Gravity & Hydrodynamics

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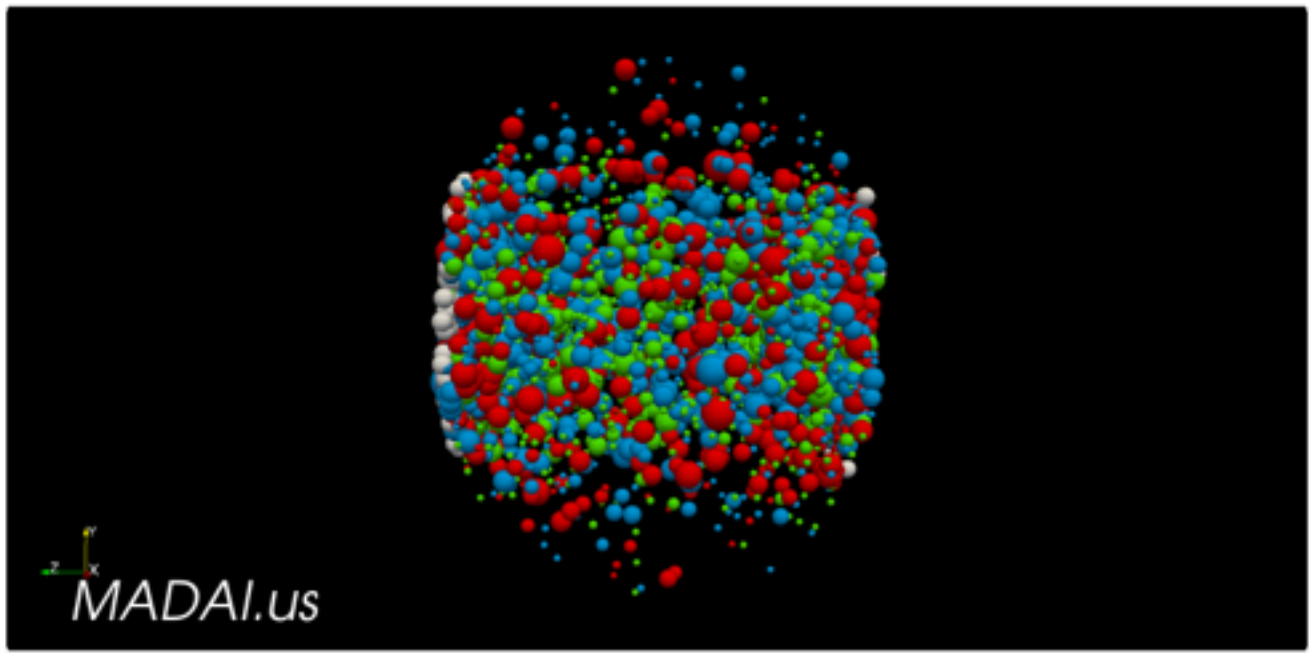
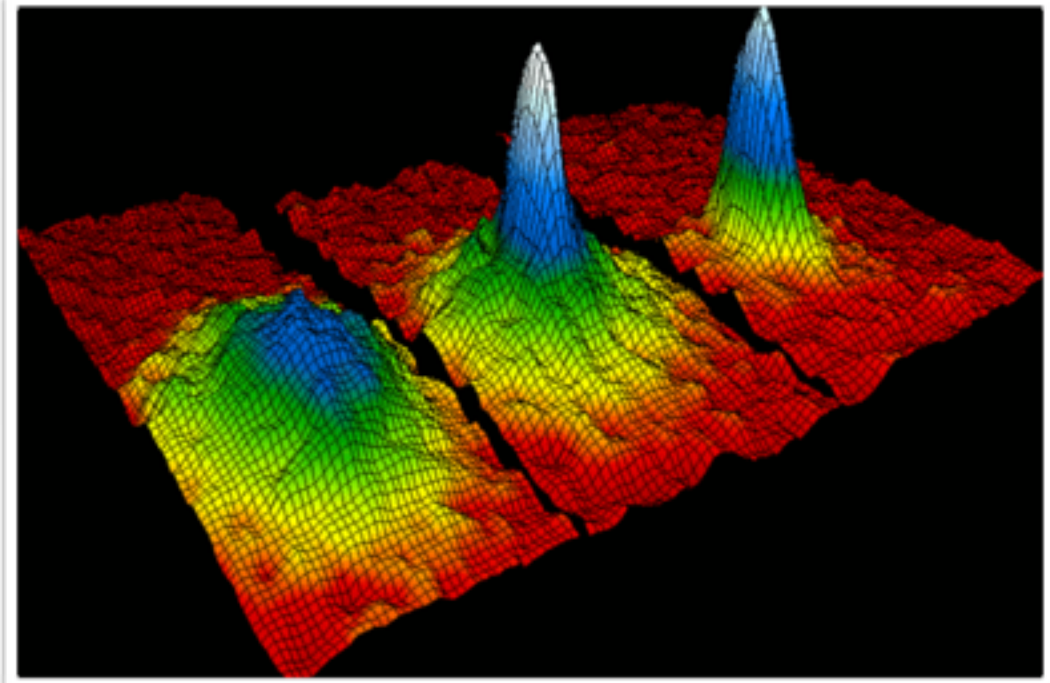
Hot quark soup, Cold atoms, and Black holes





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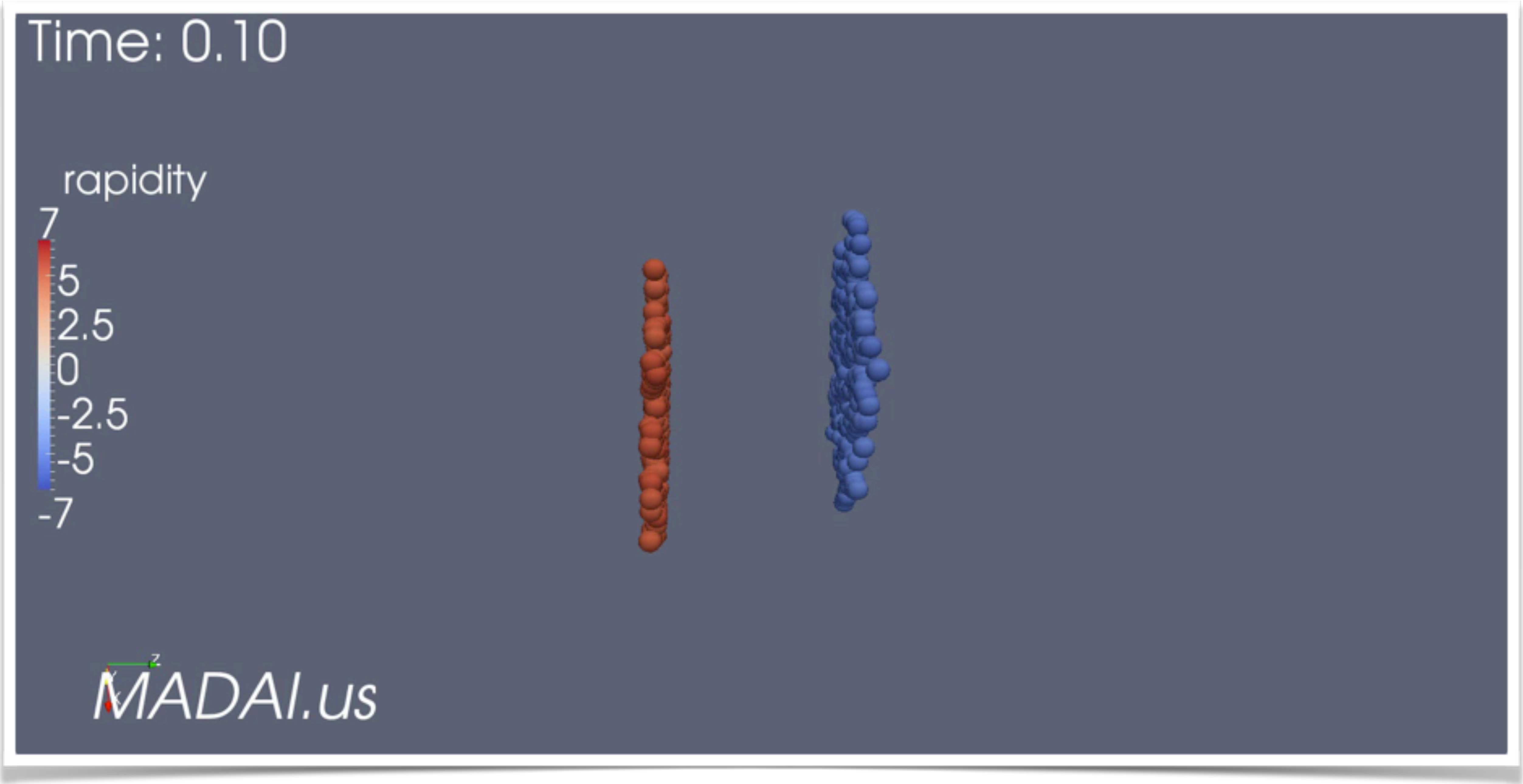
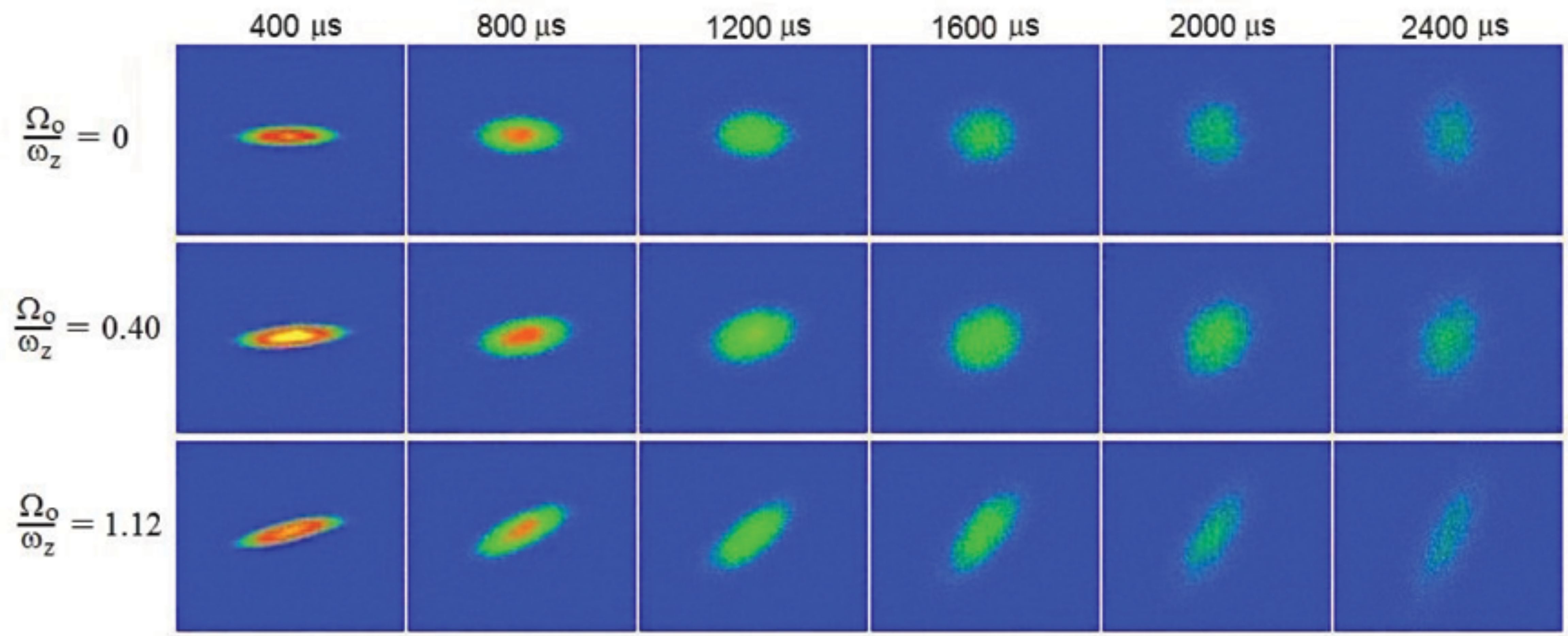
The evolution of the Quark Gluon Plasma

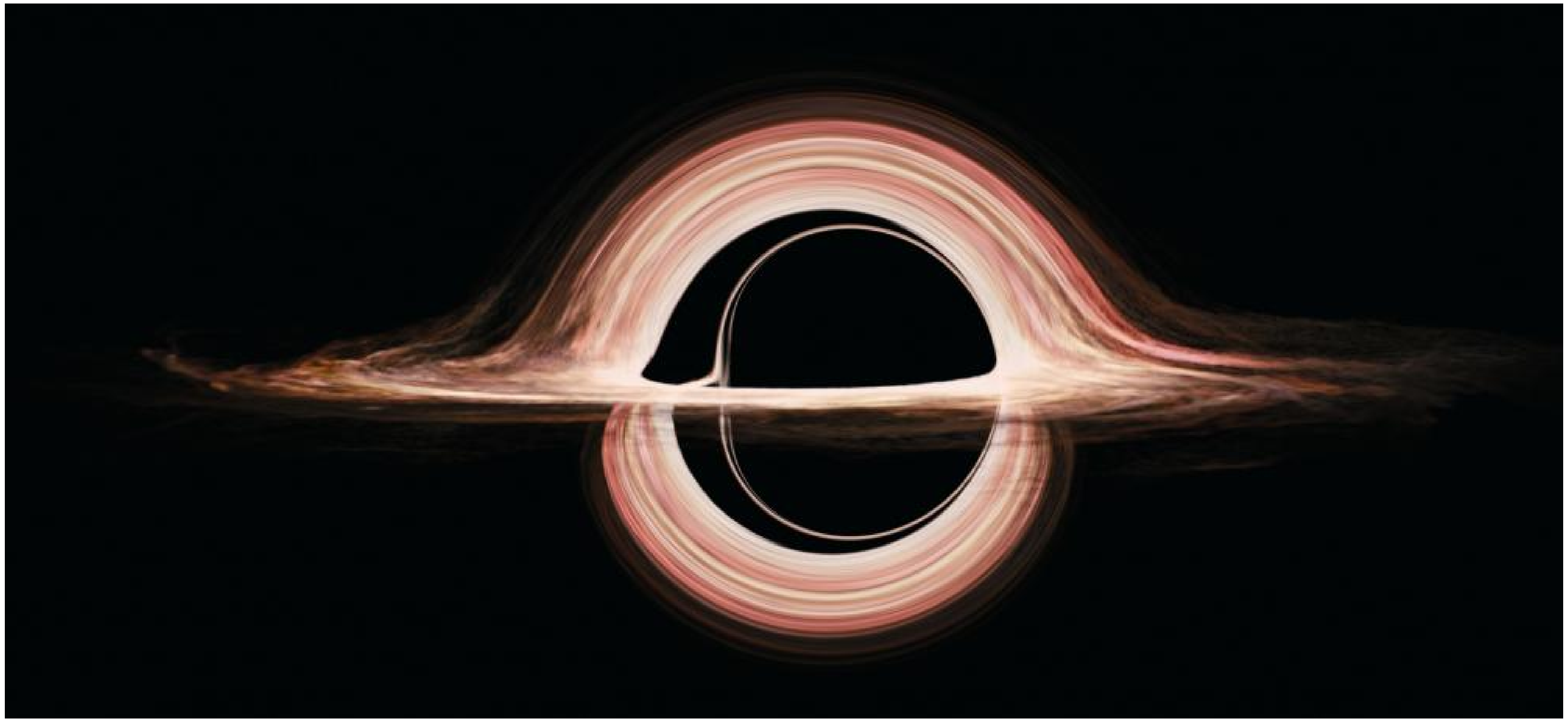


Evolution of Cold Atoms



Clancy, Luo, Thomas '07

The response of black holes



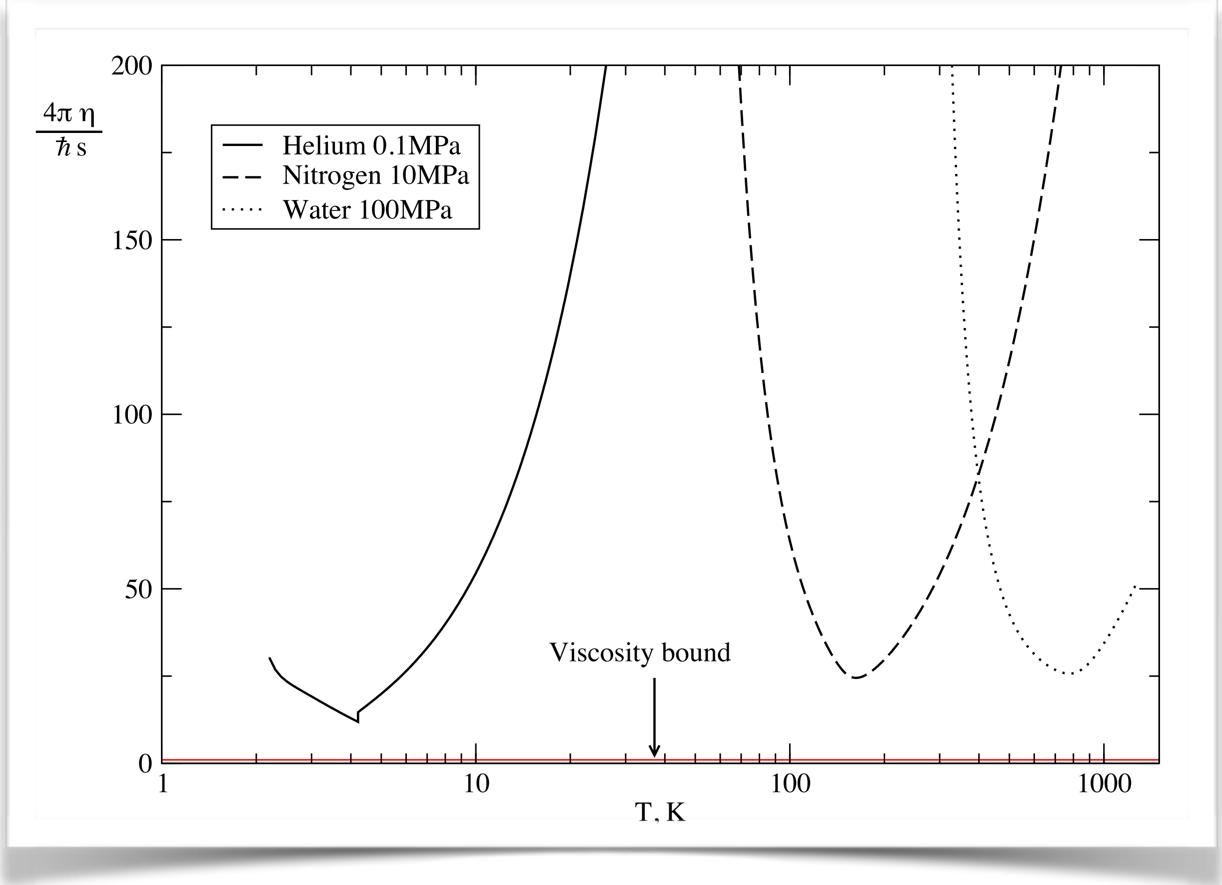
- Black holes respond to external perturbations which as we shall see extensively in the sequel is in certain cases well described by hydrodynamics.

James, von Tunzelmann, Franklin, Thorne '15

The parable of nearly ideal fluids

- The evolution of the three widely disparate systems: the hot quark gluon plasma, the cold atom gas, and perturbed black holes, is extremely well described by hydrodynamics.

- In fact, these three systems appear to be extreme examples of fluids with relatively low friction: the (shear) viscosity is minimized in these systems.



Kovtun, Son, Starinets ‘04

Gravity versus hydrodynamics

HYDRODYNAMICS

* Classical low energy theory describing dynamics on macroscopic scales for systems in local but not global thermal equilibrium.
* Intrinsically statistical and coarse-grained.
* Valid in a suitable low-energy limit, when variations are large on scale of mean free path.

GRAVITY

* Classical low energy theory describing dynamics on macroscopic scales.
* A-priori not statistical, although black hole thermodynamics hints at some underlying structure of this kind.
* Valid when curvature scales are large compared to the Planck scale.

Black hole thermodynamics

Thermodynamic Laws

- T is constant in thermal equilibrium.
- Energy conservation

$$dE = T\,dS + \text{work terms}$$

- Entropy is non-decreasing in any physical process (*Boltzmann*)

$$\delta S \geq 0$$

Laws of Black hole mechanics

- The surface gravity κ is constant over the horizon of a stationary black hole.
- Total spacetime energy conservation

$$dM = \frac{1}{8\pi}\,\kappa\,dA_H + \text{work terms}$$

- Area of the event horizon is non-decreasing in any physical process (*Hawking*)

$$\delta A_H \geq 0$$

Precursor: The membrane paradigm

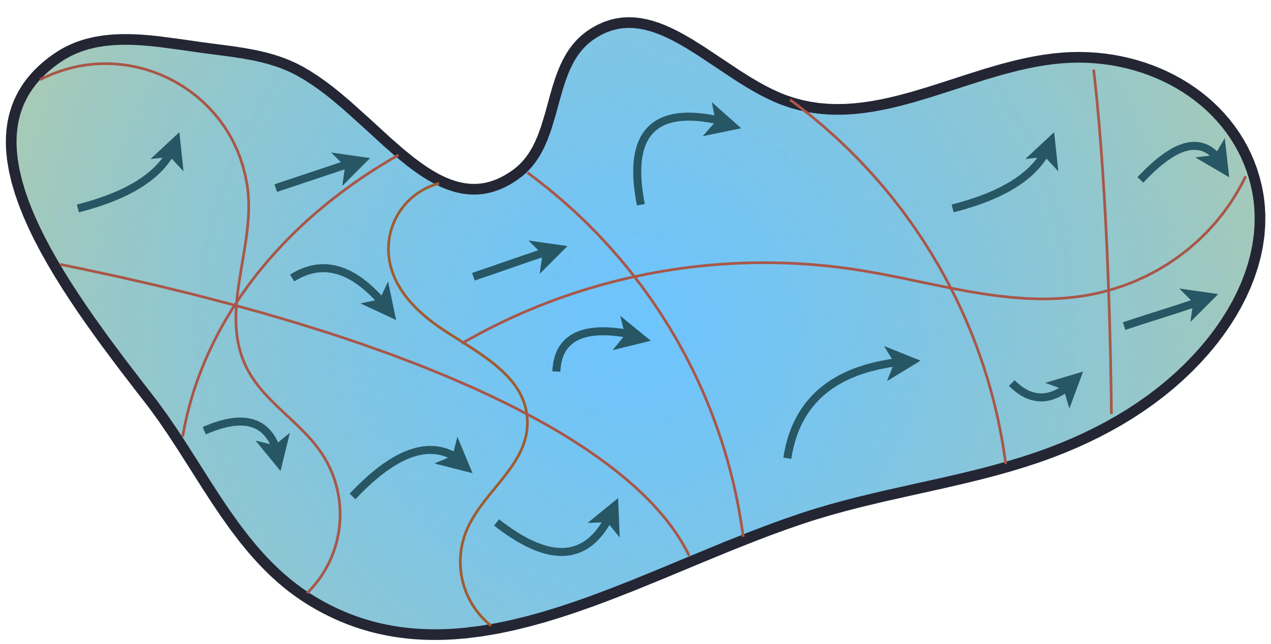
- ✦ Black hole entropy being related to the surface area suggests an effective model of the geometry in terms of the dynamics on the horizon surface.
- ✦ *Membrane paradigm*: the internal dynamics of a black hole can be modeled effectively as a membrane with electromechanical properties. The dynamics of Einstein's equations allows determination of the response of the black hole to external disturbances.
- ✦ The membrane paradigm for black holes was invented to demystify the characteristics of the black hole and to describe the associated physics as one would for "ordinary bodies".
- ✦ However, puzzles abound…
- ✳ non-relativistic, compressible fluid with negative bulk viscosity.
- ✳ no systematic low energy limit

Damour '78; Price, Thorne '86

Hydrodynamics as an effective field theory

- Relativistic fluid dynamics is best thought of as an effective field theory for quantum systems in local, but not global thermal equilibrium.

- The description in terms of fluid dynamics is valid when departures from equilibrium are on scales that are large compared to the characteristic mean free path of the underlying quantum dynamics.



$$\ell_{\mathrm{mfp}} \ll L\,, \qquad t_{\mathrm{mfp}} \ll t$$

- Local domains of equilibrated fluid can be characterized by the local temperature/energy density and conserved charges.

- Energy/charge flux exchanged across the domains: velocity field.

Axioms of Hydrodynamics I: Fields

- Hydrodynamics describes low-energy, near-equilibrium fluctuations of an equilibrium Gibbsian density matrix on scales large compared to the characteristic mean free path.

- The macroscopic description involves currents which capture energy-momentum and charge transport $T^{\mu\nu},\ J^\mu$ (and entropy current J^μ_S).

- The currents are functionals of the hydrodynamic fields, which are the intensive variables characterizing the density matrix and background sources.

  - temperature and chemical potential and a flux vector (fluid velocity) $T, \mu, u^\mu, \qquad u^\mu\, u_\mu = -1$

  - background metric and electromagnetic potential $g_{\mu\nu}, A_\mu$

Axioms of Hydrodynamics II: Data

✳ Repackage the dynamical degrees of freedom in a vector an scalar

thermal vector $\boldsymbol{\beta}^\mu = \frac{u^\mu}{T}\,, \qquad \Lambda_{\boldsymbol{\beta}} \sim \frac{\mu}{T}$ thermal twist

✳ The currents of hydrodynamics are expressed as functionals of the hydrodynamical fields and the background sources.

- currents $T^{\mu\nu}, J^\mu, J^\mu_S$
- fields $\boldsymbol{\Psi} \equiv \{g_{\mu\nu}, A_\mu, \boldsymbol{\beta}^\mu, \Lambda_{\boldsymbol{\beta}}\}$
- constitutive relations

$$
\begin{aligned}
T^{\mu\nu} &= T^{\mu\nu}\left[\boldsymbol{\Psi}\right] = T^{\mu\nu}\left[g_{\alpha\beta}, A_\alpha, \boldsymbol{\beta}^\alpha, \Lambda_{\boldsymbol{\beta}}\right] \\
J^\mu &= J^\mu\left[\boldsymbol{\Psi}\right] = J^\mu\left[g_{\alpha\beta}, A_\alpha, \boldsymbol{\beta}^\alpha, \Lambda_{\boldsymbol{\beta}}\right] \\
J^\mu_S &= J^\mu_S\left[\boldsymbol{\Psi}\right] = J^\mu_S\left[g_{\alpha\beta}, A_\alpha, \boldsymbol{\beta}^\alpha, \Lambda_{\boldsymbol{\beta}}\right] .
\end{aligned}
$$

Axioms of Hydrodynamics III: Dynamics

- The dynamical content of hydrodynamics is the statement of conservation, modulo work done by sources and anomalies:

$$\nabla_\nu T^{\mu\nu} = J_\nu \cdot F^{\mu\nu} + \mathrm{T}^{\mu\perp}_H \qquad D_\nu J^\nu = \mathrm{J}^\perp_H$$

work term

covariant anomalies

- These are effectively *Ward identities* for the one-point functions of the conserved currents in the fluctuating Gibbs density matrix.

- The task of a hydrodynamicist is to specify the currents as a functional of the hydrodynamic fields, consistent with the dynamics, constructing a current algebra of sorts, but…

Axioms of Hydrodynamics IV: Constraints

- From a macroscopic, statistical viewpoint, one has to demand that a local form of the second law of thermodynamics is upheld.

$$\exists \;\; J_S^\mu[\mathbf{\Psi}]: \;\; \forall \;\; \mathbf{\Psi}_{\text{on-shell}} \;\; \nabla_\mu J_S^\mu[\mathbf{\Psi}] \geq 0$$

- This is required to be upheld on-shell, and complicates the analysis of hydrodynamics, for without it the current algebra can be analyzed purely in terms of representation theory.

- Note that usually one only requires the existence of some entropy current.

- From a microscopic viewpoint the entropy current is rather mysterious; it is not associated with any underlying symmetry per se.

- Dealing with density matrices, currents rather than effective actions, etc., pose many questions for a first principles formulation.

Neutral fluids

- A neutral fluid is characterized by its energy-momentum stress tensor

$$T^{\mu\nu} = \epsilon(T)\, u^\mu\, u^\nu + p(T)\, P^{\mu\nu} - \eta(T)\, \sigma^{\mu\nu} - \zeta(T)\, \Theta\, P^{\mu\nu} + \cdots$$

$$P^{\mu\nu} = g^{\mu\nu} + u^\mu\, u^\nu$$

spatial metric

$$\nabla_\mu u_\nu = \sigma_{(\mu\nu)} + \omega_{[\mu\nu]} + \Theta\, P_{\mu\nu} - u_\mu\, \mathfrak{a}_\nu$$

shear　　vorticity　　expansion　　acceleration

- The second law forces some of the transport data to satisfy some inequalities, e.g., the viscosities are non-negative definite (friction)

$$J^\mu_S = s\, u^\mu + \cdots \qquad\qquad \eta, \zeta \geq 0$$

Neutral fluids entropy production

- Entropy produced during the fluid flow is determined by viscosities:

$$\nabla_\mu J^\mu_S = \eta\,\sigma^2 + \zeta\,\Theta^2 + \cdots \qquad\qquad \eta, \zeta \geq 0$$

- For the QGP and cold atom system data suggests that shear viscosity satisfies

$$\eta \approx \frac{\hbar}{4\pi\, k_B}\; s \;\approx 10^{14}\text{cp}$$

- For more common systems the shear viscosity is actually rather small

$$\eta_{\text{water}} \approx 1\text{cp}\,, \quad \eta_{\text{honey}} \approx 10^4\,\text{cp}\,, \quad \eta_{\text{peanut butter}} \approx 2.5\times 10^5\text{cp}$$

- But the right measure of idealness is relative rate of entropy production:

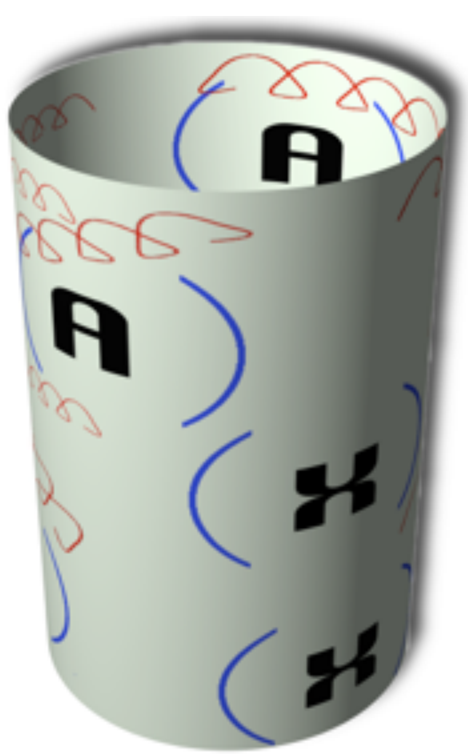
$$\nabla_\mu J^\mu_S \sim \frac{d\log s}{dt} = \frac{\eta}{s}\,\sigma^2 + \frac{\zeta}{s}\,\Theta^2$$

Gravity & Fluids: The holographic fluid

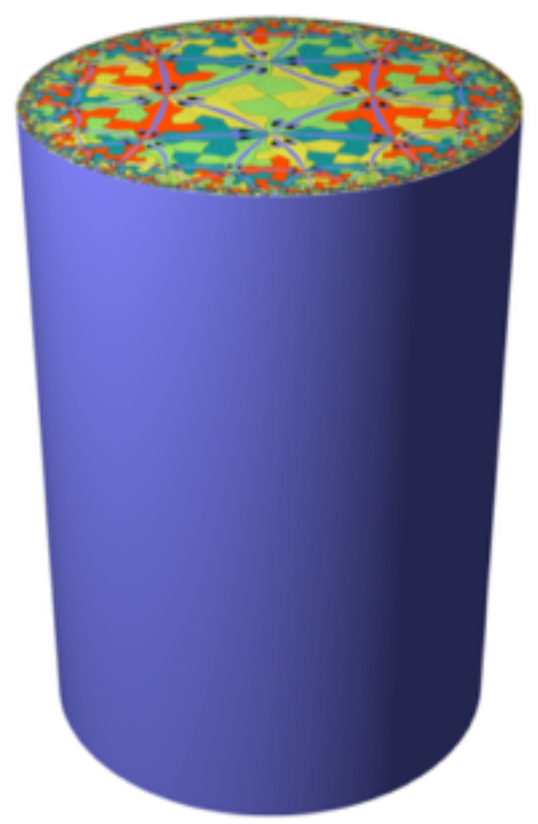
- The holographic AdS/CFT correspondence relates the dynamics of strongly coupled, planar field theories to classical gravitational dynamics.
- Since the low energy dynamics of any interacting quantum system in the near-equilibrium regime is given by hydrodynamics, one should be able to use gravity to derive the hydrodynamic equations governing such holographic field theories.
- Bonus: universality. The dynamics of energy-momentum flow is universal in a large class of holographic field theories, because of underlying gravitational description.

The gauge/gravity correspondence

String theory which includes quantum gravity is exactly equivalent (or dual) to a non-gravitational quantum theory (gauge theory).



- The quantum theory lives on the boundary of the spacetime where gravity reigns.
- All the gravitational action is captured completely on the boundary.
- Boundary dynamics holographically captures gravitational physics.



Maldacena '97

The fluid/gravity correspondence

- The fluid/gravity correspondence establishes a correspondence between Einstein's equations with a negative cc and those of relativistic conformal fluids.

Einstein's eqn with negative cosmological constant (cc)

$$E_{MN} = R_{MN} - \frac{1}{2}\,G_{MN} R - \frac{d(d-1)}{2}\,G_{MN} = 0$$

$$\updownarrow$$

Relativistic ideal fluid equations and beyond...

$$(\epsilon + p)\,\Theta + u^\mu \nabla_\mu\,\epsilon + \cdots = 0$$

$$P_\alpha{}^\mu\,\nabla_\mu p + (\epsilon + p)\,\mathfrak{a}_\alpha + \cdots = 0$$

- Given any solution to the hydrodynamic equations, one can construct, in a gradient expansion, an approximate *inhomogeneous, dynamical black hole* solution in an asymptotically AdS spacetime.

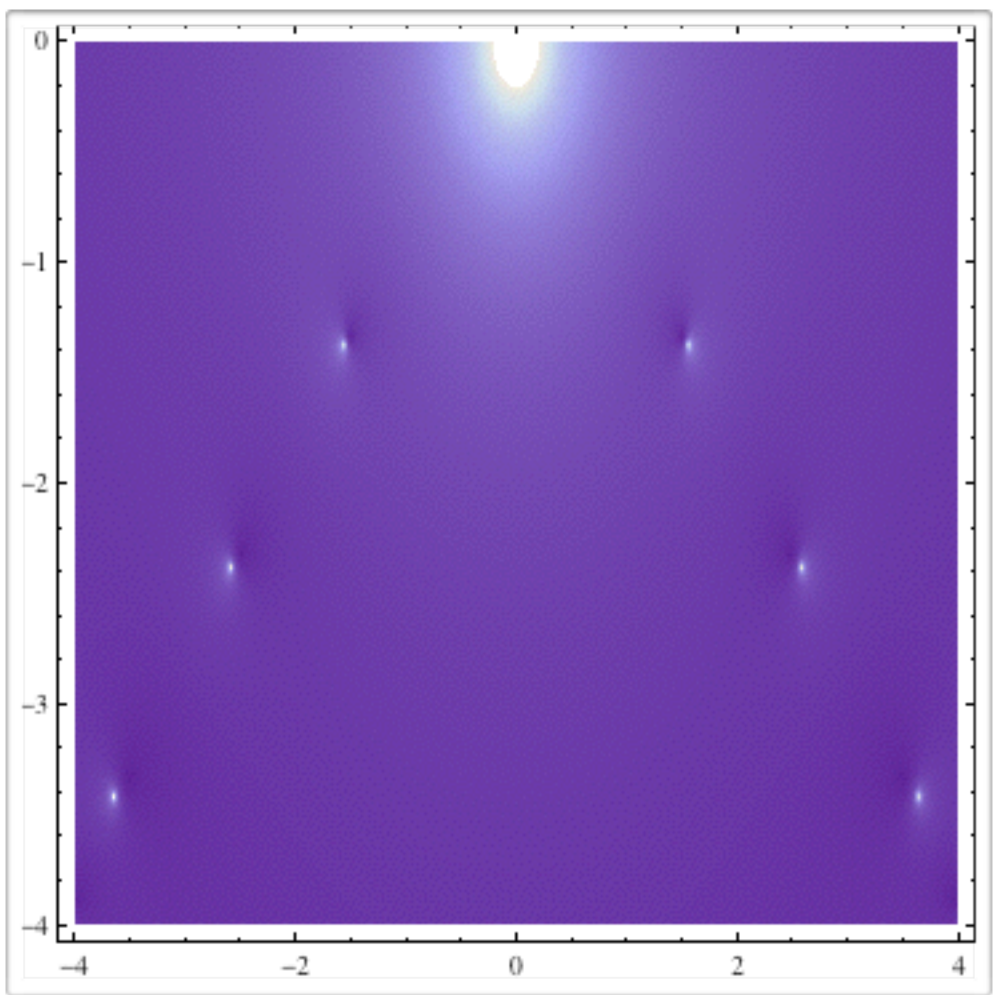
Bhattacharyya, Hubeny, Minwalla, MR '07

Long-wavelength in gravity

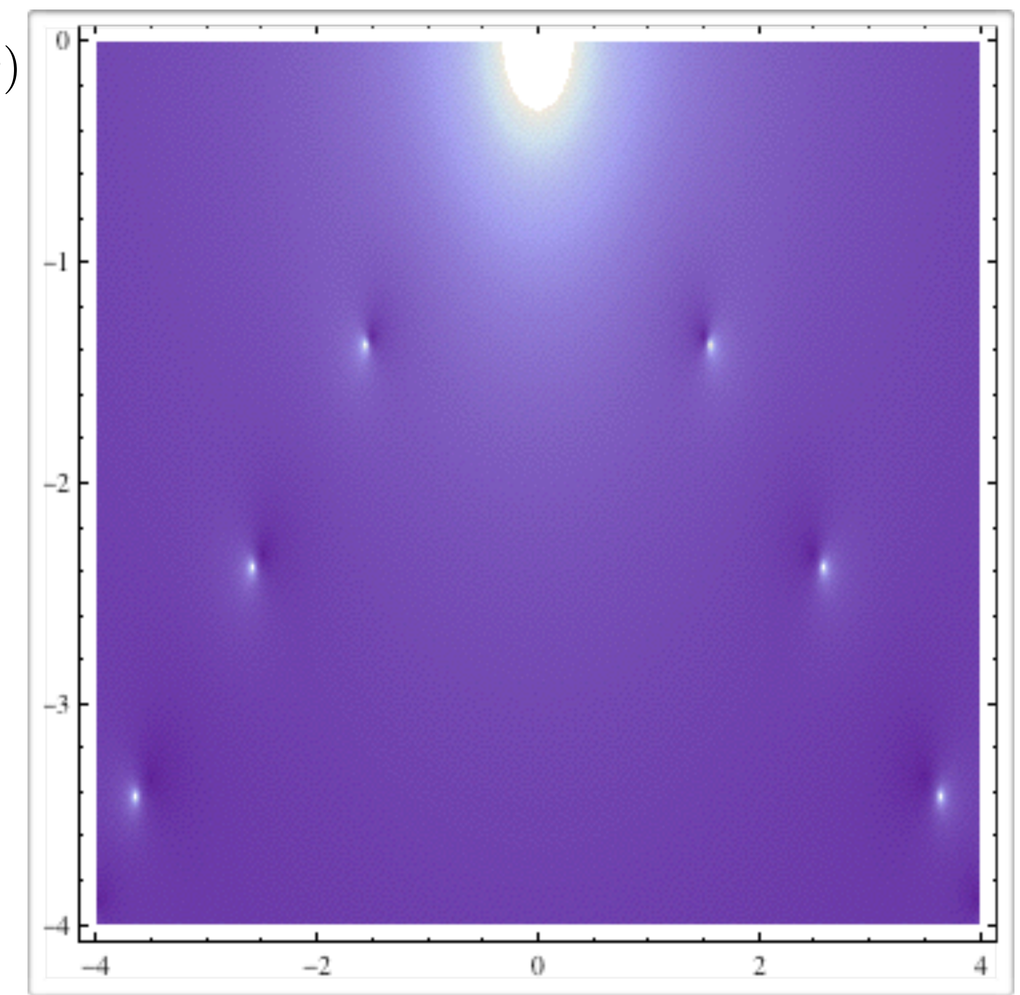
- How do we `derive' fluid dynamics from gravity?

- Intuition: perturbations of planar AdS black holes reveal long-wavelength quasinormal modes having hydrodynamic character: $\omega(k) \to 0 \;\; \text{as} \;\; k \to 0$

Im(ω)



Im(ω)

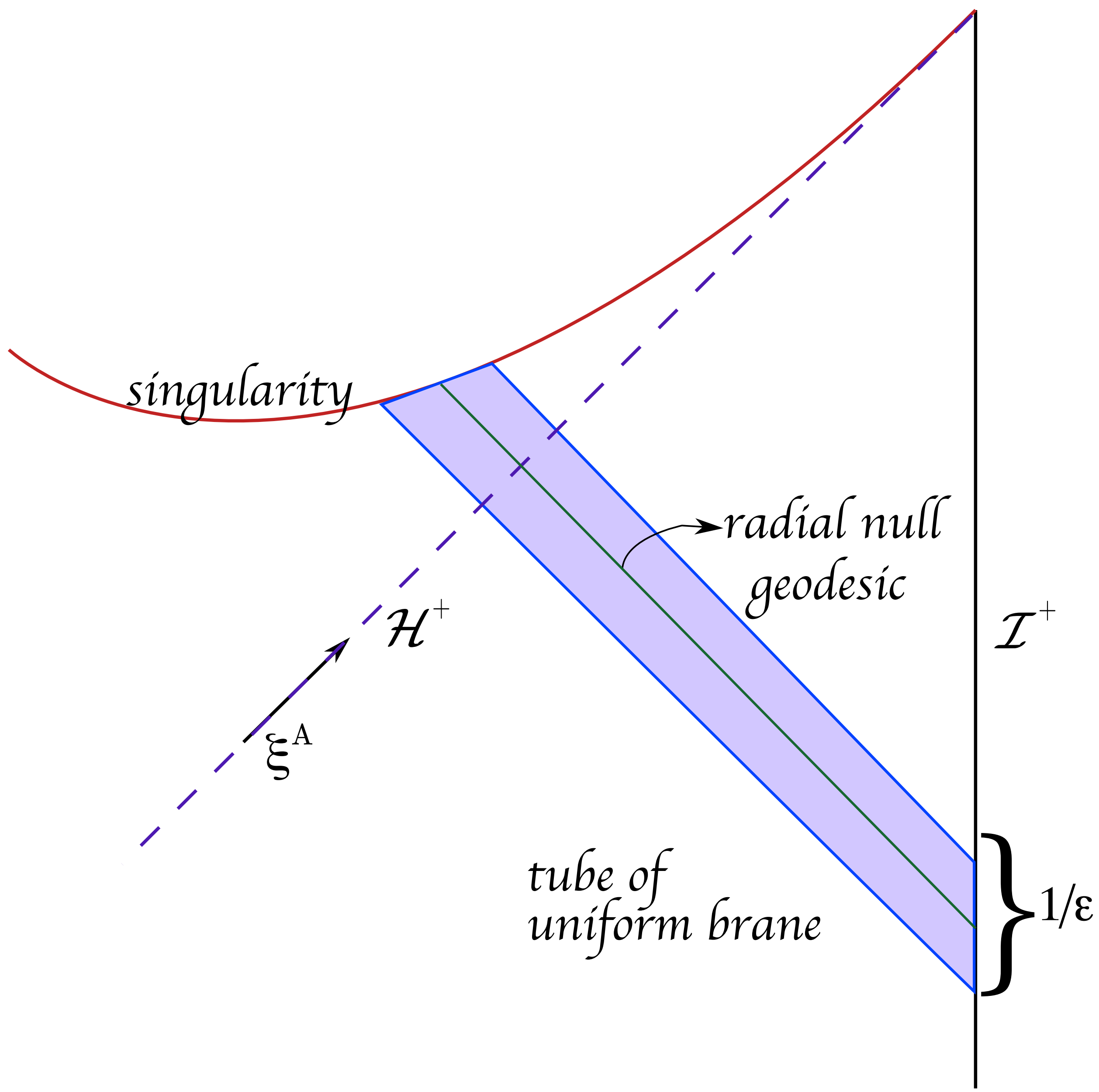


scalar/sound channel Re(ω) vector/shear channel Re(ω)

Horowitz, Hubeny '98; Policastro, Son, Starinets '02

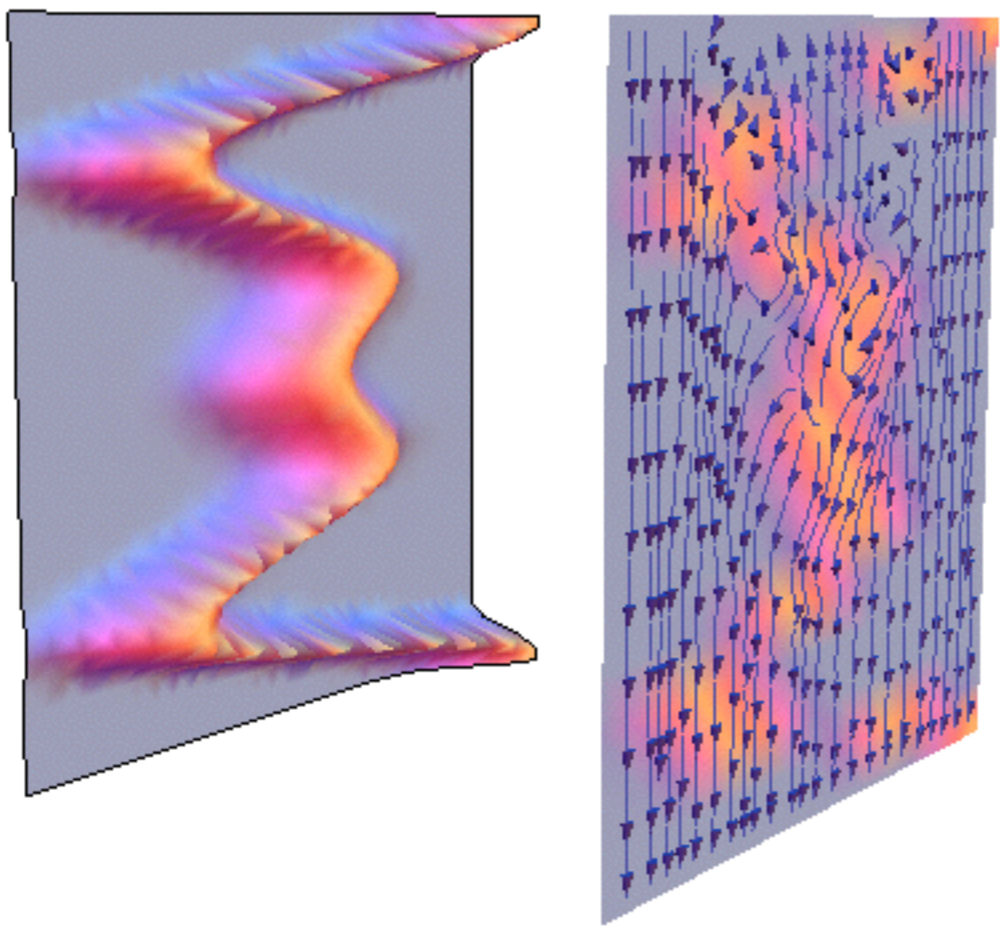
Nonlinear fluids from gravity

* ✳ Treat the light quasinormal modes as moduli of the gravitational problem.
* ✳ Hydrodynamics is the collective field theory of these modes and can be constructed systematically in a perturbation expansion.
* ✳ Intuition: patch together different bulk black hole spacetimes, along tubes of locally equilibrated fluid.



Black holes as lumps of fluid

- Black holes really behave as lumps of fluid in the low energy limit.
- In the fluid/gravity correspondence, the fluid lives at the end of the universe, on the asymptotic boundary of the spacetime where the black hole resides.
- Here the fluid is a hologram, honestly capturing all the low energy physics of the entire geometry.



Connections to physical systems: QGP

- The quark gluon plasma created in heavy ion collisions above the deconfinement temperature of QCD, whilst not quite a conformal fluid, appears to share qualitative features.

$$\eta = \frac{\pi^2}{8} N^2 T^3 \qquad \Longrightarrow \qquad \frac{\eta}{s} = \frac{1}{4\pi} \qquad\qquad \zeta = 0$$

conformal symmetry

$$\tau = \frac{2 - \log 2}{\pi T}\,\eta \qquad \kappa = \frac{2\,\eta}{\pi\,T} \qquad \lambda_1 = \frac{2\,\eta}{\pi\,T} \qquad \lambda_2 = \frac{\eta \log 2}{2\pi T} \qquad \lambda_3 = 0$$

$$u^\alpha \nabla_\alpha \sigma_{\mu\nu} \qquad C_{\mu\alpha\nu\beta}\, u^\alpha\, u^\beta \qquad \sigma_{\mu\alpha}\sigma^\alpha{}_\nu \qquad \sigma_{\mu\alpha}\omega^\alpha{}_\nu \qquad \omega_{\mu\alpha}\omega^\alpha{}_\nu$$

- Fluid/gravity transport coefficients have been used successful in hydrodynamic models for the evolution of this plasma.

Bhattacharyya, Hubeny, Minwalla, MR '07; Baier, Romatschke, Son, Starinets, Stephanov '07

Connections to physical systems: Cold Atoms

- The cold atom systems are non-relativistic compressible fluids, but their hydrodynamic descriptions can also be obtained from holography.

- The simplest modeling of these systems involves deforming the standard AdS/CFT correspondence to situations with non-relativistic (Galilean) conformal symmetry.

- Once again study of black holes in these geometric backgrounds allows computation of hydrodynamic transport coefficients in these systems.

- Eg., shear viscosity again takes on the universal value.

Son; McGreevy, Balasubramanian
Maldacena, Martelli, Tachikawa '08
Adams, McGreevy, Balasubramanian

Herzog, MR, Ross '08
MR, Ross, Son, Thompson '08

Entropy current and second law

- In the gravitational description, the entropy current which was a consequence of the statistical description, has a clean interpretation: it is simply the pull-back of the area form on the horizon onto the boundary.

- The classical area theorem in general relativity, then guarantees that the entropy current satisfies the second law of thermodynamics.

Bhattacharyya, Hubeny, Loganayagam, Mandal, Minwalla, Morita, MR, Reall '08

- So while the conserved currents and the dynamics happens on the boundary, we need to pass into the bulk and access the horizon to see the physics of entropy production and dissipation.

- NB: this picture suggests a way to tackle a notorious gravitational problem – definition of black hole entropy in higher derivative theories beyond equilibrium.

Lessons for hydrodynamics

- ✦ The fluid/gravity correspondence provides an excellent environment for analyzing structural properties of fluid dynamics.

- ✦ We have learnt from various investigations that:

  - ✳ the effective field theory is at best asymptotic

  - ✳ second law imposes non-trivial equality relations on transport coefficients

    Banerjee et. al, Jensen et. al. '12

  - ✳ quantum anomalies manifest themselves in transport (and can be amplified in a thermal medium)

    Son, Surowka '08 Jensen, Loganayagam, Yarom '12-'13

- ✦ Can one use this intuition to define an autonomous theory of hydrodynamics?

Convergence of hydrodynamic expansion

- The hydrodynamic effective field theory should be an asymptotic expansion with zero radius of convergence.
- In the fluid/gravity construction this behaviour follows from the fact that black holes have non-hydrodynamic ``massive” quasinormal modes.

Bhattacharyya, Hubeny, Minwalla, MR, ‘07

- Borel re-summation of the perturbation series should lead to a prediction for the location of the non-hydrodynamic quasinormal mode.
- Explicit analysis in gravity for a particular flow (boost invariant Bjorken flow) shows excellent agreement between the re-summed answer and the explicit determination of the first non-hydrodynamic quasinormal mode in the black hole background.

Heller, Janik, Witaszczyk ‘13

An autonomous theory of hydrodynamics

- ✦ Challenges for constructing an effective field theory for hydrodynamics:
- ✴ no obvious effective action (dissipative system); instead current algebra
- ✴ origins of entropy current somewhat mysterious; no symmetry principle
- ✴ dynamics is equivalent to current conservation
- ✴ requires understanding mixed states or density matrices (how to implement Wilsonian RG)?
- ✦ Answering this question will also illuminate some outstanding issues in gravitational dynamics.

Bhattacharyya, Bhattacharya, MR '12

Loganayagam, Haehl, MR '13-'15

Haehl, MR '13

Density matrices and doubling

$$\langle\psi| \to e^{iHt}\langle\psi|$$

$$|\psi\rangle \to e^{-iHt}\,|\psi\rangle$$

$$\rho = |\psi\rangle\langle\psi| \to e^{-iHt}\,|\psi\rangle\langle\psi|\,e^{iHt}$$

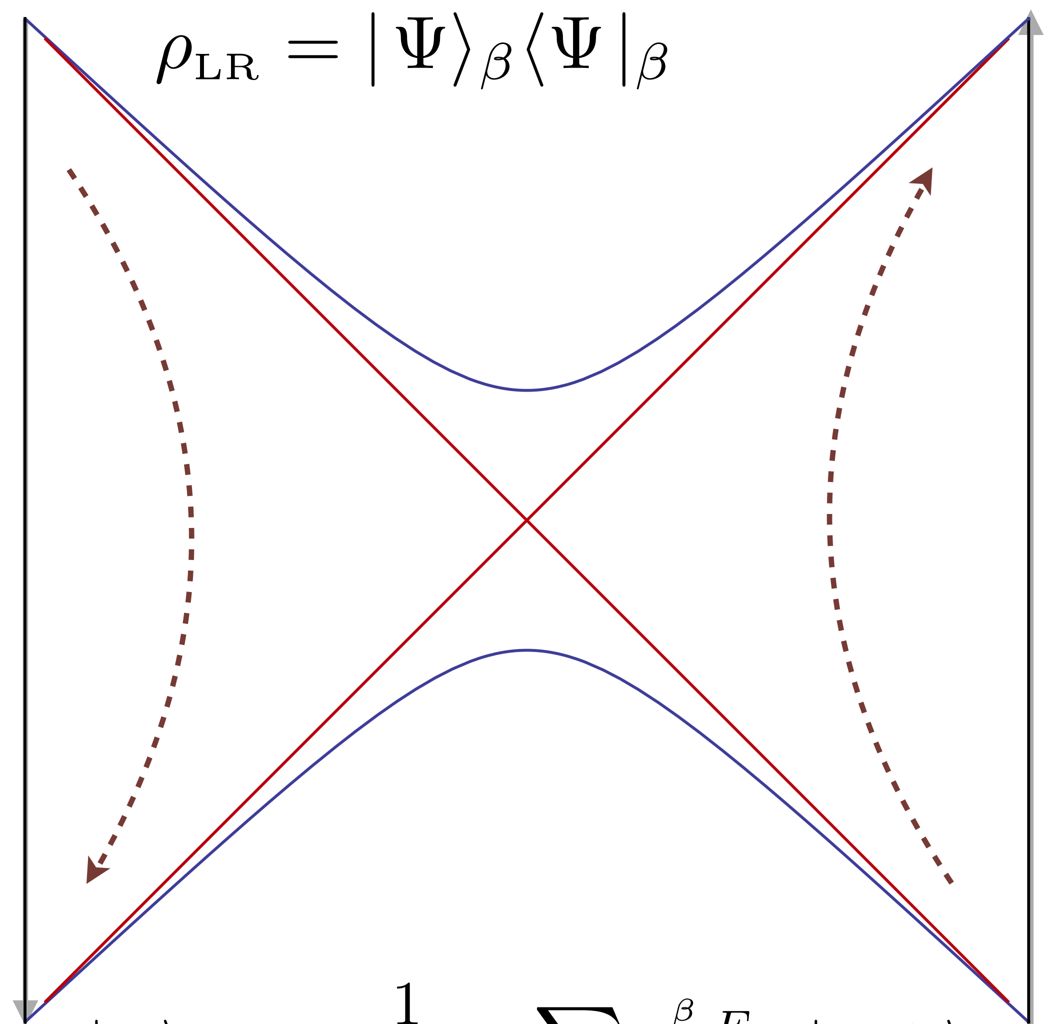
Density matrices and doubling

$$\rho_\beta = \frac{1}{Z}\, e^{-\beta H} = \mathrm{Tr}_{\mathrm{L}}\left(\rho_{\mathrm{LR}}\right)$$

$$\rho_{\mathrm{LR}} = |\,\Psi\rangle_\beta \langle\Psi\,|_\beta$$

CFT$_{\mathrm{L}}$

$|\,\mathfrak{l}_a\,\rangle$



CFT$_{\mathrm{R}}$

$|\,\mathfrak{r}_a\,\rangle$

$$|\,\Psi\rangle_\beta = \frac{1}{\sqrt{Z(\beta)}} \sum e^{\frac{\beta}{2}\,E_a}\ |\,\mathfrak{r}_a\,\mathfrak{l}_a\rangle$$

Density matrices & equilibrium dynamics

- Equilibrium QFT is well understood in this thermofield double, or Schwinger-Keldysh construction (useful for computing real time correlation functions).

- The gravitational analog for equilibrium dynamics is the eternal black hole spacetime which constructs the Hartle-Hawking thermofield state (cf., ER=EPR).

Israel '76; Maldacena '01

- What is unclear is what classes of interactions, usually called Feynman-Vernon influence functionals, are admissible?

- Hydrodynamics accords a perfect opportunity to formulate a Wilsonian construction in background density matrices.

- Take some cues from holography….

Heemskerk, Polchinski; Nickel, Son; Faulkner, Liu, MR '11

Towards a hydrodynamic effective field theory

- ✦ Understand the constraints from the second law on hydrodynamic constitutive relations.

- ✴ Chart out the space of constitutive relations that do not lead to entropy production: these are adiabatic constitutive relations. ✓

- ✴ Understand the constraints on dissipative transport and terms forbidden by the second law of thermodynamics. ✓

Bhattacharyya ’13-‘14

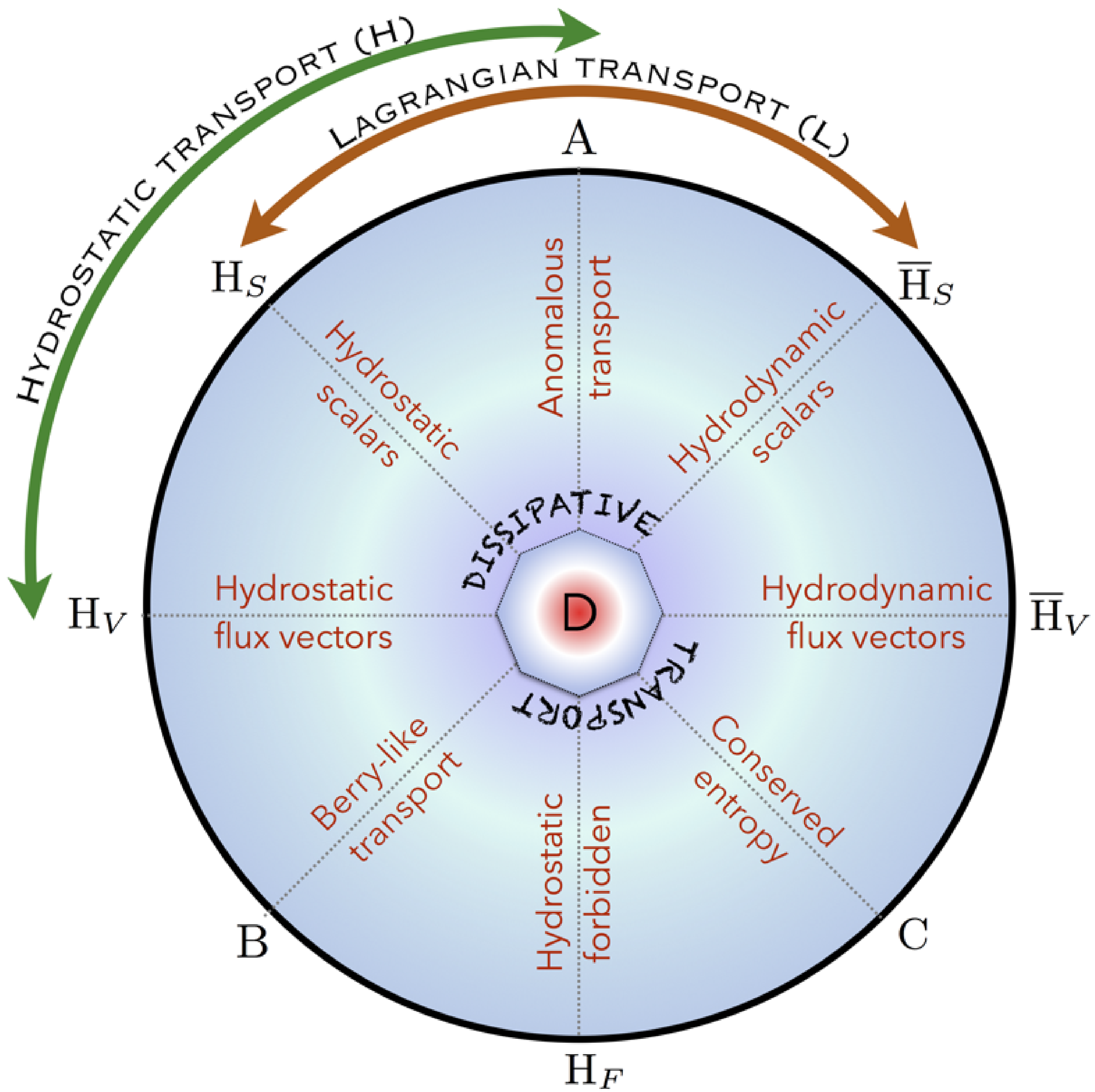
- ✦ Construct effective actions for the adiabatic sector. ✓

- ✦ This can be done once we double the degrees of freedom, and provides insight into the origins of the entropy current. ✓

- ✦ Effective actions for dissipation… $\not\checkmark$

Loganayagam, Haehl, MR ’13-‘15

Eightfold classification of hydrodynamic transport



Loganayagam, Haehl, MR ’14-‘15

Adiabatic fluids in holography

✦ There is ample evidence for the eightfold classification in holography (also in kinetic theory).

✦ At second order, for Weyl invariant neutral fluids, we find

- 5 transport coefficients: 2 $\mathrm{H_S}$, 1 $\overline{\mathrm{H}}_\mathrm{S}$, 1 D, 1 B
- Rather surprisingly, in holographic models the B and D terms vanish.
- This allows us to write down a simple effective action to compute non-viscous transport (up to second order)

$$\mathcal{L}_{\text{conformal}} \sim N^2 \left[\pi^4 \, T^4 - \frac{\pi^2}{2} T^2 \, \left({}^{\mathcal{W}}\!R + \omega^2 - \log 2 \; \sigma^2 \right) \right]$$

- Vanishing of D term ⇒ holographic fluids are more ideal than expected!

A new symmetry from the eightfold way

- The sevenfold adiabatic classification includes constitutive relations which do not admit a simple Lagrangian description (e.g., A,B,C).
- However, there exists a framework which has an enhanced symmetry and captures all of the adiabatic transport in a single Lagrangian density.

  - the background sources $\{g_{\mu\nu}, A_\mu\}$
  - the fluid fields $\{\boldsymbol{\beta}^\mu, \Lambda_{\boldsymbol{\beta}}\}$
  - partners for the sources $\{\tilde{g}_{\mu\nu}, \tilde{A}_\mu\}$ Schwinger-Keldysh like
  - KMS photon $A^{(T)}{}_\mu$ $U(1)_T$: macroscopic manifestation of KMS invariance

The Eightfold Lagrangian

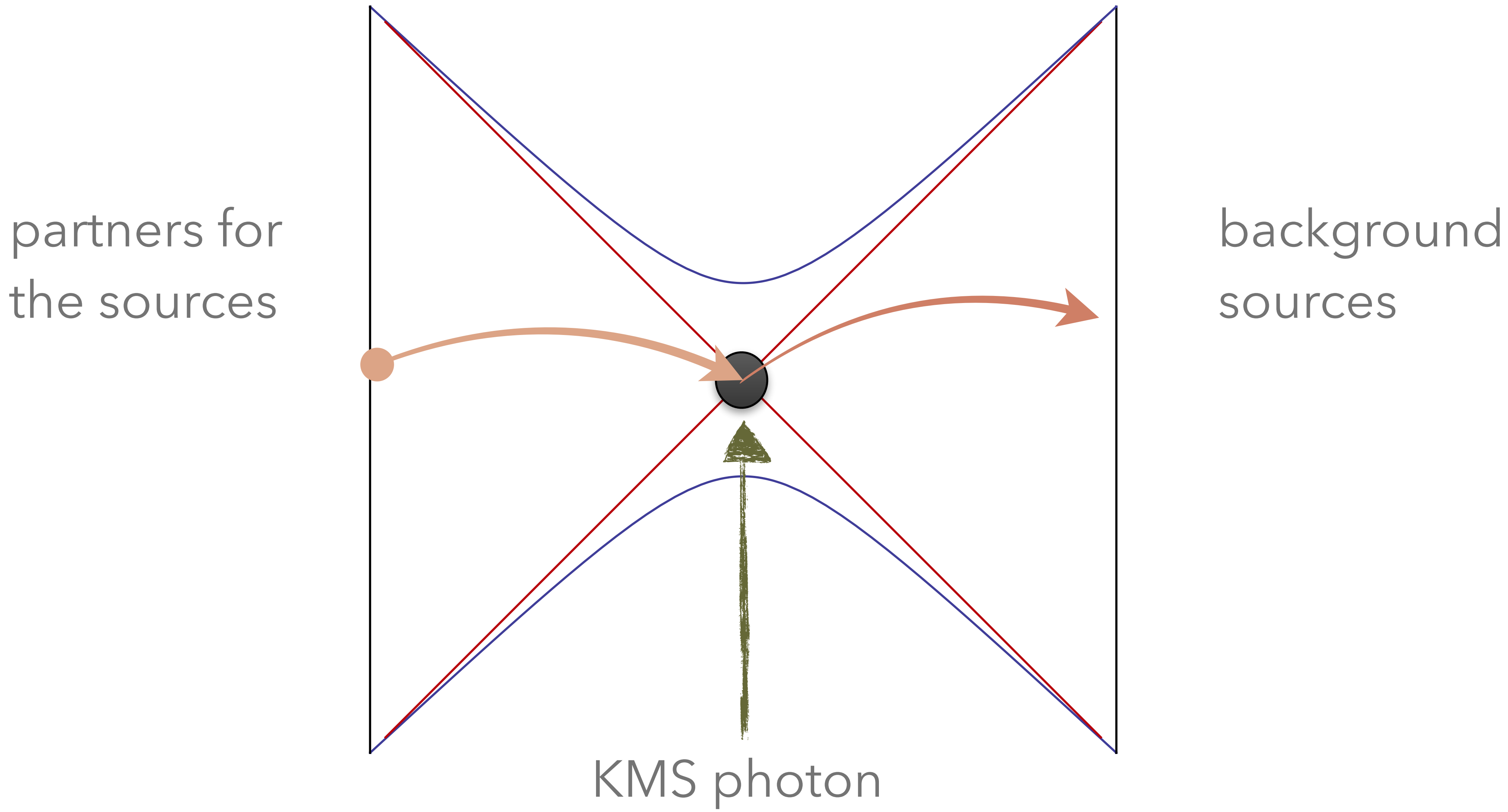
- The adiabatic constitutive relations can be derived in one swoop from a Lagrangian density that is invariant under diffeomorphisms, flavour gauge transformations and the KMS flavour $U(1)_T$ symmetry.

$$\boxed{\mathcal{L}_T = \frac{1}{2} T^{\mu\nu} \, \tilde{g}_{\mu\nu} + J^\mu \cdot \tilde{A}_\mu + (J_S^\sigma + \boldsymbol{\beta}_\nu T^{\nu\sigma} + (\Lambda_{\boldsymbol{\beta}} + \boldsymbol{\beta}^\nu A_\nu) \cdot J^\sigma) \, \mathsf{A}^{(\mathsf{T})}{}_\sigma}$$

- The $U(1)_T$ symmetry appear to ensure that the influence functionals which are allowed in the Schwinger-Keldysh construction respect the second law.

- A complete map between the Schwinger-Keldysh construction and the picture involving the partner sources and KMS photon is being developed, but there is a heuristic that is rather suggestive….

A gravitational heuristic for KMS flavour invariance



Schwinger-Keldysh like construction, with KMS photon ensuring consistency with second law (macroscopic manifestation of KMS conditions).

Haehl, Loganayagam, MR (wip)

Gravity & Hydrodynamics

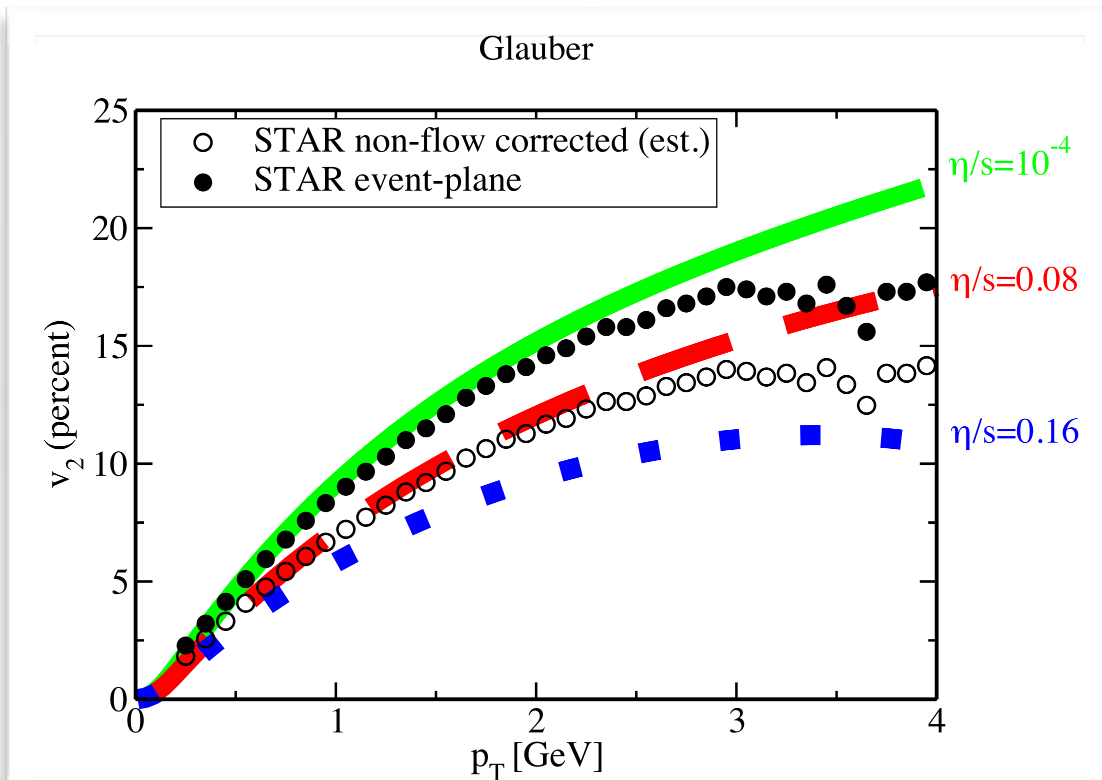
- The interplay between gravity and hydrodynamics has been quite enriching for both subjects.
- We have been able to compute explicit transport data at strong coupling for an interesting (perhaps exotic) class of QFTs.
- More importantly, aided and abetted by intuition borrowed from the fluid/gravity correspondence we have succeeded in giving a complete classification for hydrodynamic transport.
- In the process have discovered a new symmetry principle which guarantees entropy conservation.

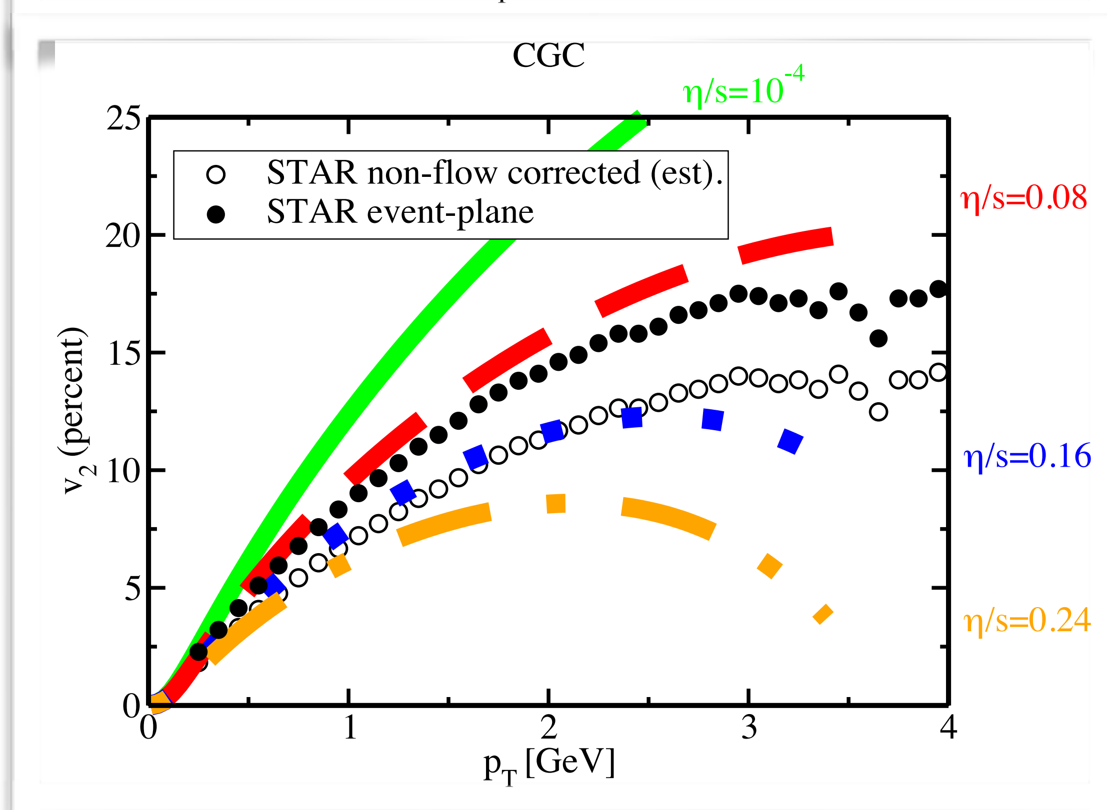
Gravity & Hydrodynamics

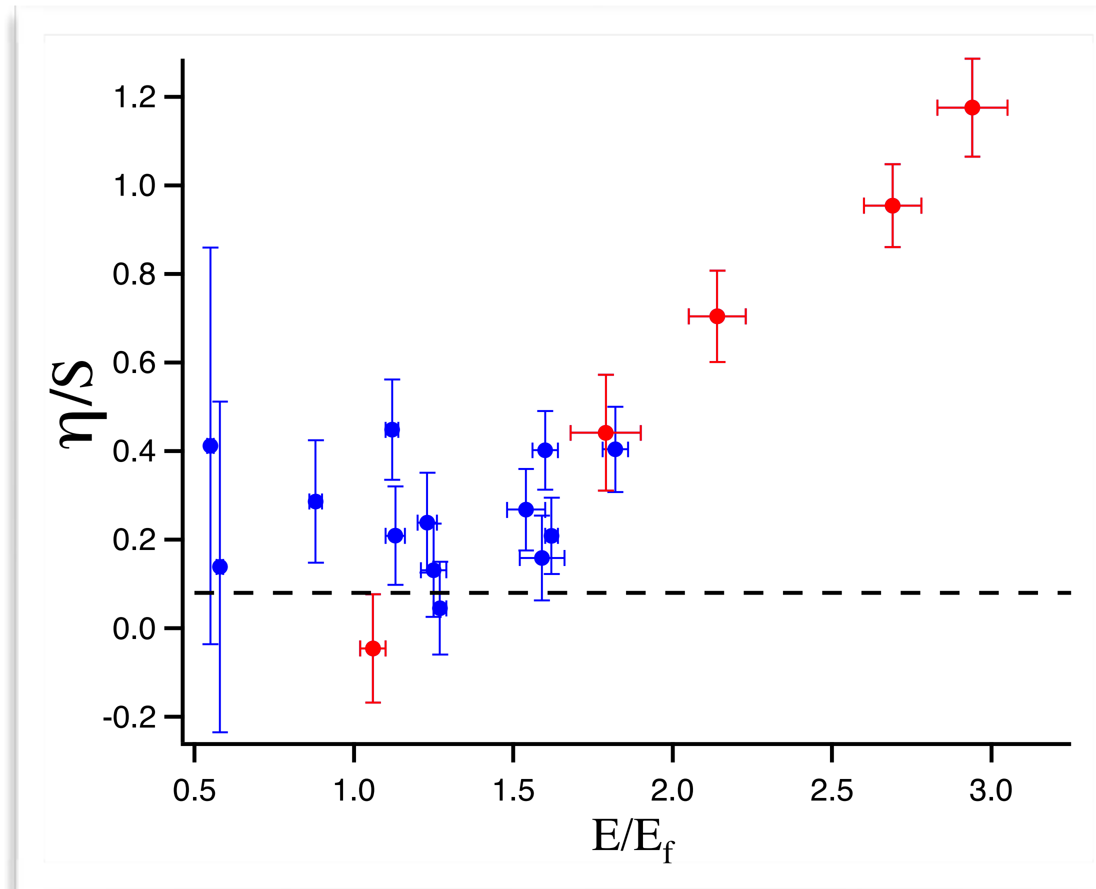
- ✦ The story is far from complete….
- ✴ Understanding the implications of the KMS gauge symmetry?
- ✴ Dissipative effective actions?
- ✴ Implications for black hole physics and spacetime emergence?
- ✴ Connections to paradigms such as ER=EPR?
- ✴ Holography and non-equilibrium (beyond hydro)?


Thank You!

Viscosity data for QGP and cold atoms



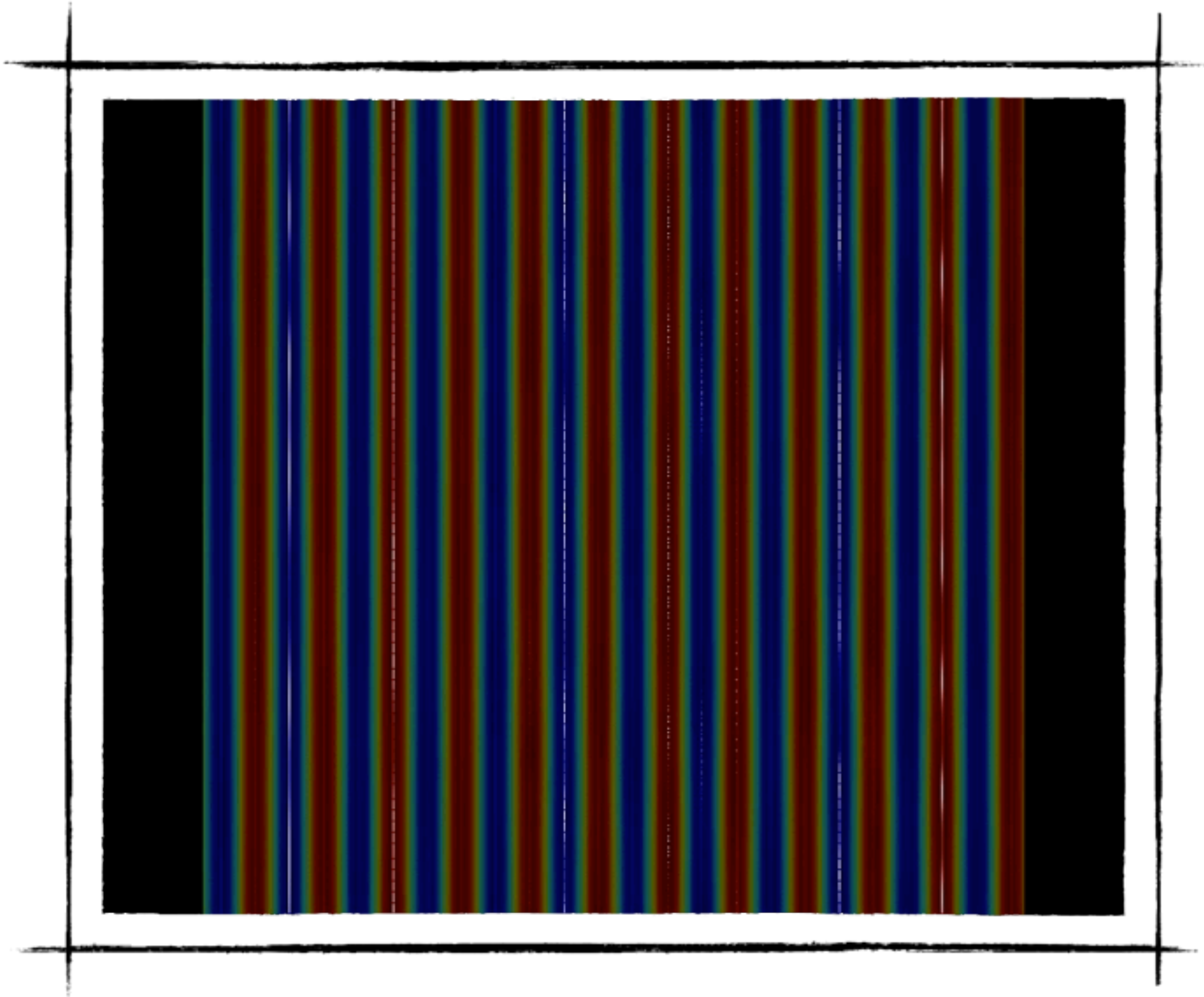




Thomas ‘09

Luzum, Romatschke ‘08

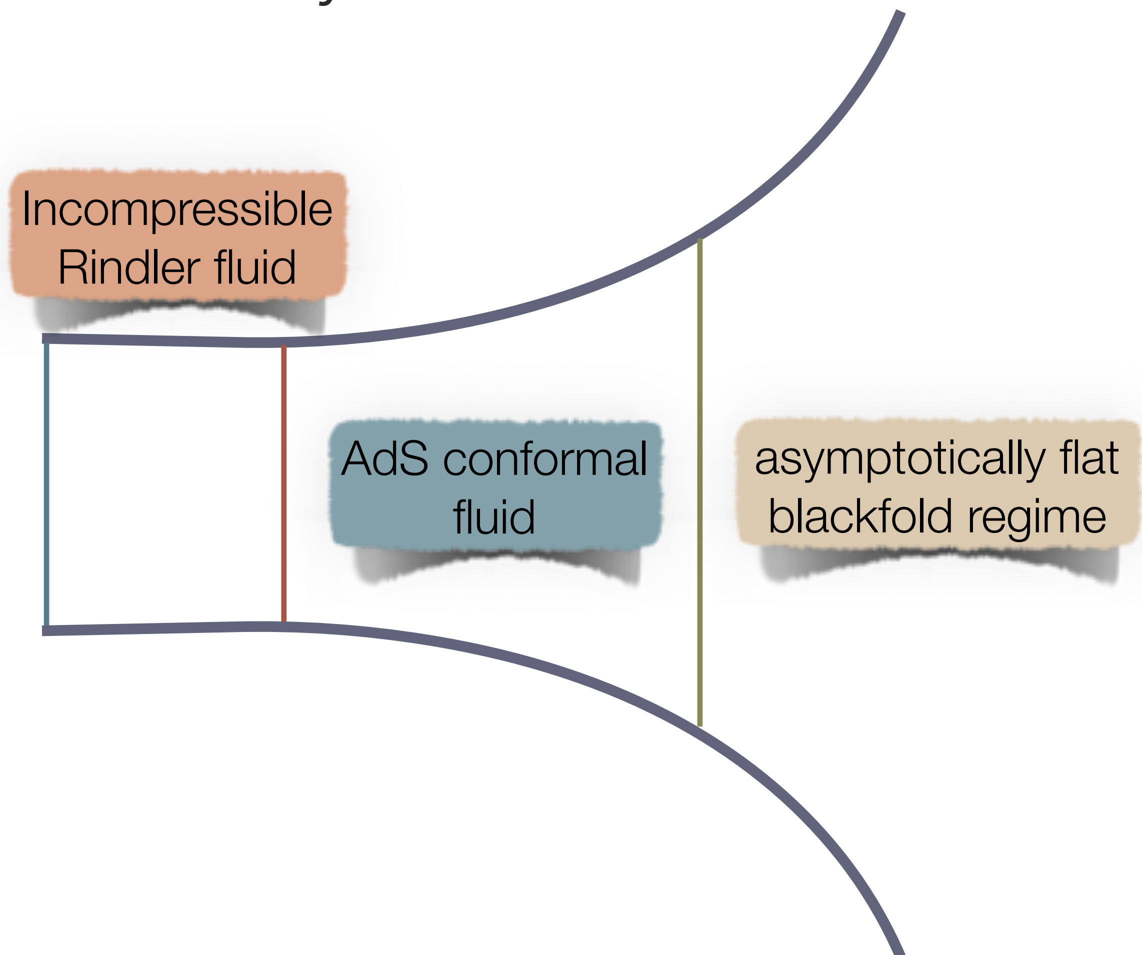
Horizon dual to a fluid flow



Adams, Chesler, Liu '13

D-branes and membranes

- In classical gravity one can explore the hypersurface dynamics in various regimes of charged black branes.



- Asymptotic region: Blackfold fluid with features described earlier.

- Throat region: Conformal fluid dual to AdS geometries. Low energy limit of a QFT.

- Rindler region: incompressible fluid

- Monitor the variation of transport properties of the fluid across the regimes: transport coefficients are pretty much determined by the throat dynamics.

Brattan, Camps, Loganayagam, MR '11
Marolf, MR '12
Emparan, Hubeny, MR '13
Erdmenger, MR, Steinfurt, Zeller '14