DIMENSIONAL REDUCTION
of
S-CONFINING DUALITIES

Cornell University

work in progress, in collaboration with **C. Csaki, Y. Shirman, F. Tanedo and J. Terning.**

Mario Martone, mcm293@cornell.edu

3D Yang-Mills

Nuclear Physics B188 (1981) 479–512

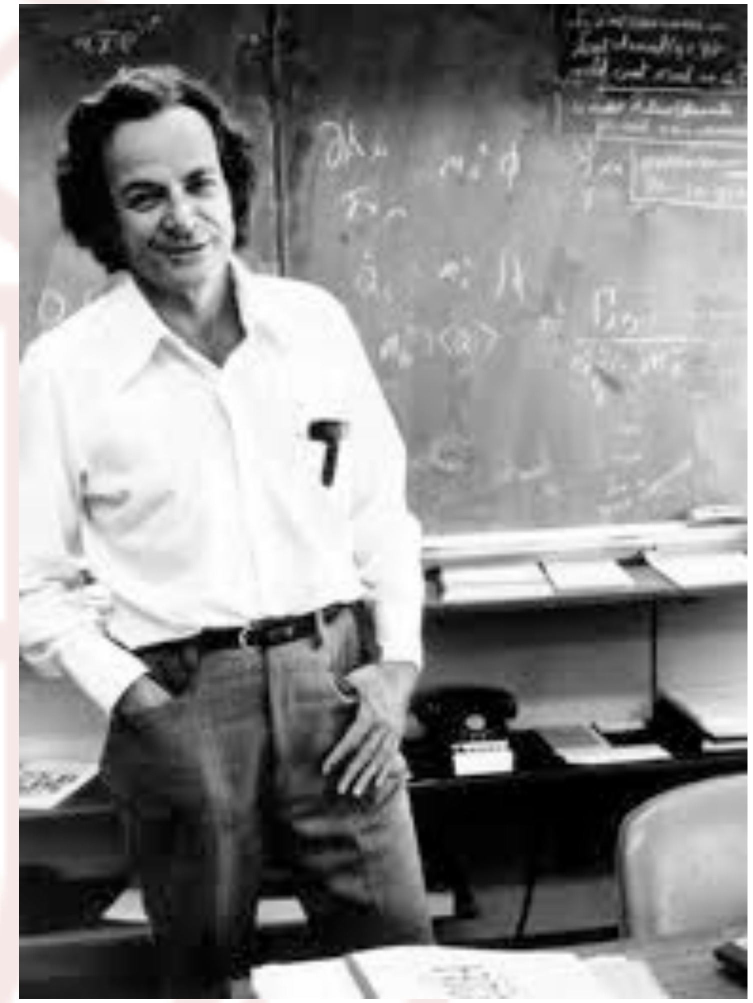
THE QUALITATIVE BEHAVIOR OF YANG–MILLS THEORY IN 2+1 DIMENSIONS

Richard P. FEYNMAN[1]

California Institute of Technology, Pasadena, California 91125, USA

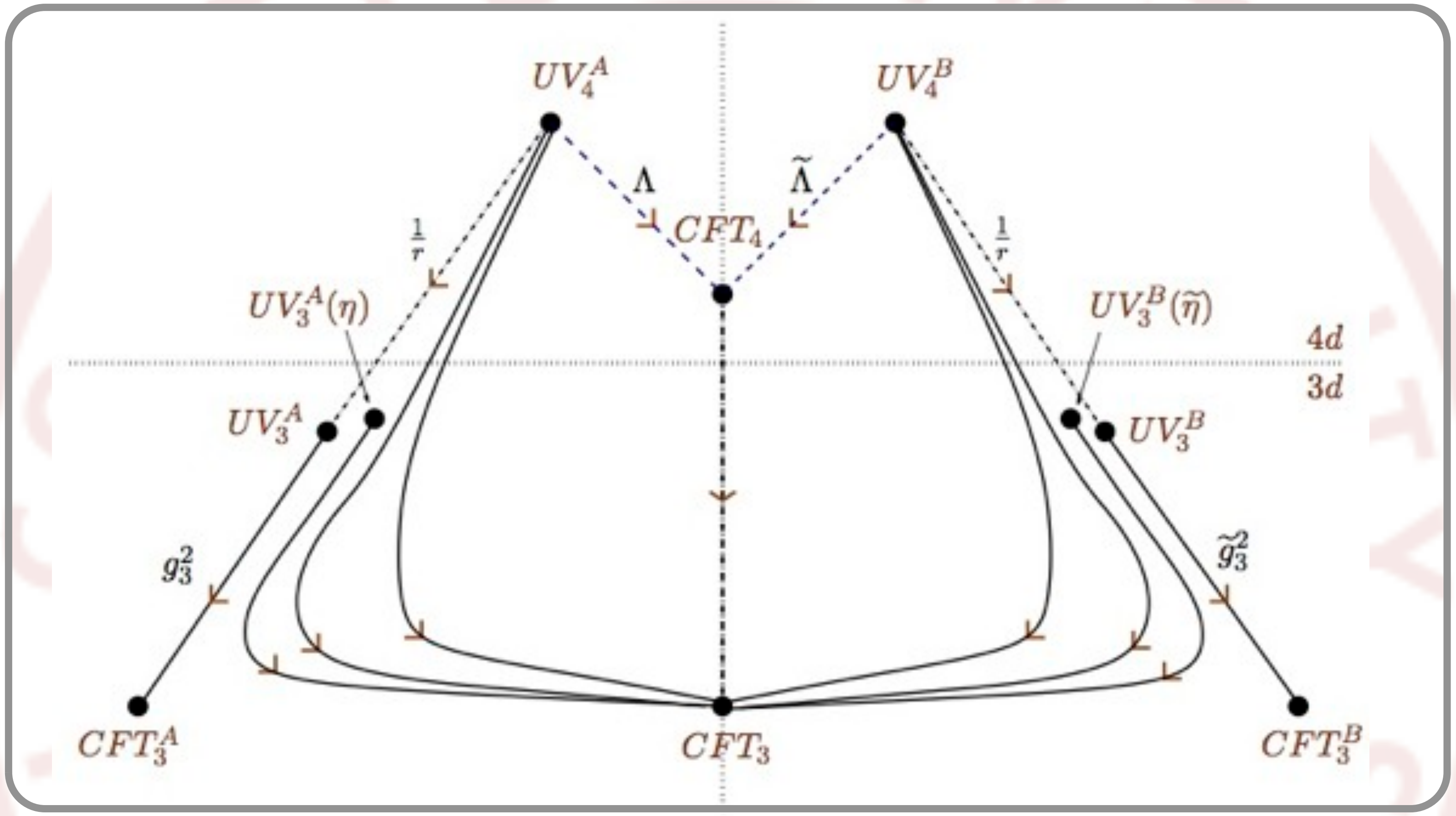
Received 11 February 1981

The SU(2) gauge theory of gluons (no quarks) is studied in two space and one time dimensions. Only qualitative or suggestive discussions are made. Starting from the quantum field equations it is argued that the necessary gauge invariance of the wave functional results, in this non-abelian case, in a finite energy for any excitation ("glueball") above the ground state. Furthermore, fluctuations in which gauging factors change sign can occur independently in regions adequately separated in space. This results in a potential between distant massive quarks rising linearly with distance (quark confinement). The situation in 3+1 dimensions is not discussed.



A. M. Polyakov, "**Quark Confinement and Topology of Gauge Groups**", Nucl. Phys. B 120 (1977) 429-458.

Why now?



New set of dualities in 3D from dimensional reduction of 4D theories and many exact results from partition function calculations.

O. Aharony, S. Razamat, N. Seiberg & B. Willet
JHEP 1307 (2013) 149 [arXiv:1305.3924]

1- S-Confining theories

2- Dimensional reduction of 4D dualities.

3- Elements of N=2 SUSY in 3D.

4- Dimensional reduction of S-Confining dualities.

S-Confinement.

"smooth confinement without chiral symmetry breaking and a non-vanishing confining superpotential"

C. Csaki, M. Schmaltz & W. Skiba *Phys. Rev. Lett.* **78** (1997) 799 [hep-th/9610139]

G. Dotti, A.V. Manohar and W. Skiba *Nucl. Phys. B* **531** (1998) [hep-th/9803087]

1- Infrared physics is described everywhere on the moduli space in terms of gauge invariant operators.

2- A non-vanishing superpotential is dynamically generated which is holomorphic function of the confined degrees of freedom.

3- The vacuum of the classical theory, where all the global symmetries are unbroken, is a vacuum of the quantum theory as well.

NATURALNESS, CHIRAL SYMMETRY, AND SPONTANEOUS

CHIRAL SYMMETRY BREAKING

G. 't Hooft

Institute for Theoretical Fysics

Utrecht, The Netherlands

ABSTRACT

A properly called "naturalness" is imposed on gauge theories. It is an order-of-magnitude restriction that must hold at all energy scales μ. To construct models with complete naturalness for elementary particles one needs more types of confining gauge theories besides quantum chromodynamics. We propose a search program for models with improved naturalness and concentrate on the possibility that presently elementary fermions can be considered as composite. Chiral symmetry must then be responsible for the masslessness of these fermions. Thus we search for QCD-like models where chiral symmetry is not or only partly broken spontaneously. They are restricted by index relations that often cannot be satisfied by other than unphysical fractional indices. This difficulty made the author's own search unsuccessful so far. As a by-product we find yet another reason why in ordinary QCD chiral symmetry must be broken spontaneously.

Why S-Confinement?

1- If there was no spontaneously chiral symmetry breaking, the proton (*baryons*) would be massless (*very light*).

2- In a s-confining theory would be *natural* for the fundamental quarks to be composite degrees of freedom.

SU(N) with N+1 flavours.

The magnetic dual has no gauge group.

$$\boxed{W = \frac{1}{\Lambda^{2N-1}}(\det M - BM\bar{B})}$$

1- ☐

2- ☐

3- ☐

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$SU(N)$ with N flavours.

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$$\boxed{W = \lambda(\det M - B\bar{B} - \Lambda^{2N})}$$

1- ☐
2- ☐
3- ☐

SU(N) with *N+1* flavours.

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1- ✓

2- ✓

3- ✓

$SU(N)$ with N flavours.

The magnetic dual has no gauge group.

$$W = \lambda(\det M - B\bar{B} - \Lambda^{2N})$$

1- ✓

2- ✓

3- ✗

How do we search for these theories?

1- A ***magnetic theory*** of baryons and mesons should match the anomalies of the ***electric theory***.

2- The dynamically generated super-potential should only involve positive powers of the composite degrees of freedom.

$$\sum_j T(\mathbf{r}_j) - T(\mathbf{Ad}) = 1$$

C. Csaki, M. Schmaltz & W. Skiba *Phys. Rev. D* 55 (1997) 7840 [hep-th/9612207]

G. Dotti and A.V. Manohar, *Nucl Phys*. **B518** (1998) 575-602 [hep-th/9710024]

A complete classification.

$SU(N)$	$(N+1)(\square+\overline{\square})\,;\ \begin{array}{\|c\|}\hline \ \\ \hline \ \\ \hline\end{array} + N\overline{\square} + 4\square;\ \begin{array}{\|c\|}\hline \ \\ \hline \ \\ \hline\end{array} + \overline{\begin{array}{\|c\|}\hline \ \\ \hline \ \\ \hline\end{array}} + 3(\square+\overline{\square})$
$SU(5)$	$3\left(\begin{array}{\|c\|}\hline \ \\ \hline \ \\ \hline\end{array} + \overline{\square}\right)\ ;\ \ 2\begin{array}{\|c\|}\hline \ \\ \hline \ \\ \hline\end{array} + 2\square + 4\overline{\square}$
$SU(6)$	$2\begin{array}{\|c\|}\hline \ \\ \hline \ \\ \hline\end{array} + 5\overline{\square} + \square;\ \begin{array}{\|c\|}\hline \ \\ \hline \ \\ \hline \ \\ \hline\end{array} + 4(\square+\overline{\square})$
$SU(7)$	$2\left(\begin{array}{\|c\|}\hline \ \\ \hline \ \\ \hline\end{array} + 3\overline{\square}\right)$

ex. $SU(4)$ with $3\ (\square+\overline{\square})$ & $\begin{array}{|c|}\hline \ \\ \hline \ \\ \hline\end{array} + \overline{\begin{array}{|c|}\hline \ \\ \hline \ \\ \hline\end{array}}$

Electric th.

Magnetic th.

$$W_{dyn} = \frac{1}{\Lambda^7}\left(T^2M_0^3 - 12TH\bar{H}M_0 - 24M_0M_2^2 - 24H\bar{H}M_2\right)$$

3D analog

There is no anomaly

$\downarrow$

How would we go about it?

1- S-Confining theories.

2- Dimensional reduction of Seiberg dualities

3- Elements of $\mathcal{N}=2$ SUSY in 3D.

4- Dimensional reduction of S-Confining dualities.

Many 3D dualities look like Seiberg dualities!

Seiberg dualities [hep-th/9411149]

Electric (*Theory A*)

$SU(N)$ with F $(\Box + \overline{\Box})$

$$\boxed{W = 0}$$

Magnetic (*Theory B*)

$SU(F-N)$ with F $(\Box + \overline{\Box})$ and F^2 mesons

$$\boxed{W = \tilde{q} M q}$$

Aharony dualities [hep-th/9703215]

Electric (*Theory A*)

$U(N)$ with F $(\Box + \overline{\Box})$

$$\boxed{W = 0}$$

Magnetic (*Theory B*)

$U(F-N)$ with F $(\Box + \overline{\Box})$ and F^2 mesons

$$\boxed{W = \tilde{q} M q + V_+ \tilde{V}_- + V_- \tilde{V}_+}$$

Although strong coupling gauge dynamics is very different in 4D and in 3D, this similarity calls for dimensional reduction.

Why doesn't naive dimensional reduction, work?

O. Aharony, S. Razamat, N. Seiberg & B. Willet
JHEP 1307 (2013) 149 [arXiv:1305.3924]

O. Aharony, S. Razamat, N. Seiberg & B. Willet
[arXiv:1307.0511]

Seiberg dualities are IR dualities

In the range of parameters where both theories are asymptotically free, *Theory A* and *Theory B* are equivalent only at low energies

$$E \lesssim \Lambda_A \lesssim \Lambda_B$$

Confinement scale for *Theory A*

$$\Lambda_A^b = \exp(-8\pi^2/g_A^2)$$

Confinement scale for *Theory B*

$$\Lambda_B^b = \exp(-8\pi^2/g_B^2)$$

Such dualities still holds true when we compactify both theories on a circle of radius *r*.

Compactification on a circle.

When we compactify one space dimension to a circle the gauge coupling satisfies:

$$\boxed{g_4^2 = 2\pi r g_3^2}$$

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In the $r \longrightarrow 0$ limit, g_3 should be kept constant

$$\Lambda_A \to 0$$
$$\Lambda_B \to 0$$

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In the $r \to 0$ limit, g_3 should be kept constant

$$\Lambda_A \to 0$$

$$\Lambda_B \to 0$$

Straightforward dimensional reduction does not work.

We can take a different limit keeping *r* fixed

$$E \lesssim \Lambda_A \lesssim \Lambda_B < 1/r$$

1- In this limit the effective low-energy behaviour of both theories is three dimensional.

2- *Theory A* and *Theory B* are still dual because of the 4D IR duality.

The 3D duality so obtained from the 4D duality, differs from the naive dimensional reduction.

How do they differ?

Because of the compact $\mathbb{S}^1$ there are extra non-perturbative corrections to the superpotential

$$W = W_{3D} + \eta Y$$

3D Super-Potential from straightforward dim. reduction.

Non perturbative correction from the compact $\mathbb{S}^1$.

Summarizing 1/2.

4D

Theory A_4		Theory B_4
$\mathcal{N}=1$	$\longleftrightarrow$	$\mathcal{N}=1$
$W_4 = 0$		$\tilde{W}_4 \neq 0$
$\downarrow$		$\downarrow$

3D

Theory A_3		Theory B_3
$\mathcal{N}=2$	$\longleftrightarrow$	$\mathcal{N}=2$
$W_3 = \eta Y$		$\tilde{W}_3 = \tilde{W}_4 + \tilde{\eta}\tilde{Y}$

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[arXiv:1307.0511]

Summarizing 2/2.

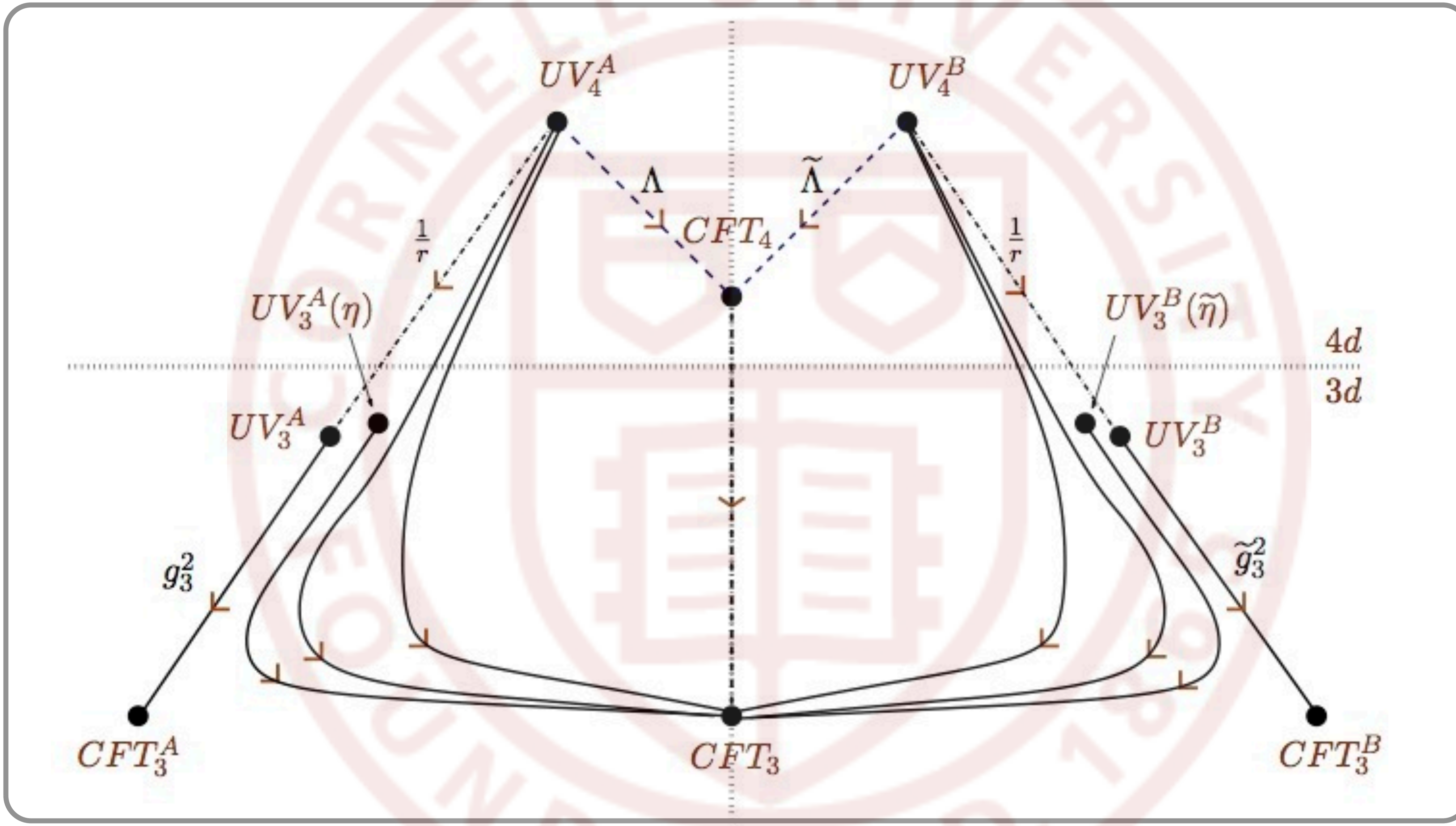

Image taken from [arXiv:1305.3924].

Matching global symmteries

No anomalies in 3D

The naive dimensionally reduced 3D theory has an extra U(1) global symmetry.

The ηY is neutral under all non-anomalous symmetries but breaks the anomalous U(1).

Through dimensional reduction more 3D dualities were conjectured.

$SU(N)$ with F $(\square + \overline{\square})$

$$W = 0$$

$\longleftrightarrow$

$U(F-N)$ with F $(\square + \overline{\square})$ and F^2 mesons

$$W = \tilde{q}Mq + Yb\tilde{b} + \tilde{X}_- + \tilde{X}_+$$

$SO(N)$ with F $\square$

$$W = 0$$

$\longleftrightarrow$

$SO(F-N+2)$ with F $\square$ and $F(F+1)/2$ mesons

$$W = \frac{1}{2}Mqq + \frac{i^{F-N}}{4}\tilde{y}Y$$

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3- Dimensional reduction of S-Confining dualities.

Dimensional reduction of 4D Supersymmetric algebra

$$i\epsilon_{\alpha\beta} = \sigma^2$$

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\gamma^i_{\alpha\beta} P_i + 2 i\epsilon_{\alpha\beta} Z$$

$$\gamma^i_{\alpha\beta} = (-\mathbf{1}, \sigma^\mathbf{1}, \sigma^\mathbf{3})$$

K. Intrilligator and N. Seiberg, "Aspects of 3d N=2 Chern-Simons-Matter Theories" [arXiv:1305.1633]

O. Aharony, A. Hanany, K. Intrilligator, N. Seiberg and M. J. Strassler, **Nucl. Phys. B 499, 67** (1997) [hep-th/9703110]

Chiral Superfields (nothing exciting...)

$$\boxed{\begin{gathered} D_\alpha \Phi = 0 \\ \Phi = \phi + \theta\psi + \theta^2 F \end{gathered}}$$

Vector superfield - $U(1)$ -

$$A^\mu = (\underbrace{A^1, A^2, A^3}_{\vec{A}}, \underset{\sigma}{\underset{\downarrow}{A^4}})$$

Vector Superfields

$$V = V^\dagger$$

$$V = -i\theta\bar{\theta}\sigma - \theta\gamma^i\bar{\theta}A_i + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D$$

$$\sigma \sim A_4$$

In 3D even the vector superfield can acquire a VEV

Coulomb Branch

What is the Topology?

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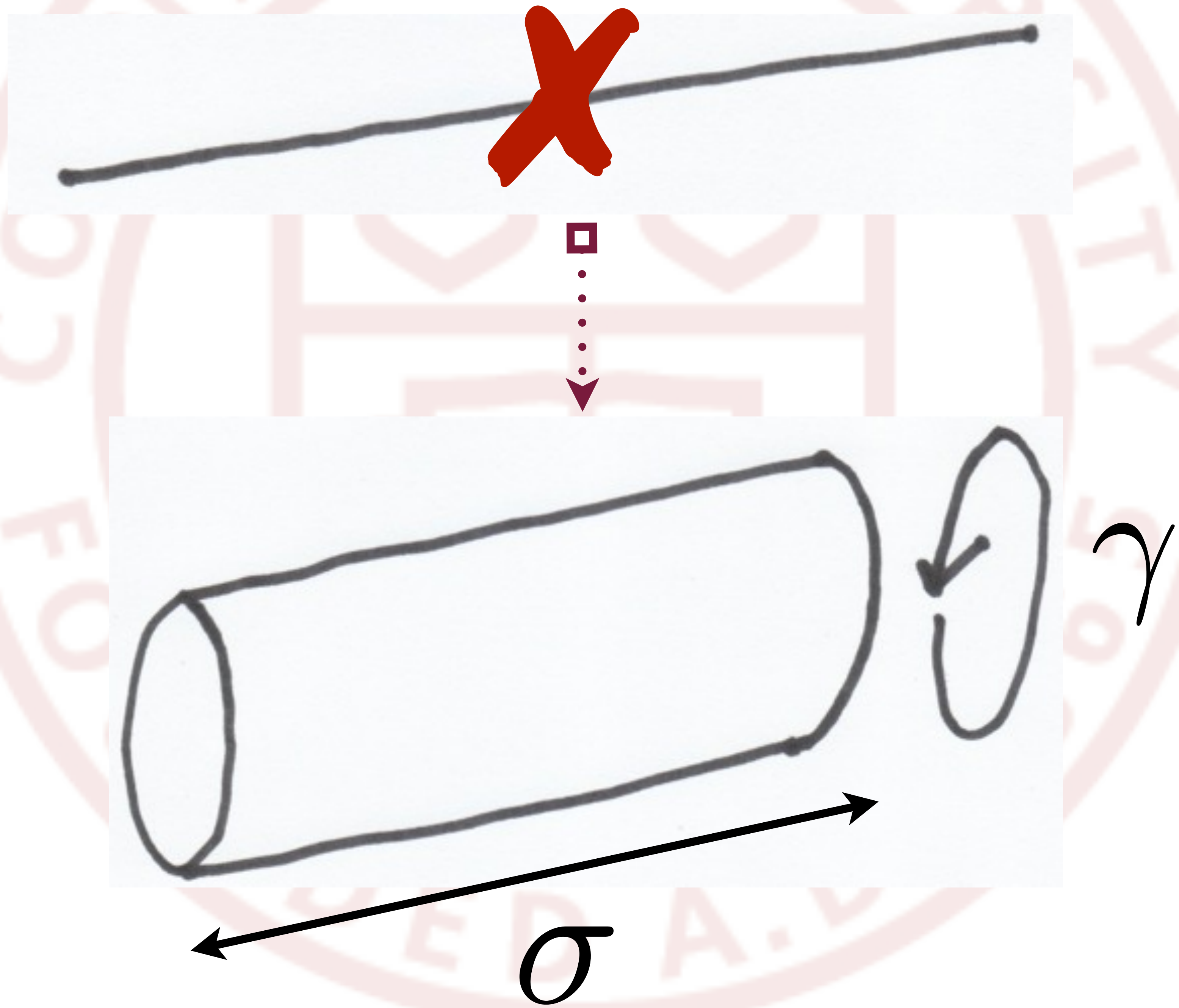

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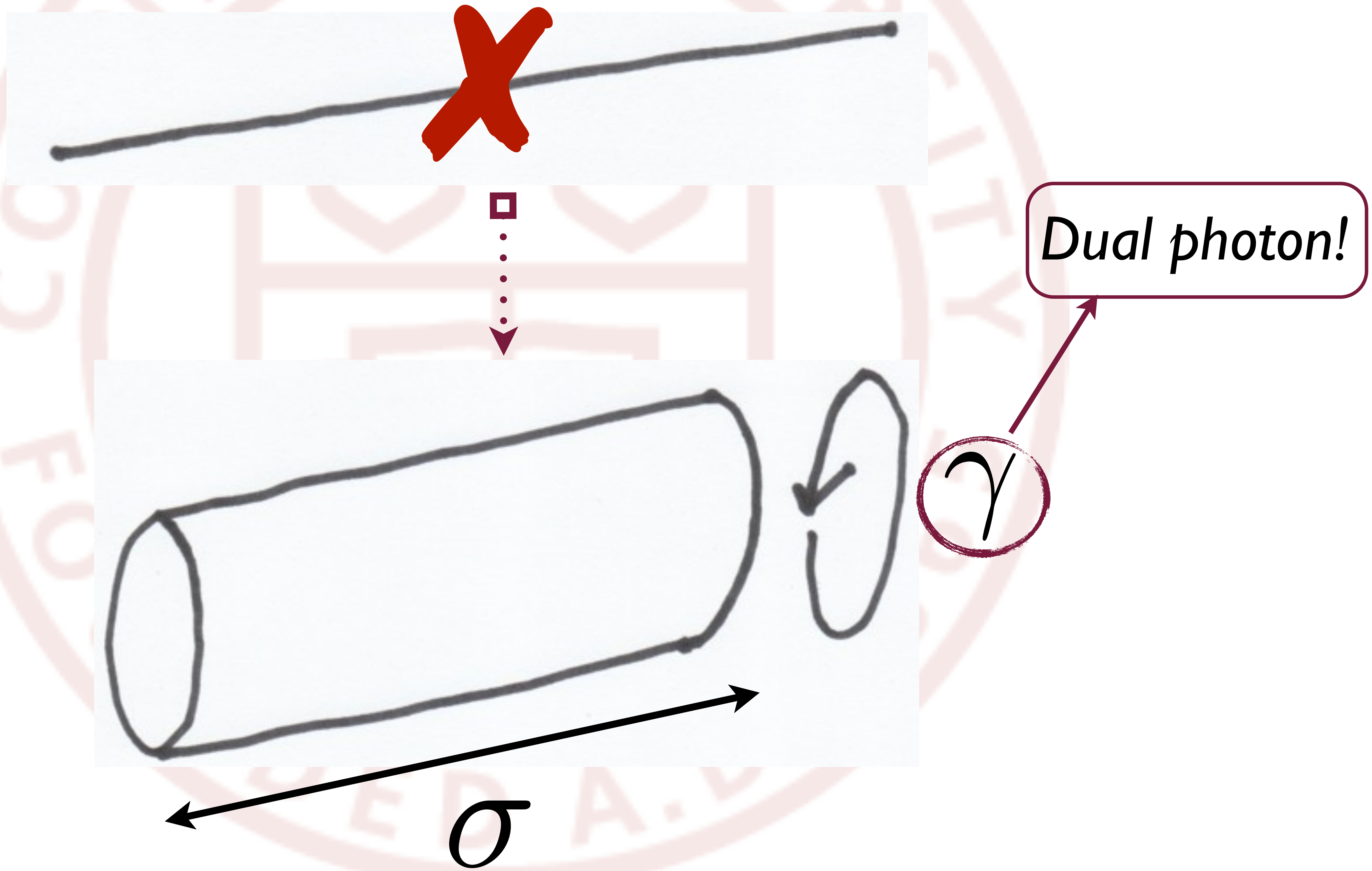

Dual photon

The vector supermultiplet has both a real and imaginary scalar component. γ arises as ***dual photon!***

$$*F \sim d\gamma$$

This construction, trivially generalises to the non-Abelian case

1- We can arrange σ and γ as lower component of a chiral superfield

$$\Phi = \sigma + i\gamma + \dots$$

2- We should take care of the periodicity of the dual photon.

$$Y = e^{\Phi}$$

It's a good coordinate for the Coulomb branch.

Operators describing the moduli space.

Higgs branch.

Nothing new:

Meson operators $M = Q\bar{Q}$

For **F>N**

Baryons $\begin{cases} B = \epsilon_{i_1 \dots i_N} Q^{i_1} \dots Q^{i_N} \\ \bar{B} = \epsilon^{i_1 \dots i_N} \bar{Q}_{i_1} \dots \bar{Q}_{i_N} \end{cases}$

Coulomb branch.

The operator Y is well defined throughout the Coulomb branch and can be used to describe it.

$$Y$$

Masses in SUSY

In 4D the only mass deformation allowed is a *complex mass:*

$$W = m_c \Phi^2$$

Masses in SUSY

In 4D the only mass deformation allowed is a *complex mass:*

$$W = m_c \Phi^2$$

m_c is protected by holomorphy.

In 3D ***real mass*** deformations are also allowed.

$$\Phi = \phi + \theta\psi + \theta^2 F$$

Real mass

$$\int d^4\theta\ \Phi^\dagger e^{m_r\theta\bar{\theta}}\Phi \sim \frac{m_r^2}{2}\phi^*\phi + m_r\bar{\psi}\psi$$

$$\boxed{\Phi = \phi + \theta\psi + \theta^2 F}$$

Real mass

$$\int d^4\theta\ \Phi^\dagger e^{m_r\theta\bar{\theta}}\Phi \sim \frac{m_r^2}{2}\phi^*\phi + m_r\bar{\psi}\psi$$

1- m_r is real.

2- m_r is a mass term.

$$\Phi = \phi + \theta\psi + \theta^2 F$$

Real mass

$$\int d^4\theta\ \Phi^\dagger e^{m_r\theta\bar{\theta}}\Phi \sim \frac{m_r^2}{2}\phi^*\phi + m_r\bar{\psi}\psi$$

1- m_r is real.

2- m_r is a mass term.

$\Big\}$ m_r is a ***Real Mass***.

$$M = \sqrt{m_r^2 + |m_c|^2}$$

Real mass

$$\int d^4\theta\ \Phi^\dagger e^{m_r\theta\bar\theta}\Phi \sim \int d^4\theta\ \Phi^\dagger e^{gV}\Phi$$

$$V = -i\theta\bar\theta\sigma - \theta\gamma^i\bar\theta A_i + i\theta^2\bar\theta\bar\lambda - i\bar\theta^2\theta\lambda + \tfrac{1}{2}\theta^2\bar\theta^2 D$$

Mapping across dualities

Real masses can be seen as background configurations of weakly gauged global symmetries

$$\sigma = \frac{m_r}{g},\ \lambda = \bar\lambda = \vec{A} = D = 0$$

As global symmetries match, real mass deformations can be easily mapped across the duality.

Moduli space deformations

$$\eta Y \left\{ \vphantom{\begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}} \right.$$

Moduli space deformations

$$\eta Y \left\{ \quad \right.$$

Complex mass

$m_c Q\bar{Q}$	$Y_{high} = m_c Y_{low}$

Moduli space deformations

$$\eta Y \begin{cases} \text{Complex mass} \\ \text{VEV} \end{cases}$$

Complex mass

$m_c Q\bar{Q}$ | $Y_{high} = m_c Y_{low}$

VEV

$\langle Q\bar{Q}\rangle = v^2$ | $Y_{high} = \dfrac{Y_{low}}{v^2}$

Moduli space deformations

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Complex mass

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VEV

$\langle Q\bar{Q}\rangle = v^2$ | $Y_{high} = \dfrac{Y_{low}}{v^2}$

Real mass

Real mass deformations depend on real parameters and real parameters cannot appear in the super-potential.

$m_r \to \infty$ | $\eta Y \to 0$

1- Dimensional reduction of 4D dualities.

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3- S-Confining theories.

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3D Dynamics

# flavours	Magnetic description	S-Confining
$N+1$	$U(1)$ gauge theory $W = Mq\tilde{q} + Yb\tilde{b} + V_+ + V_-$	yes ☐ no ☐
N	$W = Y(\det M - B\bar{B})$	yes ☐ no ☐
$N-1$	$Y \det M = 1$	yes ☐ no ☐

K. Intrilligator and N. Seiberg, "Aspects of 3d N=2 Chern-Simons-Matter Theories" [arXiv:1305.1633]

O. Aharony, A. Hanany, K. Intrilligator, N. Seiberg and M. J. Strassler, Nucl. Phys. B **499**, 67 (1997) [hep-th/9703110]

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Decoupling one flavor

There is a pattern. 3D theories with *F* flavors show a behaviour similar to 4D theories with *F+1* flavors.

4D dualities

Compactification

Decoupling one flavor

There is a pattern. 3D theories with F flavors show a behaviour similar to 4D theories with $F+1$ flavors.

4D dualities

Compactification

3D dualities $+\eta Y$

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4D dualities

$\downarrow$ ***Compactification***

3D dualities $+\eta Y$

$\downarrow$ ***Real mass deformation***

Decoupling one flavor

There is a pattern. 3D theories with F flavors show a behaviour similar to 4D theories with $F+1$ flavors.

4D dualities

Compactification

3D dualities $+\eta Y$

Real mass deformation

3D dualities without tree level S.P.

Witten Index

It can be computed in 3D using real mass deformations.

A necessary condition for a 3D theory to confine is:

$$\boxed{\mathrm{Tr}(-1)^F = 1}$$

E. Witten, "Supersymmetric index of three-dimensional gauge theory" [hep-th/9903005]

$$\mathrm{Tr}(-1)^F = 1$$

$$\sum_j T(\mathbf{r}_j) - T(\mathbf{Ad}) = 0$$

This condition is "shifted" by one compared to the 4D case

It is only a **necessary** *condition.*

K. Intrilligator and N. Seiberg, "Aspects of 3d N=2 Chern-Simons-Matter Theories" [arXiv:1305.1633]

S-Confinment in 3D

1- The dynamical generated super-potential breaks already the anomalous $U(1)$.

2- The "magnetic" version has no gauge symmetry. No instanton configurations exist.

Theory A

Dimensional reduction of the 4D electric theory + ηY

$$
\begin{array}{l|l}
SU(N) & (N+1)(\square+\overline{\square})\,;\ \begin{smallmatrix}\square\\\square\end{smallmatrix}+N\overline{\square}+4\square;\ \begin{smallmatrix}\square\\\square\end{smallmatrix}+\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}+3(\square+\overline{\square}) \\
SU(5) & 3\left(\begin{smallmatrix}\square\\\square\end{smallmatrix}+\overline{\square}\right)\,;\ 2\begin{smallmatrix}\square\\\square\end{smallmatrix}+2\square+4\overline{\square} \\
SU(6) & 2\begin{smallmatrix}\square\\\square\end{smallmatrix}+5\overline{\square}+\square;\ \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}+4(\square+\overline{\square}) \\
SU(7) & 2\left(\begin{smallmatrix}\square\\\square\end{smallmatrix}+3\overline{\square}\right)
\end{array}
\quad + \ \eta Y
$$

Duality

Theory A

Dimensional reduction of the 4D electric theory + ηY

$$
\left.\begin{array}{l|l}
SU(N) & (N+1)(\Box+\overline{\Box})\,;\ \begin{smallmatrix}\Box\\\Box\end{smallmatrix}+N\overline{\Box}+4\Box;\ \begin{smallmatrix}\Box\\\Box\end{smallmatrix}+\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}+3(\Box+\overline{\Box}) \\
SU(5) & 3\left(\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+\overline{\Box}\right)\ ;\ 2\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+2\Box+4\overline{\Box} \\
SU(6) & 2\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+5\overline{\Box}+\Box;\ \begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}+4(\Box+\overline{\Box}) \\
SU(7) & 2\left(\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+3\overline{\Box}\right)
\end{array}\right. \qquad +\ \eta Y
$$

Duality

Theory B

The dynamical generated super-potential is not corrected.

$SU(4)$ with $3\,(\Box + \overline{\Box})$ & $\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$

Dynamically generated Super-Potential

$$W_{dyn} = \frac{1}{\Lambda^7}\left(T^2 M_0^3 - 12 T H \bar{H} M_0 - 24 M_0 M_2^2 - 24 H \bar{H} M_2\right)$$

	$SU(4)$	$SU(2)$	$SU(3)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$\Box$	1	1	0	-3	0
Q	$\Box$	1	$\Box$	1	1	2	$\frac{1}{3}$
$\bar{Q}$	$\overline{\Box}$	1	1	$\Box$	-1	2	$\frac{1}{3}$
$M_0 = Q\bar{Q}$		1	$\Box$	$\Box$	0	4	$\frac{2}{3}$
$M_2 = QA^2\bar{Q}$		1	$\Box$	$\Box$	0	-2	$\frac{2}{3}$
$H = AQ^2$		$\Box$	$\overline{\Box}$	1	2	1	$\frac{2}{3}$
$\bar{H} = A\bar{Q}^2$		$\Box$	1	$\overline{\Box}$	-2	1	$\frac{2}{3}$
$T = A^2$		$\Box\Box$	1	1	0	-6	0

$SU(4)$ with $3\,(\square + \overline{\square})$ & $\begin{smallmatrix}\square\\\square\end{smallmatrix} + \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$

Real Masses

$$\sigma_3 = \begin{pmatrix} m_r & 0 \\ 0 & -m_r \end{pmatrix}$$

Dynamically generated Super-Potential

$$W_{dyn} = \frac{1}{\Lambda^7}\left(T^2 M_0^3 - 12 T H \bar{H} M_0 - 24 M_0 M_2^2 - 24 H \bar{H} M_2\right)$$

	$SU(4)$	$SU(2)$	$SU(3)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$\cancel{A}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	1	1	0	-3	0
Q	$\square$	1	$\square$	1	1	2	$\frac{1}{3}$
$\bar{Q}$	$\overline{\square}$	1	1	$\square$	-1	2	$\frac{1}{3}$
$M_0 = Q\bar{Q}$		1	$\square$	$\square$	0	4	$\frac{2}{3}$
$M_2 = QA^2\bar{Q}$		1	$\square$	$\square$	0	-2	$\frac{2}{3}$
$H = AQ^2$		$\square$	$\overline{\square}$	1	2	1	$\frac{2}{3}$
$\bar{H} = A\bar{Q}^2$		$\square$	1	$\overline{\square}$	-2	1	$\frac{2}{3}$
$T = A^2$		$\square\square$	1	1	0	-6	0

$SU(4)$ with $3\,(\square + \bar{\square})$ & $\begin{smallmatrix}\square\\\square\end{smallmatrix} + \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$

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	$SU(4)$	$SU(2)$	$SU(3)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$\cancel{A}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	1	1	0	-3	0
Q	$\square$	1	$\square$	1	1	2	$\frac{1}{3}$
$\bar{Q}$	$\bar{\square}$	1	1	$\square$	-1	2	$\frac{1}{3}$
$M_0 = Q\bar{Q}$		1	$\square$	$\square$	0	4	$\frac{2}{3}$
$M_2 = QA^2\bar{Q}$		1	$\square$	$\square$	0	-2	$\frac{2}{3}$
$H = AQ^2$		$\square$	$\bar{\square}$	1	2	1	$\frac{2}{3}$
$\bar{H} = A\bar{Q}^2$		$\square$	1	$\bar{\square}$	-2	1	$\frac{2}{3}$
$T = A^2$		$\square\square$	1	1	0	-6	0

$SU(4)$ with $3\ (\square + \bar{\square})$ & $\begin{array}{c}\square\\[-2pt]\square\end{array} + \overline{\begin{array}{c}\square\\[-2pt]\square\end{array}}$

Real Masses

$$\sigma_3 = \begin{pmatrix} m_r & 0 \\ 0 & -m_r \end{pmatrix}$$

Dynamically generated Super-Potential

$$W_{dyn} = \frac{1}{\Lambda^7}\left(T^2 M_0^3 - 12 T H \bar{H} M_0 - 24 M_0 M_2^2 - 24 H \bar{H} M_2\right)$$

	$SU(4)$	$SU(2)$	$SU(3)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$\cancel{A}$	$\begin{array}{c}\square\\[-2pt]\square\end{array}$	$\square$	1	1	0	-3	0
Q	$\square$	1	$\square$	1	1	2	$\frac{1}{3}$
$\bar{Q}$	$\bar{\square}$	1	1	$\square$	-1	2	$\frac{1}{3}$
$M_0 = Q\bar{Q}$		1	$\square$	$\square$	0	4	$\frac{2}{3}$
$M_2 = QA^2\bar{Q}$		1	$\square$	$\square$	0	-2	$\frac{2}{3}$
$\cancel{H} = AQ^2$		$\square$	$\bar{\square}$	1	2	1	$\frac{2}{3}$
$\bar{H} = A\bar{Q}^2$		$\square$	1	$\bar{\square}$	-2	1	$\frac{2}{3}$
$T = A^2$		$\square\square$	1	1	0	-6	0

$SU(4)$ with $3\,(\square + \overline{\square})$ & $\begin{smallmatrix}\square\\\square\end{smallmatrix} + \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$

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$$\sigma_3 = \begin{pmatrix} m_r & 0 \\ 0 & -m_r \end{pmatrix}$$

Dynamically generated Super-Potential

$$W_{dyn} = \frac{1}{\Lambda^7}\left(T^2 M_0^3 - 12 T H \bar{H} M_0 - 24 M_0 M_2^2 - 24 H \bar{H} M_2\right)$$

	$SU(4)$	$SU(2)$	$SU(3)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$\cancel{A}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	1	1	0	-3	0
Q	$\square$	1	$\square$	1	1	2	$\frac{1}{3}$
$\bar{Q}$	$\overline{\square}$	1	1	$\square$	-1	2	$\frac{1}{3}$
$M_0 = Q\bar{Q}$		1	$\square$	$\square$	0	4	$\frac{2}{3}$
$M_2 = QA^2\bar{Q}$		1	$\square$	$\square$	0	-2	$\frac{2}{3}$
$\cancel{H} = AQ^2$		$\square$	$\overline{\square}$	1	2	1	$\frac{2}{3}$
$\cancel{\bar{H}} = A\bar{Q}^2$		$\square$	1	$\overline{\square}$	-2	1	$\frac{2}{3}$
$T = A^2$		$\square\square$	1	1	0	-6	0

$SU(4)$ with $3\,(\square + \bar{\square})$ & $\begin{smallmatrix}\square\\\square\end{smallmatrix} + \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$

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$$\sigma_3 = \begin{pmatrix} m_r & 0 \\ 0 & -m_r \end{pmatrix}$$

Dynamically generated Super-Potential

$$W_{dyn} = \frac{1}{\Lambda^7}\left(T^2 M_0^3 - 12 T H \bar{H} M_0 - 24 M_0 M_2^2 - 24 H \bar{H} M_2\right)$$

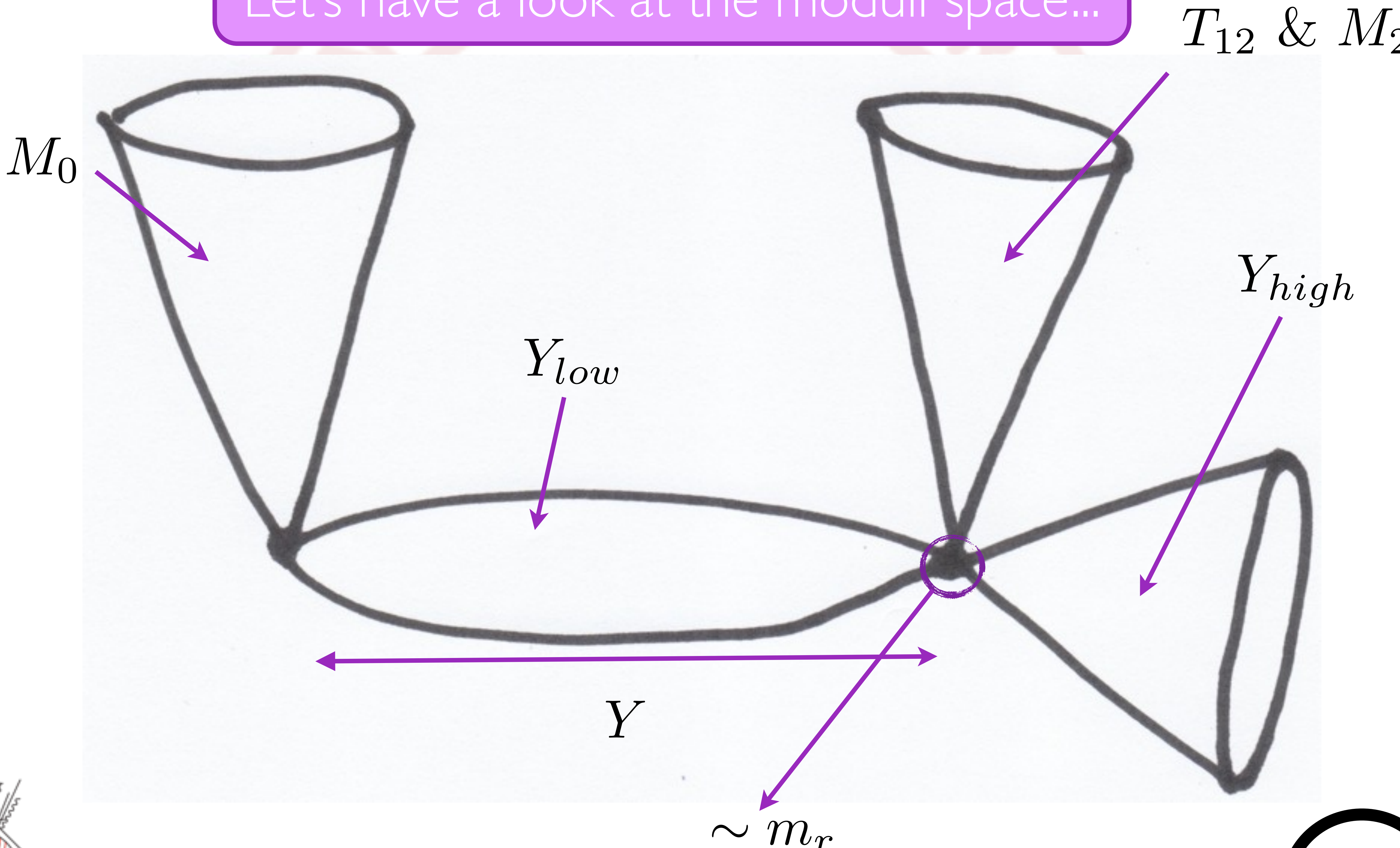
	$SU(4)$	$SU(2)$	$SU(3)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$\cancel{A}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	1	1	0	-3	0
Q	$\square$	1	$\square$	1	1	2	$\frac{1}{3}$
$\bar{Q}$	$\bar{\square}$	1	1	$\square$	-1	2	$\frac{1}{3}$
$M_0 = Q\bar{Q}$		1	$\square$	$\square$	0	4	$\frac{2}{3}$
$M_2 = QA^2\bar{Q}$		1	$\square$	$\square$	0	-2	$\frac{2}{3}$
$\cancel{T} = AQ^2$		$\square$	$\bar{\square}$	1	2	1	$\frac{2}{3}$
$\cancel{\bar{T}} = A\bar{Q}^2$		$\square$	1	$\bar{\square}$	-2	1	$\frac{2}{3}$
$\cancel{H} = A^2$		$\square\square$	1	1	0	-6	0

Only

T_{12}

$SU(4)$ with $3\,(\square + \overline{\square})$ & $\begin{smallmatrix}\square\\\square\end{smallmatrix} + \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$

Let's have a look at the moduli space...



$SU(4)$ with $3\,(\square + \overline{\square})$ & $\begin{smallmatrix}\square\\\square\end{smallmatrix} + \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$

Not S-Confining.

As we take the $m_r \to \infty$ the M_2 branch decouples and the theory develops a quantum modified constrain.

$$\boxed{T_{12}\det M_0 = 1}$$

$SU(4)$ with $3\,(\square + \overline{\square})$ & $\begin{array}{c}\square\\[-4pt]\square\end{array} + \overline{\begin{array}{c}\square\\[-4pt]\square\end{array}}$

Not S-Confining.

As we take the $m_r \to \infty$ the M_2 branch decouples and the theory develops a quantum modified constrain.

$$T_{12} \det M_0 = 1$$

Decoupling an anti-symmetric did not give us an s-confining theory as expected.

Exploring the moduli space

Exploring the moduli space

The moduli space of an s-confining theory is smooth. By exploring the moduli space we can hope to completely classify s-confining theories in 3D.

CONCLUSIONS

1- Naive dimensional reduction of 4D dualities does not work. A more involved procedure is needed to obtain 3D dualities from 4D.

2- Flowing down to different theories with less flavours or exploring the moduli space allows to decouple the ηY term and flow to S-Confining theories.

3- The Witten index provides a precious tool to look for confining theories. It is only a necessary condition.

4- In 4D, exploring the moduli space of S-Confining theories provide more S-Confining dualities. We expect the same to happen in 3D to obtain a complete classification.

THANKS!