UV fixed points - from gauge theories to quantum gravitation

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UC Davis, 5 Feb 2015

standard model

University of Sussex

local QFT for fundamental interactions

- **strong** nuclear force
- **weak** force
- **electromagnetic** force

degrees of freedom

- spin 0 (the **Higgs** has finally arrived)
- spin 1/2 (quite a few)
- spin 1

perturbatively renormalisable & **predictive**

standard model

University of Sussex

local QFT for fundamental interactions

strong nuclear force
weak force
electromagnetic force

challenges

Higgs, QED: maximum UV extension?
complete asymptotic freedom?
how does quantum gravity fits in?
...

interacting UV fixed points

UV fixed points

perturbation theory

theory with coupling α: $t = \ln \mu/\Lambda$

$$\partial_t \alpha = \qquad - B\,\alpha^2$$ $\alpha_* \ll 1$

perturbation theory

theory with coupling α**:**

$t = \ln \mu / \Lambda$

$$\partial_t \alpha = \quad - B\, \alpha^2$$

$\alpha_* \ll 1$

free fixed point $\alpha_* = 0$

perturbation theory

theory with coupling α:

$t = \ln \mu/\Lambda$

$$\partial_t \alpha = \quad - B\,\alpha^2$$

$\alpha_* \ll 1$

free fixed point

$$\alpha_* = 0$$

QED, Higgs $\quad B < 0 \quad$ IR fixed point

perturbative UV Landau pole
predictive up to maximal UV extension

asymptotic freedom

theory with coupling α:

$t = \ln \mu/\Lambda$

$$\partial_t \alpha = \quad - B\,\alpha^2$$

$\alpha_* \ll 1$

free fixed point $\alpha_* = 0$

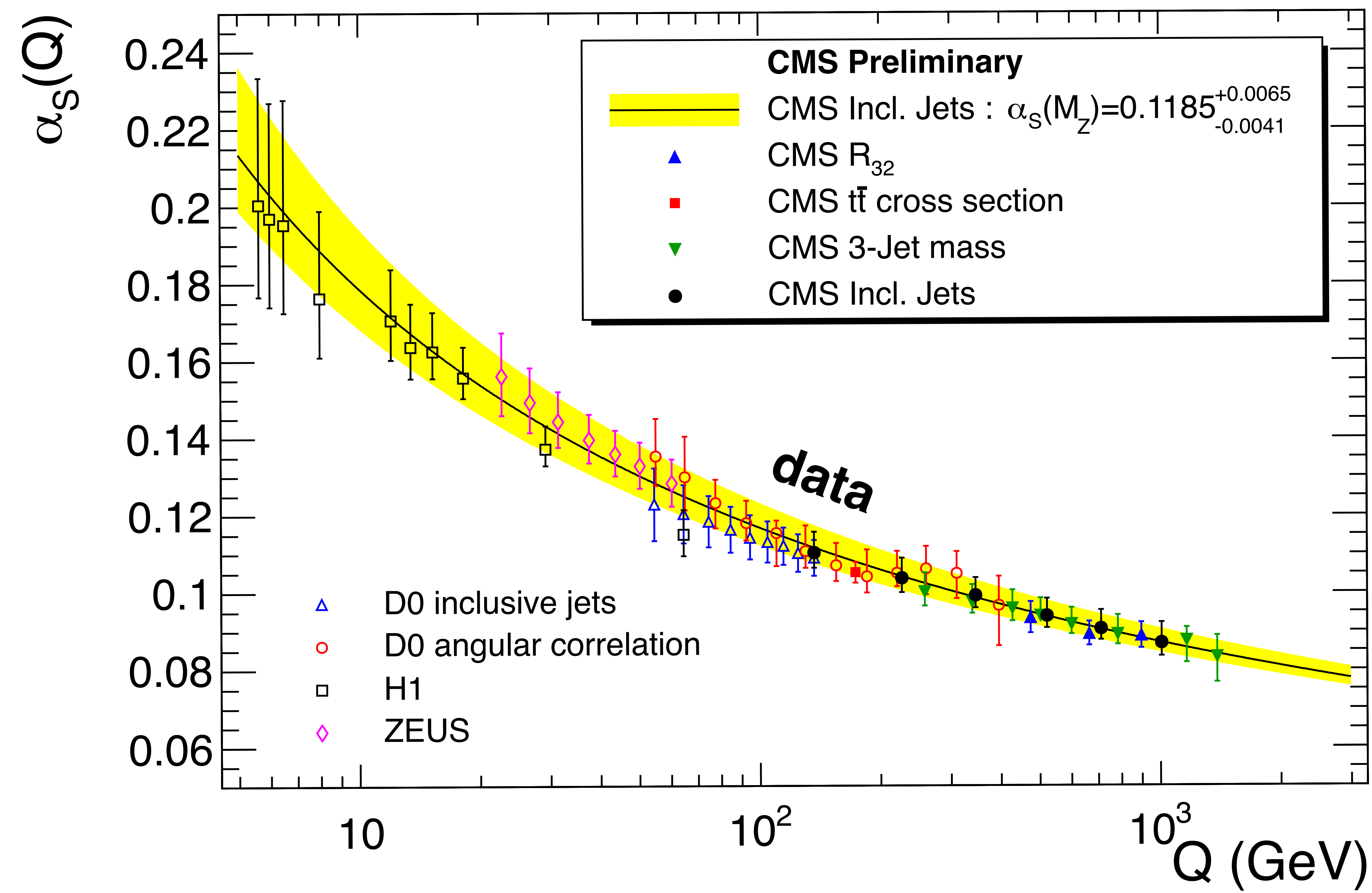
QCD $B > 0$ UV fixed point

perturbative renormalisability & asymptotic freedom
predictive up to highest energies

fundamental definition of QFT ⟷ UV fixed point

Wilson '71

asymptotic freedom



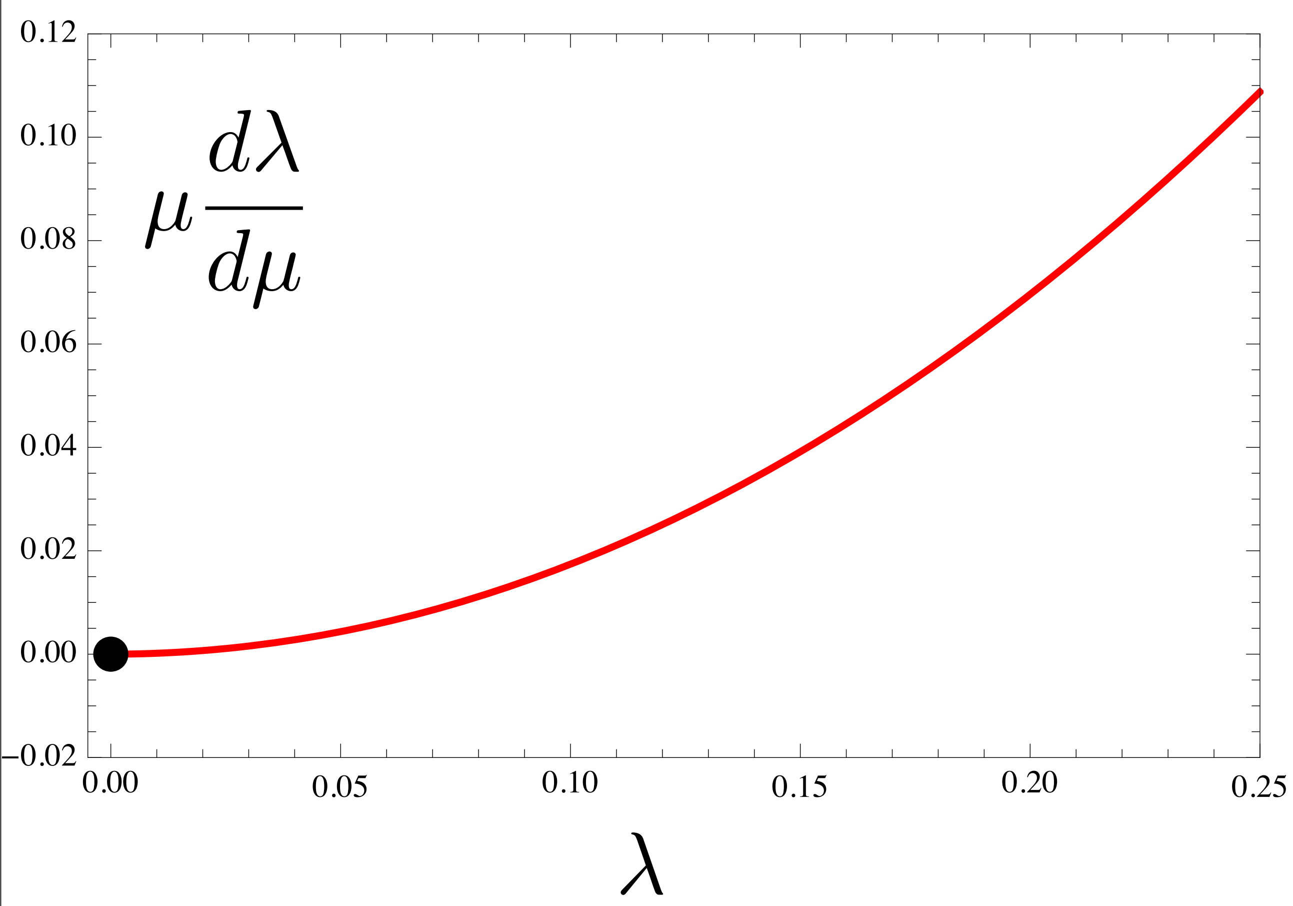
asymptotic freedom

$$\mu \frac{d\alpha_{e.m.}}{d\mu} \propto \alpha_{e.m.}^2 > 0$$

QED beta function

$$\mu \frac{d\lambda}{d\mu} \propto \lambda^2 > 0$$

Higgs self-coupling
Yukawa couplings



perturbative UV Landau pole:
maximal UV extension

cure:
complete asymptotic freedom

interacting fixed point

theory with coupling α **:**

$t = \ln \mu / \Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

perturbative non-renormalisability: $A > 0$

interacting fixed point

theory with coupling α**:**

$t = \ln \mu / \Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

fixed points $\alpha_* = 0$ $\alpha_* = A/B$

interacting fixed point

theory with coupling α**:** $t = \ln \mu/\Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

fixed points	$\alpha_* = 0$	$\alpha_* = A/B$
if $A > 0\,,\ B > 0$**:**	**IR**	**UV**

interacting fixed point

theory with coupling α **:**

$t = \ln \mu/\Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

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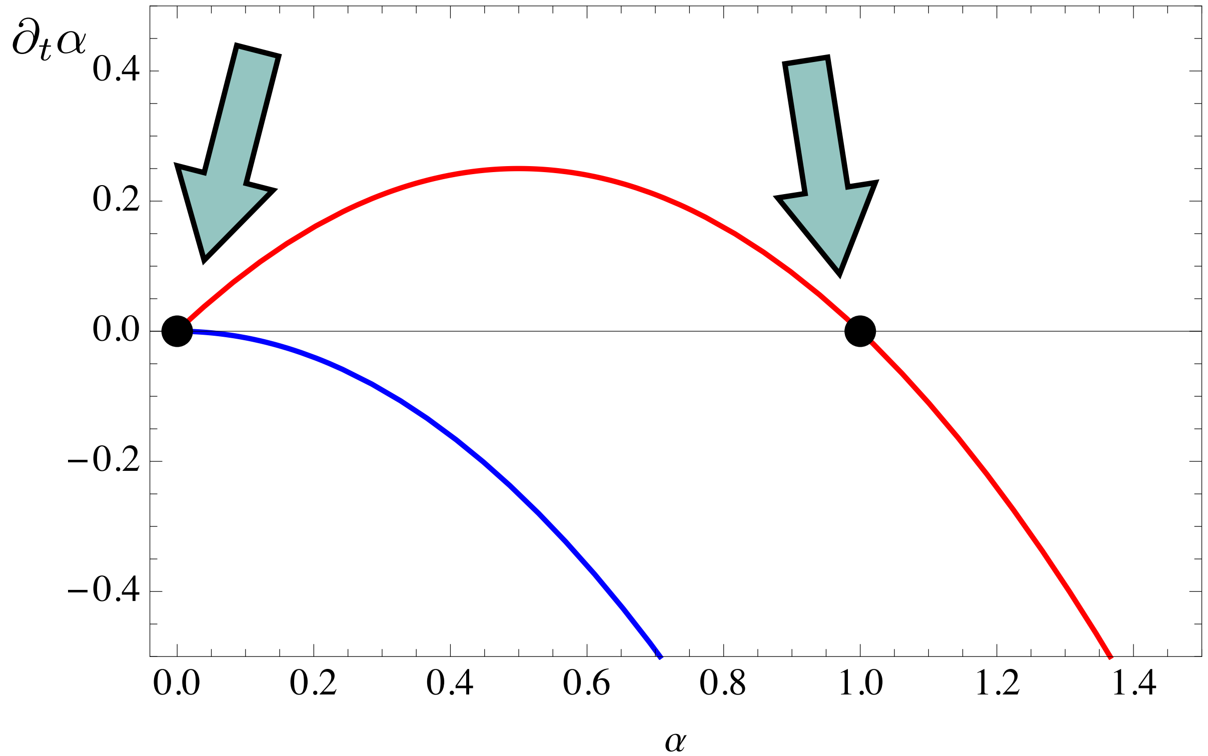
if $A > 0\,,\ B > 0$ **:** **IR** **UV**



interacting fixed point

theory with coupling α:

$t = \ln \mu / \Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

fixed points $\quad \alpha_* = 0 \qquad \alpha_* = A/B$

epsilon expansion: $\quad \epsilon = D - D_c$

large-N expansion: $\quad$ many fields

perturbation theory

theory with coupling α**:**

$t = \ln \mu/\Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

gravitons	$D = 2 + \epsilon: \quad \alpha = G_N(\mu)\mu^{D-2}$	Gastmans et al '78 Weinberg '79 Kawai et al '90
fermions	$D = 2 + \epsilon: \quad \alpha = g_{\rm GN}(\mu)\mu^{2-D}$	Gawedzki, Kupiainen '85 de Calan et al '91
gluons	$D = 4 + \epsilon: \quad \alpha = g^2_{\rm YM}(\mu)\mu^{4-D}$	Peskin '80 Morris '04
scalars	$D = 2 + \epsilon: \quad \alpha = g_{NL}(\mu)\mu^{D-2}$	Brezin, Zinn-Justin '76 Bardeen, Lee, Shrock '76

non-perturbative renormalisability

$$A = \epsilon \ll 1\,, \quad B = \mathcal{O}(1) > 0$$

UV fixed points in 4D quantum gauge theories

DL, F Sannino, JHEP1214(2014)178 arXiv:1406.2337
DL, M Mojaza, F Sannino, arXiv:1501.03061

gauge theory with fermions

SU(NC) YM with NF fermions:

$$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$

$\alpha_* \ll 1$

gauge theory with fermions

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$$\alpha_* = 0 \qquad \alpha_g^* = B/C$$

large-NF,NC (Veneziano) limit:

ϵ **continuous**

$$\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

Veneziano '79

we consider

$$0 < -B \equiv -B\,(\epsilon) \ll 1$$

gauge theory with fermions

SU(NC) YM with NF fermions: $\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$ $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$

$\alpha_* \ll 1$

$$\alpha_* = 0 \qquad \alpha_g^* = B/C$$

interacting fixed points:

$B > 0 \;\;\&\;\; C > 0:$ **Caswell - Banks-Zaks**
IR fixed point

Caswell '74
Banks, Zaks '82

$B < 0 \;\;\&\;\; C < 0:$ **UV fixed point**
no asymptotic freedom

gauge theory with fermions

SU(NC) YM with NF fermions: $\alpha_g = \dfrac{g^2\, N_c}{(4\pi)^2}$ $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$

$\alpha_* \ll 1$

$$\alpha_* = 0 \qquad \alpha_g^* = B/C$$

we are in the regime $0 < \epsilon \ll 1$

here: $B = -\dfrac{4\epsilon}{3} < 0 \;\; \& \;\; C > 0$

hence:
no physical fixed point

Caswell '74

gauge theory with fermions

SU(NC) YM with NF fermions: $$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$ $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$ $\alpha_* \ll 1$

$\alpha_* = 0$ $\alpha_g^* = B/C$

→ **scalar fields & Yukawa couplings required**

gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \qquad \alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$\alpha_* \ll 1$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \qquad \alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

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gauge-Yukawa theory

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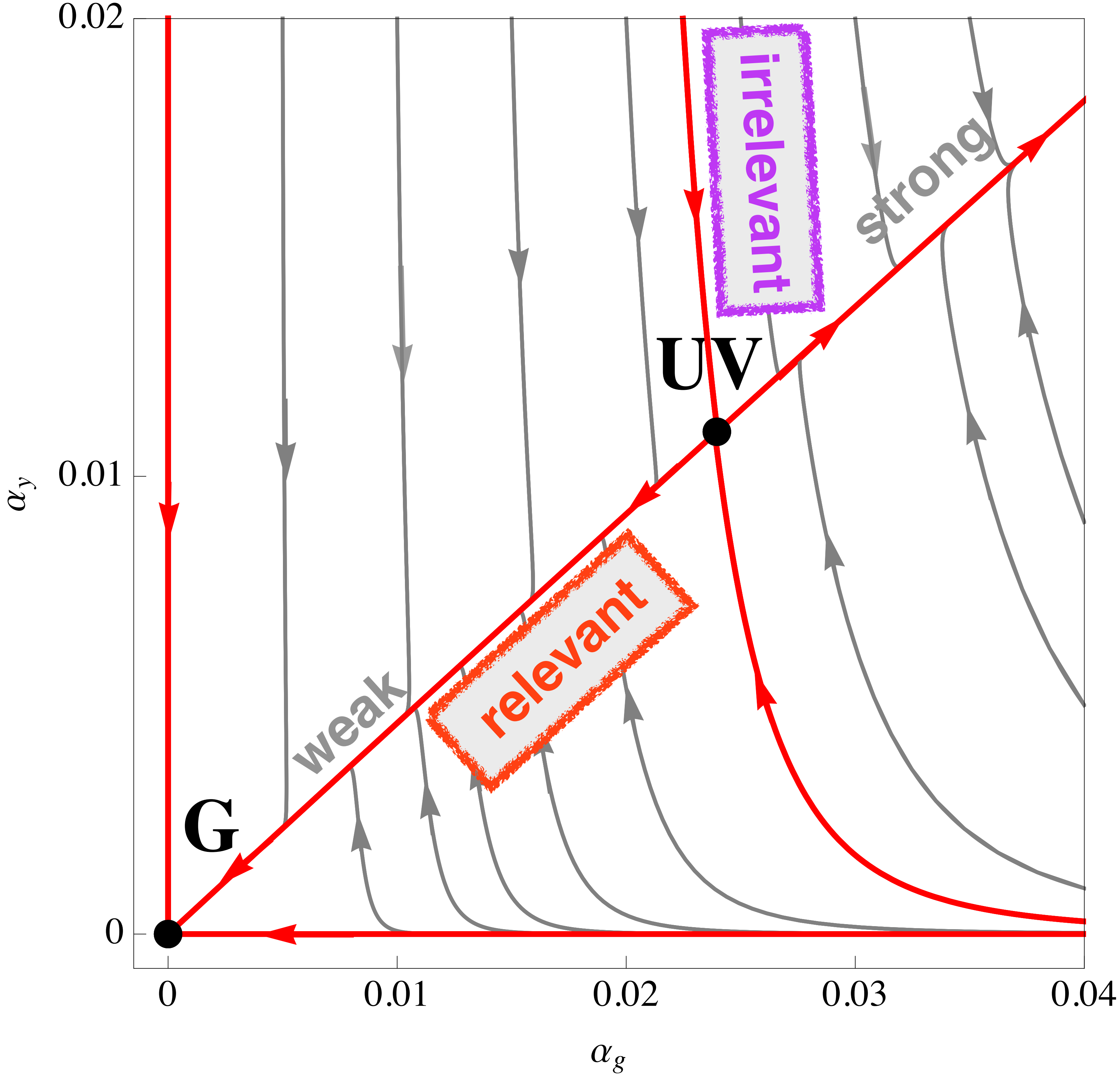
$$\partial_t \alpha_y = E\,\alpha_y^2 - F\,\alpha_g\,\alpha_y$$

→ **sensible interacting UV fixed point**

$$D\,F - C\,E \geq 0$$

phase diagram



UV finite theories
(weak & strong)

exact UV FP
strict perturbative control

template gauge-Yukawa theory

Lagrangean

$$
\begin{aligned}
L_{\rm YM} &= -\frac{1}{2}{\rm Tr}\, F^{\mu\nu}F_{\mu\nu} \\
L_F &= {\rm Tr}\left(\overline{Q}\, i\not{D}\, Q\right) \\
L_Y &= y\,{\rm Tr}\left(\overline{Q}\, H\, Q\right) \\
L_H &= {\rm Tr}\,(\partial_\mu H^\dagger\, \partial^\mu H) \\
L_U &= -u\,{\rm Tr}\,(H^\dagger H)^2 \\
L_V &= -v\,({\rm Tr}\, H^\dagger H)^2\,.
\end{aligned}
$$

couplings

$$
\alpha_g = \frac{g^2\, N_C}{(4\pi)^2}\,, \quad \alpha_y = \frac{y^2\, N_C}{(4\pi)^2}
$$

$$
\alpha_h = \frac{u\, N_F}{(4\pi)^2}\,, \quad \alpha_v = \frac{v\, N_F^2}{(4\pi)^2}\,.
$$

no asymptotic freedom

small parameter

$$
0 < \epsilon \ll 1 \qquad \epsilon = \frac{N_f}{N_c} - \frac{11}{2}
$$

global symmetry

$$
SU(N_F) \times SU(N_F) \times U(1)
$$

template gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right\}$$

$$\beta_y = \alpha_y \left\{ (13 + 2\epsilon)\,\alpha_y - 6\,\alpha_g \right\} .$$

$$\beta_h = -(11 + 2\epsilon)\,\alpha_y^2 + 4\alpha_h(\alpha_y + 2\alpha_h)$$

$$\beta_v = 12\alpha_h^2 + 4\alpha_v\,(\alpha_v + 4\alpha_h + \alpha_y) .$$

up to 3-, 2-, 1-loop order in the gauge, Yukawa and scalar couplings

coupling	order in perturbation theory		
α_g	1	2	3
α_y	0	1	2
α_h	0	0	1
α_v	0	0	1
approximation level	LO	NLO	NNLO

template gauge-Yukawa theory

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$$\beta_v = 12\alpha_h^2 + 4\alpha_v\, (\alpha_v + 4\alpha_h + \alpha_y) \; .$$

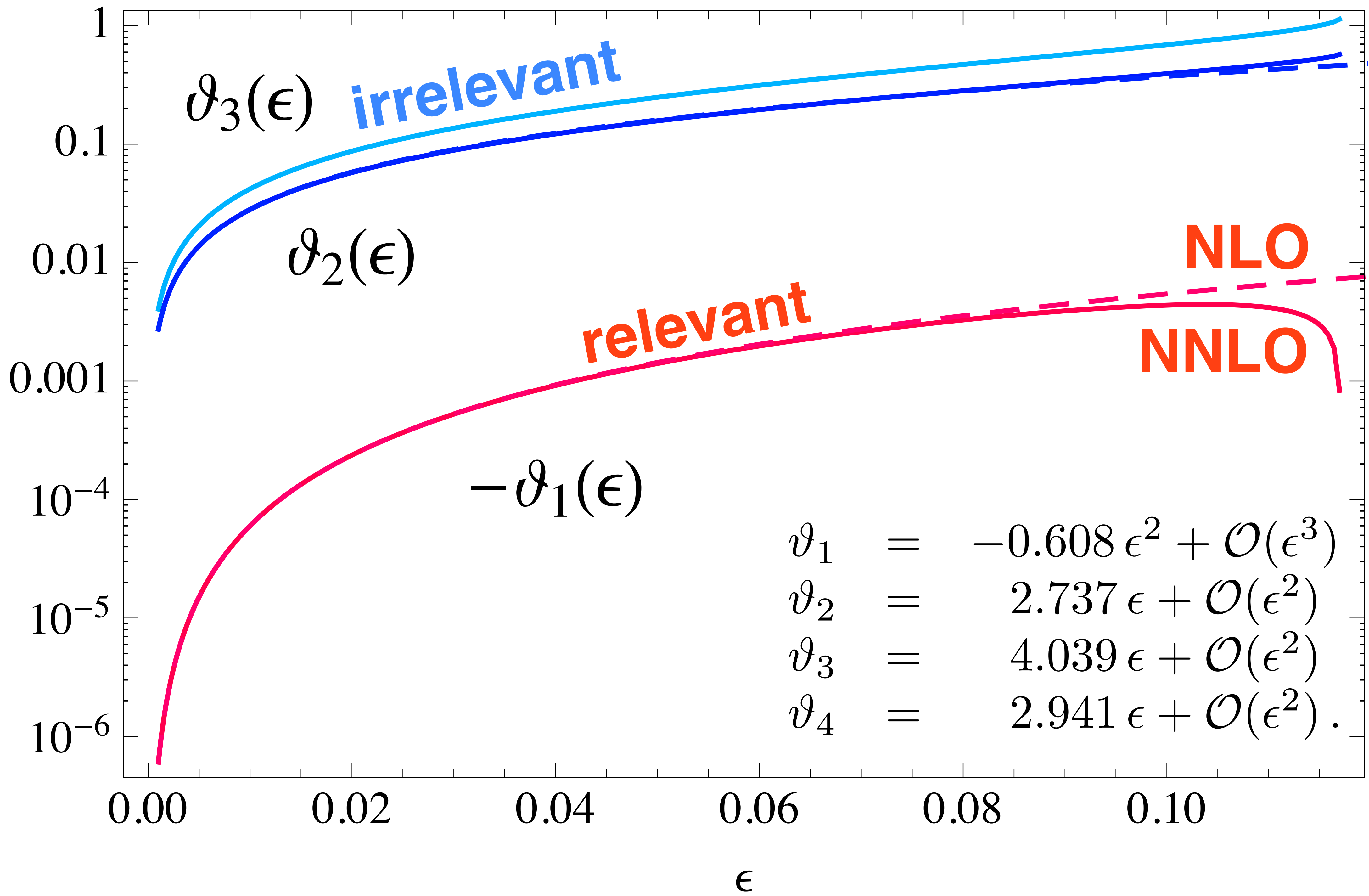
universal
UV fixed point

$$\begin{aligned} \alpha_g^* &= 0.4561\,\epsilon + 0.7808\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_y^* &= 0.2105\,\epsilon + 0.5082\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_h^* &= 0.1998\,\epsilon + 0.5042\,\epsilon^2 + \mathcal{O}(\epsilon^3) \, . \\ \alpha_v^* &= -0.1373\,\epsilon + \mathcal{O}(\epsilon^2) \end{aligned}$$

results

UV scaling exponents

$$\vartheta_1 < 0 < \vartheta_2 < \vartheta_4 < \vartheta_3$$



$$\begin{aligned} \vartheta_1 &= -0.608\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \vartheta_2 &= 2.737\,\epsilon + \mathcal{O}(\epsilon^2) \\ \vartheta_3 &= 4.039\,\epsilon + \mathcal{O}(\epsilon^2) \\ \vartheta_4 &= 2.941\,\epsilon + \mathcal{O}(\epsilon^2)\,. \end{aligned}$$

results

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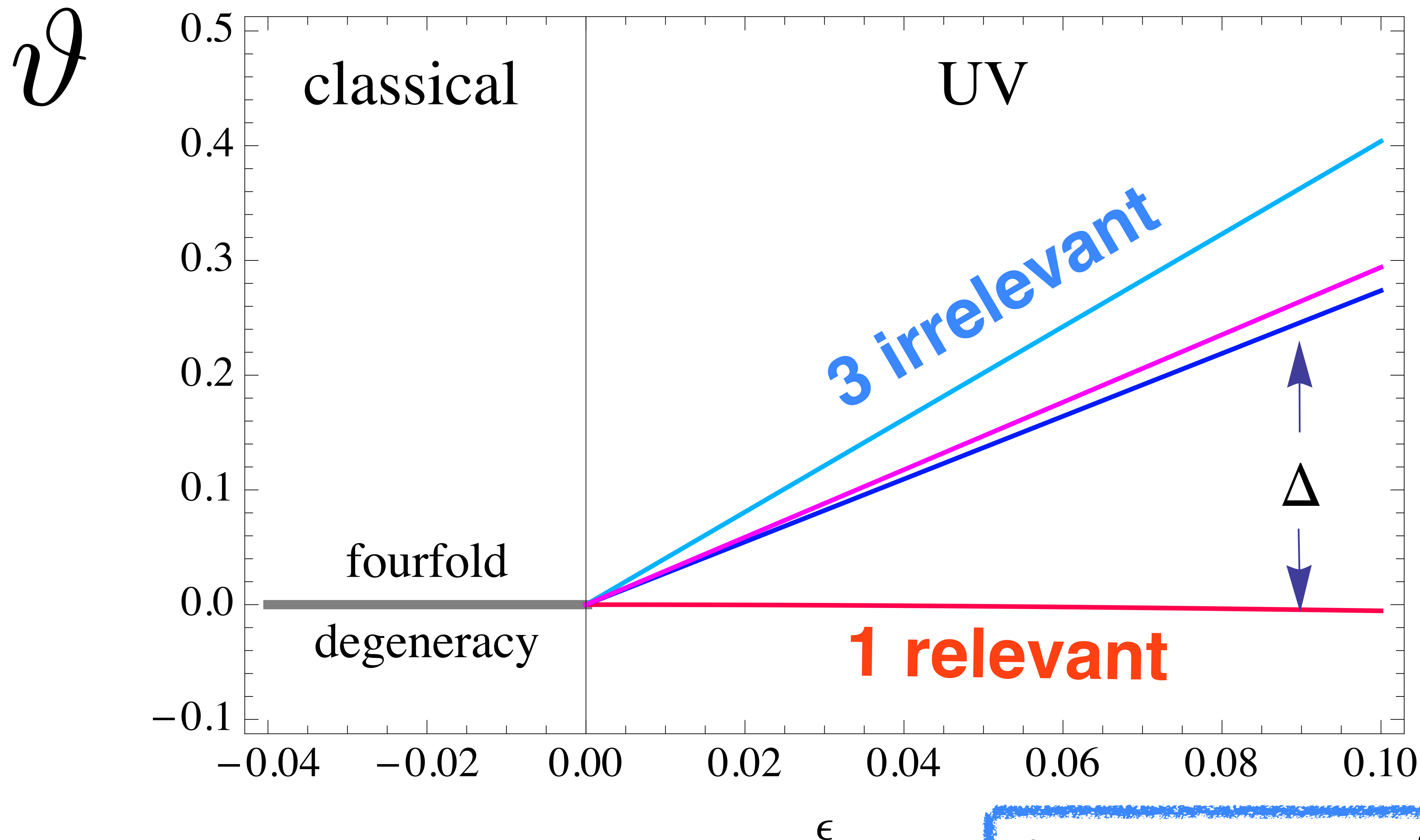

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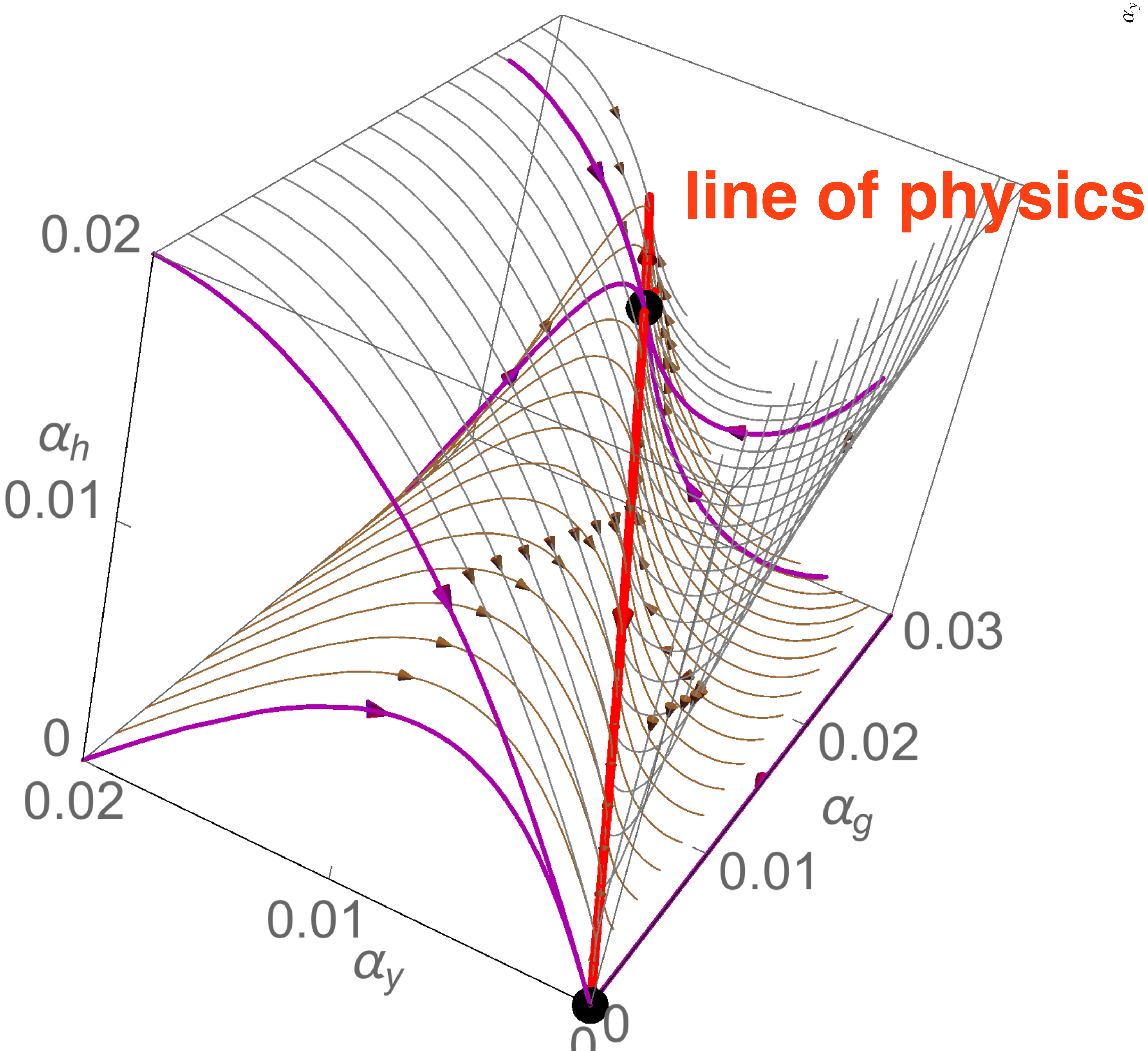
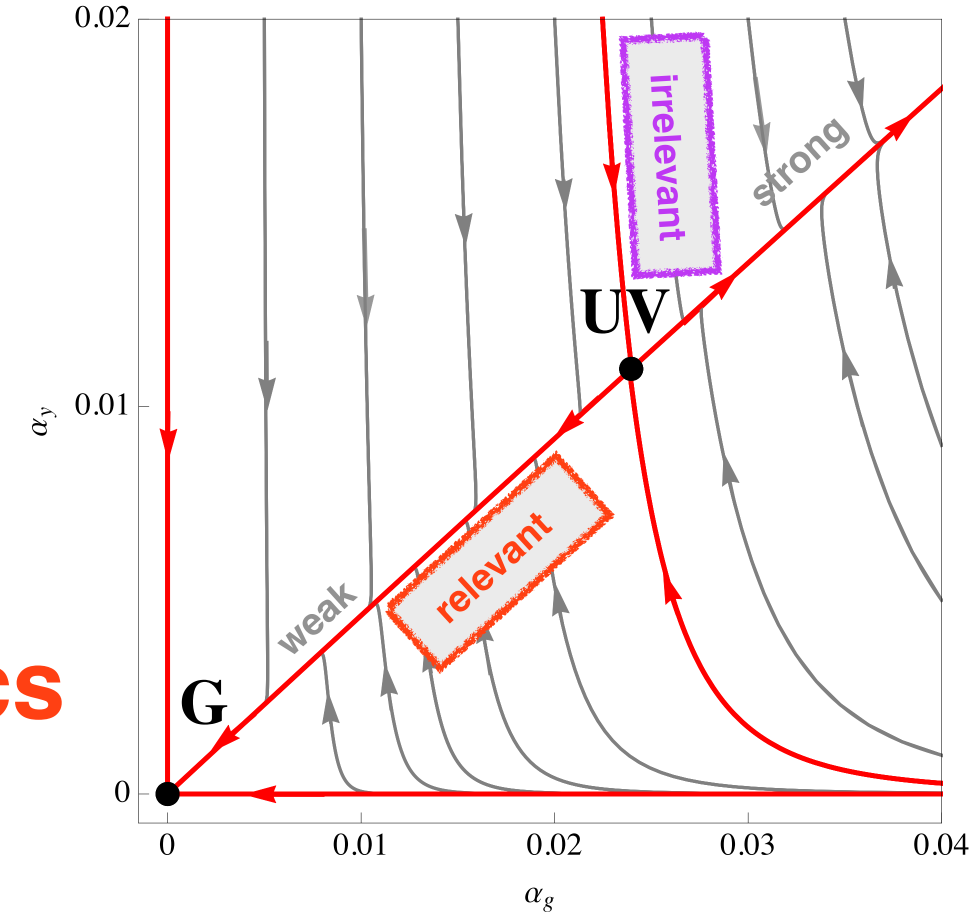
phase diagram



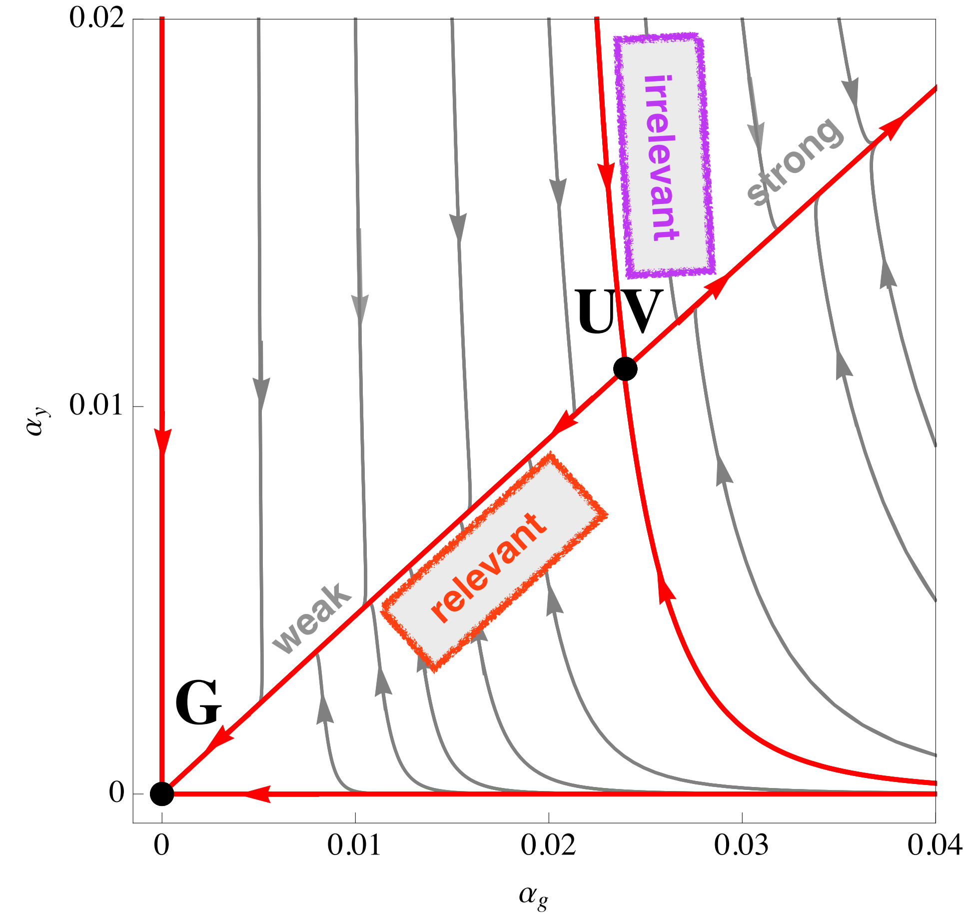


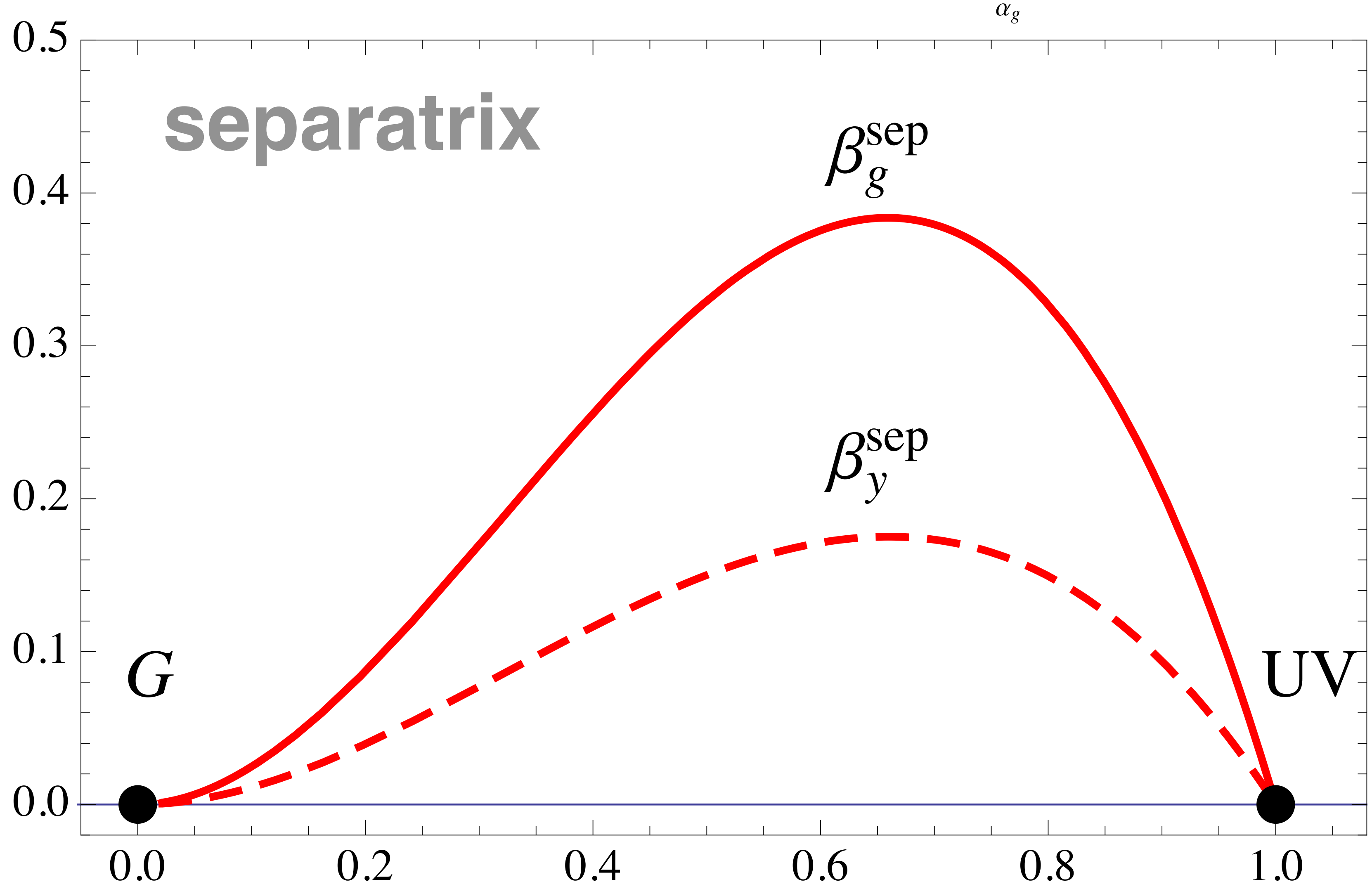
lines of physics

$$\alpha(\mu) = \frac{\alpha_*}{1 + W(\mu)}$$

$$W(\mu) = W_{\text{Lambert}}[z(\mu)]$$

$$z = \left(\frac{\mu_0}{\mu}\right)^{-B\cdot\alpha_*} \left(\frac{\alpha_*}{\alpha_0} - 1\right) \exp\left(\frac{\alpha_*}{\alpha_0} - 1\right) .$$





vacuum stability

vacuum must be stable classically
and quantum-mechanically

$$V \propto \alpha_v \mathrm{Tr}(H^\dagger H)^2 + \alpha_h (\mathrm{Tr} H^\dagger H)^2$$

stability

$$\alpha_h > 0 \quad \text{and} \quad \alpha_h + \alpha_v \geq 0 \qquad H_c \propto \delta_{ij}$$

$$\alpha_h < 0 \quad \text{and} \quad \alpha_h + \alpha_v / N_F \geq 0 \qquad H_c \propto \delta_{i1}$$

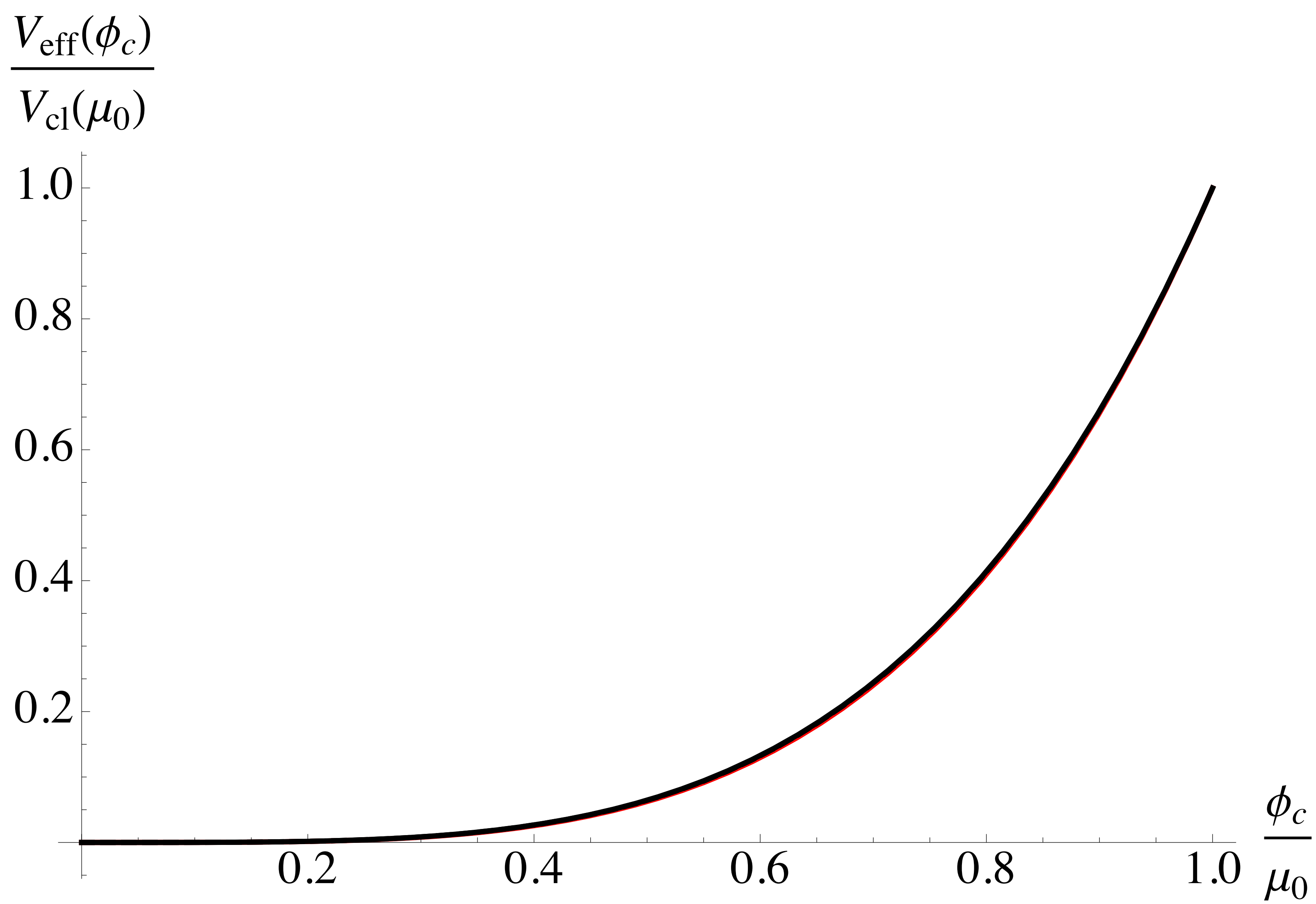
UV FP: $0 < \alpha_h^* + \alpha_v^*$ **ok**

quantum effects: integrate exact RG

$$\left(\mu_0 \frac{\partial}{\partial \mu_0} - \gamma(\alpha_j)\, \phi_c \frac{\partial}{\partial \phi_c} + \sum_i \beta_i(\alpha_j) \frac{\partial}{\partial \alpha_i}\right) V_{\mathrm{eff}}(\phi_c, \mu_0, \alpha_j) = 0$$

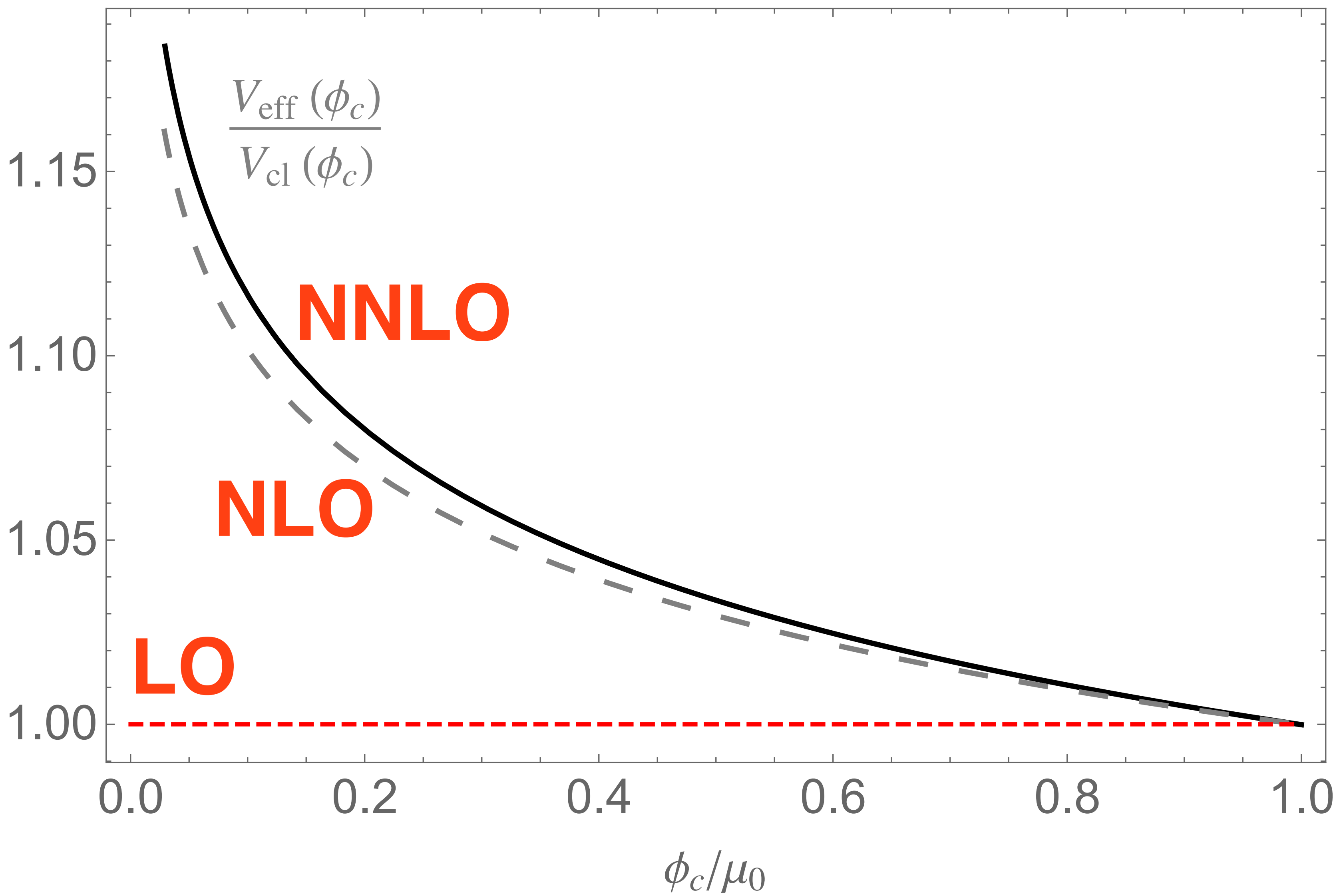
vacuum stability

Coleman-Weinberg potential



vacuum stability

quantum stability: resummation of logarithms



scalar effective potential defined for all scales
quantum vacuum is stable

summary

QFTs beyond asymptotic freedom

4D matter-gauge theories

exact **perturbative** proof of existence
all types of fields required
sensible interacting & UV finite theory
no supersymmetry

UV fixed points
in 4D quantum gravity

gravitation

physics of classical gravity

Einstein's theory $G_N = 6.7 \times 10^{-11} \frac{m^3}{\mathrm{kg}\, s^2}$

physics of quantum gravity

Planck length $\ell_{\mathrm{Pl}} = \left(\frac{\hbar G_N}{c^3}\right)^{1/2} \approx 10^{-33}\,\mathrm{cm}$

Planck mass $M_{\mathrm{Pl}} \approx 10^{19}\mathrm{GeV}$

Planck time $t_{\mathrm{Pl}} \approx 10^{-44}\,\mathrm{s}$

Planck temperature $T_{\mathrm{Pl}} \approx 10^{32}\,\mathrm{K}$

expect **quantum modifications** at energy scales M_{Pl}

perturbation theory

- **structure of UV divergences**

  gravity: $[g_{\mu\nu}] = 0\,, \quad [\mathrm{Ricci}] = 2\,, \quad [G_N] = 2 - d$

  effective expansion parameter: $g_{\mathrm{eff}} \equiv G_N\, E^2 \sim \frac{E^2}{M_{\mathrm{Pl}}^2}$

  N-loop Feynman diagram $\sim \int dp\, p^{A-[G]N}$

  $[G] > 0:$ superrenormalisable

  $[G] = 0:$ renormalisable

  $[G] < 0:$ **dangerous** interactions

- **perturbative non-renormalisability**

  gravity with matter interactions

  pure gravity (Goroff-Sagnotti term)

perturbation theory

- **effective theory for gravity** (Donoghue '94)

  quantum corrections computable for energies $E^2/M_{\rm Pl}^2 \ll 1$

  knowledge of UV completion not required

- **higher derivative gravity I** (Stelle '77)

  R^2 gravity perturbatively renormalisable

  unitarity issues at high energies

- **higher derivative gravity II** (Gomis, Weinberg '96)

  all higher derivative operators

  gravity 'weakly' perturbatively renormalisable

  no unitarity issues at high energies

quantum gravity

running coupling $g(k) = G_N(k) k^{D-2}$

$$\partial_t g = (D - 2 + \eta_N)\, g \qquad t = \ln k/\Lambda_c$$

quantum gravity

running coupling $g(k) = G_N(k) k^{D-2}$

$$\partial_t g = (D - 2 + \eta_N)\, g \qquad t = \ln k/\Lambda_c$$

$g_* \neq 0$ — UV

$g_* = 0$ — IR

fixed points

non-trivial anomalous dimension

quantum gravity

running coupling $g(k) = G_N(k) k^{D-2}$

$$\partial_t g = (D - 2 + \eta_N)\, g \qquad t = \ln k/\Lambda_c$$

$g_* \neq 0$ — UV

$g_* = 0$ — IR

fixed points

large anomalous dimension $\eta_N = \eta_N(g, \text{all other couplings})$

large UV scaling exponents $\vartheta \approx \mathcal{O}(1)$

strong coupling effects $g_* \approx \mathcal{O}(1)$

relevant vs **irrelevant**

invariants not known a priori

evidence for UV fixed point

overviews: DL **0810.3675** and **1102.4624**

gravitation

Einstein-Hilbert (Souma '99, Reuter, Lauscher '01, DL '03)

higher dimensions, dimensional reduction (DL '03, Fischer, DL '05)

f(R), polynomials in R (Lauscher, Reuter, '02, Codello, Percacci, Rahmede '08, Machado, Saueressig '09 Benedetti,Caravelli'12, Dietz, Morris'12, Falls, DL, Nikolakopoulos, Rahmede '13)

local potential approximation (Benedetti,Caravelli'12, Dietz, Morris, '12, Demmel, Saueressig, Zanusso '12, Falls, DL, Nikolakopoulos, Rahmede '13, Benedetti '13, Benedetti, Guarnieri '13)

higher-derivative gravity (Codello, Percacci '05) (Benedetti, Saueressig, Machado '09, Niedermaier '09) (DL, Rahmede, in prep.)

conformally reduced gravity (Reuter, Weyer '09, Machado, Percacci '10, DL, Satz '12)

Holst action + Immirzi parameter (Daum, Reuter '10, Benedetti, Speciale '11)

signature effects (Manrique, Rechenberger, Saueressig '11)

gravitation + matter

matter (Percacci '05, Perini, Percacci '05, Narain, Percacci '09, Narain, Rahmede '09, Codello '11, Eichhorn et al '13)

Yang-Mills gravity

1-loop: (Robinson, Wilzcek '05, Pietrokowski, '06, Toms '07, Ebett, Plefka, Rodigast '08)

beyond: (Manrique, Reuter, Saueressig '09, Folkerts, DL, Pawlowski `11, Harst, Reuter '11)

asymptotic freedom vs asymptotic safety

asymptotic freedom	asymptotic safety
$g_* = 0$	$g_* \neq 0$
anomalous dimensions	anomalous dimensions
$\eta_A = 0$	$\eta_N \neq 0$
canonical power counting	**non-canonical** power counting
$\{\vartheta_{\mathrm{G},n}\}$ are known	$\{\vartheta_n\}$ are **not** known
F^{256} irrelevant !	R^{256} relevant / marginal / irrelevant ?

bootstrap search strategy

hypothesis relevancy of invariants follows canonical dimension

strategy

Step 1 retain invariants up to mass dimension D

Step 2 compute $\{\vartheta_n\}$ (eg. RG, lattice, holography)

Step 3 enhance D, and iterate

convergence (no convergence) of the iteration:

hypothesis supported (refuted)

f(R)

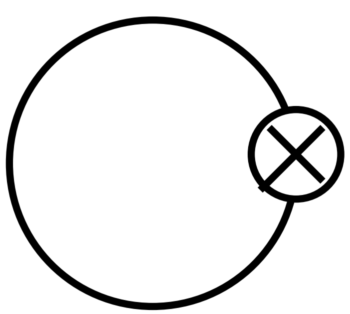
$$\Gamma_k \propto f(R)$$

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n \, k^{d_n} \int d^4x \sqrt{g}\, R^n$$

effective action with invariants up to mass dimension $D = 2(N-1)$

technicalities: functional renormalisation

$$k\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2}\,\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right] = \frac{1}{2}$$



here:

M Reuter hep-th/9605030

DL hep-th/0103195

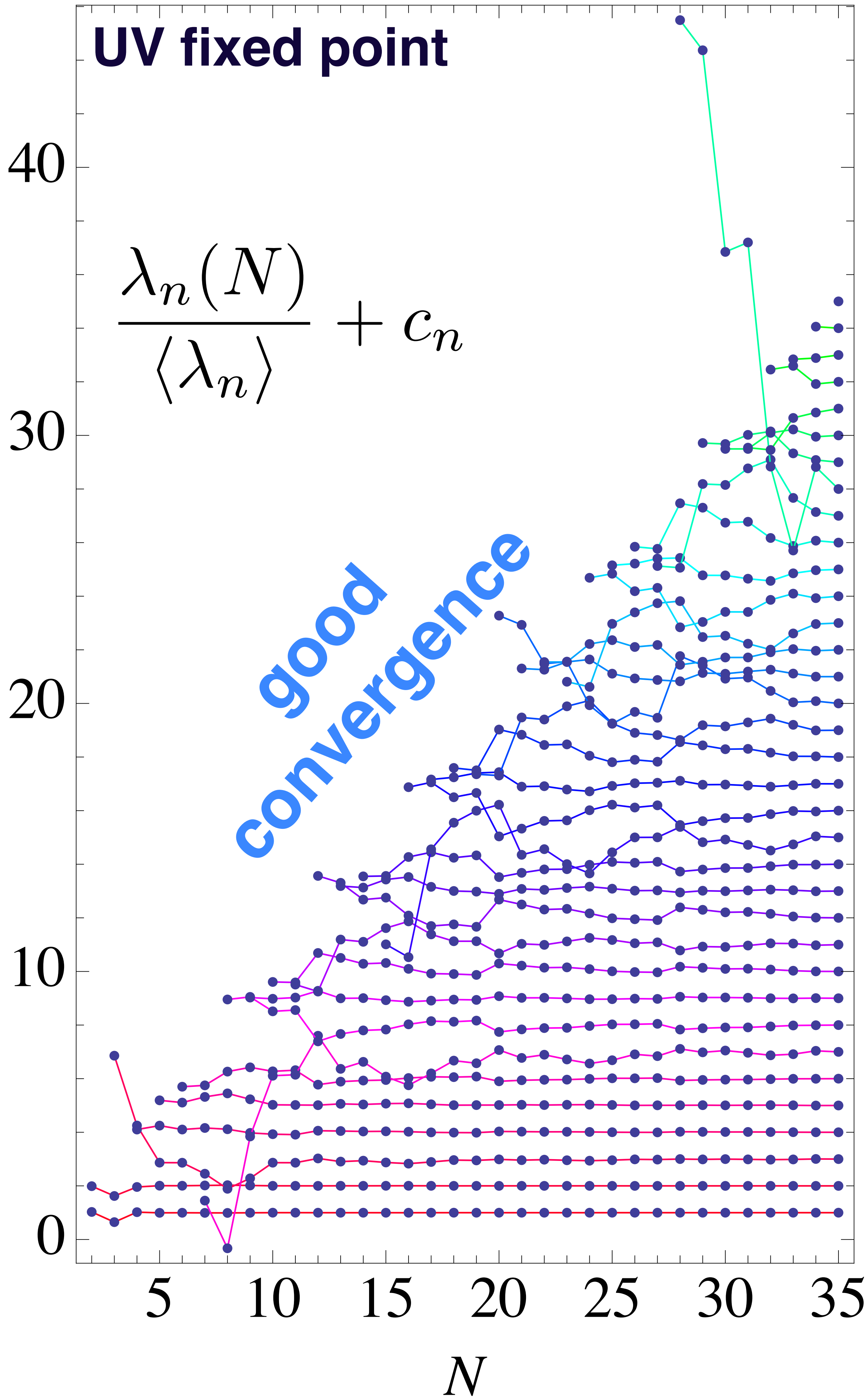
hep-th/0312114

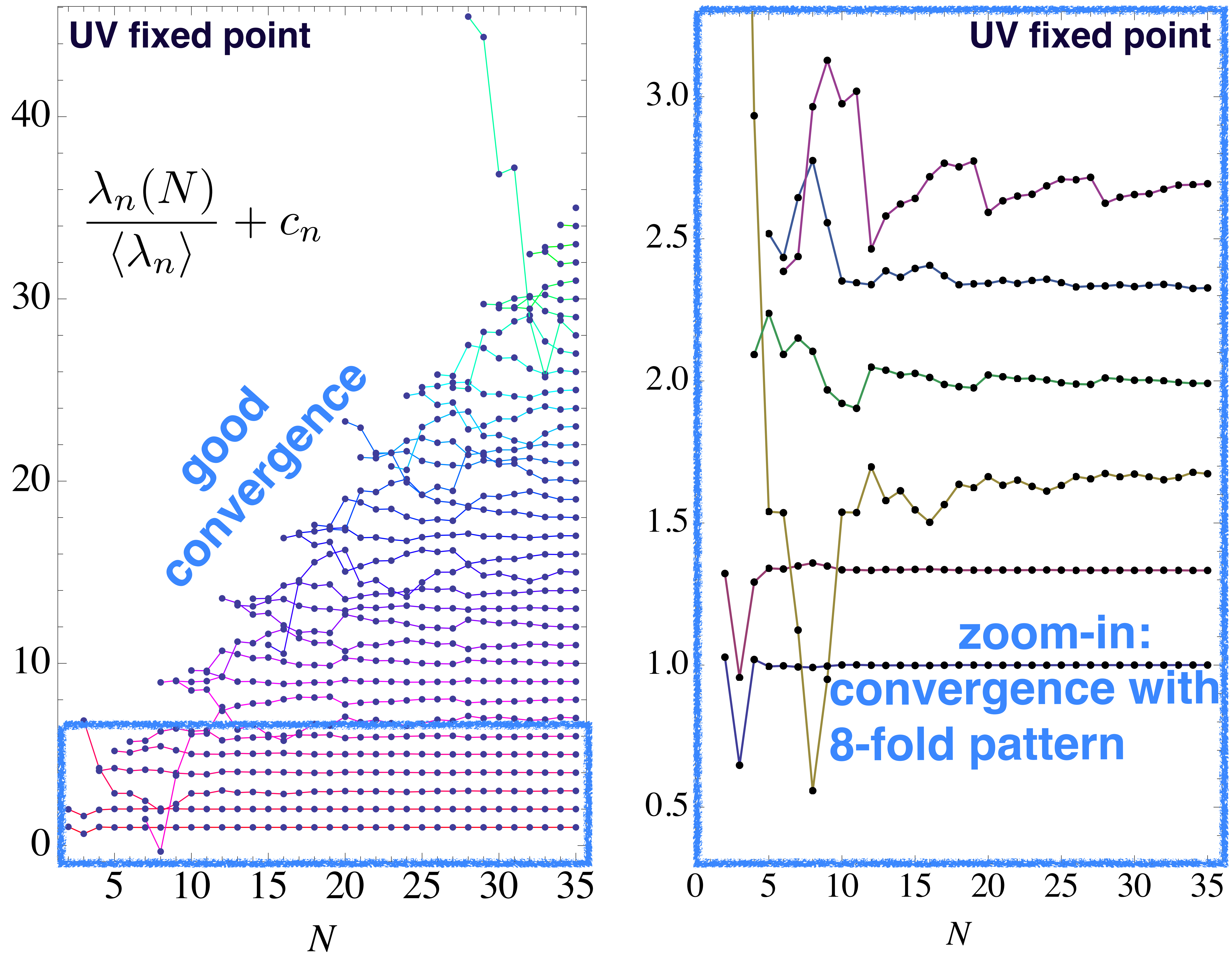
Falls, DL, Nikolakopoulos, Rahmede 1301.4191.pdf

Falls, DL, Nikolakopoulos, Rahmede 1410.4815

A Codello, R Percacci, C Rahmede 0705.1769, 0805.2909

P Machado, F Saueressig 0712.0445

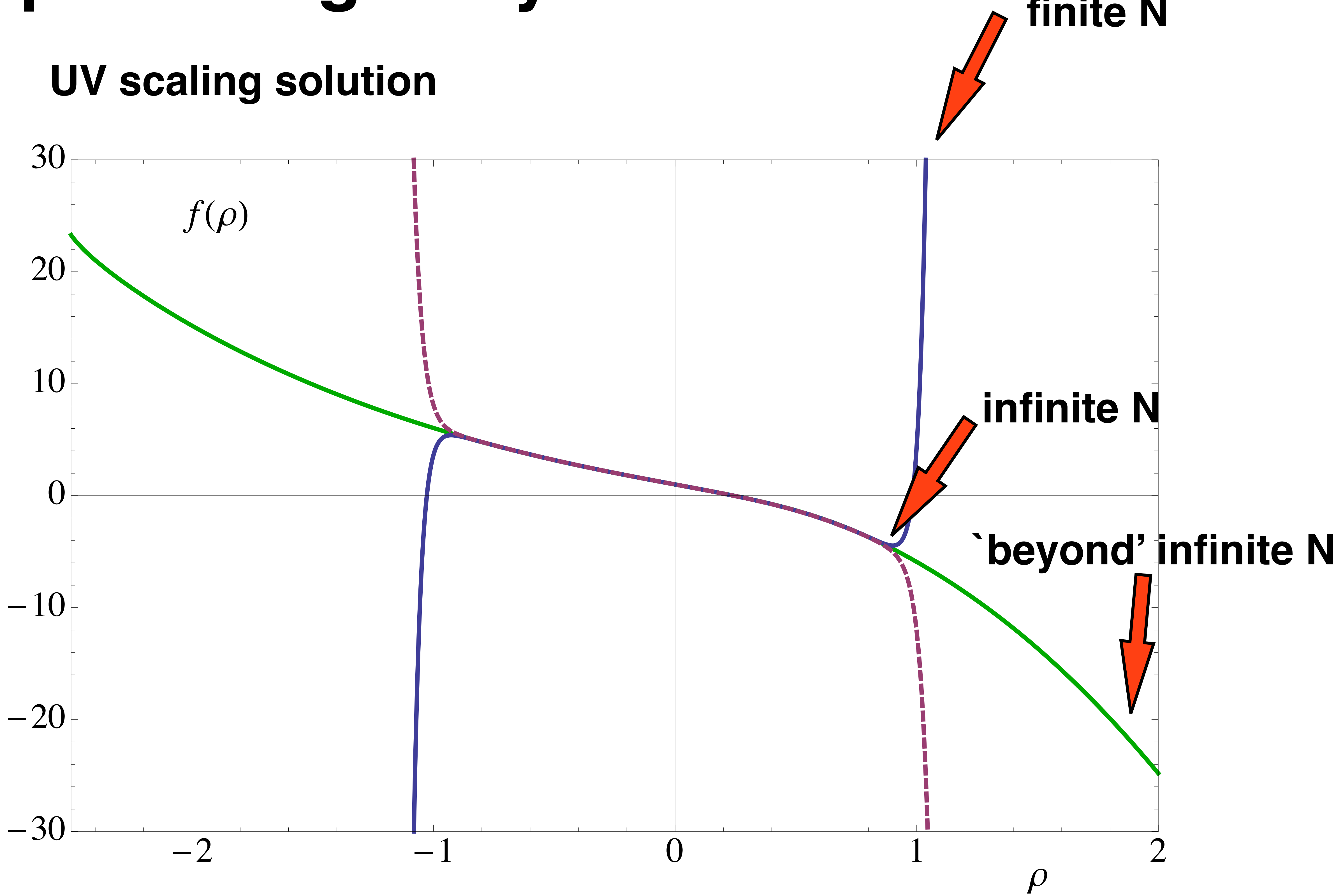

UV fixed point
$\frac{\lambda_n(N)}{\langle\lambda_n\rangle} + c_n$
good convergence
40
30
20
10
0
5
10
15
20
25
30
35
N



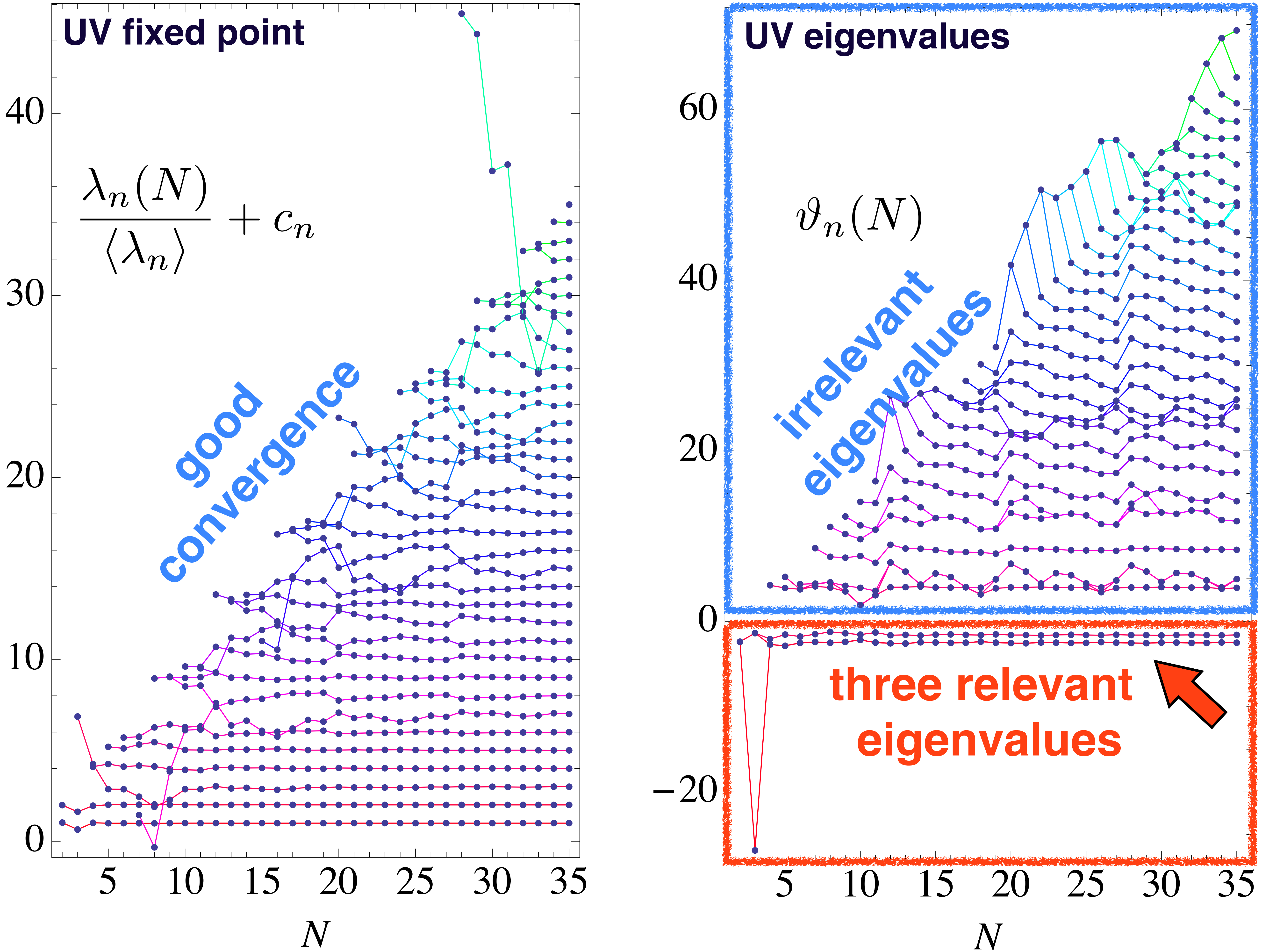
UV fixed point
$\frac{\lambda_n(N)}{\langle\lambda_n\rangle} + c_n$
good convergence
N
UV fixed point
zoom-in: convergence with 8-fold pattern
N

f(R) quantum gravity

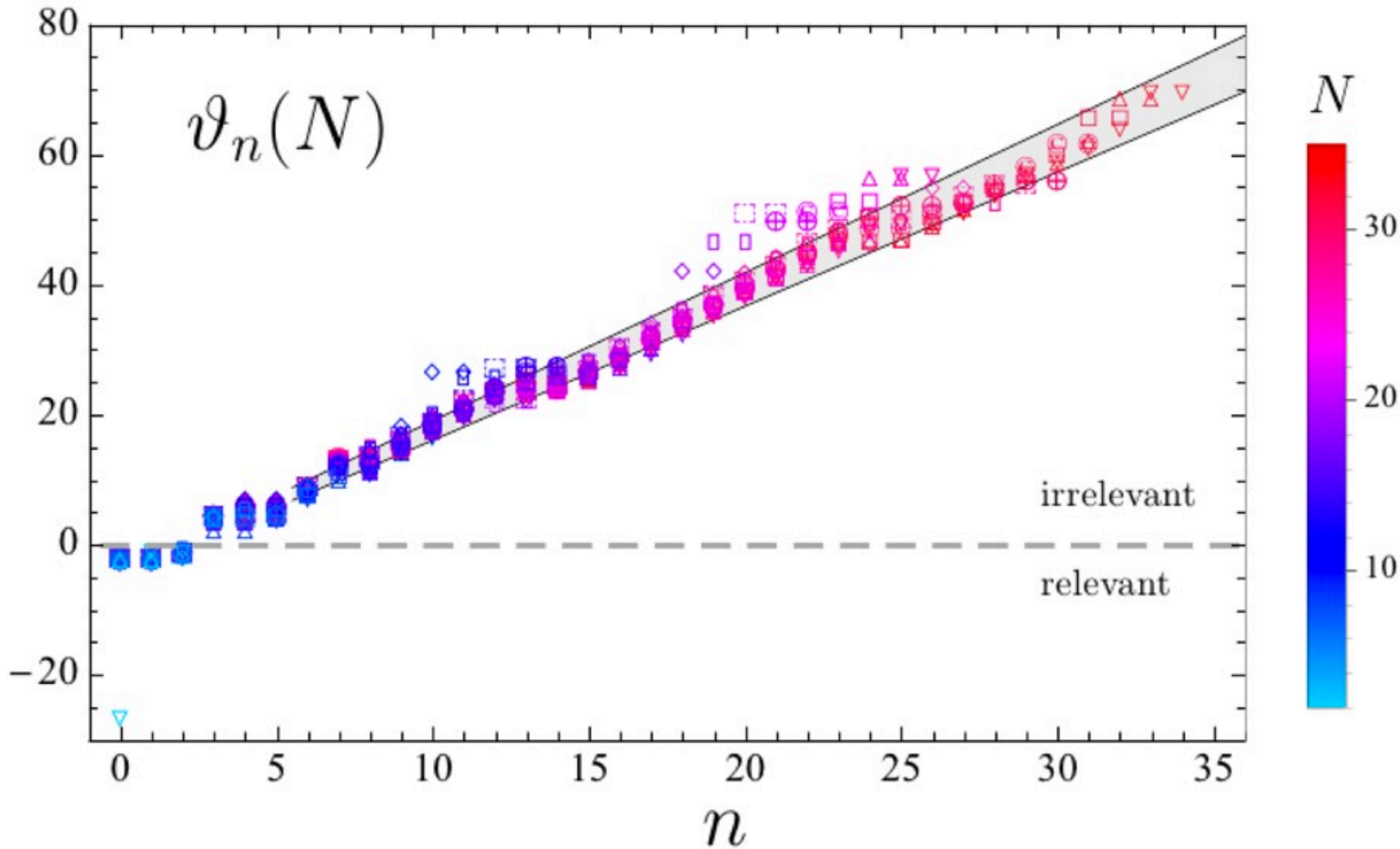
UV scaling solution



radius of convergence $\rho_c \approx 0.82 \pm 5\%$


UV fixed point
$\frac{\lambda_n(N)}{\langle\lambda_n\rangle} + c_n$
good convergence
40
30
20
10
0
5
10
15
20
25
30
35
N
UV eigenvalues
$\vartheta_n(N)$
irrelevant eigenvalues
three relevant eigenvalues
60
40
20
0
−20
5
10
15
20
25
30
35
N

near-Gaussian



f(Ricci)

$$\Gamma_k \propto \int d^d x \sqrt{g}\,[f_k(R_{\mu\nu}R^{\mu\nu}) + R \cdot z_k(R_{\mu\nu}R^{\mu\nu})]$$

$$\begin{aligned}\partial_t\Gamma[\bar{g},\bar{g}] =& \frac{1}{2}\mathrm{Tr}_{(2T)}\left[\frac{\partial_t\mathcal{R}_k^{h^T h^T}}{\Gamma^{(2)}_{h^T h^T}}\right] + \frac{1}{2}\mathrm{Tr}'_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\xi\xi}}{\Gamma^{(2)}_{\xi\xi}}\right] + \frac{1}{2}\mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\sigma\sigma}}{\Gamma^{(2)}_{\sigma\sigma}}\right] + \frac{1}{2}\mathrm{Tr}_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{hh}}{\Gamma^{(2)}_{hh}}\right]\\ &+ \mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\sigma h}}{\Gamma^{(2)}_{\sigma h}}\right] - \mathrm{Tr}_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{C}^T C^T}}{\Gamma^{(2)}_{\bar{C}^T C^T}}\right] - \mathrm{Tr}_{(0)'}\left[\frac{\partial_t\mathcal{R}_k^{\bar{\eta}\eta}}{\Gamma^{(2)}_{\bar{\eta}\eta}}\right] - \mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{\lambda}\lambda}}{\Gamma^{(2)}_{\bar{\lambda}\lambda}}\right]\\ &+ \frac{1}{2}\mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\omega\omega}}{\Gamma^{(2)}_{\omega\omega}}\right] - \mathrm{Tr}'_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{c}^T c^T}}{\Gamma^{(2)}_{\bar{c}^T c^T}}\right] + \frac{1}{2}\mathrm{Tr}'_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\zeta^T\zeta^T}}{\Gamma^{(2)}_{\zeta^T\zeta^T}}\right] + \mathrm{Tr}'_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{s}s}}{\Gamma^{(2)}_{\bar{s}s}}\right]\end{aligned}$$

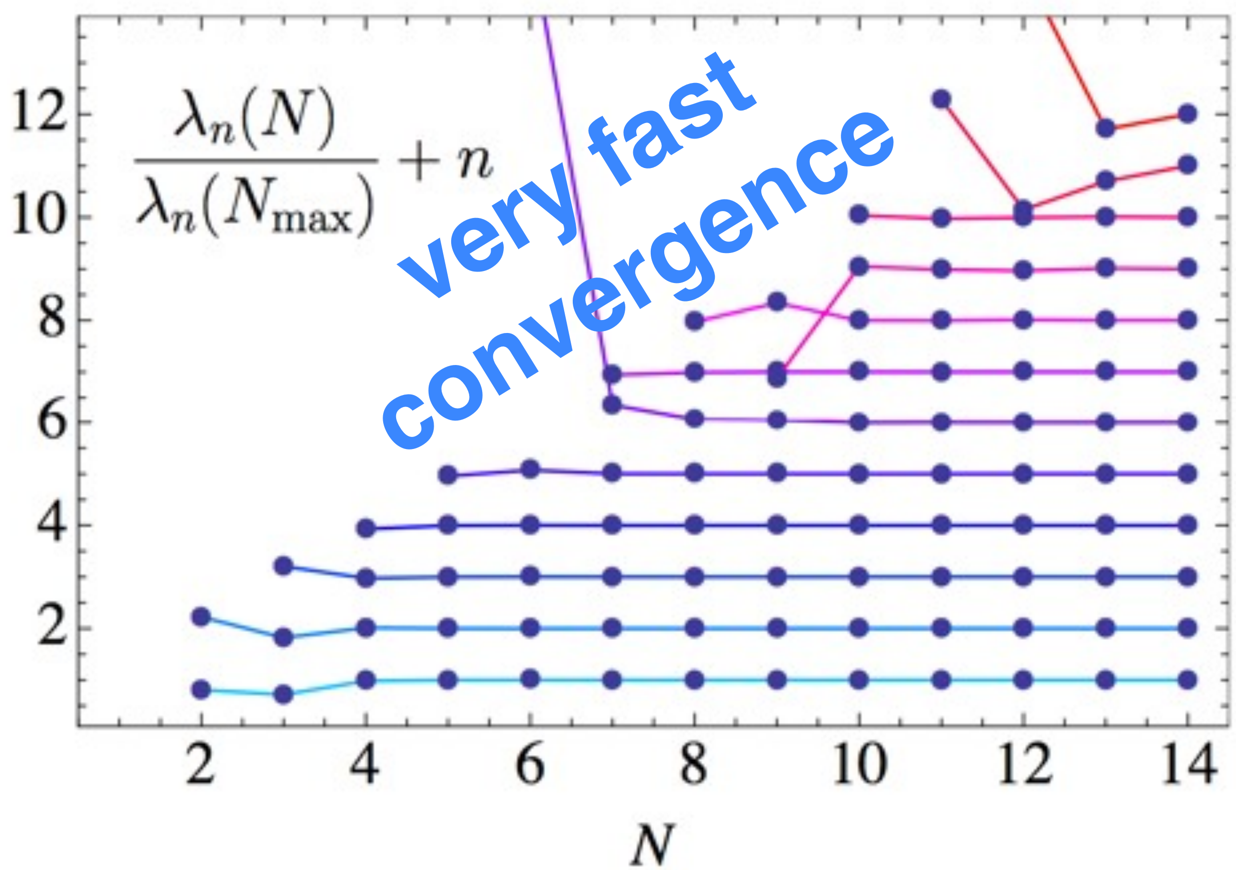
K Falls, DL, K Nikolakopoulos & C Rahmede, (to appear)

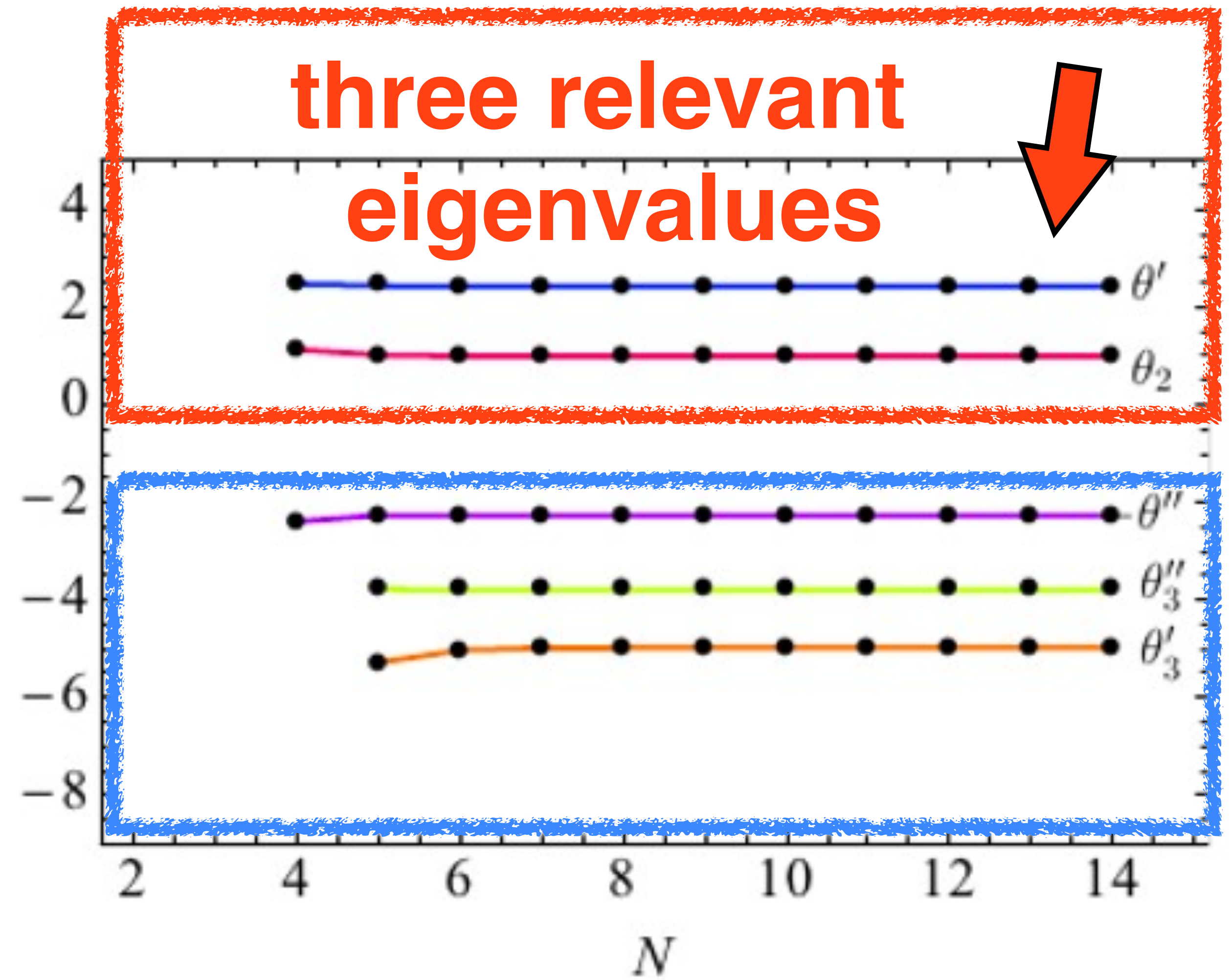
f(Ricci)

K Falls, DL, K Nikolakopoulos & C Rahmede, (to appear)

$$\Gamma_k \propto \int d^dx\sqrt{g}\,[f_k(R_{\mu\nu}R^{\mu\nu}) + R\cdot z_k(R_{\mu\nu}R^{\mu\nu})]$$

fixed point





conclusions

QFTs beyond asymptotic freedom

4D matter-gauge theories

exact proof of existence
requires elementary scalars, fermions, vectors
no additional (super)symmetry

4D quantum gravity

systematic non-perturbative search strategies
strong hints towards interacting UV fixed point
field-dependent anomalous dimension

conclusions

what's next?

4D matter-gauge theories

composite operators
UV FP beyond perturbation theory?
realistic models beyond SM?

4D quantum gravity

test further curvature invariants
include matter fields
combined FP for gravity-matter theories?