L3 1

Presusy Summer School Lectures on Scattering Amplitudes : Aug 20-22, 2015

Lecture 3

Y'day : BCFW shift $[1,2\rangle$-shift

$$|\hat{1}] = |1] + z|2] \quad , \quad |\hat{2}\rangle = |2\rangle - z|1\rangle .$$

& no other spinor brackets shift.

Also little grp determines 3pt, ex.

$$A_3(g_1^- \tilde{q}_2 \tilde{q}_3^*) = g \frac{\langle 12\rangle\langle 13\rangle}{\langle 23\rangle}$$

$$A_3(g_1^+ \tilde{q}_2 \tilde{q}_3^*) = g \frac{[12][13]}{[23]} .$$

[One can show $[-, \text{anything}\rangle$ - shift is always good.]

So let's use the $[-,\pm\rangle$ shift to calculate

$$A_4(g_1^- g_2^\pm \tilde{q}_3 \tilde{q}_4^*)$$

$$= A_4[g_1^- g_2^\pm \tilde{q}_3 \tilde{q}_4^*](T^a T^b)_{ij}$$

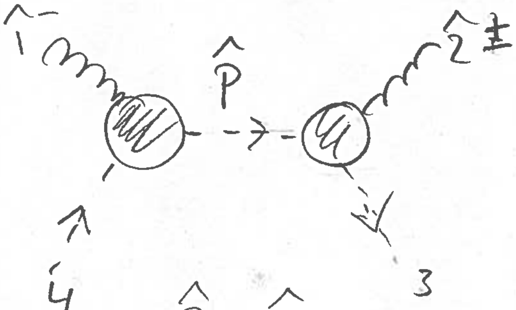
$$+ A_4[g_2^\pm g_1^- \tilde{q}_3 \tilde{q}_4^*](T^b T^a)_{ij} = 1 + 2$$

$$\sum_{\substack{\text{color} \\ \text{hel.}}} |A_4|^2 = \sum_{\text{helicities}} \left((1)^2 + (2)^2 \right) \mathrm{Tr}(T^a T^b T^b T^a)$$

$$+ \left(1\, 2^* + 1^*\, 2 \right) \mathrm{Tr}(T^a T^b T^a T^b)$$

L23 (2)

$A_4[g_1^- g_2^\pm \tilde{q}_3 \tilde{q}_4^*] =$ BCFW $[1,2\rangle$



no other diagrams!

$\hat{P} = \hat{P}_{24}$.

$$0 = \hat{P}_{14}^2 = \hat{P}_{23} = \langle \hat{2} 3\rangle [23]$$

$\Rightarrow \langle \hat{2} 3 \rangle = 0$. But then $A_3(g_2^-, 3, \hat{P}) = 0$

so diagram vanishes.

We conclude $A_4[g_1^\pm g_2^\pm \tilde{q}_3 \tilde{q}_4^*] = 0$

→ so now consider opposite helicity gluons:

$$A_4[g_1^- g_2^+ \tilde{q}_3 \tilde{q}_4^*] = A_3[g_1^- \tilde{q}_{-\hat{P}} \tilde{q}_4^*] \frac{1}{P_{14}^2} A_3[g_2^+ \tilde{q}_3 \tilde{q}_{\hat{P}}^*]$$

$$= g^2 \frac{\langle \hat{1} \hat{P} \rangle \langle \hat{1} 4 \rangle}{\langle \hat{P} 4 \rangle} \cdot \frac{1}{P_{14}^2} \frac{[\hat{2} 3][\hat{2} \hat{P}]}{[3 \hat{P}]}$$

- $\langle 1 \hat{P} \rangle [2 \hat{P}] = \langle 1 | \hat{P} | 2] = \langle 1 | \hat{1} + 4 | 2] = \langle 1 | 4 | 2]$
  $= \langle 14 \rangle [24]$

- $\langle \hat{P} 4 \rangle [3 \hat{P}] = -\langle 4 | \hat{P} | 3] = -\langle 4 | \hat{1} + 4 | 3] = -\langle 41 \rangle [\hat{1} 3]$.

location of pole: $\langle \hat{2} 3 \rangle = 0 = \langle 23 \rangle - z \langle 13 \rangle \Rightarrow z = \frac{\langle 23 \rangle}{\langle 13 \rangle}$

so $[\hat{1} 3] = \frac{1}{\langle 13 \rangle} \left[[13]\langle 13 \rangle + [23]\langle 23 \rangle \right] = \frac{1}{\langle 13 \rangle} \left(-\langle 34 \rangle [34] \right)$

$2p_1 \cdot p_3 + 2p_2 \cdot p_3 = 2p_3(p_1+p_2) = -2p_3 \cdot \ldots$

So

- $\langle \hat{p}4\rangle [3\hat{p}] = \dfrac{\langle 14\rangle \langle 34\rangle [34]}{\langle 13\rangle}$

So we get

$$A_4[g_1^- g_2^+ \tilde{q}_3 \tilde{q}_4^*] = g^2 \frac{\cancel{\langle 14\rangle}[23]\left(\cancel{\langle 14\rangle}[24]\right)}{\cancel{\langle 14\rangle}[14] \cdot \dfrac{\cancel{\langle 14\rangle}\langle 34\rangle[34]}{\langle 13\rangle}}$$

$$= g^2 \frac{[23][24]}{[34][14]} \frac{\langle 13\rangle \quad [12]}{\cancel{\langle 34\rangle}\;[12]} \to -\cancel{\langle 43\rangle}[42]$$

$$= g^2 \frac{[23]^2[24]^2}{[12][23][34][41]} \qquad \text{ok.}$$

c.c. $\Rightarrow$

$$A_4[g_1^+ g_2^- \tilde{q}_3 \tilde{q}_4^*] = \frac{\langle 23\rangle^2 \langle 24\rangle^2}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}.$$

Now

$$|①|^2 = |A_4(g_1^- g_2^+ \tilde{q}_3 \tilde{q}_3^*)|^2 = g^4 \frac{u^2 t^2}{s\,u\,s\cdot u} = g^4 \frac{t^2}{s^2}$$

$$|②|^2 = |①|^2(1 \leftrightarrow 2) = g^4 \frac{u^2}{s^2}$$

$$|①|^2 + |②|^2 = g^4 \frac{t^2+u^2}{s^2} = g^4\left(1 - 2\frac{tu}{s^2}\right) \qquad \text{Using } s+t+u=0.$$

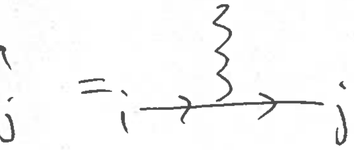
$$① \times ②^* + \text{c.c} = 2\frac{tu}{s^2}.$$

So

helicity sum

$$\sum_{\substack{\text{color}\\ \text{hel.}}} |A_4|^2 = 2g^4\left[\left(1-2\frac{tu}{s^2}\right)\mathrm{Tr}(T^aT^bT^bT^a) + 2\frac{tu}{s^2}\mathrm{Tr}(T^aT^bT^aT^b)\right]$$

Now, an aside: how to diagrammatically calculate color traces:

Def. $T^a_{ij} =$  $\Rightarrow$  $= 0$ $(\mathrm{Tr}\,T^a = 0)$

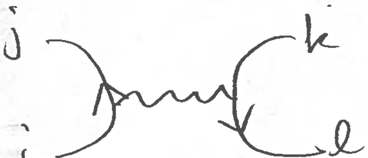
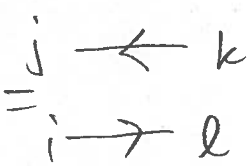
 $=$ 

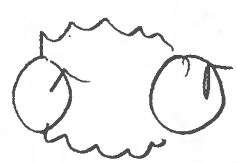
$\mathrm{tr}(T^aT^b) = \delta^{ab}$

 $= N$,  $= N^2-1$

 $=$  $=$  $= N^2-1.$

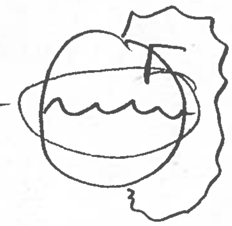
$$T^a_{ij}T^a_{kl} = \delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}$$

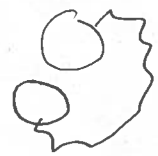
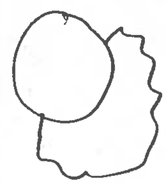
 $=$  $- \frac{1}{N}$ 

So $\mathrm{Tr}(T^aT^b)\,\mathrm{Tr}(T^aT^b) =$  $=$  $= N^2-1.$

$\mathrm{Tr}(T^aT^bT^bT^a) =$  $=$  $- \frac{1}{N}$  $= \left(N-\frac{1}{N}\right)(N^2-1)$

$$= \frac{(N^2-1)^2}{N}$$

$$\mathrm{Tr}(T^a T^b T^a T^b) =$$ 

$$=$$  $$-\frac{1}{N}$$  $$= 0 - \frac{1}{N}(N^2-1) = -\frac{N^2-1}{N}.$$

So for massless squarks

$$\sum_{\substack{\text{hel}\\ \text{color}}} |A_4|^2 = 2g^4 \frac{(N^2-1)}{N}\left[(N^2-1)\left(1-2\frac{tu}{s^2}\right) - 2\frac{tu}{s}\right]$$

$$(N^2-1) + N^2\left(-2\frac{tu}{s^2}\right)$$

$$= 2g^2\left[N(N^2-1)\left(1-2\frac{tu}{s^2}\right) - \frac{N^2-1}{N}\right]$$

as in literature.

If the squarks are massive, one can repeat BCFW argument. Now also $A_4(g^{\pm} g^{\pm} \tilde{q}\, \tilde{q}^{*})$ nonvanishing so one more contribution

Answer

$$\sum |A_4|^2 = 2g^4\left\{N(N^2-1)\left(1-2\frac{u_1 t_1}{s^2}\right) - \frac{N^2-1}{N}\right\}\left[1-2\frac{m_{\tilde q}^2 s}{u_1 t_1}\left(1-\frac{m_{\tilde q}^2 s}{u_1 t_1}\right.\right.$$

Perspective:

When does recursion work?

Back to gluon scattering:

$$F_{\mu\nu}F^{\mu\nu} \supset AA\partial A + A^4$$

Can calculate all tree-level gluon on-shell amplitudes using only information about  ; not .

The role of  is to insure <u>off-shell</u> gauge inv of the Lagrangian. Does not carry indep information.

Scalar QED $\quad |D\phi|^2 \ni A\phi\partial\phi^*$ 

$AA\phi\phi^*$ 

$A_4(\gamma\gamma\phi\phi^*)$ can be calculated recursively w/ input from just  ; b/c $AA\phi\phi^*$ is fixed by gauge inv.

$A_4(\phi\phi^*\phi\phi^*)$ can NOT be calc. recursively in scalar QED.

B/c we could also have contrib. from $\lambda|\phi|^4$. gen. λ & gauge coupl. <u>indep</u>. So there is <u>indep</u> gauge inv. information in $\lambda|\phi|^4$.

If we, nonetheless, forged ahead and calculated $A_4(\phi\phi^*\phi\phi^*)$ via BCFW, using only  as input, what do we get?

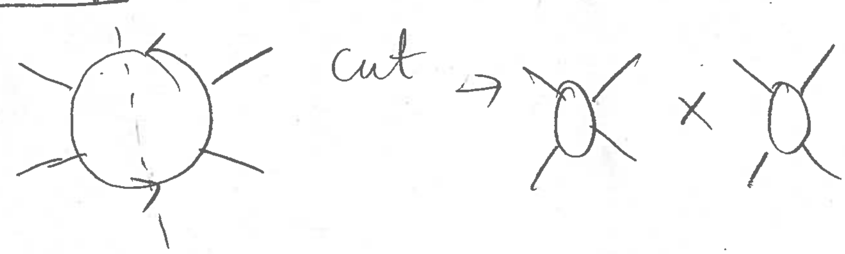
Well, the answer is pretty nice: it turns out we would get $A_4(\phi,\phi^*;\phi\phi^*)$ as in supersymmetric theory w/ the quartic coupling λ fixed in terms of e: $\lambda = 2e^2$! Secret SUSY!!

In general, however, we cannot expect to get something out of nothing ∴ must specify the needed indep. <u>gauge invariant</u> information.

So BCFW and recursion makes gauge inv. automatic. Can be enhanced to also encode SUSY → super-BCFW. With that, all amplitudes in tree-level $\mathcal{N}=4$ SYM have been solved (MHV, NMHV, etc).

<u>What about loops?</u>

Unitarity



only extracts Im-part. However, using multiple cuts the full amplitude can be extracted (<u>generalized</u> unitarity) (up to rational terms)

Loop methods

- Generalized unitarity (trees → loops)
- loop-level recursion (integrand only; susy only; (also trees → loops) planar only)
- Grassmannians, polytopes, amplituhedrons (geometrizing).
- "Bootstrap"
- Integrability...

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and interesting applications in (quantum) gravity, string theory, ...