Charting the Space of Quantum Field Theories

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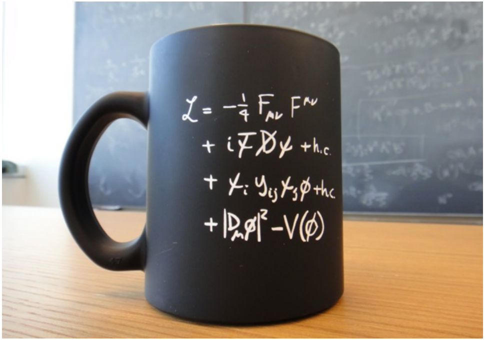
Quantum Field Theory in Fundamental Physics

Quantum mechanics + special relativity

$$\Downarrow$$

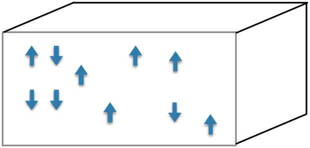
Local quantum fields $\{\varphi_i(x)\}$ $\quad x = (t, \vec{x})$

The language of particle physics



Quantum Field Theory for Collective Behavior

Modelling $N \to \infty$ degrees of freedom.

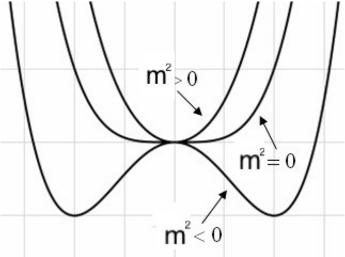


Ferromagnet
(Ising model)

Coarse-grained magnetization $M(\vec{x})$

$$H = \int d^3x \left[-\vec{\nabla} M \cdot \vec{\nabla} M + m^2 M^2 + g^2 M^4 + \dots \right]$$

$$m^2 \sim T - T_c$$



QFT $\equiv$ "Theory of quantum fields" (duh!)

$$\int \prod_x d\varphi(x)\; e^{-\frac{S[\varphi]}{\hbar}}$$

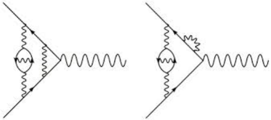
Infinite-dimensional integral handled by

- Introducing a cut-off (e.g., $x \in$ Lattice)
- Renormalization theory

Mathematicians may get a little nervous, but we think we know what we are doing...

$$S = \int d^D x \, \mathcal{L} \, , \quad \mathcal{L} = \text{quadratic} + g^2 \varphi^4 + \ldots$$

- "Easy" when $g \to 0$. Perturbative expansion:



$$\text{Rescaling } \ \varphi \to \varphi/g \text{ gives } e^{-\frac{S}{g^2 \hbar}}$$

$g \to 0$ equivalent to classical limit $\hbar \to 0$

- Hard for large g. Lattice simulations, ...

“QFT is about Fields and Lagrangians then ...” But is it?

- Hidden simplicity of perturbative scattering amplitudes. E.g. MHV amplitude for n gluons

$$A_n[1^+ \dots i^- \dots j^- \dots n^+] = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$

$\sum$ many Feynman diagrams = ultrasimple answer.

- Strong/weak coupling dualities $g \leftrightarrow 1/g$.

- Existence of non-Lagrangian QFTs.

Inadequacy of “fields”: dualities

In happy cases, as $g \to \infty$ an equivalent dual description emerges.

Pair of dual theories

$$\mathcal{T}[\varphi_i; g] \Leftrightarrow \mathcal{T}'[\varphi_i'; g'], \quad g' = \frac{1}{g}$$

$\mathcal{T}$ and $\mathcal{T}'$ different classical limits of the same quantum theory.

φ and φ' not fundamental objects.

Some QFTs are even dual to quantum gravity theories
(in higher spacetime dimensions)!

All that is solid melts into air
Fields, gauge symmetries, spacetime itself..not fundamental?

Paradigm: S-duality of $\mathcal{N} = 4$ SYM

$\mathcal{N} = 4$ supersymmetric Yang-Mills theory in 3+1 dimensions.
Maximally symmetric cousin of QCD.

Complexified gauge coupling $\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$.

$SL(2, \mathbb{Z})$ duality symmetry

$$\tau \to \frac{a\tau + b}{c\tau + d}, \quad \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix}.$$

with a, b, c, d integers and $ad - bc = 1$.
Infinitely many semiclassical limits!

$\tau \to -1/\tau$: electric-magnetic duality $\mathbf{E} \to \mathbf{B}$, $\mathbf{B} \to -\mathbf{E}$.

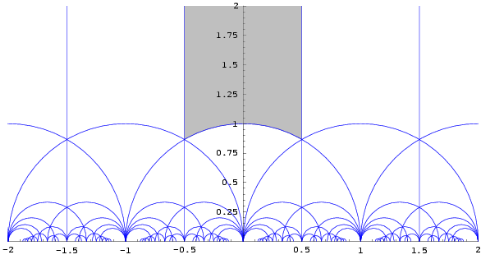
No path-integral derivation *remotely* in sight.

Abstract statement of S-duality

$$\langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_n(x_n) \rangle_\tau \,, \quad \mathcal{O}_i = \text{gauge-invariant operator}$$

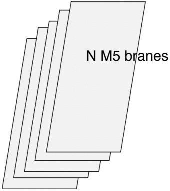
have good transformation properties under $SL(2,\mathbb{Z})$.

"Theory space" parametrized not by $\tau \in H$, but by $\tau \in H/SL(2,\mathbb{Z})$



Inadequacy of "fields": non-Lagrangian QFTs

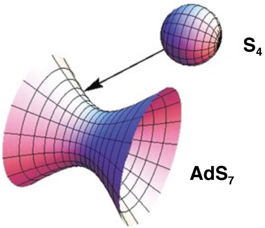
$d = 6$ maximally SUSY theory, known as the (2, 0) theory.



$(2,0)_N$ theory governs low-energy fluctuations of N five-branes in M-theory

Discrete parameter N. For finite N, intrinsically quantum.

As $N \to \infty$
11d supergravity on



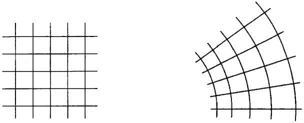
Beyond Lagrangian field theory

We *can* do better in (at least) two overlapping classes of QFTs:

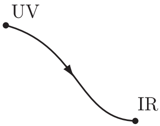
- Conformal field theories.
  Defined by an abstract, intrinsically quantum, operator algebra.
- Supersymmetric theories.
  Some observables fixed by internal consistency alone.

Conformal symmetry

Physics simplifies when intrinsic mass scales can be neglected: large/low energy regimes of QFTs and statistical systems near T_c.
Scale invariance. "Generically" enhanced to conformal invariance.
A conformal transformation acts *locally* as rotation and dilatation:



CFTs are signposts in the space of QFTs.



(Conjecture) Generic behavior of (unitary) QFT: an RG flow between two CFTs.
$\text{DOF}_{UV} > \text{DOF}_{IR}$

Abstract CFT

A CFT is defined by the correlation functions

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle$$

of a set of local operators $\{\mathcal{O}_k(x)\}$.

E.g., in Ising CFT we have the spin operator σ, the energy operator ϵ and infinitely many more.

Scaling dimensions Δ_i: $\langle \mathcal{O}_i(x) \mathcal{O}_i(y) \rangle = |x-y|^{-2\Delta_i}$

Operator Product Expansion

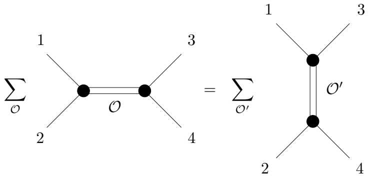
$$\text{OPE}: \quad \mathcal{O}_i(x)\mathcal{O}_j(0) = \sum_k c_{ijk} x^{\Delta_k - \Delta_i - \Delta_j} \left(\mathcal{O}_k(0) + \dots \right) .$$

The sum converges (unlike in a general QFT).

Conformal bootstrap

Old aspiration (1970s) Polyakov, Ferrara Gatto Grillo:
use crossing symmetry to solve the theory.

For a 4-point function:



Vastly over-constrained system of equations for $\{\Delta_i, c_{ijk}\}$.

Famous success story in $2d$:
conformal symmetry $z \to f(z)$ is infinite-dimensional.
Exact solution of many models.

The modern bootstrap program

2008 breakthrough in $d > 2$ Rattazzi Rychkov Tonni Vichi

Crossing + unitarity $\Rightarrow$ inequalities for $\{\Delta_i, c_{ijk}\}$.

(Unitarity: Δ_i bounded from below, c_{ijk} real.)

Bootstrap inequalities obtained numerically but perfectly rigorous: they may not be optimal but they are true.

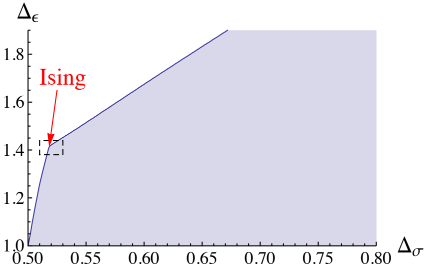
Very flexible tool: any dimension, any global symmetry.

Bound in $d = 3$ from single correlator

El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, PRD **86**, 025022

CFT$_3$ with $\mathbb{Z}_2$ symmetry. σ odd, ϵ even, $\sigma \times \sigma = \mathbf{1} + \epsilon + \ldots$

Exclusion plot from crossing of $\langle \sigma\sigma\sigma\sigma \rangle$:

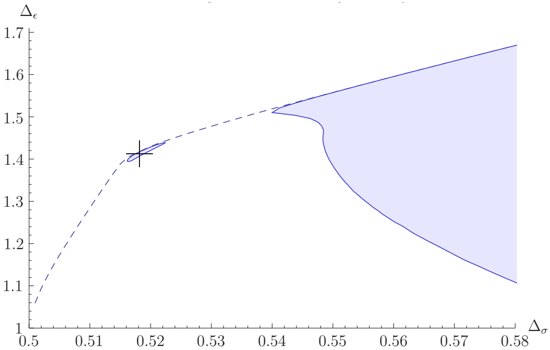


Two empirical surprises:

- $3d$ Ising appears to lie just on the exclusion curve
- $3d$ Ising appears to sit at a special kink on the exclusion curve.

Multiple Correlators Kos, Poland, Simmons-Duffin, '14

System of correlators $\langle\sigma\sigma\sigma\sigma\rangle$, $\langle\sigma\sigma\epsilon\epsilon\rangle$, $\langle\epsilon\epsilon\epsilon\epsilon\rangle$.
Assuming that σ, ϵ are the only operators with $\Delta < 3$ (physically very well-motivated):



$3d$ Ising gets cornered!

Most precise determination ever of Ising critical exponents, with rigorous error bars.

Bootstrap of $(2,0)_N$ Theory Beem Lemos LR van Rees

Abstract approach is all we have.

- Great news: Crossing constraints solvable for a subalgebra!

A closed subsector of SUSY operators, with meromorphic correlators, isomorphic to the $2d$ W_N algebra.

Exact 3-point functions of for any N.
For $N \to \infty$, striking agreement with supergravity on $AdS_7 \times S^4$.
One recovers non-linear SUGRA purely from algebraic consistency.

$1/N$ corrections $\Rightarrow$ quantum M-theory corrections.

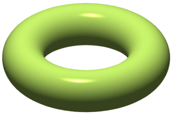
- Non-SUSY spectrum can be constrained numerically.

$6 = 4 + 2$

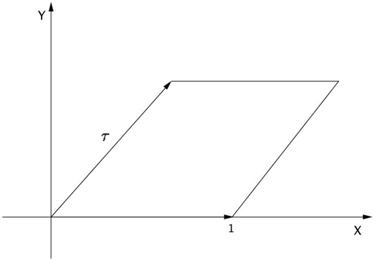
Put $(2,0)_N$ on $\mathbb{R}^4 \times T^2$. Flow to the IR

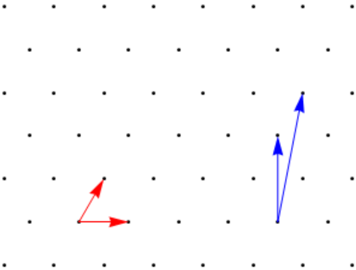
$$\Downarrow$$

$SU(N)$ $\mathcal{N} = 4$ super Yang-Mills on $\mathbb{R}^4$
with coupling $\tau \equiv$ modular parameter of T^2.



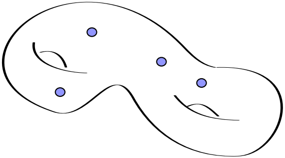
This picture "explains" S-duality.





$6 = 4 + 2$: class $\mathcal{S}$(ix) theories Gaiotto

Put $(2,0)$ on $\mathbb{R}^4 \times \mathcal{C}$.
$\mathcal{C} \equiv$ Riemann surface with punctures.
$\Downarrow$
$\mathcal{N} = 2$ SUSY CFT on $\mathbb{R}^4$.



$4d$ **SCFT** $\mathcal{T}[\mathcal{C}]$	$2d$ **data on** $\mathcal{C}$
Gauge couplings $\{\tau_i\}$	Complex moduli of $\mathcal{C}$
Global symmetry	Puncture
Generalized S-duality	Modular transformation of $\mathcal{C}$

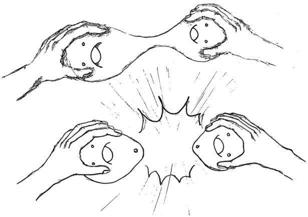
Theory space interpreted as a "real" geometric space, the surface $\mathcal{C}$.

Only "measure zero" subset of class $\mathcal{S}$ has a Lagrangian description!

How can we approach these theories?

- Conformal bootstrap:
  both analytic (for SUSY subsector) and numeric (for the rest).
  Beem Lemos Liendo Peelaers LR van Rees

- Consistency conditions in theory space.
  Degeneration of $\mathcal{C}$ $\Rightarrow$ Theory $\mathcal{T}[\mathcal{C}]$ splits into decoupled theories.
  Very powerful constraint.



[Roy Sato's drawing, from Tachikawa's webpage]

"$\mathcal{N} = 2$ Theories labelled by Riemann surfaces"

$\mathcal{T}[\mathcal{C}]$ may not be yet a well-defined mathematical object,
but many of its observables are, e.g.

- Partition function of theory $\mathcal{T}$ on manifold $\mathcal{M}$: a number.
- Higgs branch of vacua of $\mathcal{T}$: Hyperkäler manifold.

"Bootstrap in theory space":

Gluing of surfaces translates into gluing rules for these observables.
Enough to fix them, provided some minimal physical input.

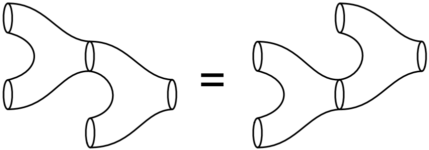
Example: $S^3 \times S^1$ partition function

Witten index, encoding the SUSY spectrum.

Very complicated function $\mathcal{I}(p, q, t; a_i)$
(p, q, t) geometric parameters of the twist.
a_i parameters associated to the flavor symmetry of $\mathcal{T}$.

For a Lagrangian theory, $\mathcal{I}[\mathcal{T}]=$ elliptic hypergeometric integral.

Theory space bootstrap fixes it uniquely for all $\mathcal{T}[\mathcal{C}]$!
Computed by a Topological QFT living on $\mathcal{C}$.
Gadde Gaiotto Razamat LR Yan



Conclusions

We're still learning what QFT is.

- New insights into the meaning of QFT.
- New practical tools, such as the revived conformal bootstrap.
- New mathematics.

I've emphasized two heuristic principles:

- "Bootstrap" approach:
  Use general principles, as opposed to detailed dynamical models.
- Enlarge the view to the whole space of QFTs.

Concrete models...

As physicists, we often build detailed dynamical models:

Identify relevant degrees of freedom $\{\varphi_i\}$

$\downarrow$

Write a model $H[\varphi_i]$

$\downarrow$

Solve it

...versus abstract symmetries

A "meta" question:
Which theories are in principle allowed?
Not *anything* goes!

Quantum mechanics + Spacetime symmetries
and Specific symmetries of the problem ⇒ very constraining

Could it be that only one theory is possible,
given some minimal physical input?

- Complex systems at a phase transition (boiling H_2O, magnets) have universal behaviors completely fixed by symmetries.
- Could there be only one consistent theory of quantum gravity?

(From the Salt Lake Tribune)

Pull yourself up from the mud of theory space!

Corner and solve your theory by leveraging internal consistency rules.