Exact Solutions of 2d Supersymmetric gauge theories

Abhijit Gadde, IAS

w. Sergei Gukov and Pavel Putrov

UV to IR

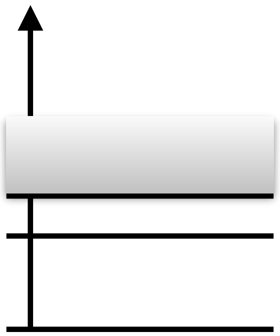
- Physics at long distances can be strikingly different from the physics at short distances
- Even the notion of "fundamental" particles may be different
  - QED and cooper pairs
  - Kinks in the Ising model in strong magnetic fields
  - Yang Mills theory and mass gap
  - Massless QCD and pions
- Given a microscopic theory, finding its manifestation at long distances is of great practical importance.

Low energy theory

- At very low energy scales (i.e. when all the mass scales are taken to infinity), the spectrum could be of one out of two types:

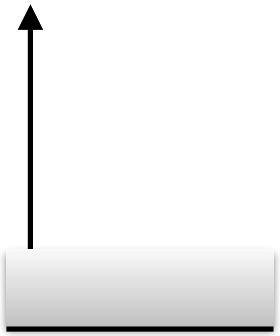
  - Gapped

    Degenerate vacuum



  - Gapless

    Nontrivial theory of gapless modes; No scale



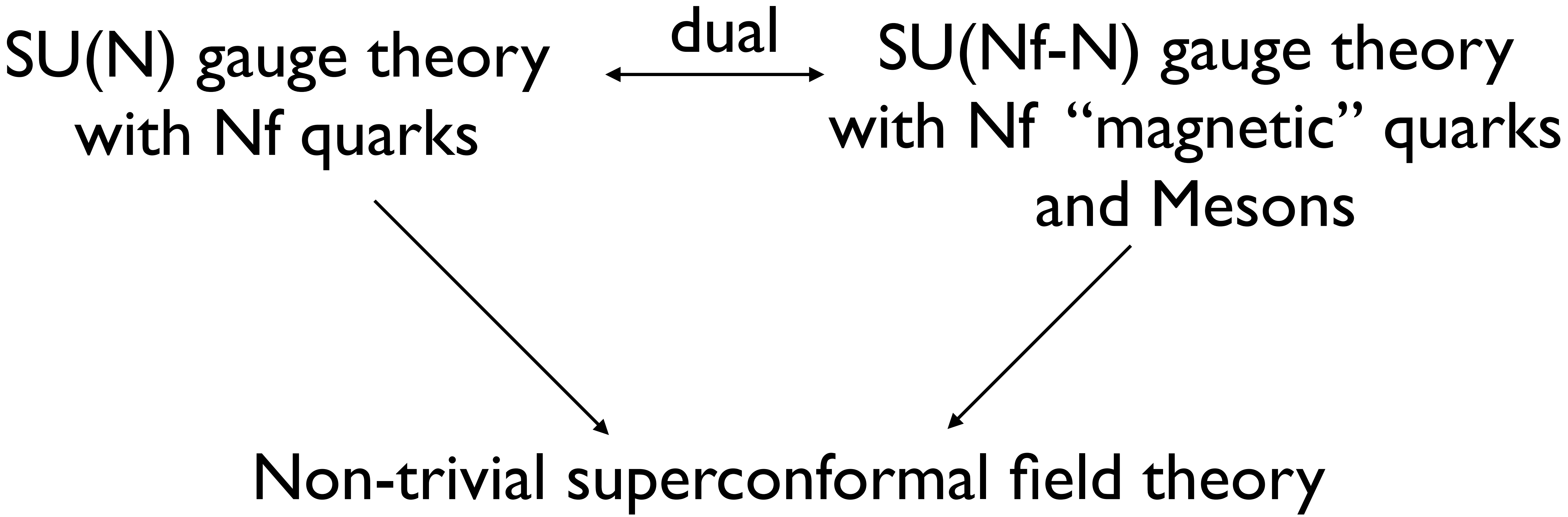
- Nontrivial CFT

Tool: Anomaly matching

Seiberg duality

A success of anomaly matching

Supersymmetric QCD



SU(N) gauge theory
with Nf quarks

dual

SU(Nf-N) gauge theory
with Nf "magnetic" quarks
and Mesons

Non-trivial superconformal field theory

- Only supersymmetric checks

(0,2) Supersymmetric QCD

- Generically, have global symmetries with non-vanishing anomalies → CFT
- As we will see, they exhibit Seiberg type duality (actually a triality).
- Power of infinite dimensional conformal invariance, anomaly matching and modular invariance → Solution of the theory
- Hence “proving” triality.

Motivation from 4-manifolds

- Compactification of 6d (2,0) theory on d-manifold ⟶ 6-d dim SCFT
  - For d=2, complex structure of the Riemann surface becomes the coupling constant 4d N=2 field theory
  - Partition functions can be computed from 2d

- For d=4

4-manifolds ⟷ 2d (0,2) theories

Vafa Witten partition function ⟷ Elliptic genus

(0,2) Multiplets

- Chiral multiplet: $\bar{D}_+\Phi = 0$

$$\Phi = \phi + \sqrt{2}\theta^+\psi_+ - i\theta^+\bar{\theta}^+\partial_+\phi$$

  - Complex scalar
  - Complex right-moving fermion

- Fermi multiplet: $\bar{D}_+\Psi = 0$

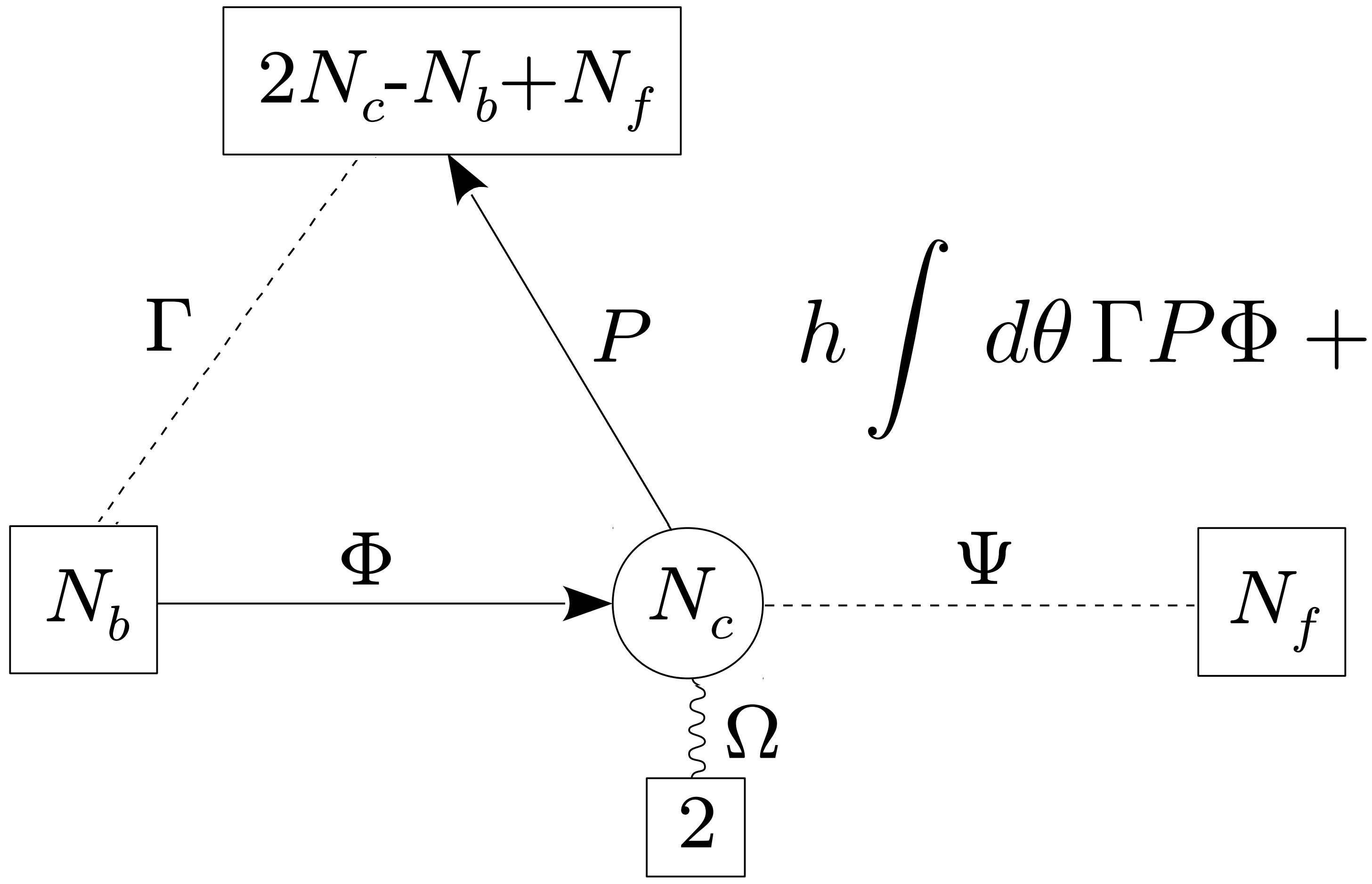
$$\Psi = \psi_- - \sqrt{2}\theta^+G - i\theta^+\bar{\theta}^+\partial_+\psi_-$$

  - Complex left-moving fermion

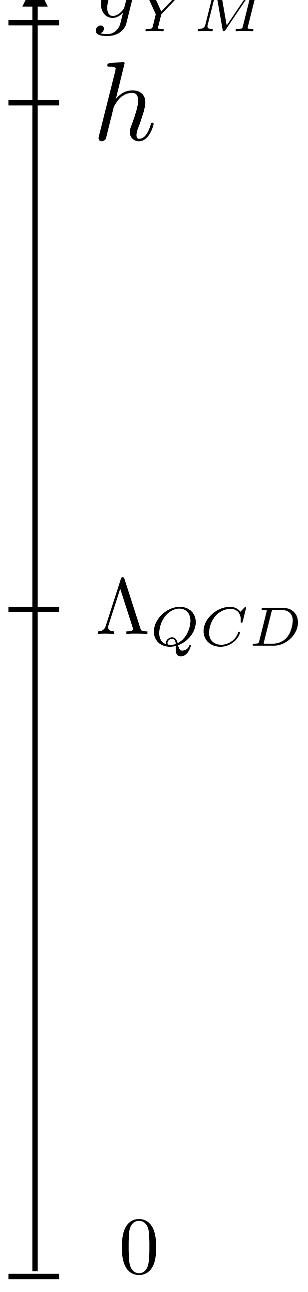
- Vector multiplet:
  - Gauge invariant d.o.f.: Fermi multiplet Λ

(0,2) SQCD

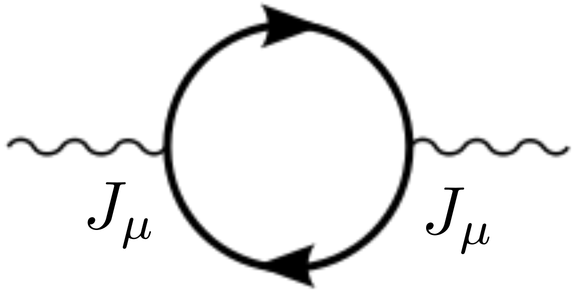
- Similar to 4d N=1 SQCD, but 2 types of matter
- U(Nc) gauge theory with Nb Chiral and Nf Fermi
- + Gauge anomaly cancellation
- + Normalizable vacuum

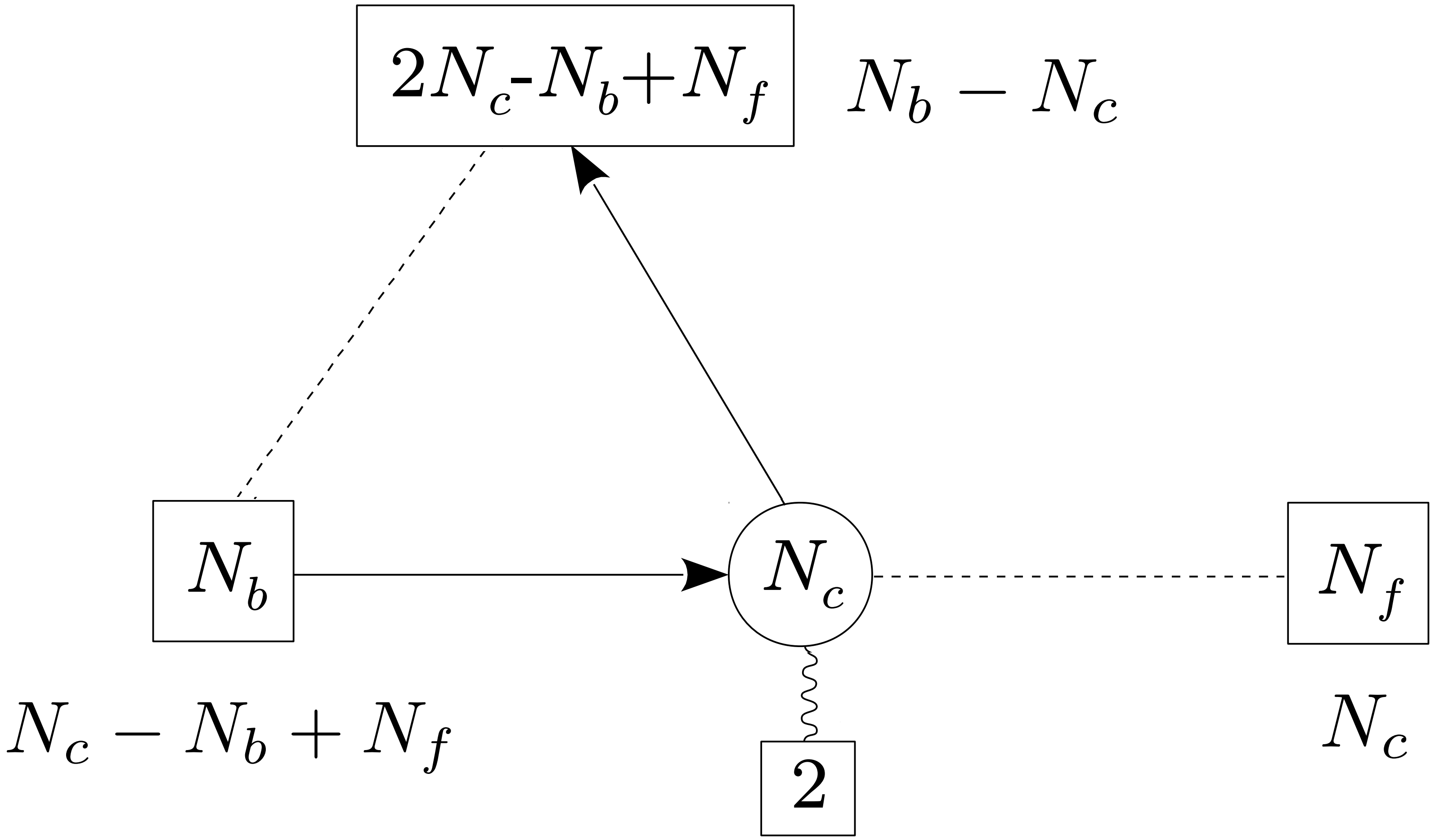


$$h\int d\theta\,\Gamma P\Phi + t\int d\theta\,\Lambda$$



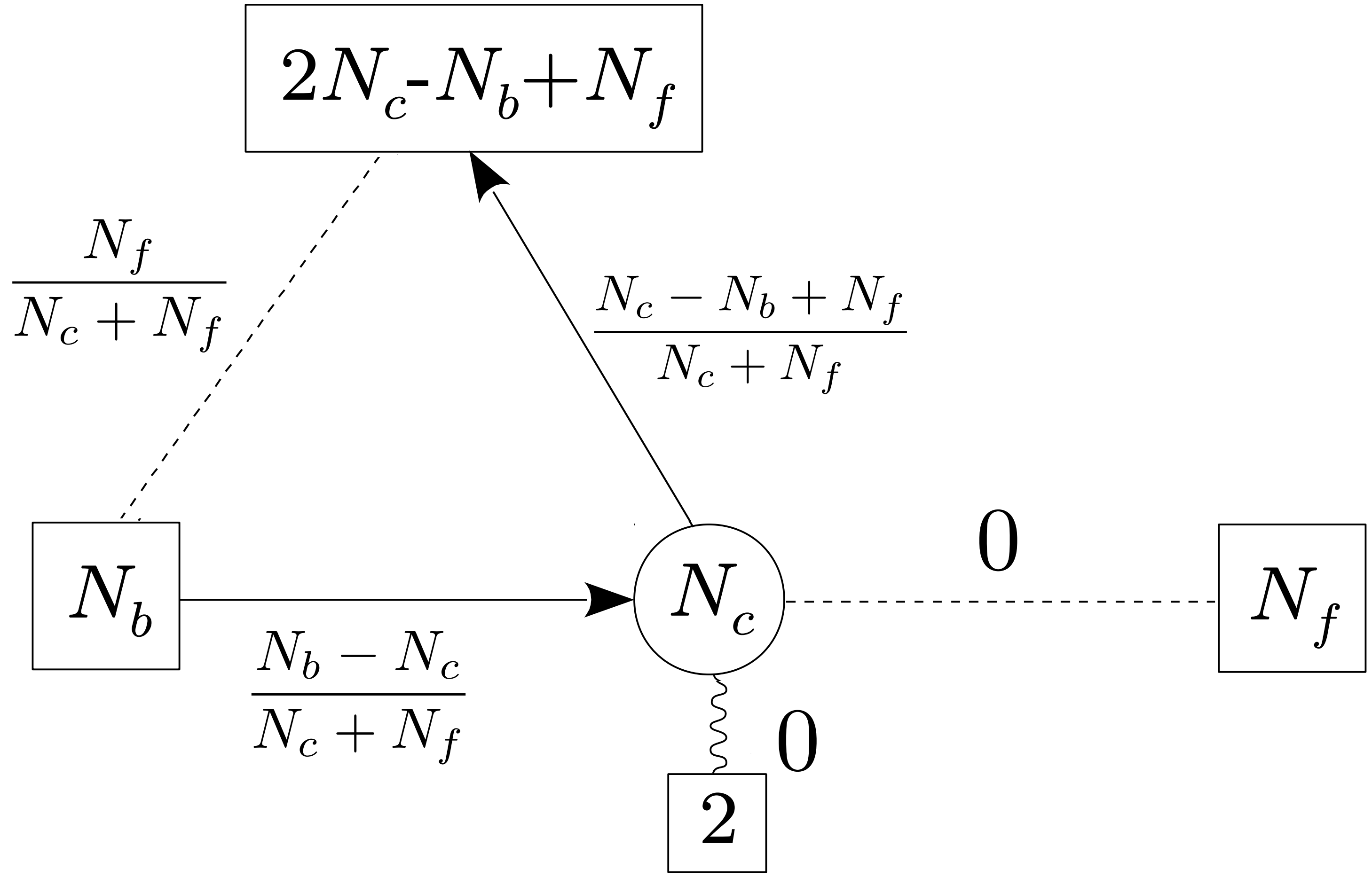
Anomalies





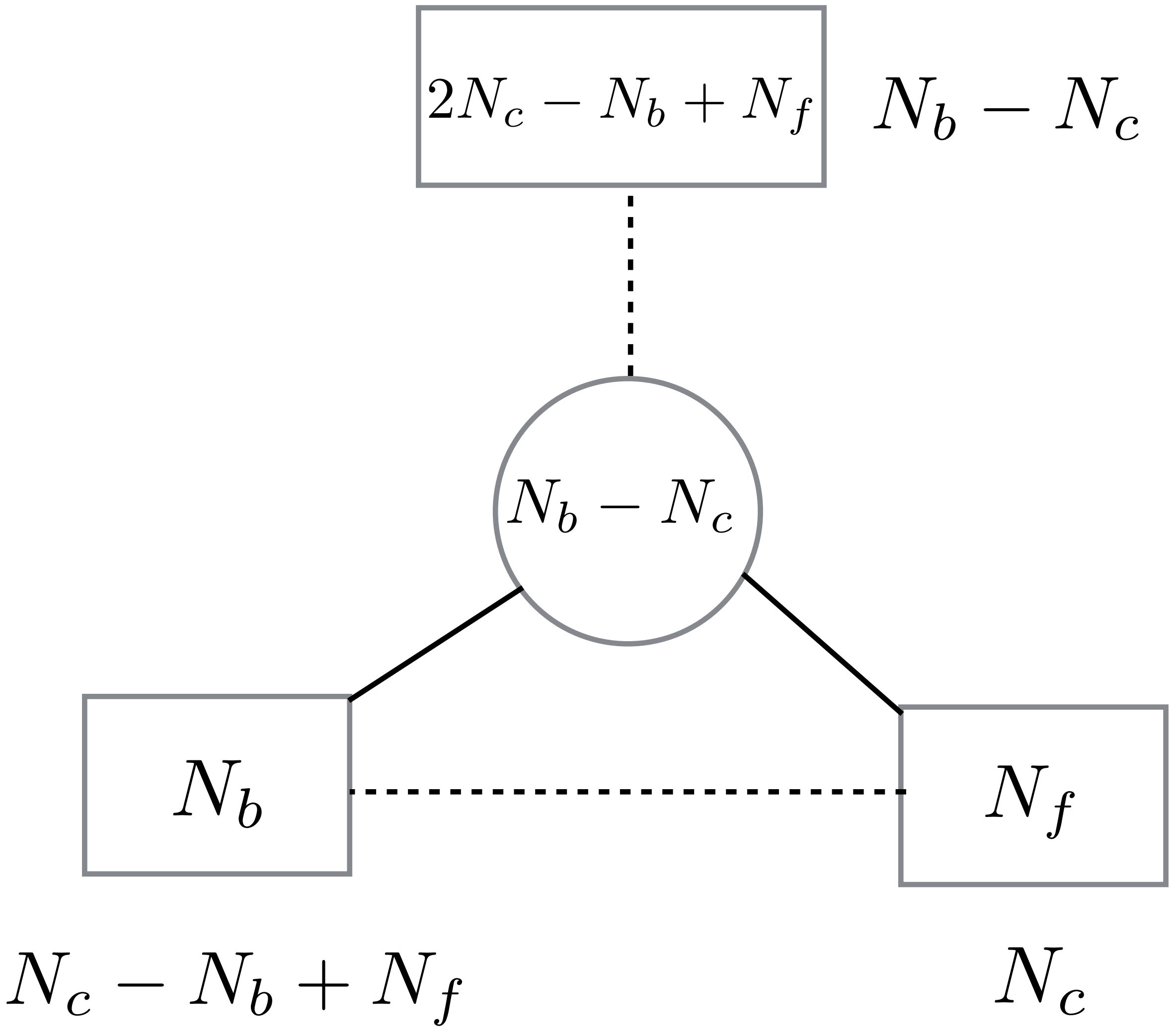
Central charge

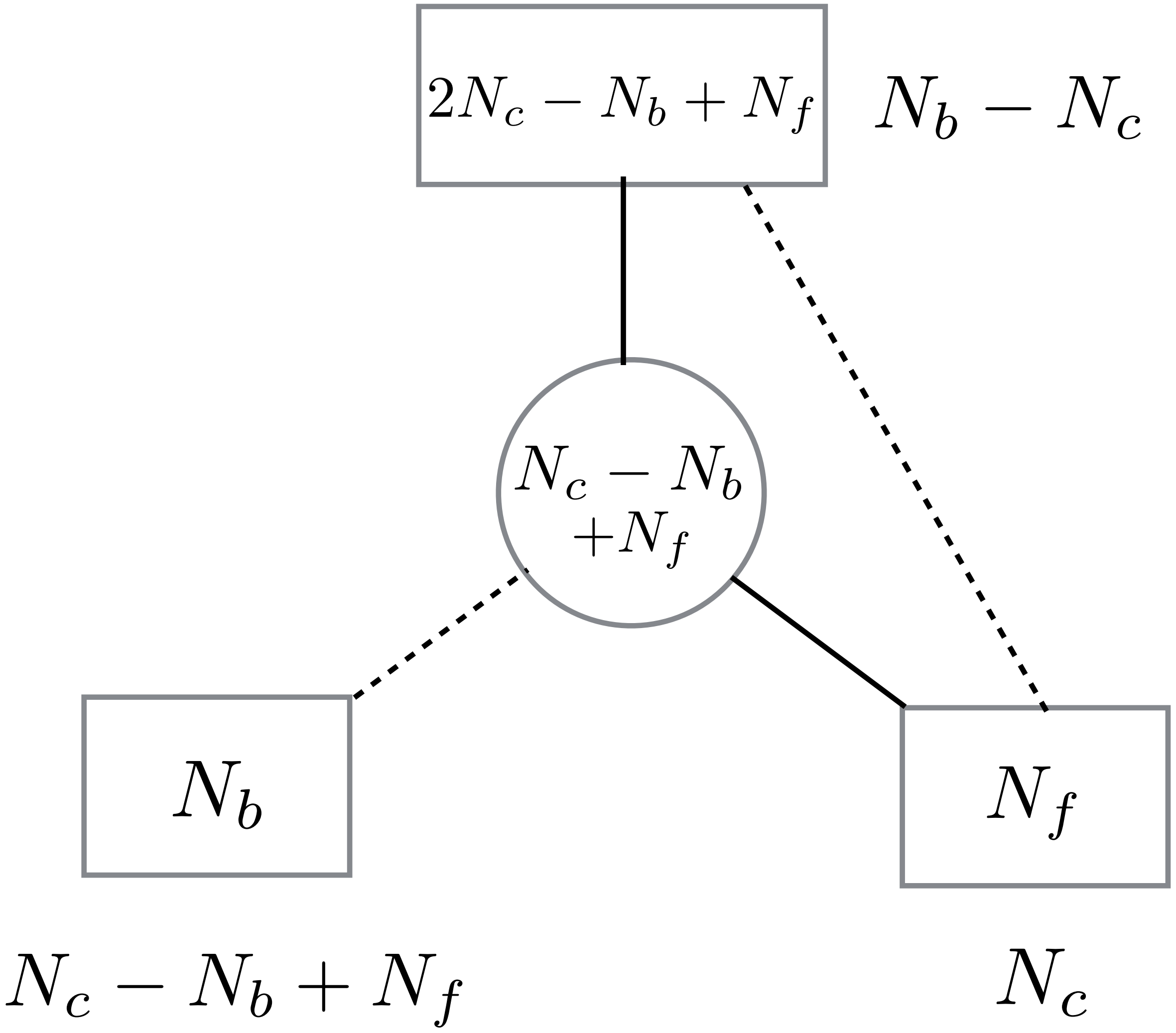
$$c_R = 3\,\mathrm{Tr}R^2$$



$$c_R = 3\,\frac{N_c(N_b-N_c)(N_c-N_b+N_f)}{N_c+N_f}$$

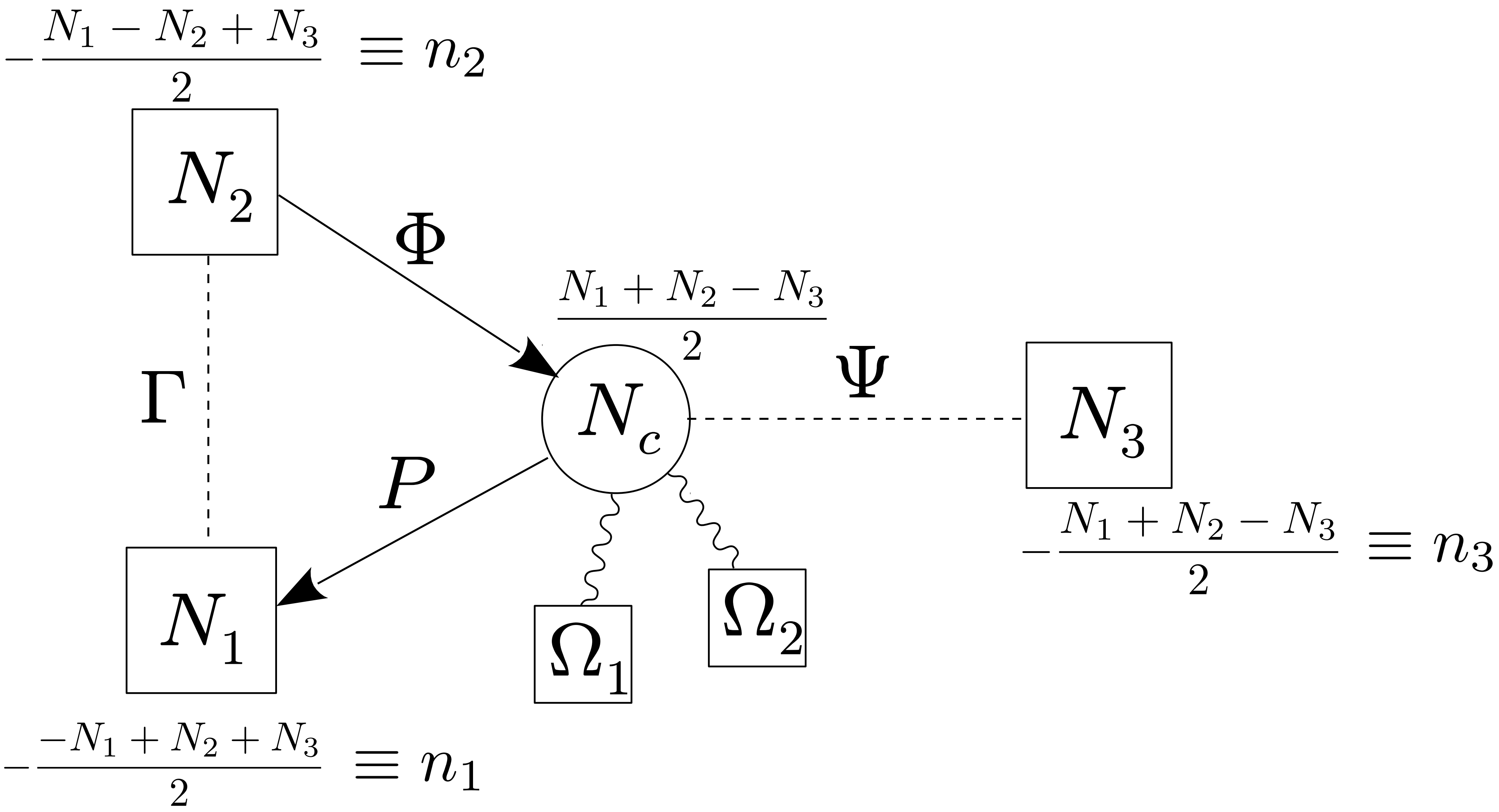
Dual frames


$2N_c - N_b + N_f$
$N_b - N_c$
$N_b - N_c$
N_b
N_f
$N_c - N_b + N_f$
N_c


$2N_c - N_b + N_f$
$N_b - N_c$
$N_c - N_b$
$+N_f$
N_b
N_f
$N_c - N_b + N_f$
N_c

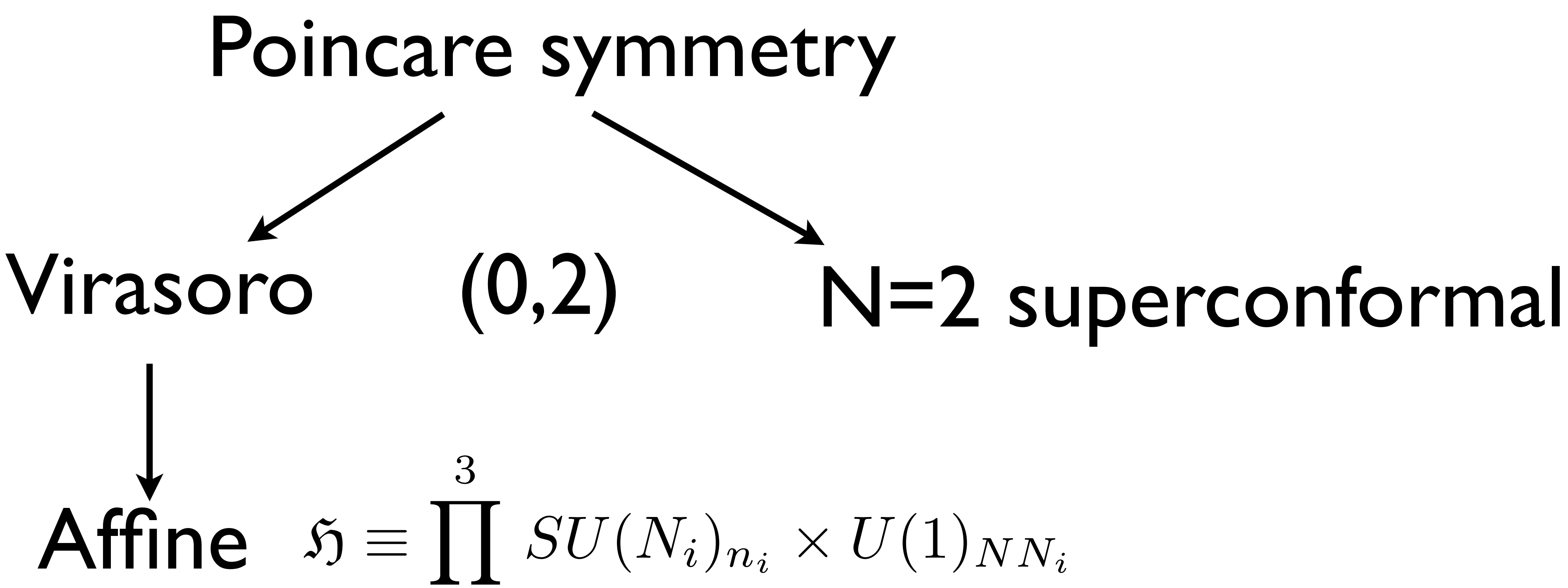
(0,2) SQCD

Symmetric labeling



- Triality is invariance of the fixed point under permutations of N's
- In addition, there are 3 abelian symmetries

Low energy physics



Poincare symmetry

Virasoro (0,2) N=2 superconformal

Affine $\mathfrak{H} \equiv \prod_{i=1}^{3} SU(N_i)_{n_i} \times U(1)_{N N_i}$

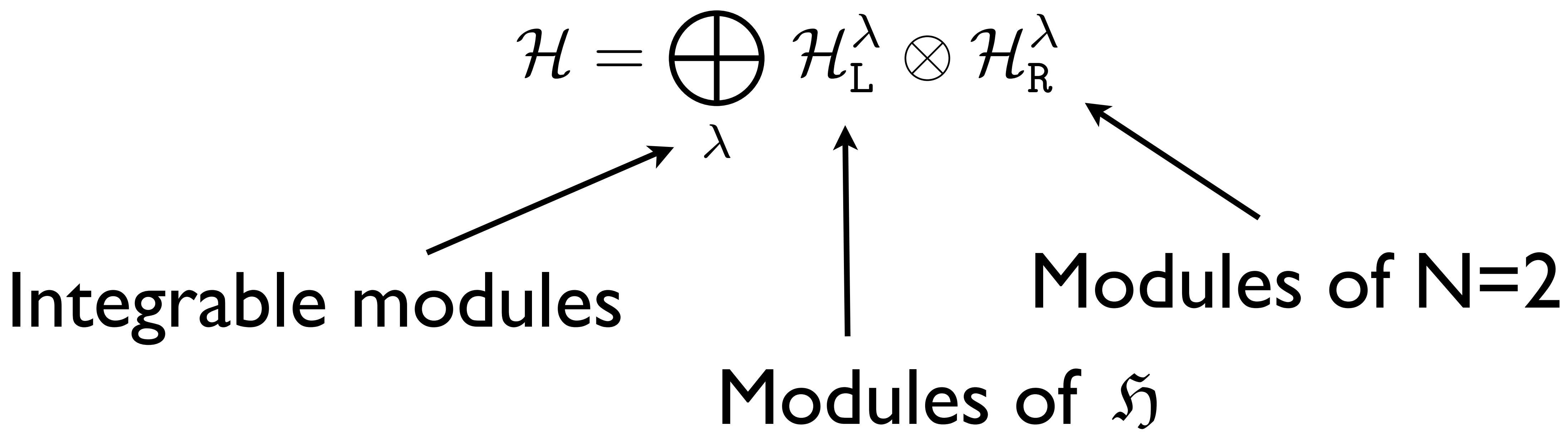
- The central charges can be determined from c-extremization and gravitational anomaly

$$c_{\mathtt{R}} = 3\mathrm{Tr}\gamma^3 RR\,, \qquad c_{\mathtt{R}} - c_{\mathtt{L}} = \mathrm{Tr}\gamma^3$$

$$c_{\mathtt{R}} = \frac{3}{4}\frac{(-N_1+N_2+N_3)(N_1-N_2+N_3)(N_1+N_2-N_3)}{N_1+N_2+N_3}$$

$$c_{\mathtt{L}} = c_{\mathtt{R}} - \frac{1}{4}(N_1^2+N_2^2+N_3^2-2N_1N_2-2N_2N_3-2N_3N_1)+2$$

Low energy solution



$$\mathcal{H} = \bigoplus_{\lambda} \mathcal{H}_{\mathtt{L}}^{\lambda} \otimes \mathcal{H}_{\mathtt{R}}^{\lambda}$$

Integrable modules

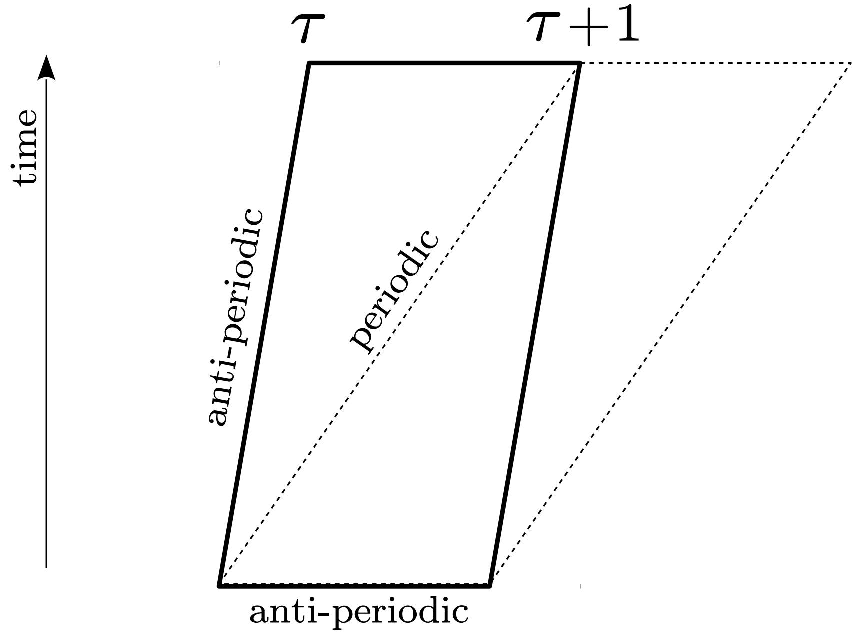
Modules of N=2

Modules of $\mathfrak{H}$

- Sugawara central charge = $c_{\mathtt{L}}$
- Immense simplification: rational CFT
- Modular invariance of the partition function helps fix $\mathcal{H}_{\mathtt{R}}^{\lambda}$

NS-NS partition function

$$Z(\tau,\bar{\tau}) := \mathrm{Tr}_{\mathcal{H}} e^{2\pi i(\tau L_0 - \bar{\tau}\bar{L}_0)}$$



Affine character

N=2 character

- Invariant under S and T^2

$$Z(\tau,\bar{\tau}) = \sum_{\lambda} \chi_{\lambda}(\tau) K_{\lambda}(\bar{\tau})$$

Partition function (contd)

Use S invariance: $Z(\tau, \bar{\tau}) = Z(-\frac{1}{\tau}, -\frac{1}{\bar{\tau}})$

$$\chi_\lambda \to S_{\lambda\,\mu}\,\chi_\mu \qquad S\bar{S} = I$$

$$\Rightarrow\; K_\lambda \to \bar{S}_{\lambda\,\mu}\,K_\mu$$

- K is NOT the anti-holomorphic affine character of $\mathfrak{H}$
- Note that characters of level-rank dual $\mathfrak{H}^t$ transform with $\bar{S}$

$$\mathfrak{H}^t = \prod SU(n_i)_{N_i} \times U(1)_{Nn_i}$$

To summarize

- K is an N=2 character with central charge cR
- It transforms as a character of holomorphic $\mathfrak{H}^t$ under modular S-transformation
- Singlet under all affine symmetries

⇓

- K is a character of the Kazama- Suzuki coset $[\mathfrak{G}]/[\mathfrak{H}^t]$

(For appropriate $\mathfrak{G}$)

Intermission: SUSY WZW

- $[\mathfrak{g}]_k$ is SUSY extension of WZW model $\mathfrak{g}$ at level k
- It is obtained by adding free adjoint fermions to $\mathfrak{g}$

$$J^a = J^a_{\mathrm{bos}} - \frac{i}{k} f^a_{bc} \psi^a \psi^c$$

$$k = k_{\mathrm{bos}} + h^\vee$$

$$c_{[\mathfrak{g}]} = c_{\mathfrak{g}} + \frac{1}{2} \dim \mathfrak{g}$$

- Matching the right-moving central charge with that of the coset:

$$c_{[\mathfrak{G}]} = N^2$$

- Combined with the condition $\mathfrak{G} \supset \mathfrak{H}^t$

$$[\mathfrak{G}] = [U(N)]_N = [U(1)]_{N^2} \times [SU(N)]_N$$

Bosonic level 0

Only bosonic part $U(1)_{N^2}$

- Coset character C is a branching function

$U(1)_{N^2}$ module

$SO(\dim \mathfrak{G}/\mathfrak{H}^t)$ module

$$C^{\Lambda,\upsilon}_{\lambda^t}$$

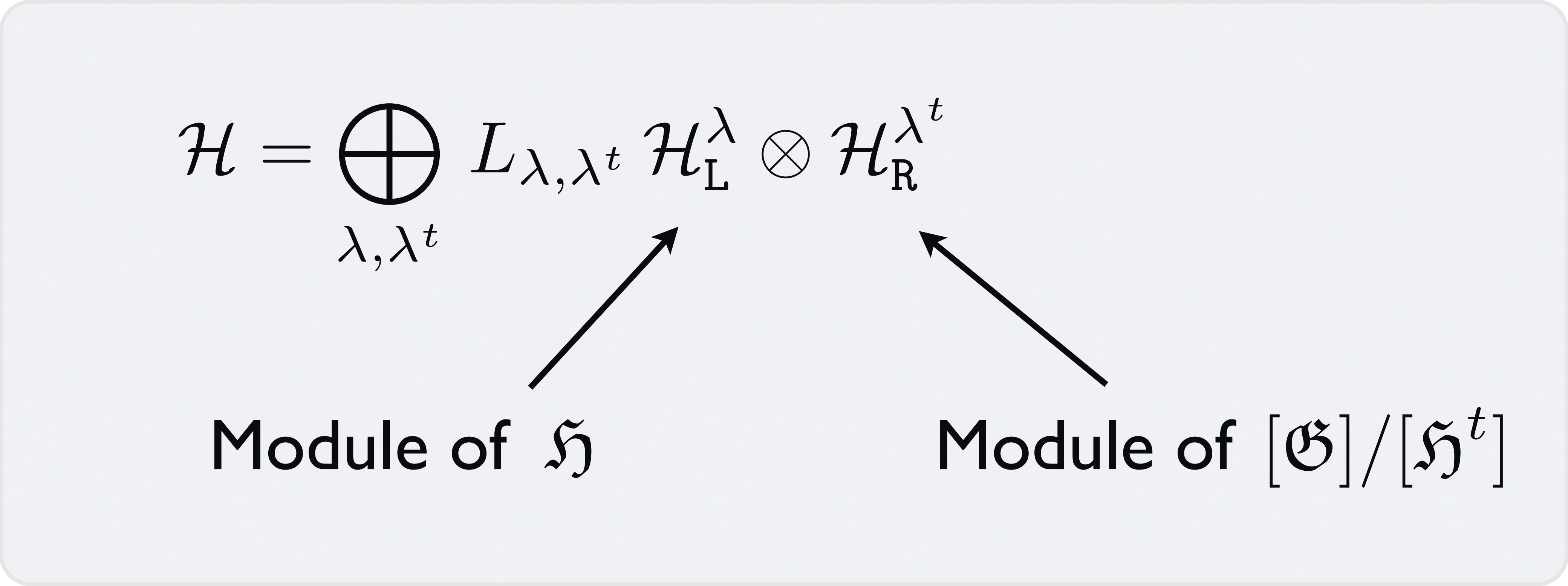
$\mathfrak{H}^t$ module

Solution

- We pick modular invariant combinations (Λ_0, v_0)

Then $$K_\lambda = \sum_{\lambda^t} L_{\lambda,\lambda^t}\, C^{\Lambda_0,v_0}_{\lambda^t}$$

has all the desired properties!



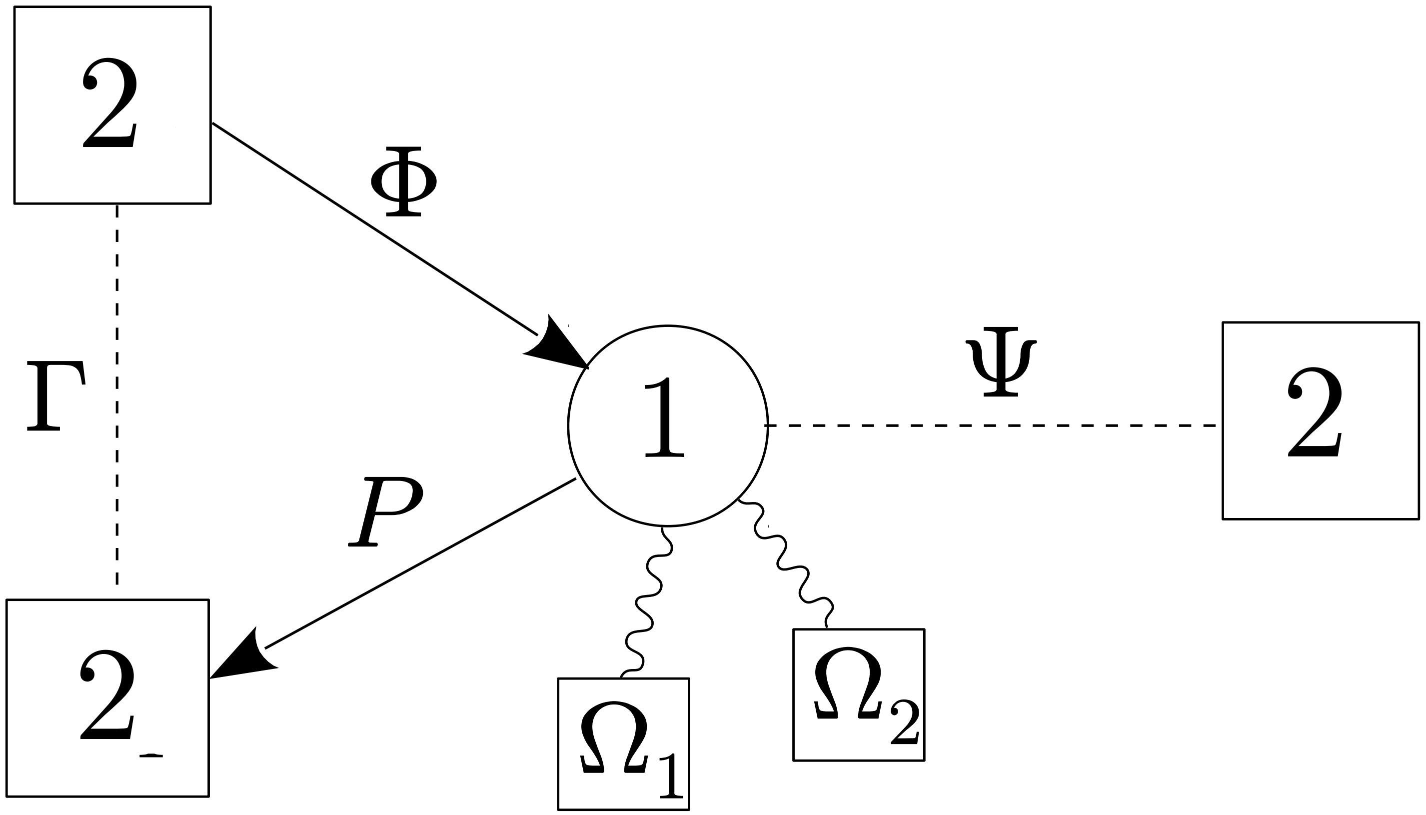
$$\mathcal{H} = \bigoplus_{\lambda,\lambda^t} L_{\lambda,\lambda^t}\, \mathcal{H}^{\lambda}_{\mathtt{L}} \otimes \mathcal{H}^{\lambda^t}_{\mathtt{R}}$$

Module of $\mathfrak{H}$

Module of $[\mathfrak{G}]/[\mathfrak{H}^t]$

- Matches with the UV computation of the index

Example



$$\mathfrak{H} = \Big(SU(2)_1 \times U(1)_6\Big)^3 \qquad [\mathfrak{G}]/[\mathfrak{H}^t] = [U(3)]_3/[U(1)_3]^3$$

↑

c=1 minimal model

$$
\begin{aligned}
Z_{\mathcal{T}_{222}} &= \chi^{\mathcal{N}=2}_{(0,0)}(\overline{\tau},\overline{\eta})\Big(\Xi_{0,0,0}(\tau)+\Xi_{1,1,1}(\tau)+\Xi_{-1,-1,-1}(\tau)\Big)\\
&+\chi^{\mathcal{N}=2}_{(\frac{1}{6},\frac{1}{3})}(\overline{\tau},\overline{\eta})\Big(\Xi_{1,0,-1}(\tau)+\Xi_{-1,1,0}(\tau)+\Xi_{0,-1,1}(\tau)\Big)\\
&+\chi^{\mathcal{N}=2}_{(\frac{1}{6},-\frac{1}{3})}(\overline{\tau},\overline{\eta})\Big(\Xi_{-1,0,1}(\tau)+\Xi_{1,-1,0}(\tau)+\Xi_{0,1,-1}(\tau)\Big)
\end{aligned}
$$

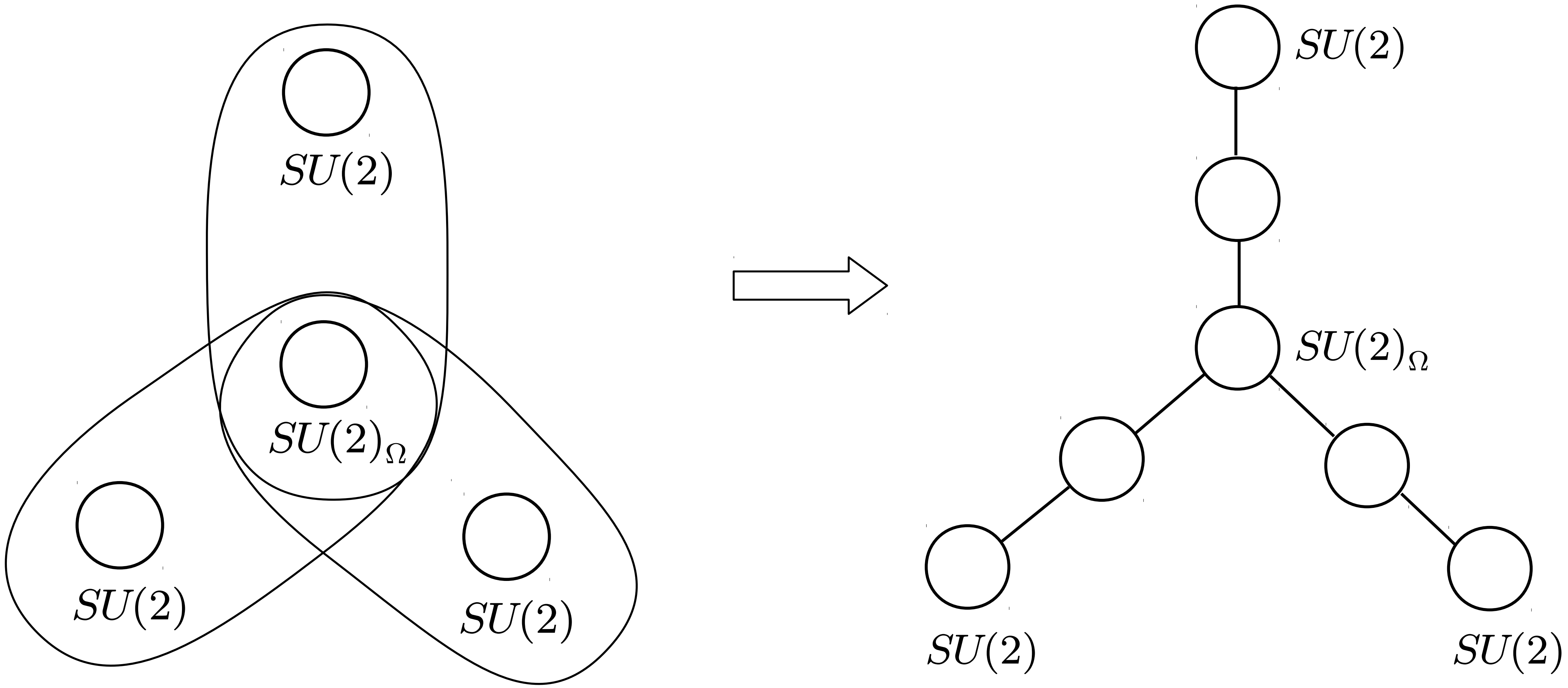
where

$$
\begin{aligned}
\Xi_{a,b,c}(\tau,\xi_1,\xi_2,\xi_3) &:= \Xi_a(\tau,\xi_1)\Xi_b(\tau,\xi_2)\Xi_c(\tau,\xi_3)\\
\Xi_{-1}(\tau,\xi) &:= \chi^{\mathrm{SU}(2)_1\times\mathrm{U}(1)_6}_{(\Box,-1)}(\tau,\xi)+\chi^{\mathrm{SU}(2)_1\times\mathrm{U}(1)_6}_{(\cdot,2)}(\tau,\xi)\\
\Xi_{0}(\tau,\xi) &:= \chi^{\mathrm{SU}(2)_1\times\mathrm{U}(1)_6}_{(\cdot,0)}(\tau,\xi)+\chi^{\mathrm{SU}(2)_1\times\mathrm{U}(1)_6}_{(\Box,3)}(\tau,\xi)\\
\Xi_{1}(\tau,\xi) &:= \chi^{\mathrm{SU}(2)_1\times\mathrm{U}(1)_6}_{(\Box,1)}(\tau,\xi)+\chi^{\mathrm{SU}(2)_1\times\mathrm{U}(1)_6}_{(\cdot,-2)}(\tau,\xi).
\end{aligned}
$$

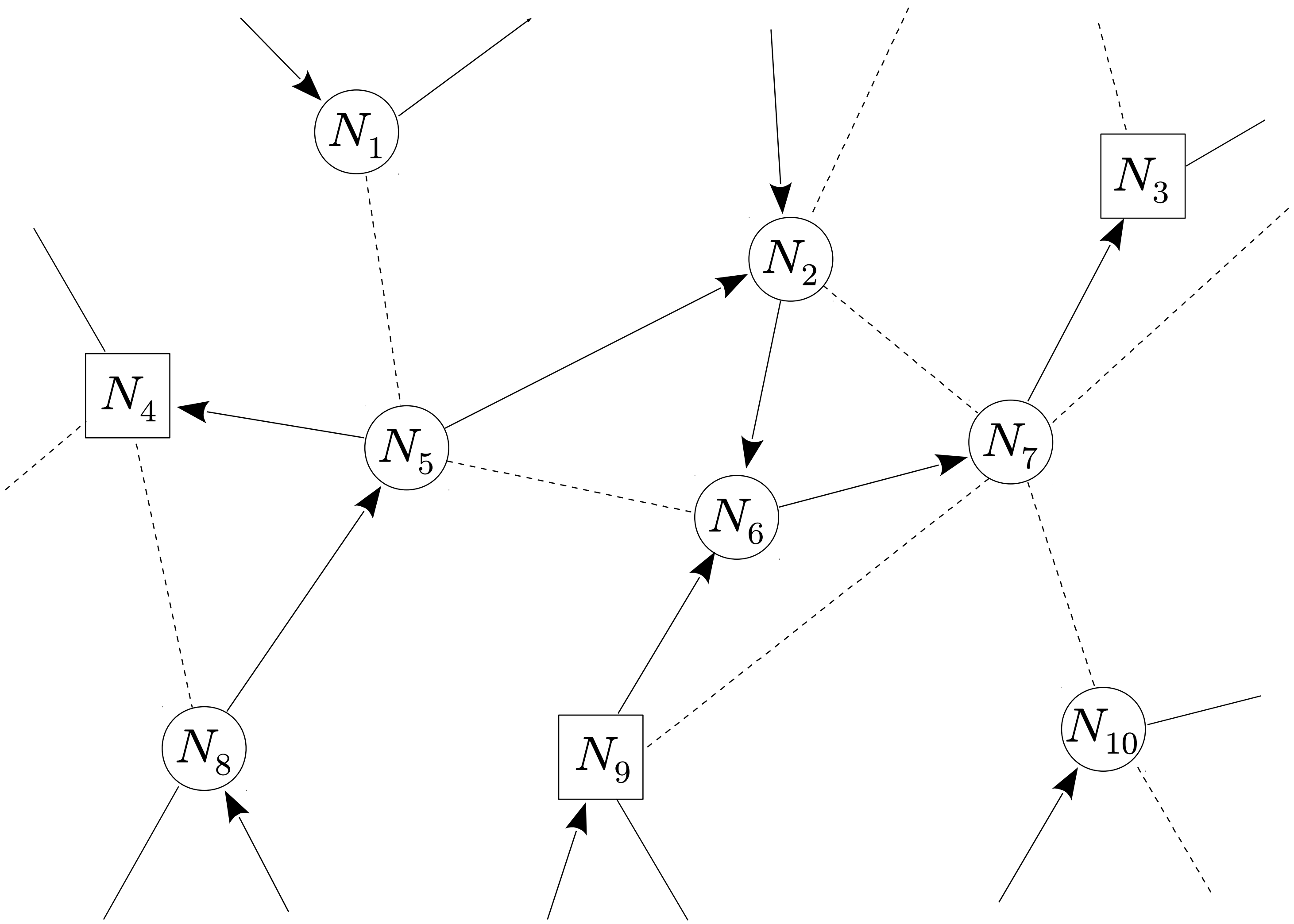
Remarkably

$$
Z_{\mathcal{T}_{222}}=\chi^{\mathcal{N}=2}_{(0,0)}(\overline{\tau},\overline{\eta})\chi^{(\mathrm{E}_6)_1}_{\bullet}(\tau,\xi_i)+\chi^{\mathcal{N}=2}_{(\frac{1}{6},\frac{1}{3})}(\overline{\tau},\overline{\eta})\chi^{(\mathrm{E}_6)_1}_{\Box}(\tau,\xi_i)+\chi^{\mathcal{N}=2}_{(\frac{1}{6},-\frac{1}{3})}(\overline{\tau},\overline{\eta})\chi^{(\mathrm{E}_6)_1}_{\overline{\Box}}(\tau,\xi_i)
$$

Triality and enhancement



Solution to a general quiver



Thank you!