

On The Hierarchy Problem

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Sanity



— BORING

Three parts of the story:

* Dynamics of QCD flux tubes

* Integrable quantum gravity

* (**Bad**) idea for solving the EW hierarchy problem

— CRAZY



Hierarchy problem: first iteration

* We saw 125 GeV Higgs

* Quadratic divergencies indicate that for a *generic* new physics

$$\delta m_H^2 = \left(\frac{g^2}{16\pi^2} \right)^\# \Lambda_{NP}^2$$

* Gravity attests the presence of new physics coupled to the Standard Model at least at the Planck scale

* Of course also

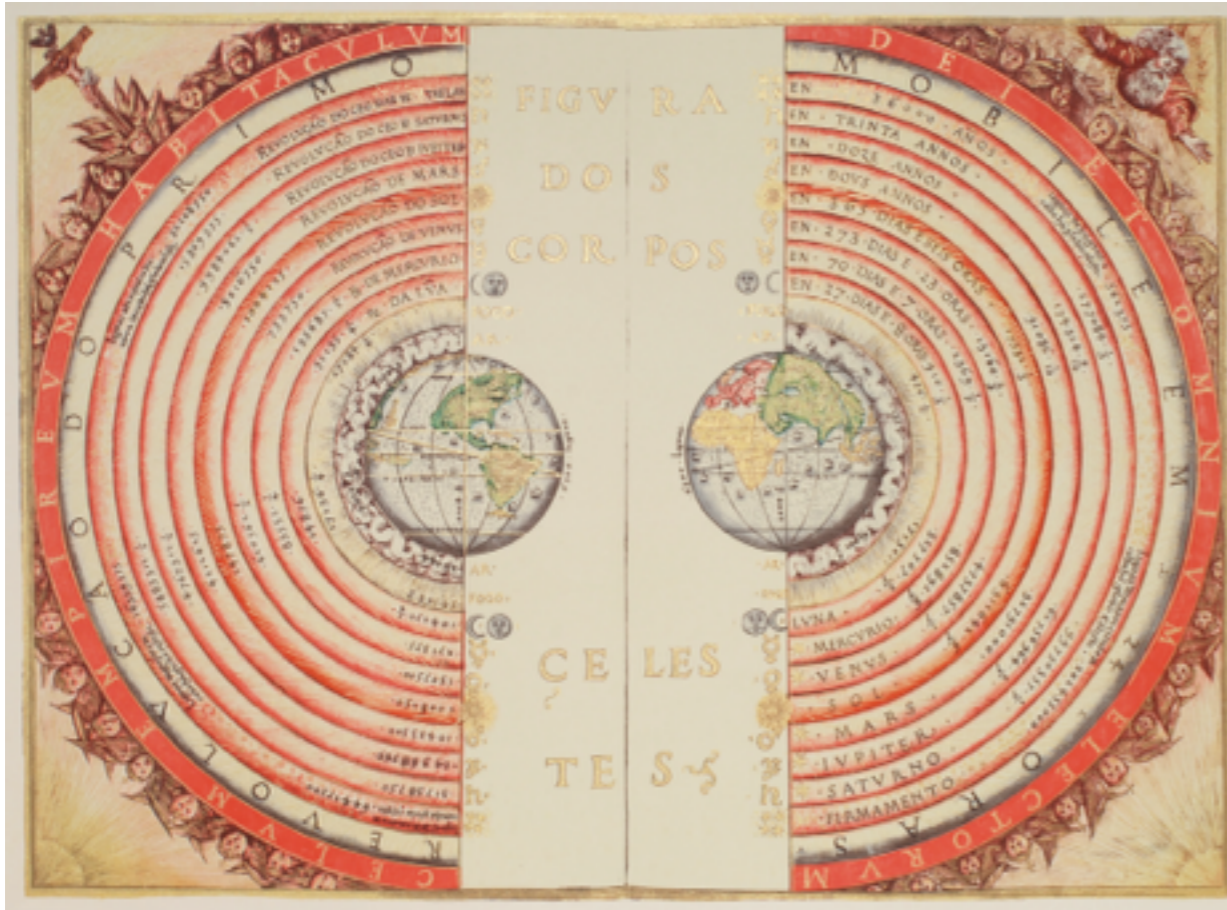
$$\delta \rho_{vac} \propto \Lambda_{UV}^4$$

(“Good”) Conservative ideas:

- * Quadratic divergencies are cancelled by new TeV scale physics
- * Electroweak scale (and/or CC) is tuned as a consequence of anthropic selection

Is there a third way?

For the solar system



Changes the way we think about our own solar system

Some people got fired for these ideas

Popular View of the Situation:



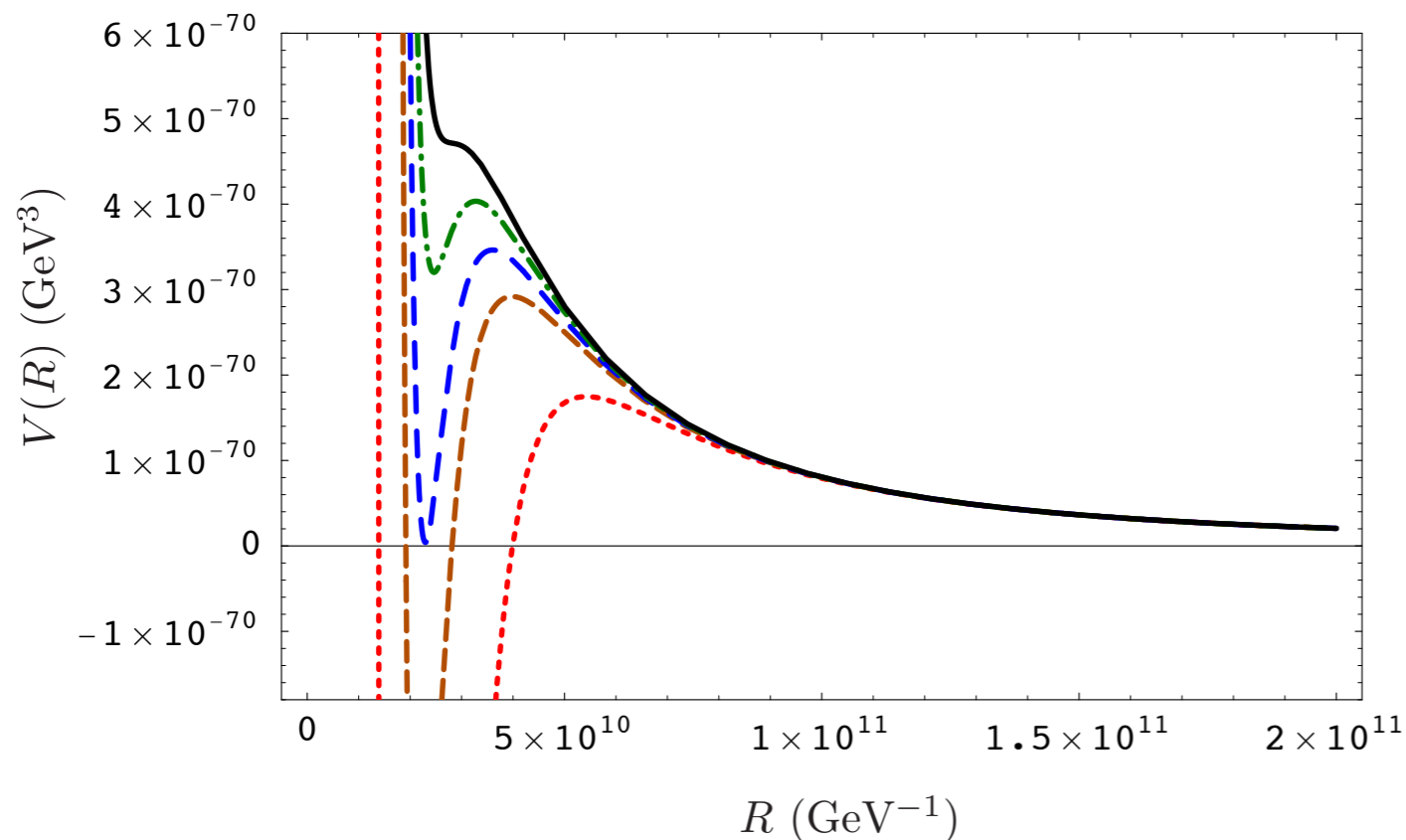
The moment we talk about naturalness we are in the Landscape/Multiverse!

A circular image showing a mountain range with a valley and a lake. The mountains are rugged and brownish, with a prominent peak in the center. The valley below is green and brown, with a dark blue lake in the foreground. The sky is blue with white clouds.

How do we test that the Landscape is there?

Landscape is a *generic* feature of gravitational theories rather than an evil product of string theory

Standard Model $AdS_3 \times S_1$ vacuum:



$$\Delta m_{\odot}^2 = 8.0 \cdot 10^{-5} \text{eV}^2 \text{ (actual value)}$$

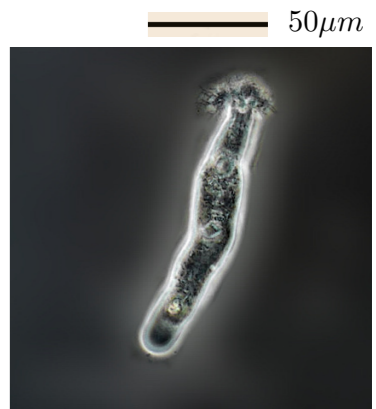
$$\Delta m_{\odot}^2 = 2.0 \cdot 10^{-5} \text{eV}^2$$

$$\Delta m_{\odot}^2 = 1.5 \cdot 10^{-5} \text{eV}^2$$

$$\Delta m_{\odot}^2 = 1.2 \cdot 10^{-5} \text{eV}^2$$

$$2\pi R_0 \approx 20 \mu\text{m}$$

$$l_{AdS} \approx 3.7 \cdot 10^{27} \text{ cm}$$



Trichamoeba sp.

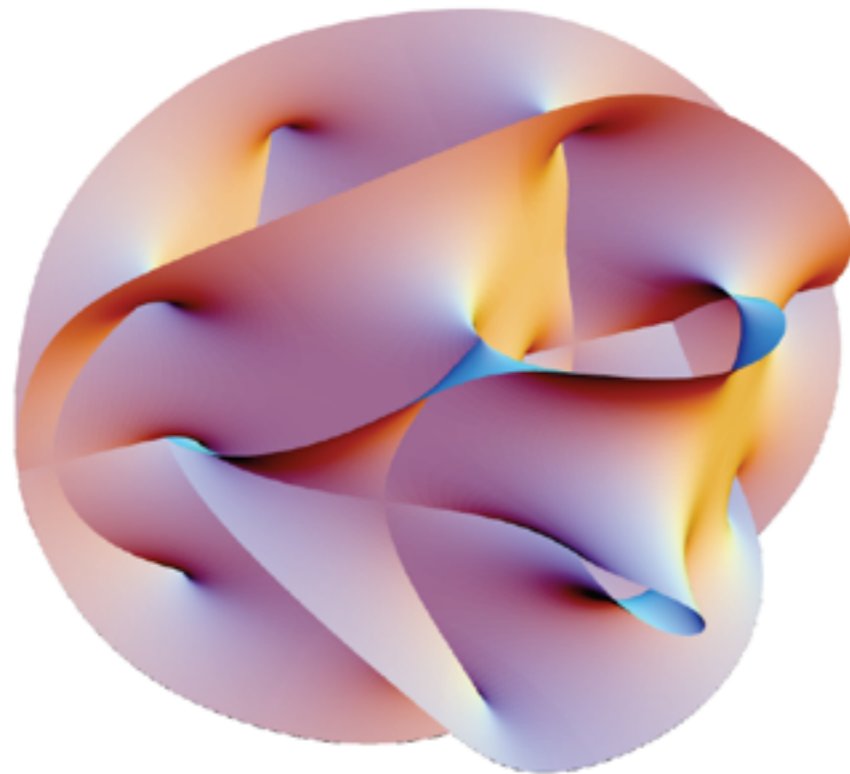
typical inhabitant of the SM Landscape

Photon Wilson line gives rise to a valley with a slope

$$\propto e^{-m_e/m_\nu} \sim e^{-10^8}$$

Where numbers like 10^{500} come from?

MULTIVERSE = Xdimensions
+
Non-trivial Topology
+
Gauge Fields



So perhaps instead of

How do we test that the Landscape is there?

a more accurate question is

How do we test the richness and dynamical
relevance of the Landscape?

Seeing traces of bubble nucleation would be
a rare example of a direct test

Minimalism

vs

Plenitude



William of Ockham



Gottfried Wilhelm Leibniz

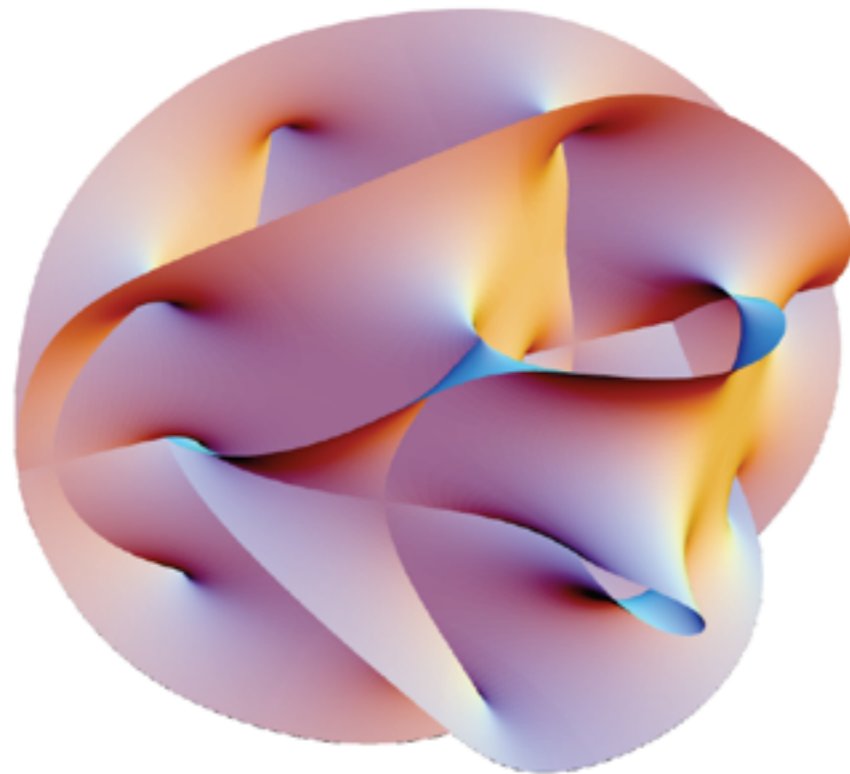
*“entia non sunt multiplicanda
praeter necessitatem”*

entities must not be multiplied
beyond necessity

*“This best of all possible worlds
will contain all possibilities,
with our finite experience of
eternity giving no reason to
dispute nature's perfection.”*

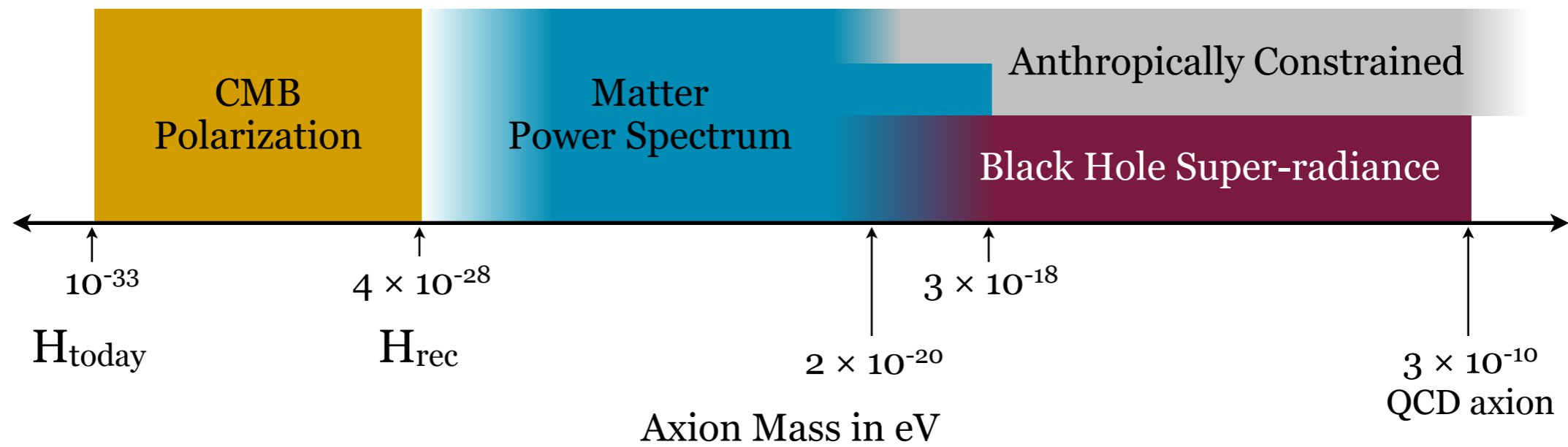
Example of model building inspired by *Plenitude*:

$$\mathbf{AXIVERSE} = \begin{array}{l} \text{Xdimensions} \\ + \\ \text{Non-trivial Topology} \\ + \\ \text{Gauge Fields} \end{array}$$



AXIVERSE=a plenitude of light axions

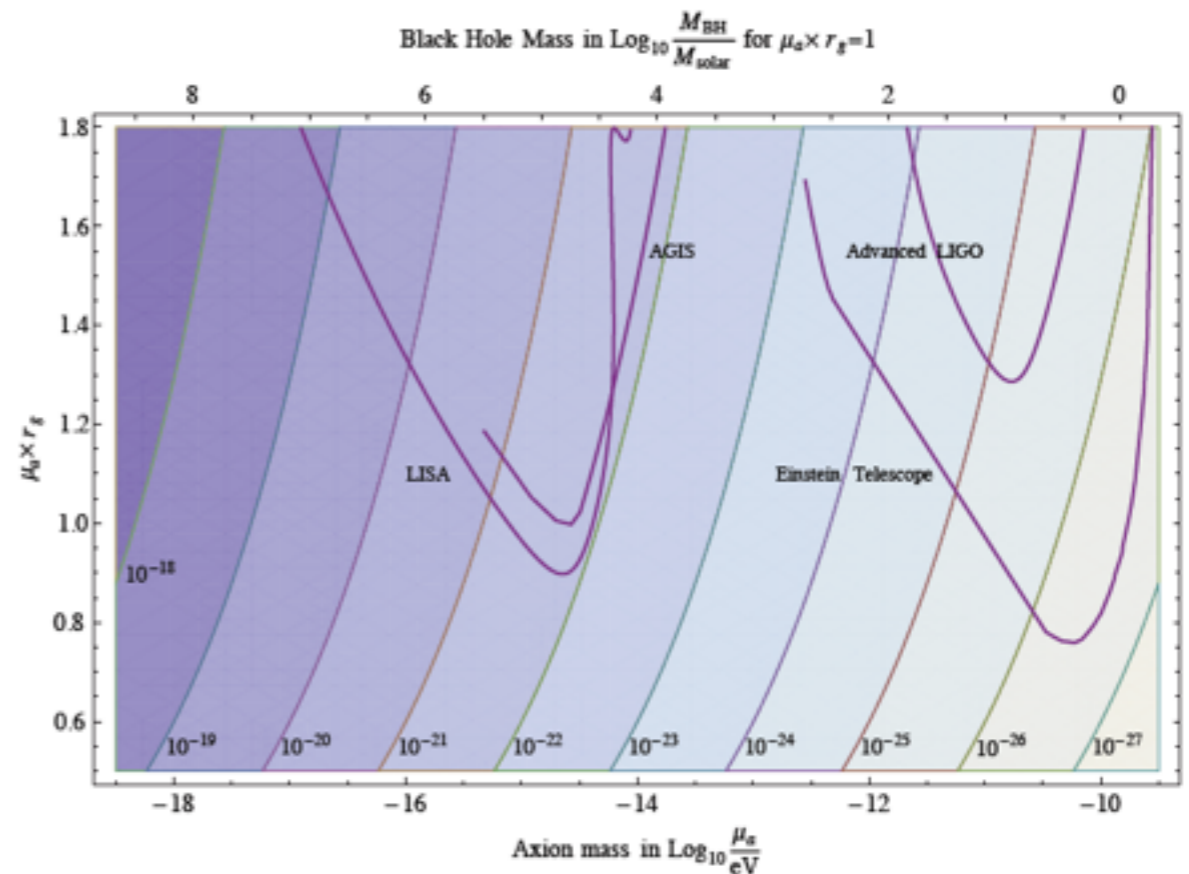
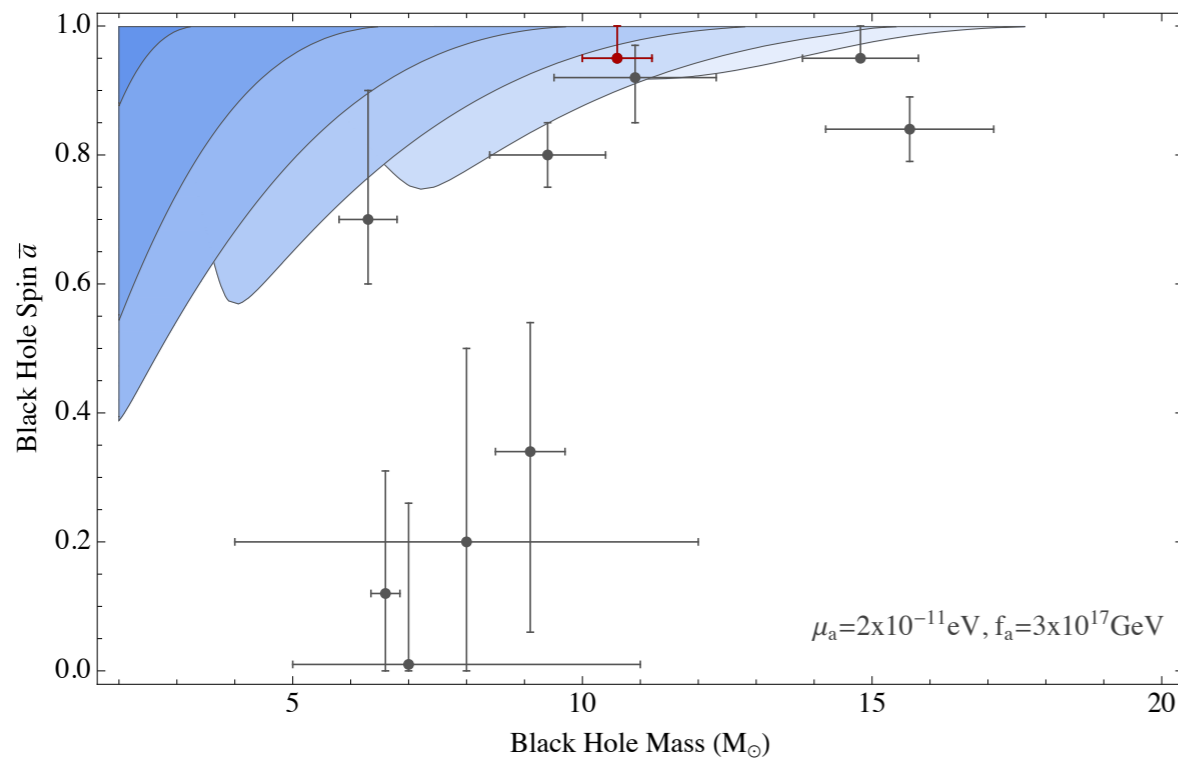
- * Axions are generic on the Landscape, reflecting the topological complexity of compactification manifold (*c.f. SM Landscape*)
- * QCD provides a strong hint at least for one axion
- * QCD axion is **not** anthropic, so it would be strange if it were alone
- * Axion masses are exponentially sensitive to detail of compactification (*c.f. SM Landscape*)



Astrophysical signatures over 23 orders of magnitude in length scale

* Black holes as probes of axions

* Advanced LIGO is a discovery machine for the QCD axion?!



(“Good”) Conservative ideas:

- * Quadratic divergencies are cancelled by new TeV scale physics
- * Electroweak scale is tuned as a consequence of anthropic selection

(“Bad”) Radical idea: *NATURAL TUNING*

- * Nature does not calculate in the Wilsonian way

Which way it calculates???

Proof of Concept

NB: The construction will be in (1+1)d. 2d theories are special in many respects, but not as far as the hierarchy problem goes

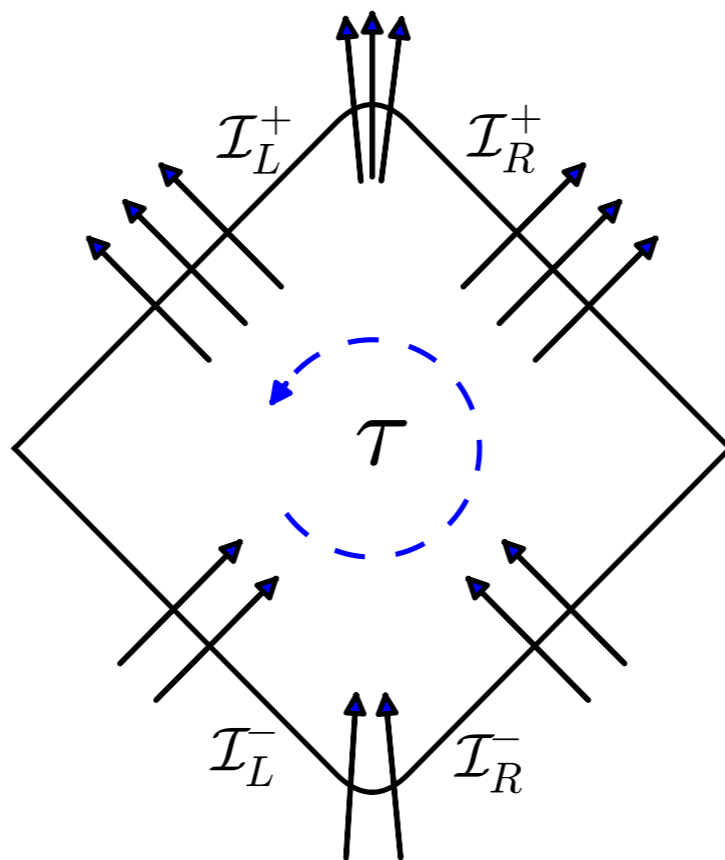
Start with a UV complete natural QFT $\mathcal{L}(\psi, H)$
Non-protected scalars are allowed as soon as they are heavy



Calculate S-matrix $S_n(p_i)$



“Gravitational dressing” gives $\hat{S}_n(p_i, \ell)$



$$\hat{S}_n(p_i) = e^{i\ell^2/4 \sum_{i < j} p_i^* p_j} S_n(p_i)$$

Properties of gravitational dressing

- * Results in a well-to-do S-matrix
- * Physical spectrum remains the same
- * Low energy EFT description, tuned for $m\ell \ll 1$

$$\mathcal{L}(\psi, H) + \sum_{\Delta_i > 2} \ell^{\Delta_i - 2} \mathcal{O}_i$$

free massive scalar:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{\ell^2}{8} \left((\partial\phi)^4 - m^4\phi^4 \right) + \dots$$

***THIS CONSTRUCTION SHOULD NOT
BE POSSIBLE !!!**

Am I cheating?

We never see fine-tuning at the S-matrix level...

I feel the construction is interesting:

- * Normally, one has to go through the Lagrangian to construct the theory, that's where the fine-tuning enters. Here we escaped this path.
- * Even stronger: we are not aware of the Wilsonian path to define the theory at all energies, and it appears very likely that it does not exist.
- * New asymptotic behavior at large energies. No mass thresholds at the scale ℓ^{-1} .

Hierarchy Problem

*Directly in terms of properties of the RG flow,
without ever mentioning quadratic divergencies*

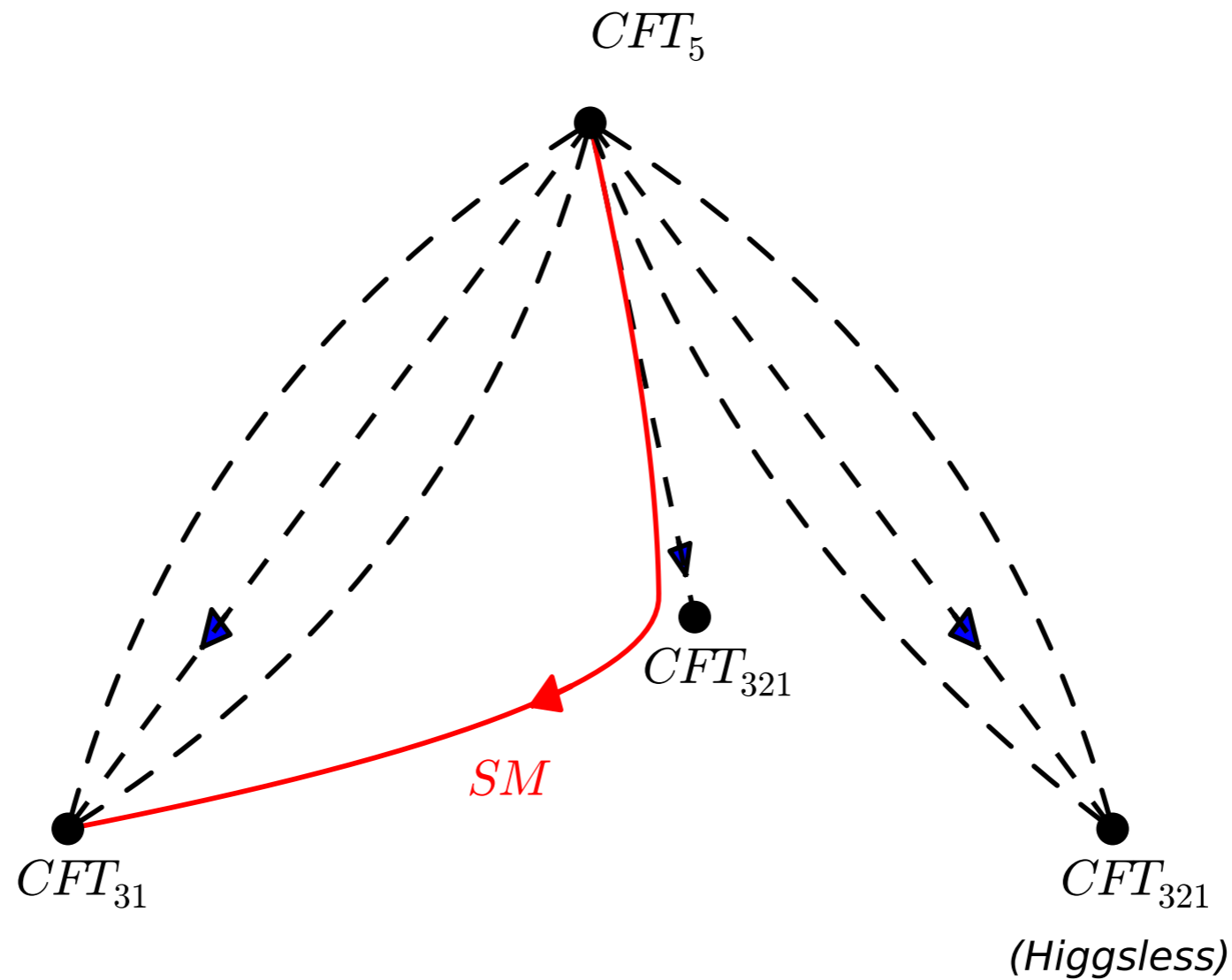
For concreteness, let us place the discussion in the context
of non-SUSY GUTs

$$m_H \ll E \ll m_{GUT} : \mathcal{L} = CFT_{321} + \underbrace{m_H^2 H^2}_{\text{relevant}} + \sum_i \frac{\mathcal{O}_i}{\underbrace{M_{GUT}^{\Delta_i - 4}}_{\text{irrelevant}}}$$

How comes $m_H \ll m_{GUT}$ given no symmetry?

However, fine-tuning is truly manifest only as
seen from higher energies:

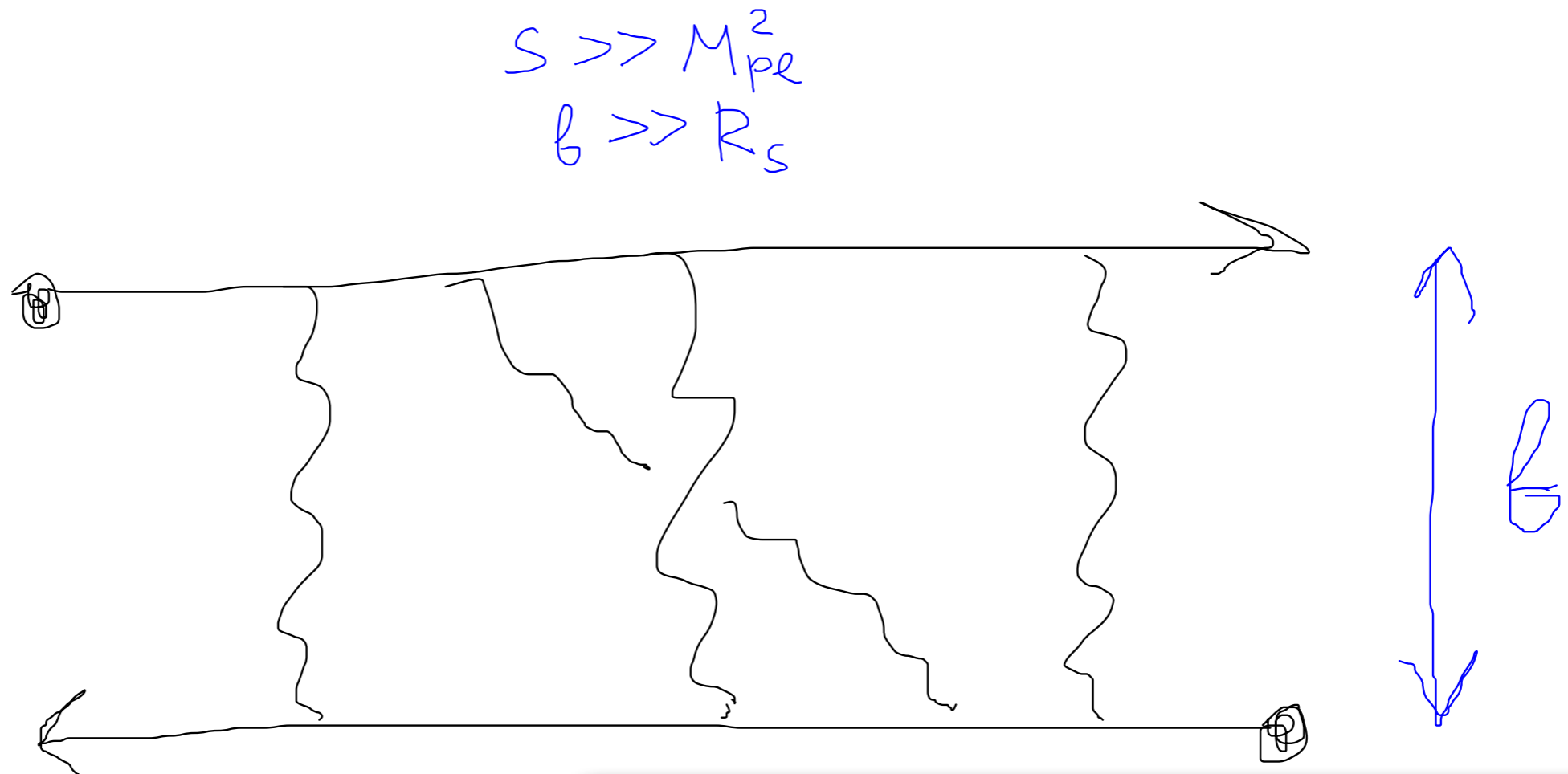
$$m_{GUT} \ll E : \mathcal{L} = CFT_5 + \underbrace{g_h m_{GUT}^2 H^2}_{\text{relevant}} + \underbrace{g_\Sigma m_{GUT}^2 \Sigma^2}_{\text{relevant}} + \dots$$



No picture like that in our example. Energy scale does not correspond to a threshold. No scale invariance and no Wilsonian RG above the scale.

Integrable QG rather than QFT

Gravitational shock waves:



Eikonal phase shift:

$$e^{i2\delta_{eik}(s)} = e^{i\ell^2 s/4}$$

$$\ell^2 \propto G_N b^{4-d}$$

Possible lesson:

Should we be more serious about thinking on-shell when gravity is involved?

CC:

*Off-shell: nothing special about zero vacuum energy

*On-shell: zero CC is extremely special:

AdS:CFT, Minkowski:S-matrix, de Sitter: ???

Is there a place for this scenario within the “standard” picture/string theory?

Two canonical regions in the Landscape capable of producing a light Higgs:

* An island where the Higgs mass is protected by a symmetry (SUSY...)

* Among “ 10^{100} ” or so of random vacua with randomly distributed values of the Higgs mass

Is there a third one?

* **Dragonland:** A (small) set of strongly coupled vacua: $g_s = 1$ and Planckian extra dimensions

Another possible lesson/alternative definition of naturalness:

Every natural QFT is an answer to some question.

Perhaps we should learn to ask more questions.

c.f. the following naturalness problem:

31415926535897932384626433832795028841971693993...

is this sequence of digits “natural”?