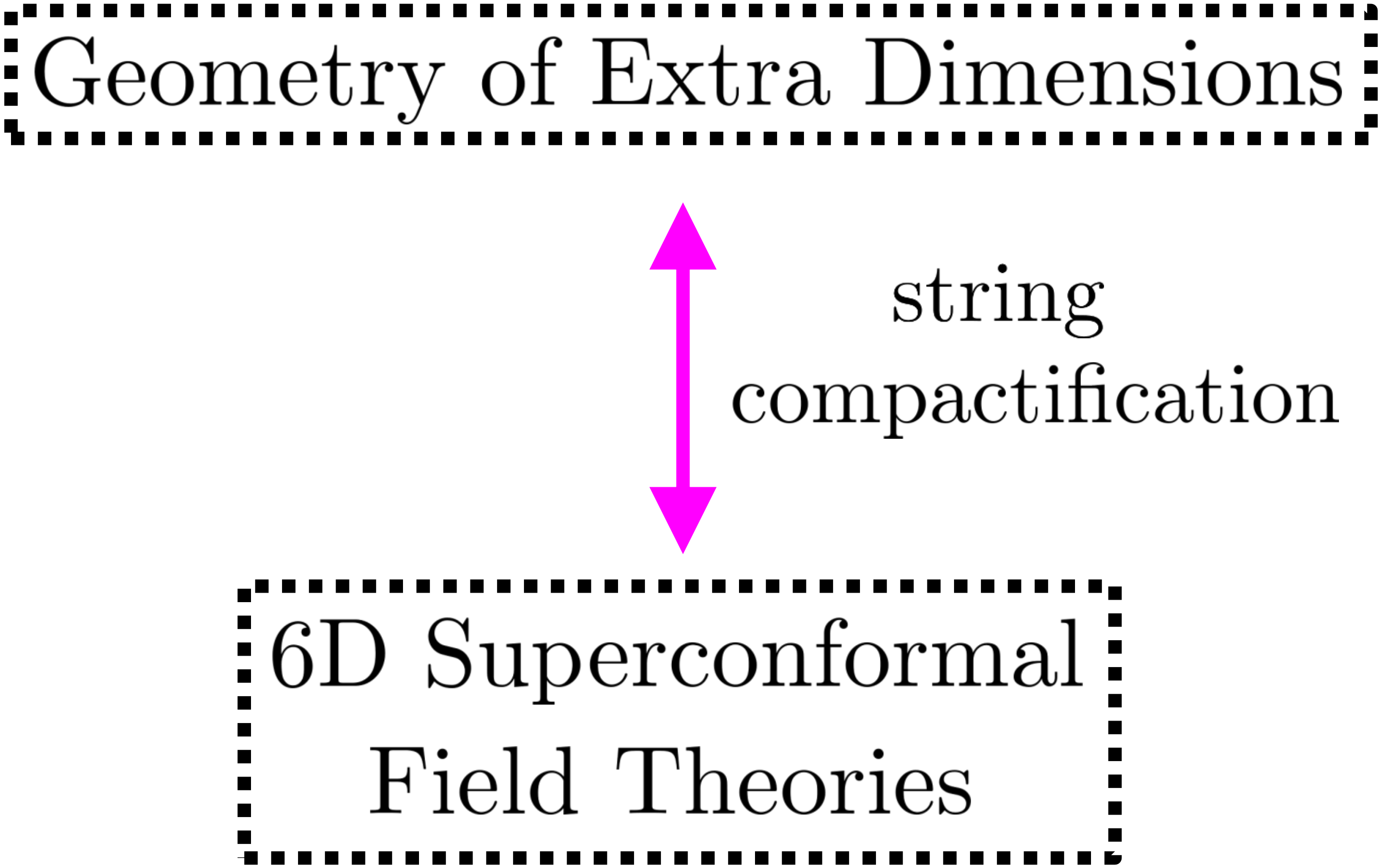


Geometry of Extra Dimensions

“compactification”

Effective Field Theories

Illustrative Example



Geometry of Extra Dimensions

string
compactification

6D Superconformal
Field Theories

So What's a 6D CFT?

A physical theory in $5 + 1$ Dimensions

(basically no distances, only angles)

Spacetime Symmetries: $SO(6, 2)$

Lorentz (i.e. $SO(5, 1)$) + a few more:

Includes Translations and Scaling + ...

So What's a 6D SCFT?

A 6D CFT + supersymmetry:

$$Q^2 = 0$$

$$QQ' + Q'Q = \text{6D Translation}$$

Two Possibilities:

- 8 Independent Q's: The "(1,0) Theories"
- 16 Independent Q's: The "(2,0) Theories"

Why Are They Interesting?

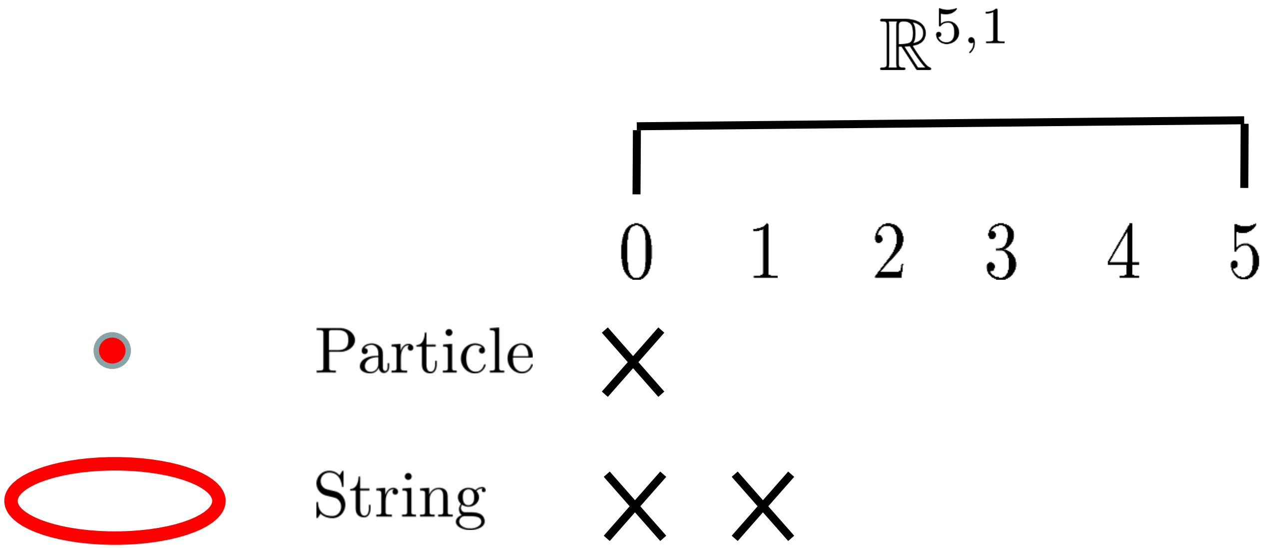
Intrinsic Reasons:

Nahm: SCFTs only for D ≤ 6

No examples until mid 1990's!
(required input from string theory)

Constituents involve *strings* (not just particles)
Note: These are *effective* strings

Why They Are Interesting



$\mathbb{R}^{5,1}$

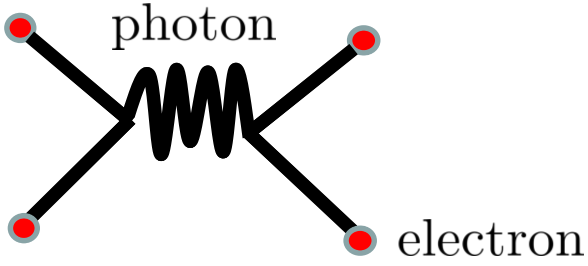
	0	1	2	3	4	5
Particle	X					
String	X	X				

Particles have mass, Strings have tension (mass / length)

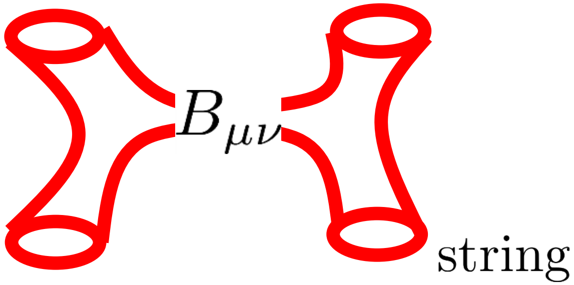
SCFT ⇒ Tensionless Strings

Conceptually Challenging!

Force carriers for particles: vector bosons A_μ



Force carriers for strings: tensor bosons $B_{\mu\nu}$



Conceptually Important

Major Issue: ¿¿¿ *Define* Quantum Field Theory???

More Examples ⇒ More Clues to the Definition

Why They Are Interesting

"Practical" Reasons:

6D $(2,0)$ Theories on $\mathbb{R}^{3,1} \times \Sigma_{2D}$ "solve":

4D $\mathcal{N} = 2$ Theories with Electrons and Monopoles

(c.f. Seiberg and Witten '94 + Witten '97 + Gaiotto '09 + ...)

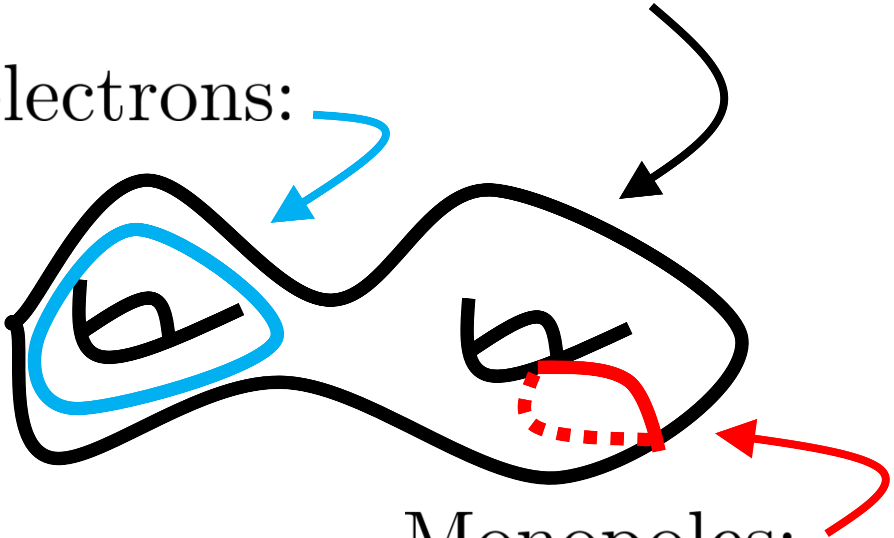
i.e. $\overrightarrow{\nabla} \cdot \overrightarrow{E} = \rho_{electric}$ and $\overrightarrow{\nabla} \cdot \overrightarrow{B} = \rho_{magnetic}$

Why They Are Interesting

“Practical” Reasons:

$(2,0)$ Theory on $\mathbb{R}^{3,1} \times \Sigma_{2D}$ “solves” 4D Theory

Electrons:



Monopoles:

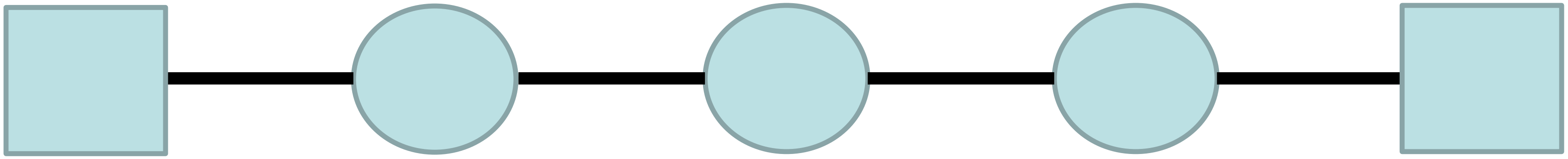
In this Talk

Use String Theory to

Explain what all 6D SCFTs “look like”

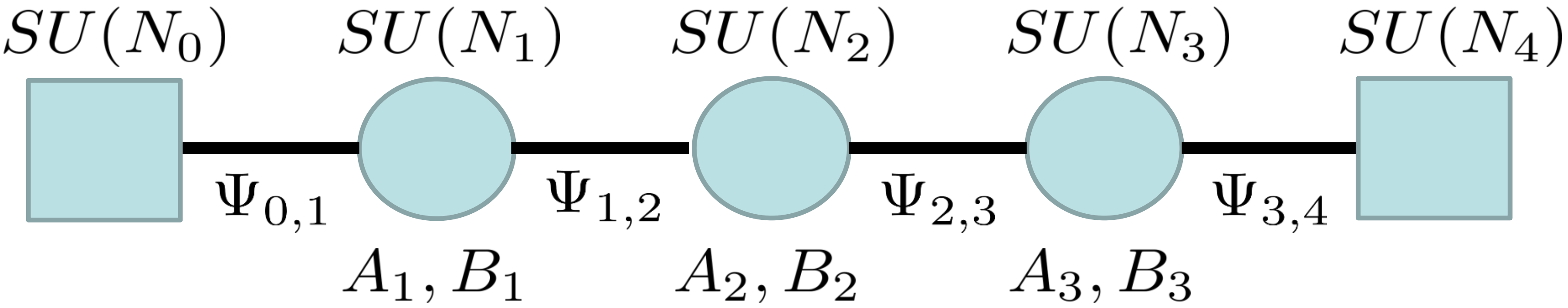
Bottom Up Perspective

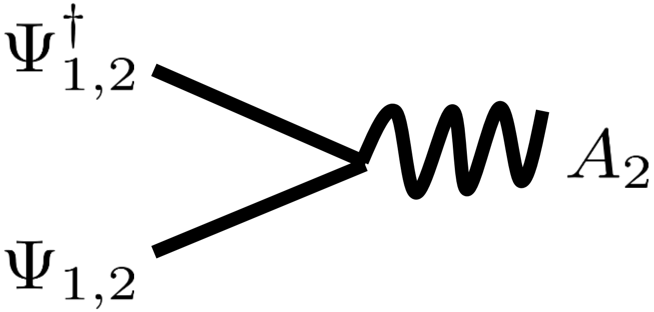
Draw a Picture!

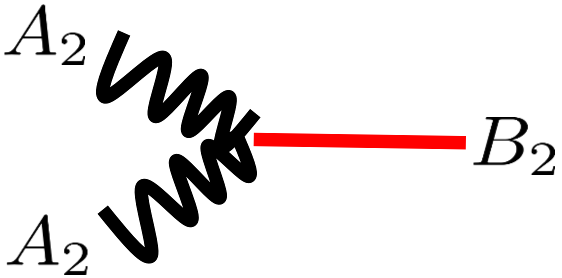


What the Picture Says

Quantum Consistency $\Rightarrow N_i = N$ for all i

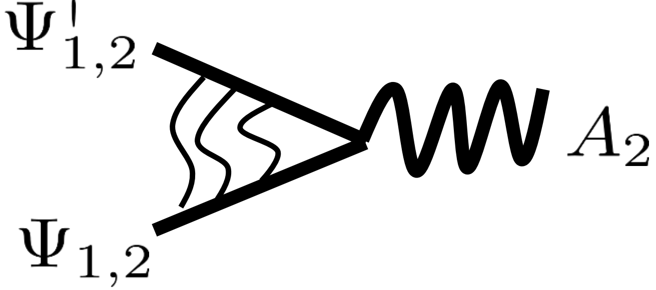






SCFT Limit?

Strong Interactions → Scale Invariance



String Tension → 0

Note: Now it's hard to compute anything!

¿¿¿So how do we know there's an SCFT at all???

Stringy Description

Need: Tensionless Strings

Can we use Superstrings?

No, Superstring Tension $\neq 0$

Effective Strings

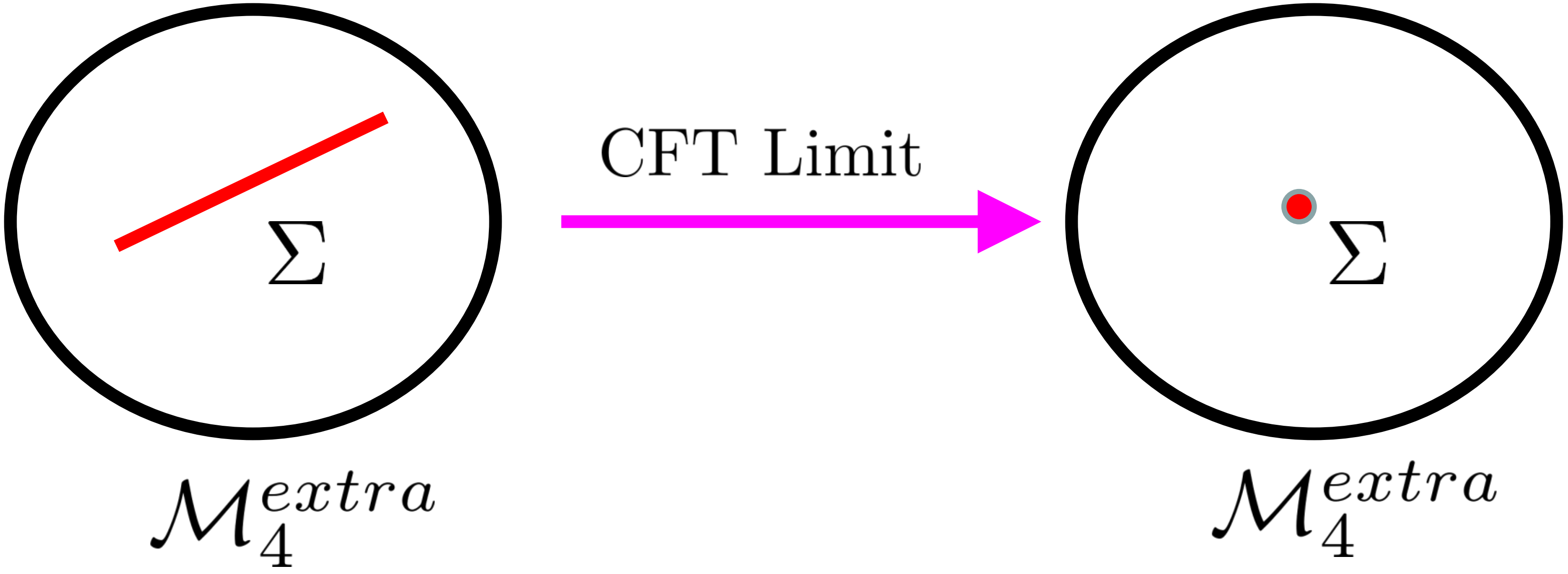
But, we can use 3-branes:

	$\mathbb{R}^{5,1}$						$\mathcal{M}_4^{extra}$			
	0	1	2	3	4	5	6	7	8	9
Effective String	×	×					×	×		

$$\boxed{\text{Tension} = \text{Vol}(\Sigma_{6,7}) \to 0}$$

Name of the Game

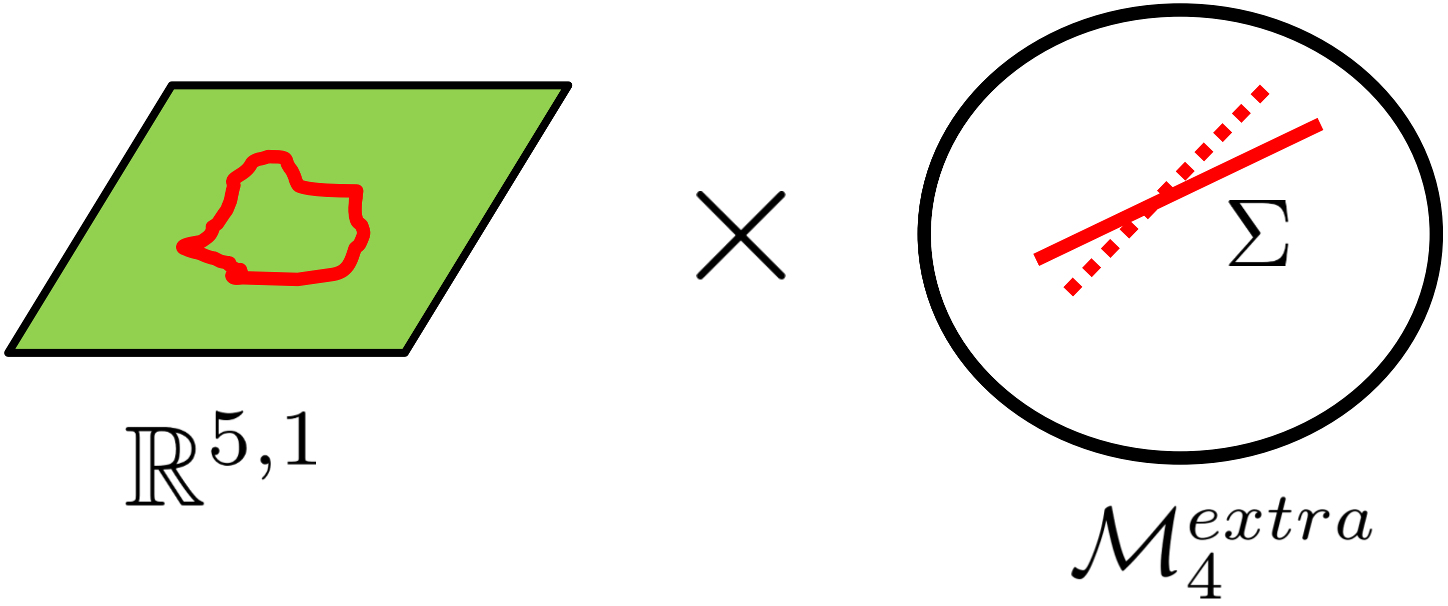
Find Σ's and shrink to zero size



Name of the Game

Physics: $-\Sigma \cap \Sigma =$ String Charge

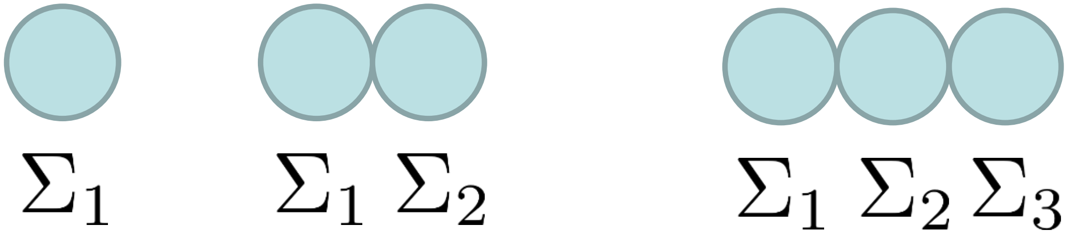
(which must be integer > 0)



Special Case: $(2, 0)$ Theories

These have 16 $Q's$, and $\Sigma_i \cap \Sigma_i = -2$

A priori, more than one Σ, e.g.



Special Case: $(2,0)$ Theories

Introduce $A_{ij} = -\Sigma_i \cap \Sigma_j$

CFT Conditions: A is positive definite

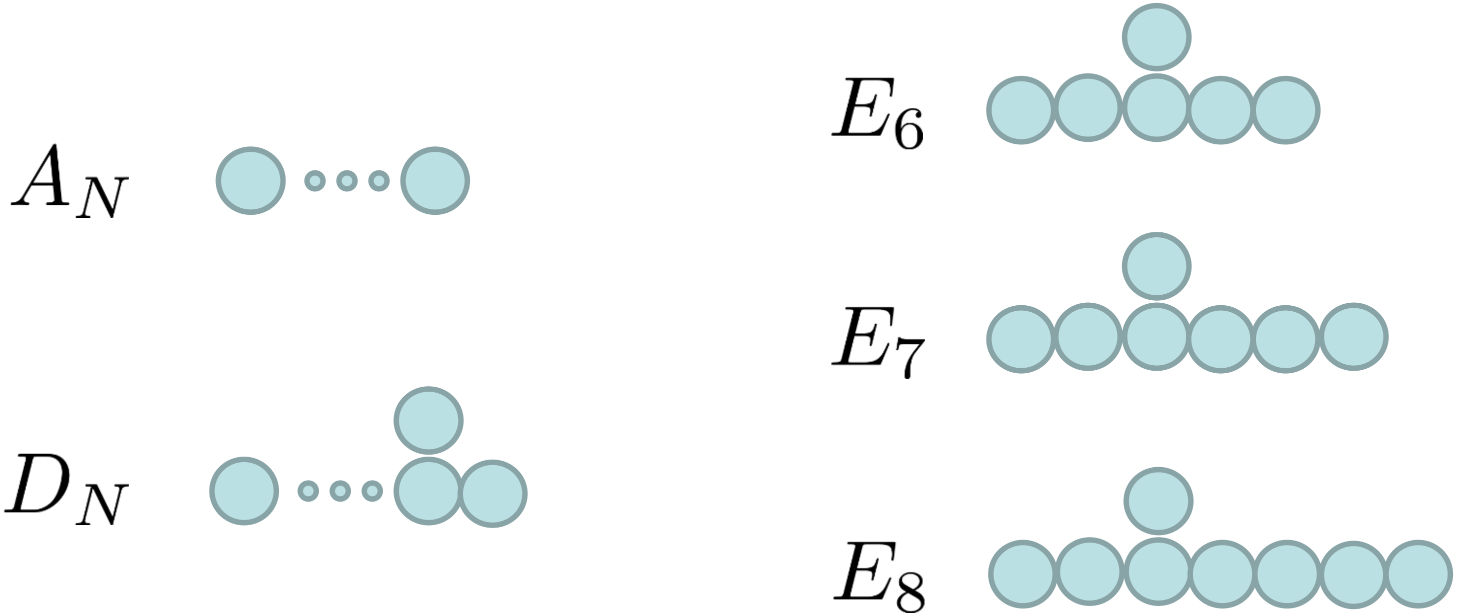
$$\Sigma_i \cap \Sigma_j = 0 \text{ or } 1 \text{ if } i \neq j$$

Already Classified!

$(2, 0)$ ADE Classification

We Can Borrow a Famous Result

Coxeter ’34, Witt ’41, Dynkin ’46, ’47; Witten ’95



$(1,0)$ Theories?

These theories have 8 Q's (the minimum)

What are the possible $\mathcal{M}_4^{extra}$'s?

What kinds of string charges?

Studied since the 1990's

Many groups:

Witten '95; Strominger '95; Ganor and Hanany '96;

Seiberg and Witten '96; Bershadsky and Johansen '96;

Brunner and Karch '96; Blum and Intriligator '97;

Intriligator '97; Hanany Zaffaroni '97;

+ ....

But: Even now, still viewed as "mysterious"...

How to Build More 6D SCFTs
(Use F-theory)

JJH et al. '13 - '15

A Hidden Assumption

So far, we have assumed $\mathcal{M}_4^{extra}$ Ricci-Flat

Einstein Field Equations in 10D vacuum:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

This limits us to $\Sigma \cap \Sigma = -2$

"Non-Compact Calabi-Yau two-folds"

More Generally

We can allow $\mathcal{M}_4^{extra}$ *not* Ricci-Flat

Einstein Field Equations with sources:

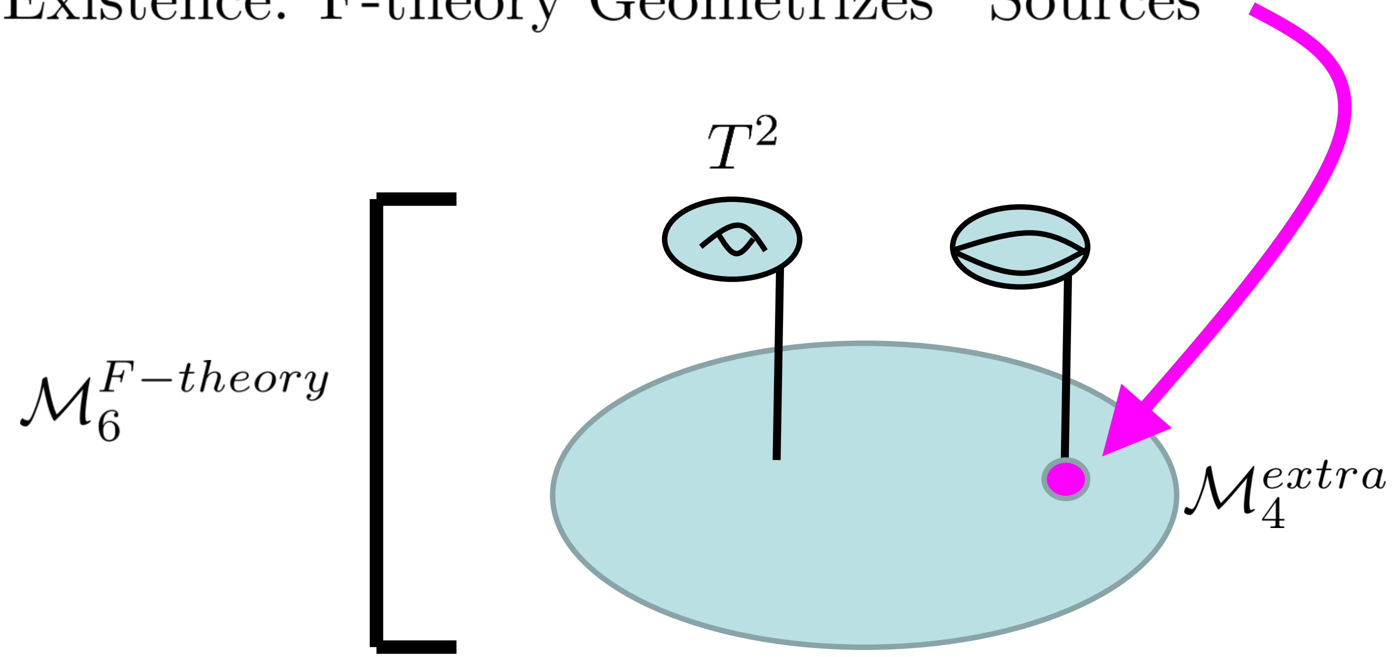
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \text{Sources}$$

This allows us to consider $\Sigma \cap \Sigma \neq -2$

F-theory Compactification

Vafa '96

Existence: F-theory Geometrizes "Sources"



$\mathcal{M}_6^{F-theory}$ is Ricci-Flat (a Calabi-Yau three-fold)

F-theory is Useful!

- Dualities: Vector Bundles on $K3 \leftrightarrow$ Elliptic CY_3

  Vafa '96, Morrison Vafa et al., '97, Friedman Morgan Witten '97 + ...

  JJH, Morrison, Rudelius, Vafa '15

- Particle Physics: Most Realistic Stringy Models to Date

  SM spectrum, flavor ($\theta^{\nu}_{13} \sim 0.2$), funparticle extra sectors + ...

  JJH et al. '08 - '15 + ...

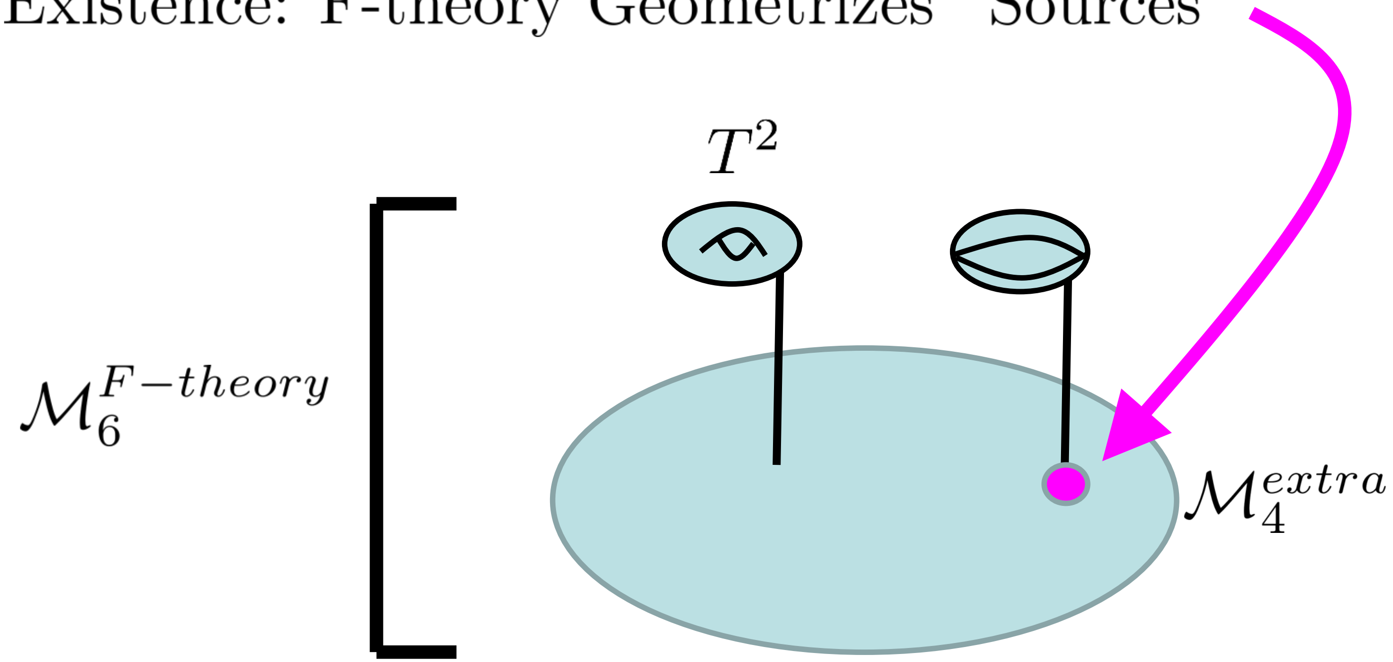
- 6D SCFTs: Novel Theories without a mass scale in 6D

  JJH et al. '13 - '15 + (some examples from '90's)

F-theory Compactification

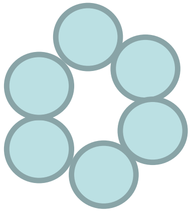
Vafa '96

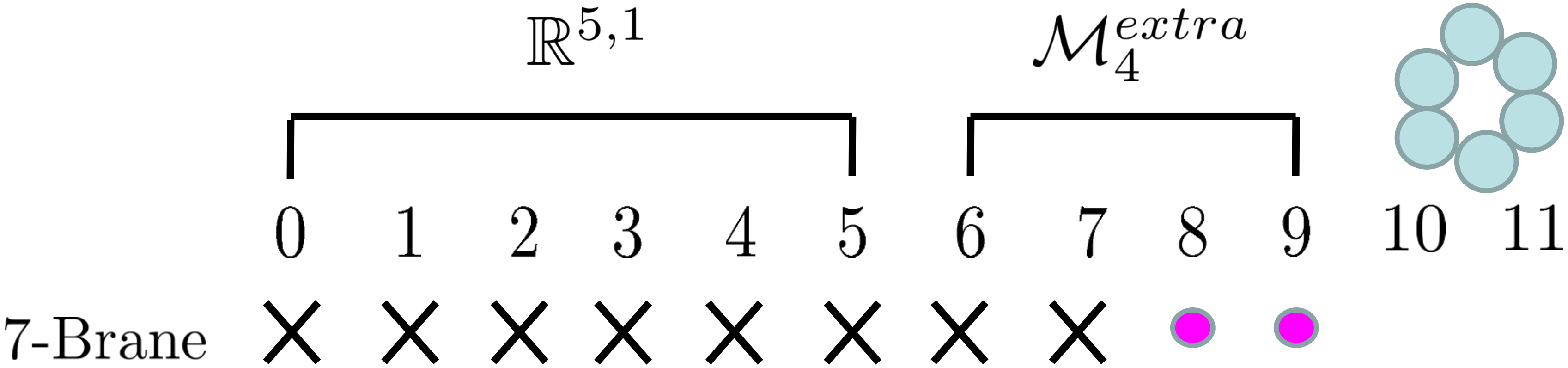
Existence: F-theory Geometrizes "Sources"



$\mathcal{M}_6^{F-theory}$ is Ricci-Flat (a Calabi-Yau three-fold)

The Sources

T^2 can pinch off:  $\Rightarrow$ Extra Physics



$\mathbb{R}^{5,1}$

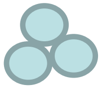
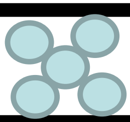
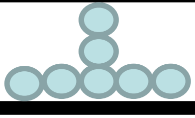
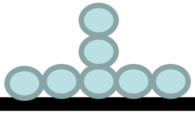
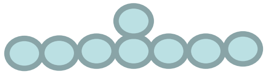
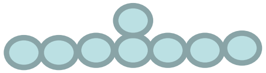
$\mathcal{M}_4^{extra}$

	0	1	2	3	4	5	6	7	8	9	10	11
7-Brane	×	×	×	×	×	×	×	×	●	●		

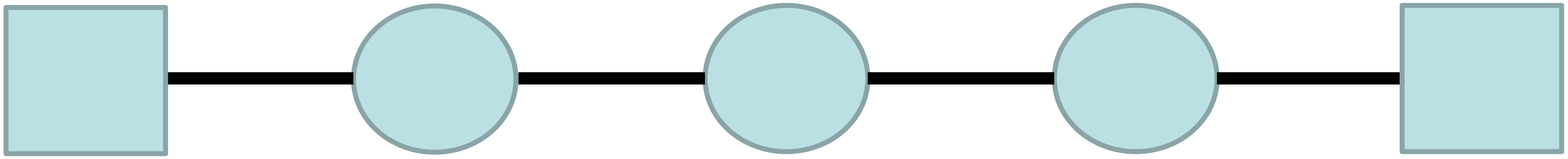
Gauge Coupling: $\frac{1}{g^2} = \mathrm{Vol}(\Sigma_{67})$

Fibers and String Charge

c.f. Kodaira '64, '66, Morrison Taylor '12

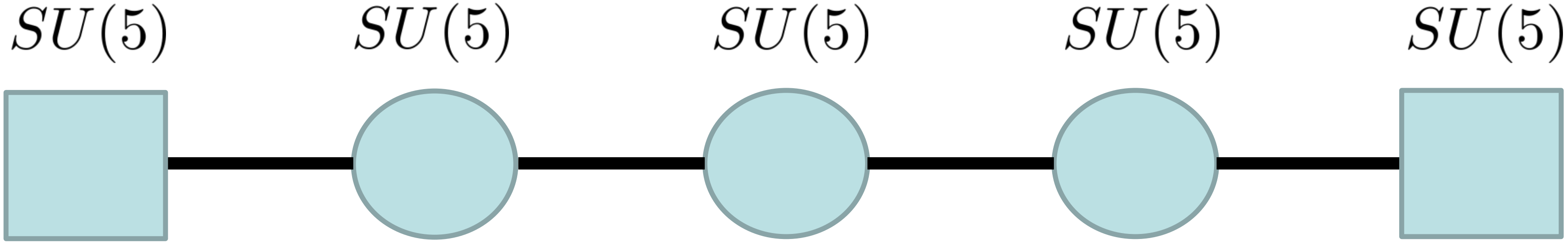
charge $= n$	pinching of T^2	gauge symmetry
3		$\mathfrak{su}_3$
4		$\mathfrak{so}_8$
5		$\mathfrak{f}_4$
6		$\mathfrak{e}_6$
7		$\mathfrak{e}_7$
8		$\mathfrak{e}_7$
12		$\mathfrak{e}_8$

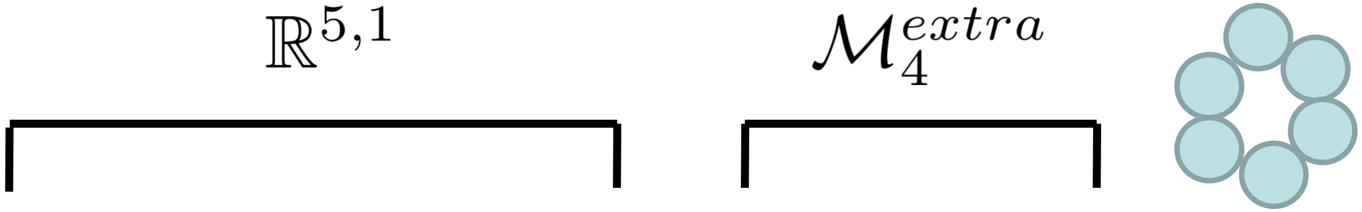
No Pinch-Off $\Rightarrow$ $(2,0)$ Theory



	$\mathbb{R}^{5,1}$						$\mathcal{M}_4^{extra}$			
	0	1	2	3	4	5	6	7	8	9
3-Brane	×	×					×	×		

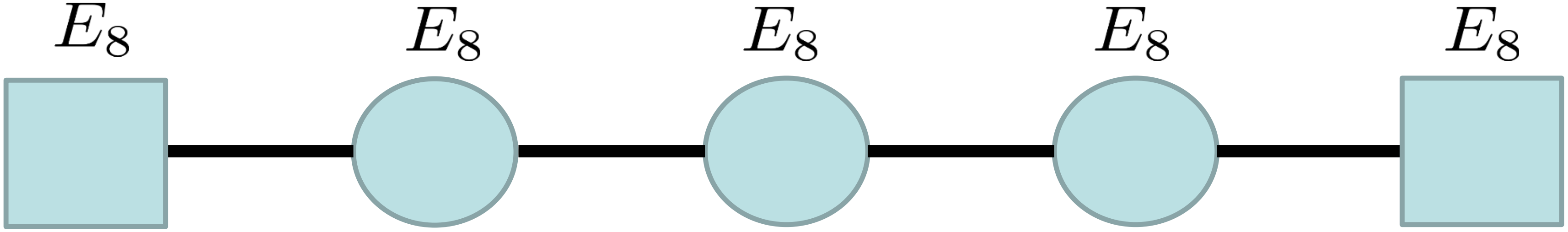
Sources ⇒ $(1,0)$ Theory





	0	1	2	3	4	5	6	7	8	9	10	11
3-Brane	×	×					×	×				
7-Brane	×	×	×	×	×	×	×	×				

Generalizations...

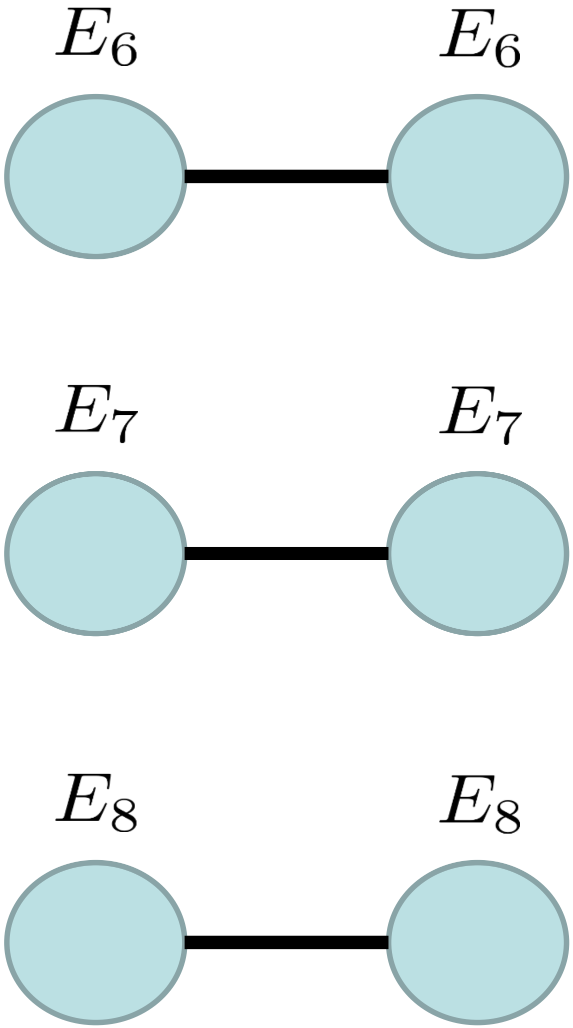


$\mathbb{R}^{5,1}$ $\mathcal{M}_4^{extra}$

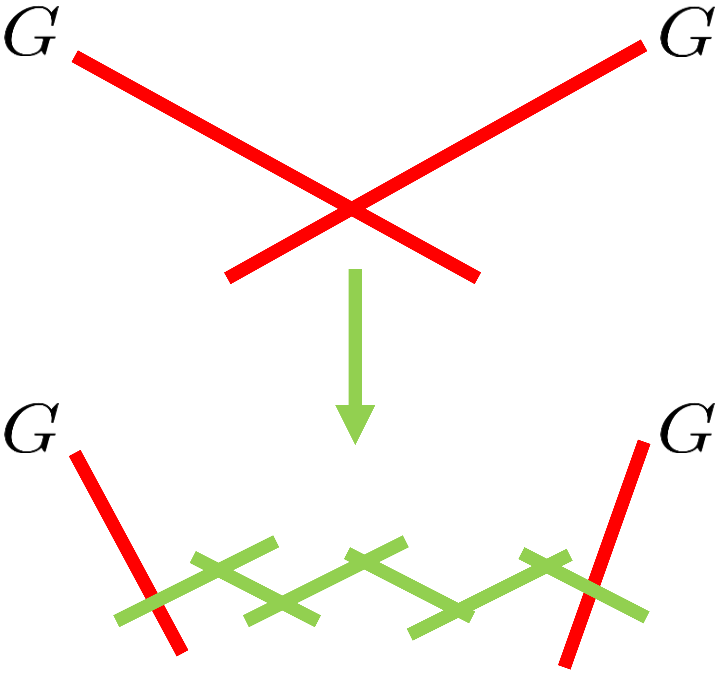
	0	1	2	3	4	5	6	7	8	9	10	11
3-Brane	×	×					×	×				
7-Brane	×	×	×	×	×	×	×	×				

Links $\neq$ Particles

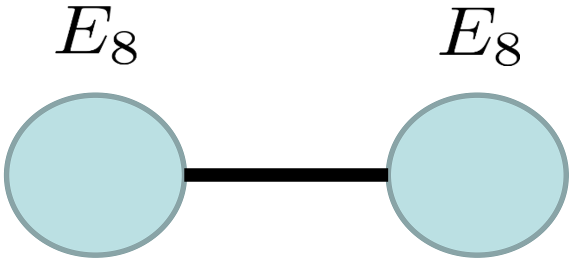
JJH et al. '14

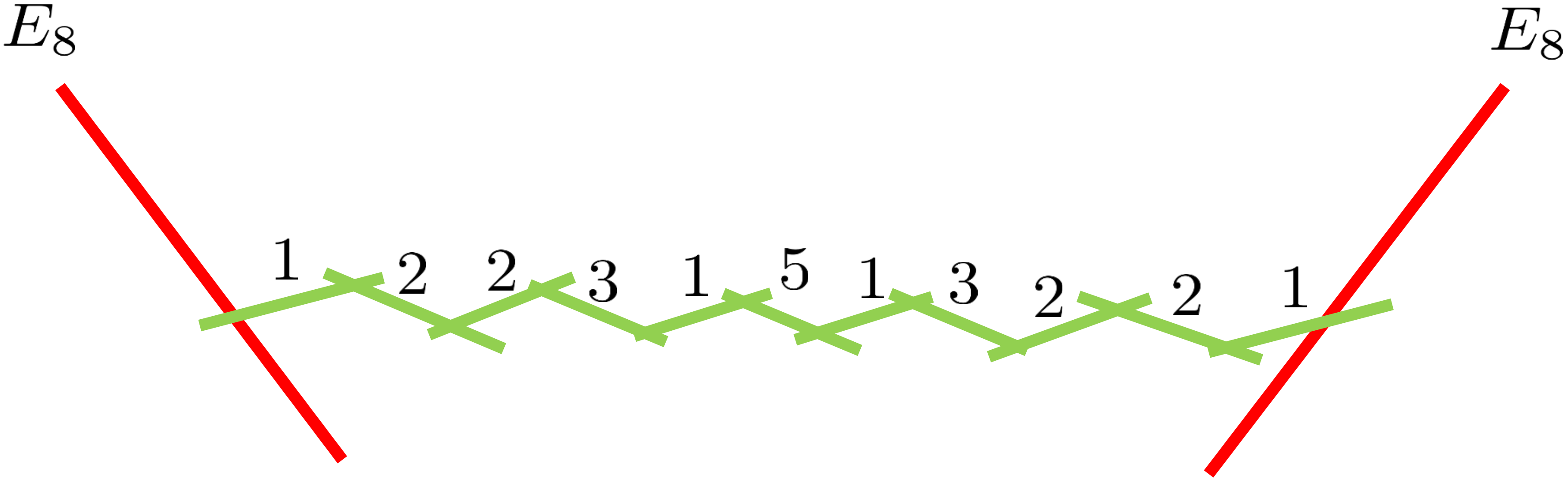


Use F-theory!



The Link is *also* an SCFT!





How to Build *All* 6D SCFTs

JJH et al. '13 - '15

Rules of the Game?

In 6D, things are quite rigid...

Can we enumerate every possible theory?

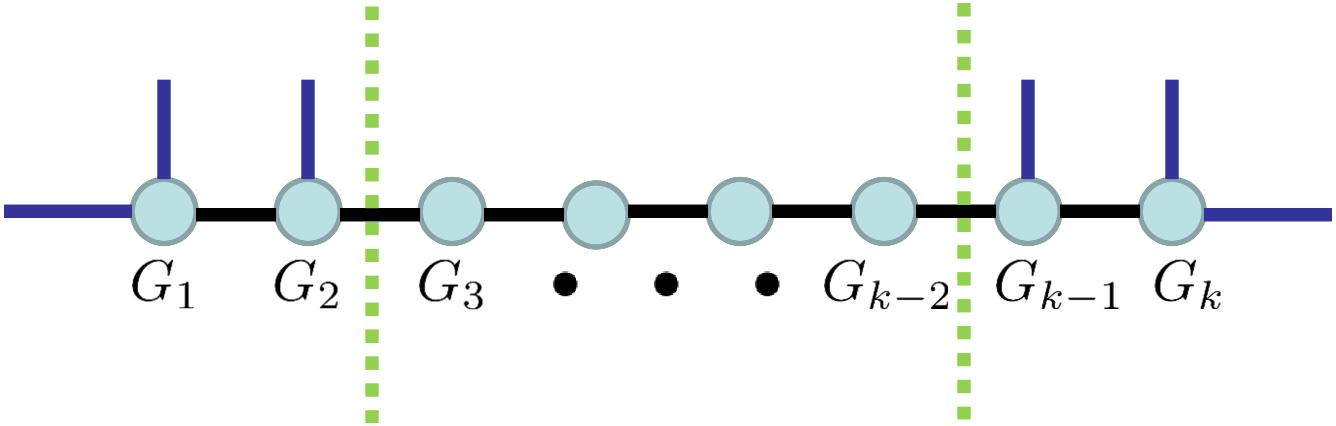
This is a purely geometric question!

Strategy

1) Find all $\mathcal{M}_4^{extra}$'s which could support an SCFT

2) Find all ways to add sources (pinched T^2)

Punchline #1



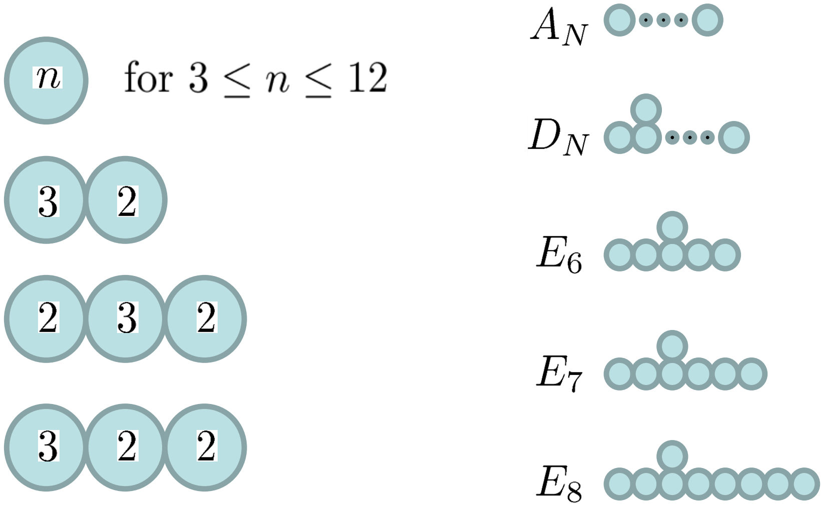
6D SCFTs = Generalized Quivers

Punchline #2

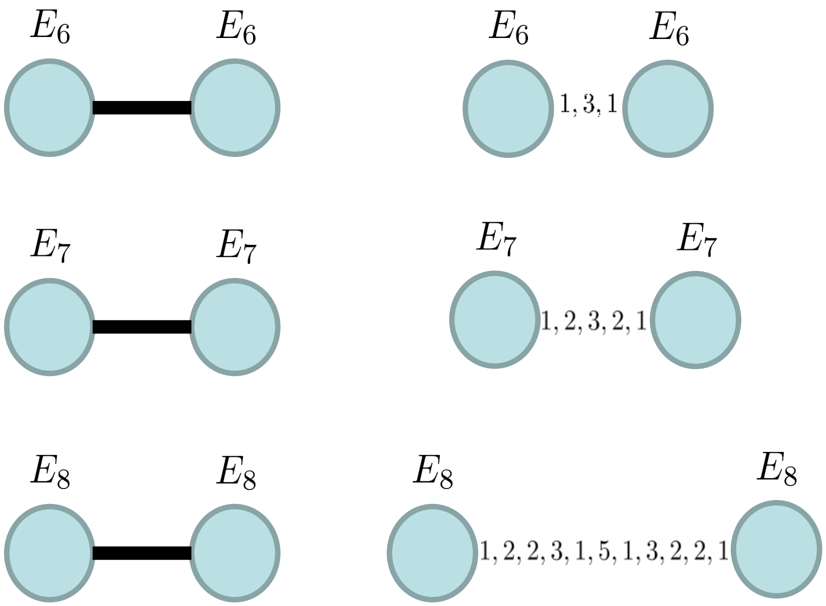
Looks Like Chemistry

“Atoms”

c.f. Morrison and Taylor ’12

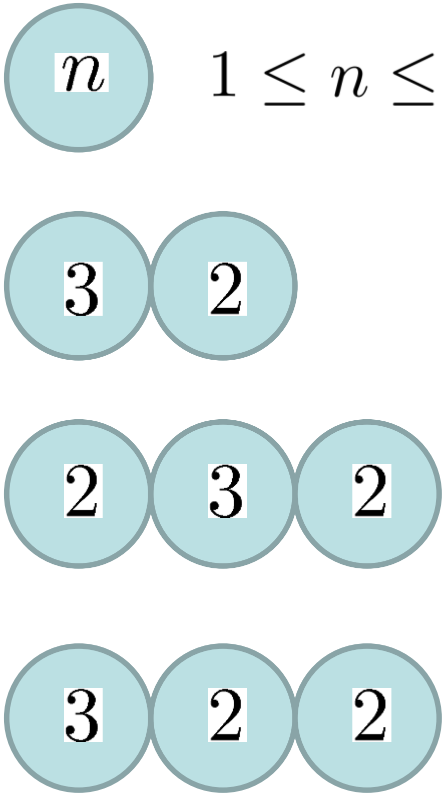


“Radicals”

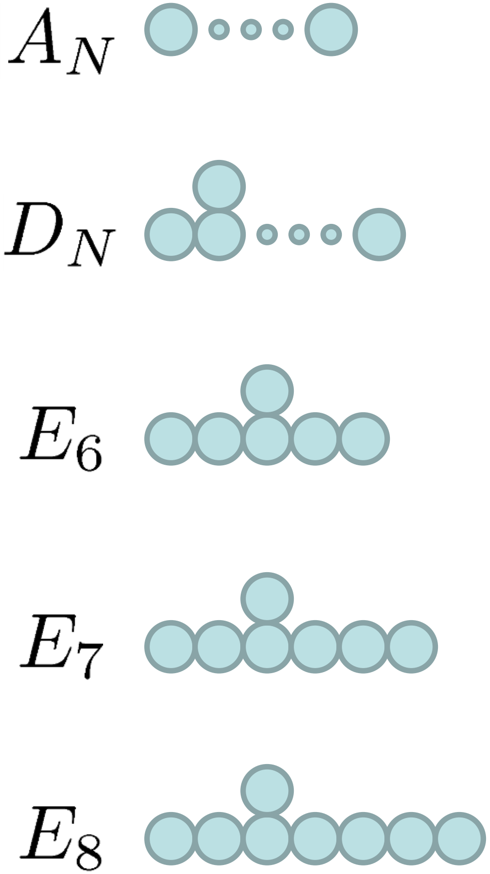


Building Blocks

c.f. Morrison and Taylor '12

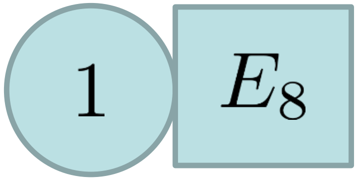


$1 \leq n \leq 12$



The -1 Building Block

Basic Unit of String Charge $= 1$

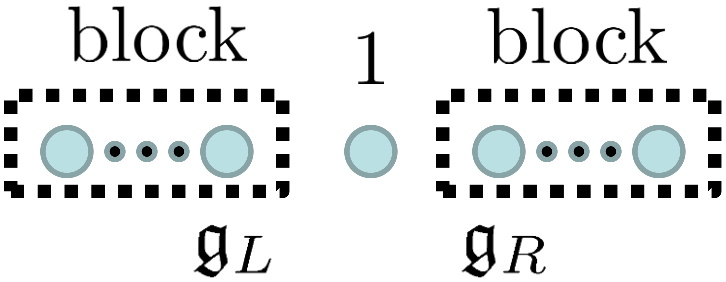


This theory has an E_8 Flavor Symmetry

States transform in multiplets of E_8 irreps

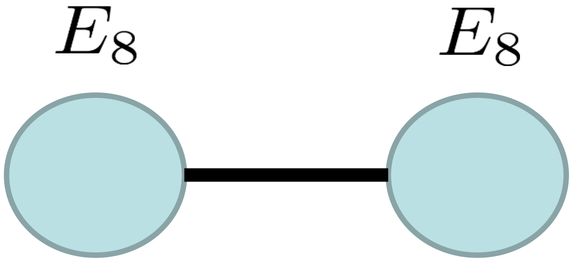
Gluing

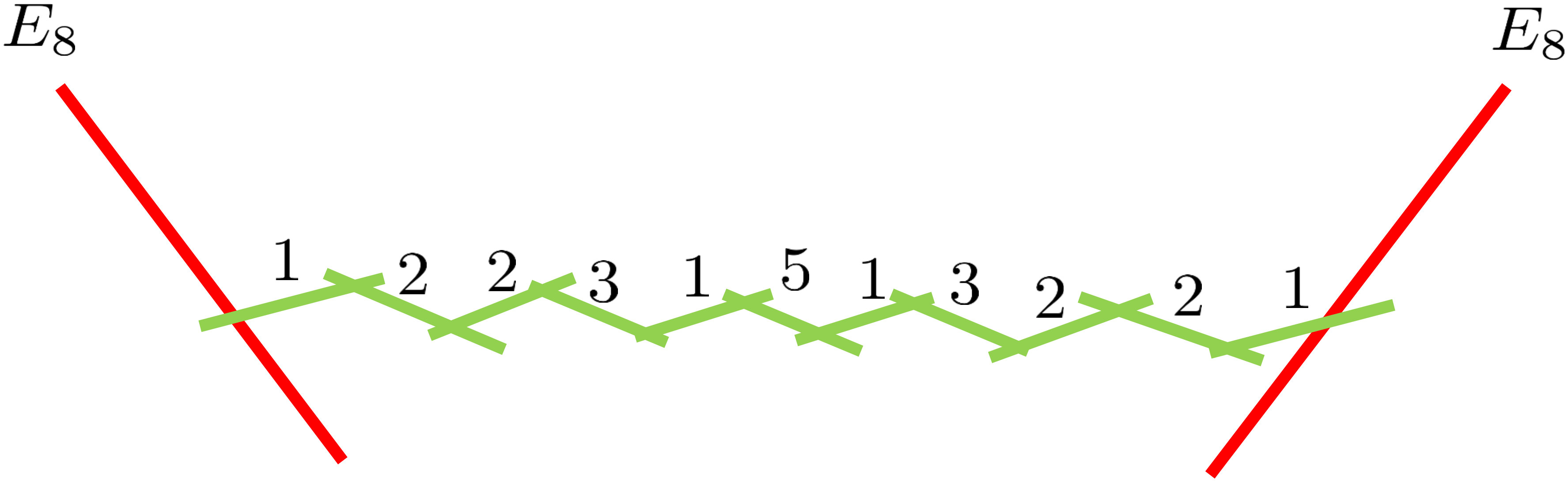
"Gauging a flavor symmetry"



$\mathfrak{g}_L \times \mathfrak{g}_R \subset \mathfrak{e}_8$

An Example of Gluing





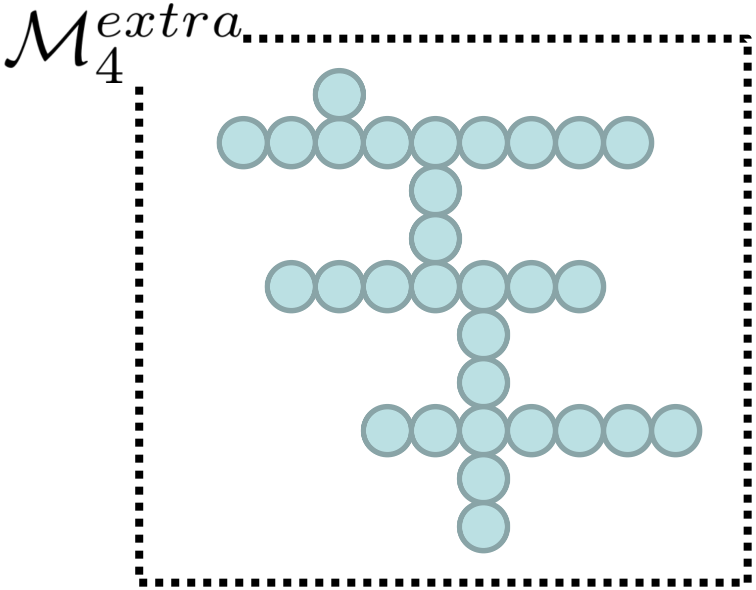
Constraints on Gluing

JJH, Morrison, Vafa ’13

1) No Closed Loops

Corollary: $\Sigma_i \cap \Sigma_j = 0$ or 1 if $i \neq j$

2) Trees are enough, e.g.



Useful Terminology: I / II

Split up building blocks into two groups:

$$\begin{array}{lcccccccc} & \mathfrak{so}_8 & \mathfrak{e}_6 & \mathfrak{e}_7 & \mathfrak{e}_7 & \mathfrak{e}_8 & \mathfrak{e}_8 & \mathfrak{e}_8 & \mathfrak{e}_8 \\ \text{DE-type:} & 4, & 6, & 7, & 8, & 9, & 10, & 11, & 12 \end{array}$$

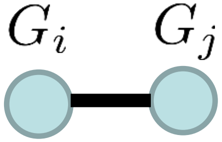
non-DE-type: 1, 2, 3, 23, 232, 223, 5

Useful Terminology: II / II

Define a Base Quiver:

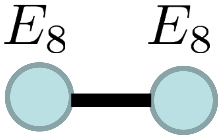
Nodes: DE-type curves

Links: Connecting DE-type curves



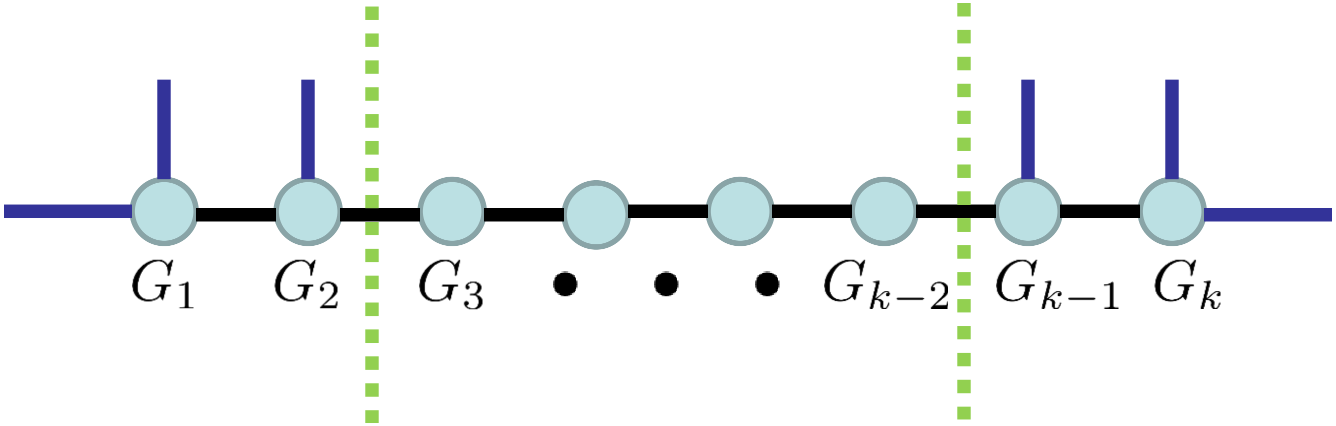
Example:

$$(12), 1, 223, 1, 5, 1, 322, 1, (12)$$



The Big Surprise

All 6D SCFTs have a *very* simple structure!

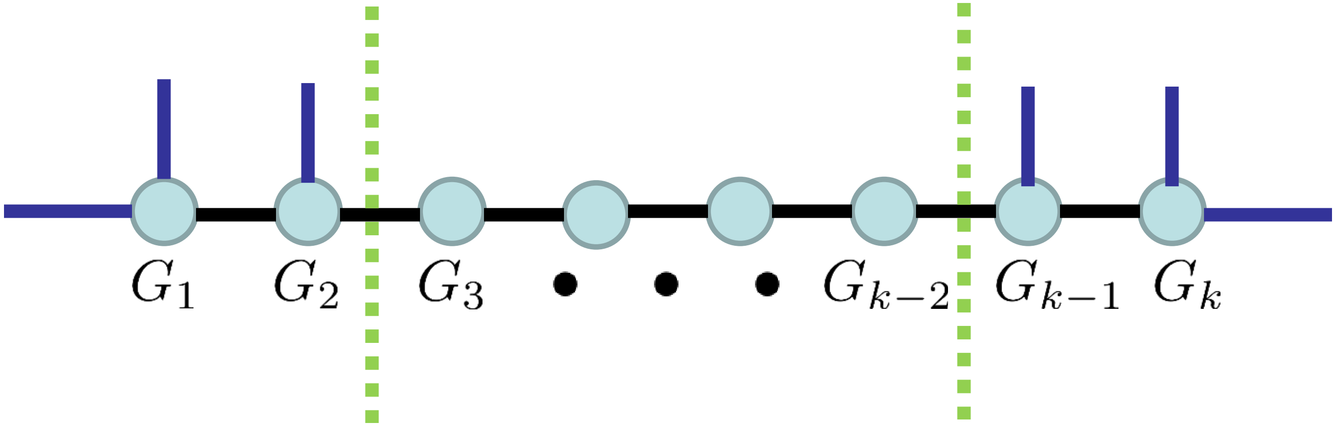


$$G_1 \subseteq G_2 \subseteq \cdots \subseteq G_m \supseteq \cdots \supseteq G_{k-1} \supseteq G_k$$

More Results...

JJH et al. '15

1)



2) Classification of all possible links

3) Classification of all possible gauge groups

So What’s Next?

Near Term Goals

Short-Term: Classical Geometry → QFT

- Compactify to $\mathbb{R}^{4,1} \times S^1$, on $\mathbb{R}^{3,1} \times \Sigma$

  In Progress: JJH et al.

- Operator Content?

  Some scaling dimensions now known (JJH '14)

- $\Lambda^*_{string}/\Lambda_{string}$ and spectrum of 6D defects?

  To Appear: JJH et al. '15

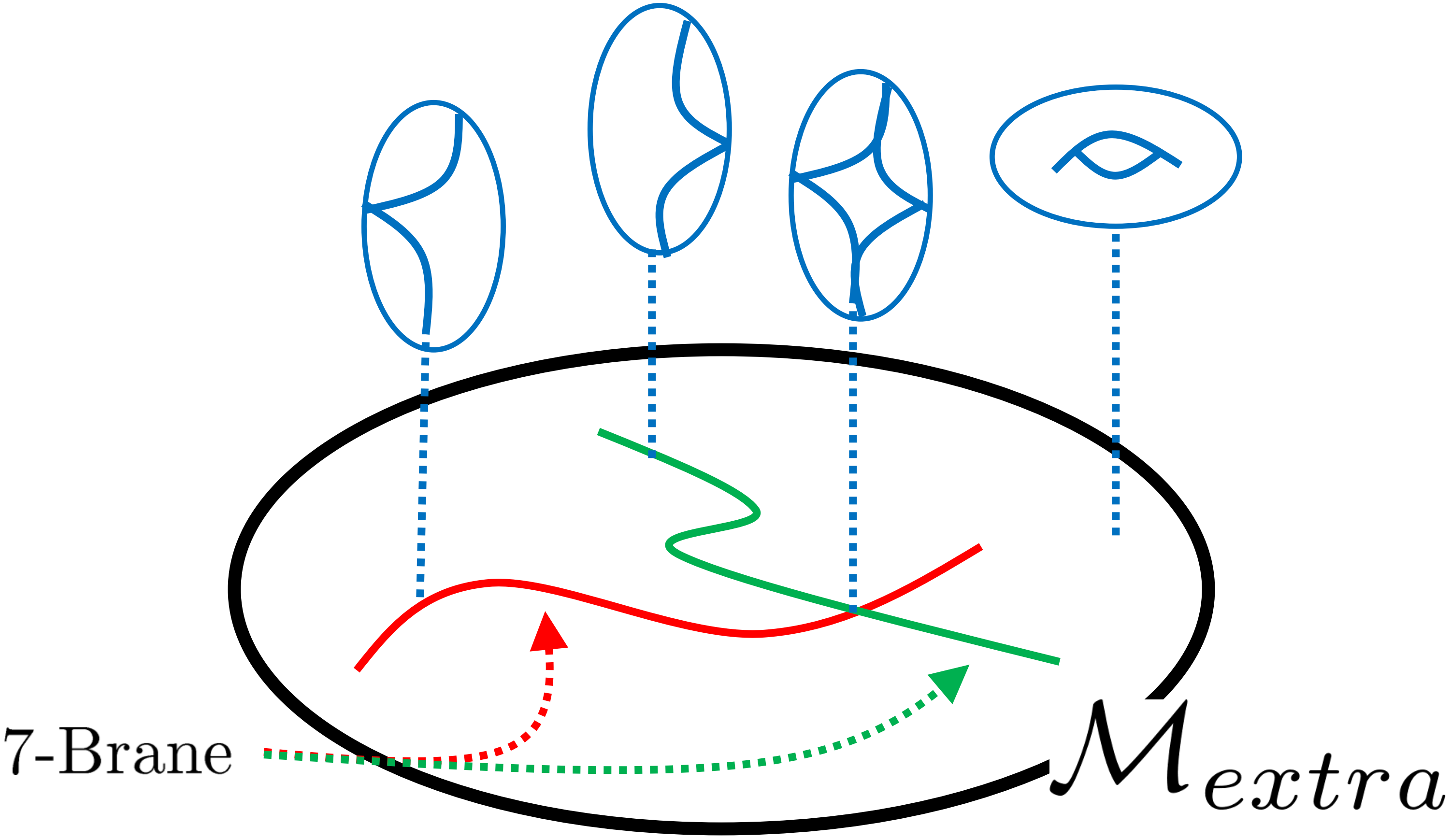
Broader Goals

Short-Term: Classical Geometry $\rightarrow$ EFT

- What are the rules of the game?
- Spectrum of physical states?
- Physical / Mathematical Structures?

Example: Doughnuts

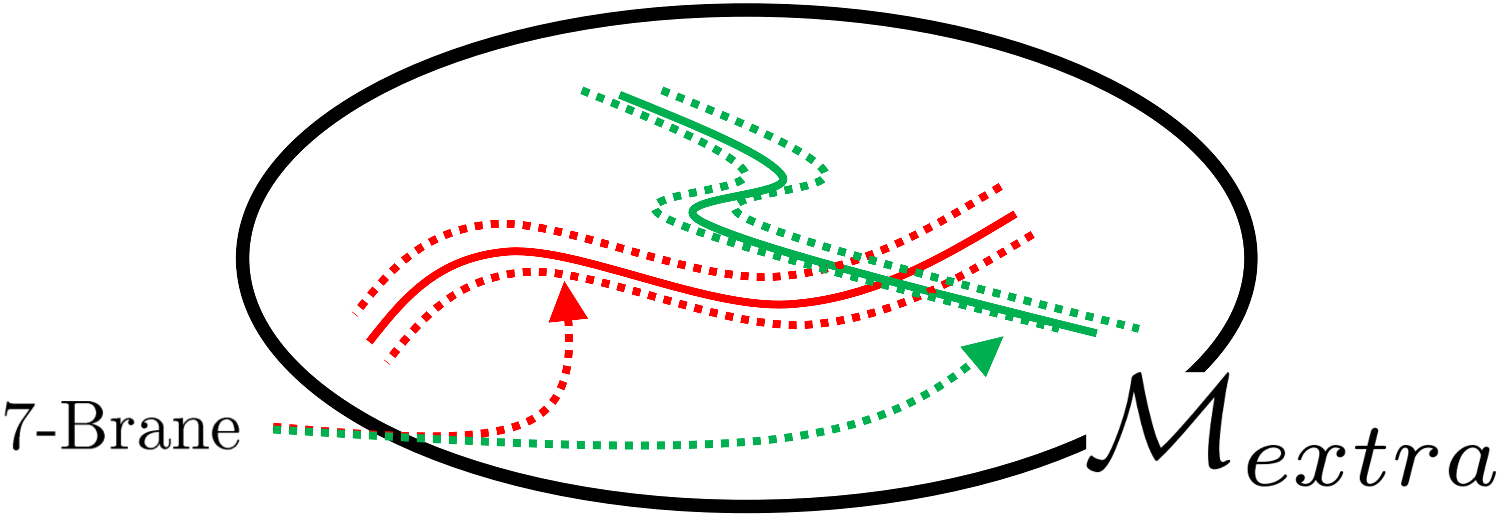
- Pinching Doughnut ⇒ Charge from 7-Brane



But...

- How About Dipole Moments?

''T-Branes'': Cecotti, Cordova, JJH, Vafa '10



Beyond Doughnuts

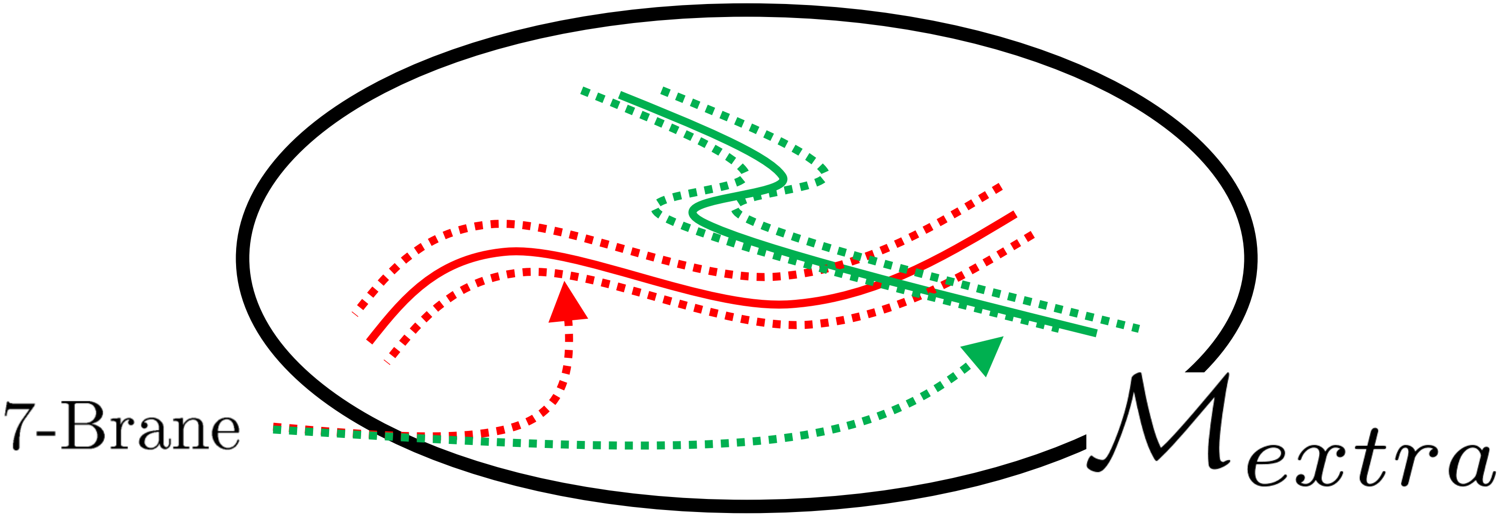
- How About Dipole Moments?

''T-Branes'': Cecotti, Cordova, JJH, Vafa '10

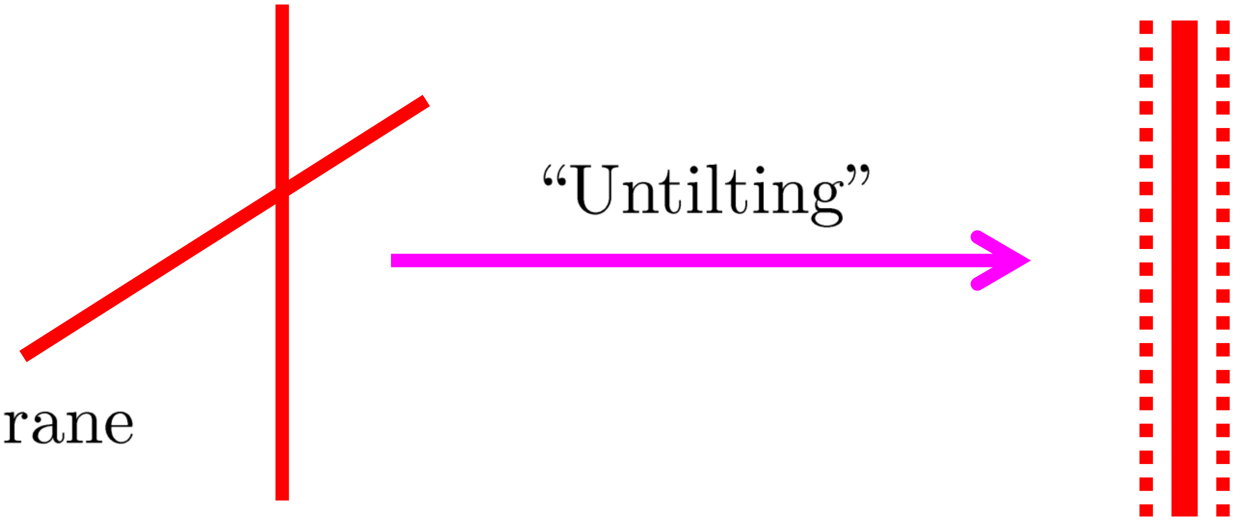
- Recent Progress involves heavy machinery:

"Theory of Limiting Mixed Hodge Structure"

Anderson, JJH, Katz '13, In Progress: Anderson, JJH Katz Schaposnik '15



Making a Dipole



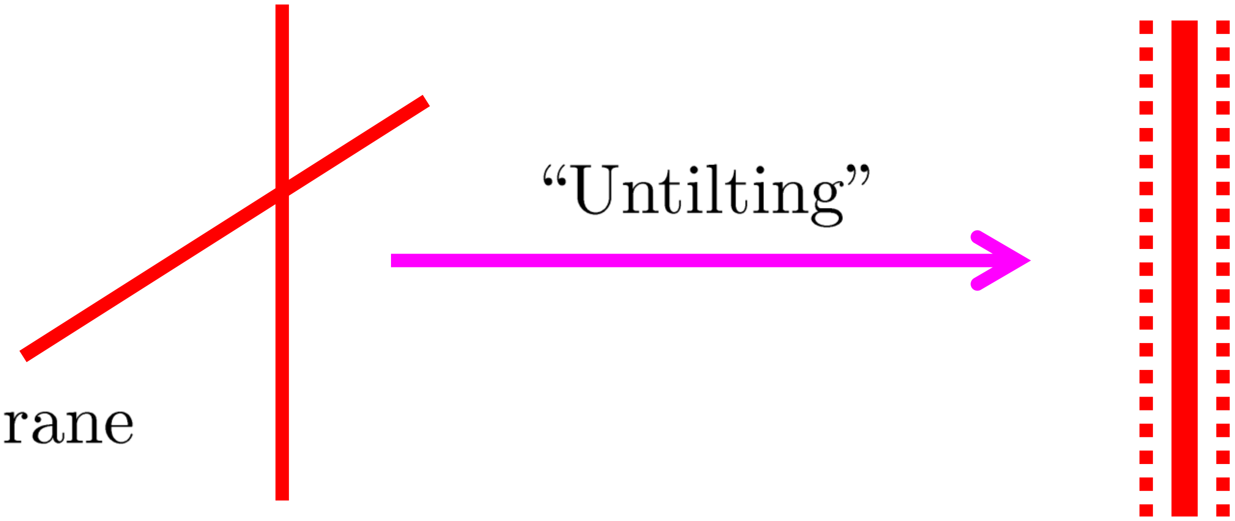
7-Brane

7-Brane

“Untilting”

7-Branes + Dipole

Math Description



7-Brane

7-Brane

“Untilting”

7-Branes + Dipole

$$\begin{tikzcd} \mathcal{J}(CY) \arrow[r] & \mathcal{M}_{HK} \arrow[d] \\ & \mathcal{M}_{cplx}(CY) \end{tikzcd}$$

“Hitchin System”

$$\overline{D}\Phi = 0$$

$$F + [\Phi, \overline{\Phi}] = 0$$

Broader Goals

Short-Term: Classical Geometry → QFT

Longer Term: Quantum Geometry → ???

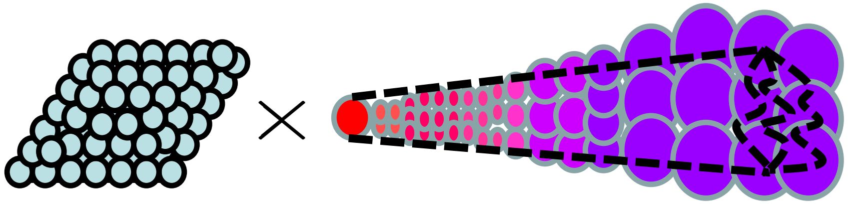
- Quantum Gravity + QFT?
- Emergent Spacetime?
- ...?

Emergence of Geometry

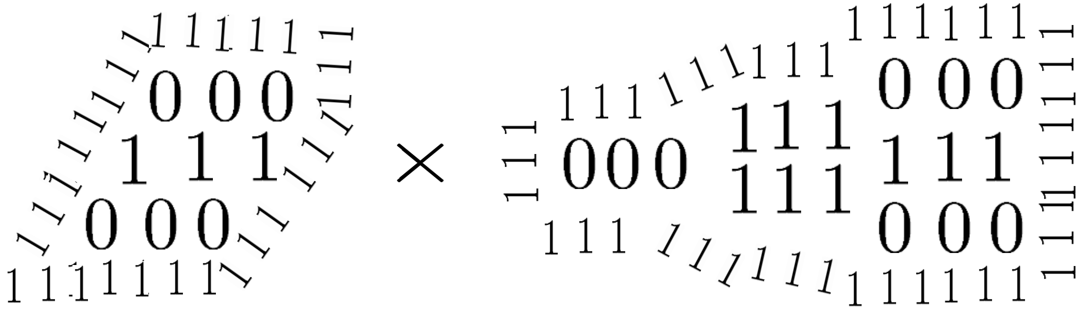
Beyond Classical Geometry:

- Non-Commutative Spacetimes

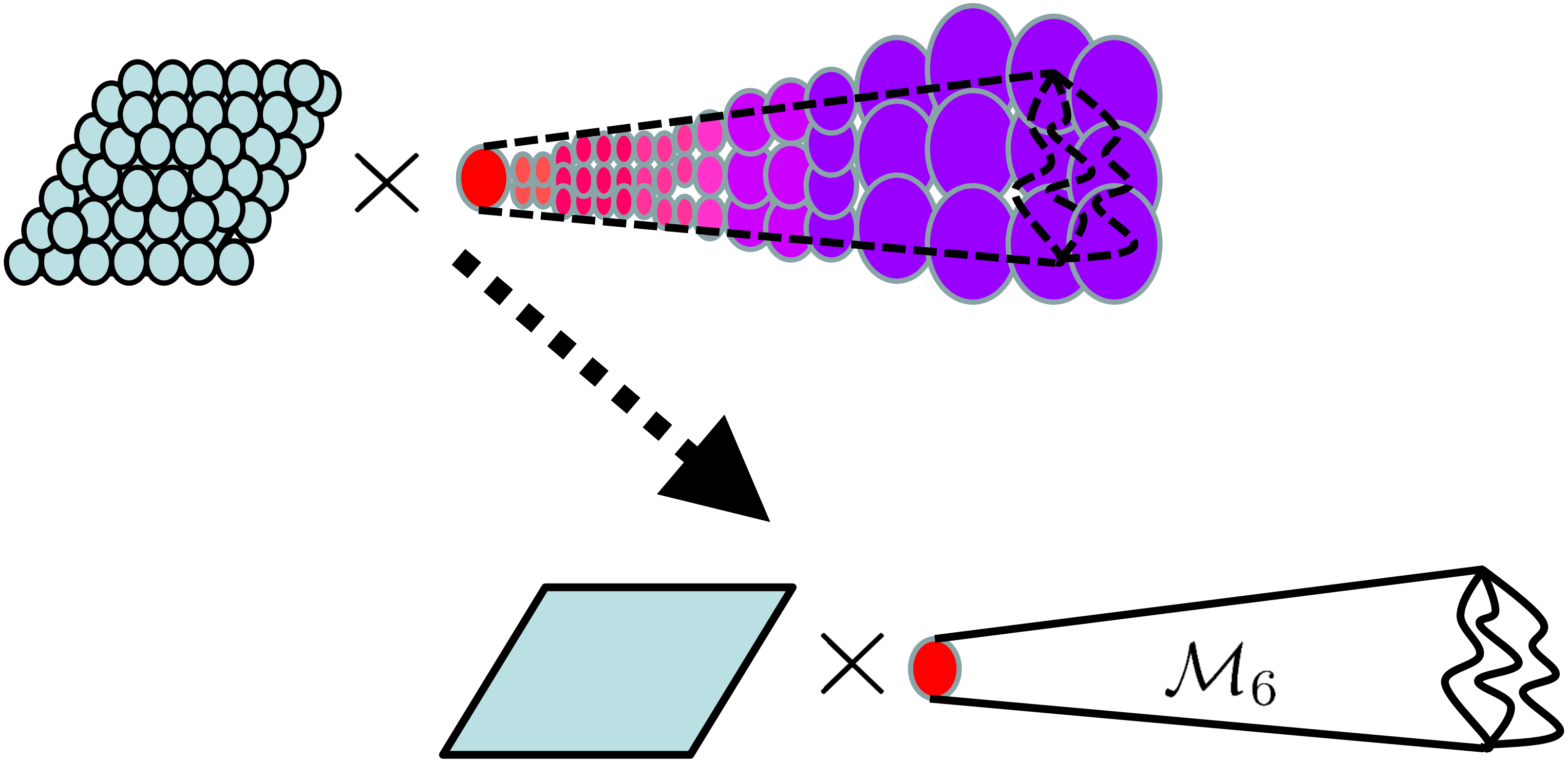
$[x, x'] \neq 0$:



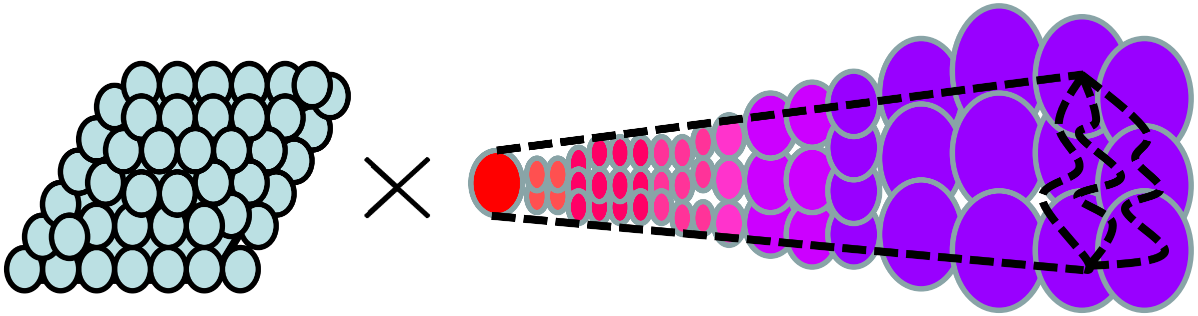
- Information Geometry



Emergence of Geometry



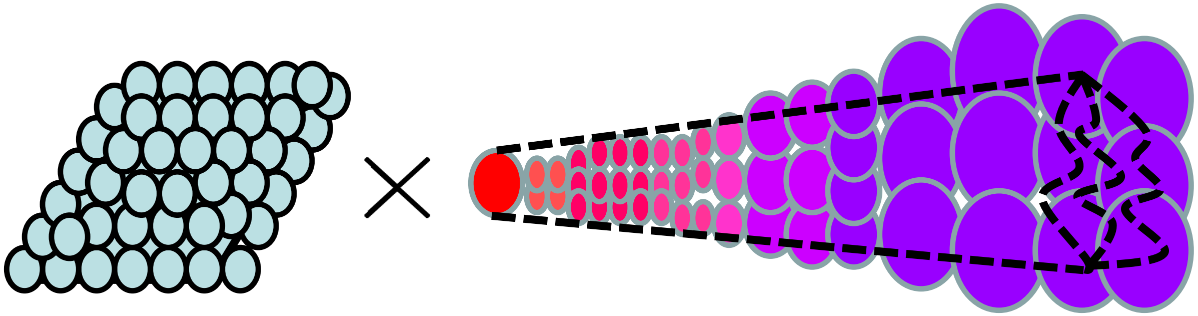
Emergence of Geometry



- Pixelize: $[x_{4D}, x'_{4D}] \neq 0$ and $[x_{6D}, x'_{6D}] \neq 0$

JJH, Verlinde '10 - '14

Emergence of Geometry



- Pixelize: $[x_{4D}, x'_{4D}] \neq 0$ and $[x_{6D}, x'_{6D}] \neq 0$

JJH, Verlinde '10 - '14

- Quantify: How many bits of data in $\mathcal{M}^{extra}$?

How about in 4D? JJH '13 + In progress

Balasubramanian, JJH, Maloney '14

What This Talk Was About

- Geometry → Effective Field Theories

- DONE: Classification of 6D SCFTs

- Next Up: Extract Universal Features
  Compactify to 5D/4D/3D/2D