Seiberg duality versus hidden local symmetry

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based on *S.Abel, JB* - 1202.2863

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Talk outline

What is a hidden local symmetry?

- The hidden local symmetry (HLS) formalism is an old idea
- Developed mainly to understand the chiral Lagrangian of QCD
- Provides an effective description of Goldstone bosons (GBs)

For any theory with flavour symmetry group G broken to subgroup H the HLS description has:

- same flavour symmetry G
- spontaneously broken gauge symmetry H

H is a hidden local symmetry: emerges from low energy dynamics.

Talk outline

Famous example is chiral Lagrangian for QCD.

- QCD with two massless flavours has $G = SU(2)_L \times SU(2)_R$
- Quark condensate breaks symmetry to $H = SU(2)_{\mathrm{diag}}$
- Leads to HLS description with broken $SU(2)$ gauge symmetry
- Provides excellent description of pions and ρ-mesons
- Pions are GBs
- ρ-mesons are massive gauge fields of HLS description

Talk outline

How does this apply to Seiberg duality?

- Take SQCD with $N + n$ flavours and N colours
- Non-Abelian flavour symmetry $SU(N+n)_L \times SU(N+n)_R$
- Generically broken to $SU(n)_L \times SU(n)_R$ by quark VEVs
- But $SU(n)$ is the gauge group of the Seiberg dual!

Talk outline

More specifically:

- The dual gauge group for SQCD can be interpreted as an HLS
- Dual gauge fields arise from 'vector mesons'
- Unlike QCD, SQCD has a limit in which symmetry is restored
- Allows for full duality, not just broken HLS description

HLS interpretation allows for:

- a more dynamical understanding of Seiberg duality
- mapping of charged states across the duality
- a possible approach for a non-supersymmetric extension

Overview

The original HLS formalism only describes GBs.[1]

$\mathcal{N} = 1$ SUSY $\implies$ each real GB has a real scalar and Weyl fermion partner.

- Superpartners are necessarily massless
- Must appear in any effective theory

Scalar partners especially important.

- Could be GBs themselves
- Otherwise provide additional massless scalars: quasi-GBs
- Quasi-GBs not just massless – appear as flat directions

[1] See *Bando, Kugo, Yamawaki* - Phys.Rept. 164 for a comprehensive review, including the supersymmetric generalisation

Crucial observation

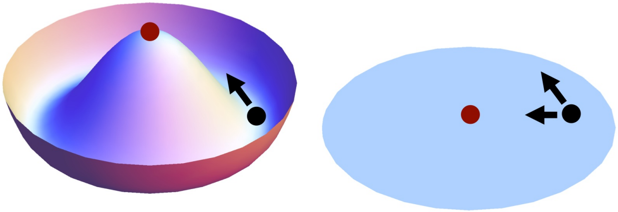
What if a quasi-GB expectation provides the order parameter for flavour symmetry breaking?

Quasi-GBs appear as flat directions so we can take their VEVs to zero $\implies$ points with enhanced symmetry.

But the exact same order parameter spontaneously breaks gauge symmetry in HLS description.

Allows gauge symmetry to be restored in supersymmetric HLS descriptions.

Non-supersymmetric vs. supersymmetric vacua



Left: non-supersymmetric theory – order parameter is stable.

Right: supersymmetric theory – order parameter is flat direction, moduli space contains point of enhanced symmetry

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   - Implications
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First step in finding HLS description is to non-linearly realise G.

How do we incorporate quasi-GBs?

Complexify flavour symmetry

$$G \longrightarrow G^c$$

Follows from holomorphy in the superpotential.

- Unbroken subgroup also complexified to $\hat{H} \supseteq H^c$
- Results in more symmetry generators
- Extra broken symmetry generators $\implies$ quasi-GBs

Attach one chiral superfield Π^a to each broken G^c generator $\hat{T}^a$.

Assemble into representatives of coset space $G^c/\hat{H}$

$$\xi(\Pi) = e^{\sum_a \Pi^a \hat{T}^a}$$

Contains all GBs and associated degrees of freedom in square matrix representation of G^c.

Non-linear transformation of ξ

$$\xi \longrightarrow g\,.\,\xi\,.\,h^{-1}(\Pi, g)$$

Note: freedom to choose g, h to act on opposite sides

Corresponds to left $\leftrightarrow$ right coset spaces $G^c/\hat{H}$.

Now to build an effective action for ξ.

First need to find projection operators satisfying

$$h \, . \, \eta = \eta \, . \, h \, . \, \eta$$

Purpose is to project components of ξ into $\hat{H}$-covariant subspaces:

$$\xi_\eta \longrightarrow g \, . \, \xi_\eta \, . \, h_\eta^{-1}(\Pi, g)$$

where

$$\xi_\eta = \xi \, . \, \eta \qquad\qquad h_\eta^{-1} = \eta \, . \, h^{-1} \, . \, \eta$$

η-projection prevents factorisation of $\det\left(\xi_\eta^\dagger \xi_\eta\right)$.

Now define:

- symmetry breaking scale v_η
- $\hat{H}$ invariant scalar d_η

Non-linear σ-model description

Action generated by Kähler potential

$$K_\eta^\sigma = d_\eta v_\eta^2 \,\mathrm{Tr}\left[\ln\left(\xi_\eta^\dagger \xi_\eta\right)\right]$$

is invariant under $\xi_\eta \to g \,.\, \xi_\eta \,.\, h_\eta^{-1}(\Pi, g)$.

- $d_\eta v_\eta^2$ is order parameter of symmetry breaking
- Will be allowed to vary when order parameter is quasi-GB VEV

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Warning!

η-projection implicitly assumed in the following

Consider ξ as a function of superspace coordinates.

Then

$$K^{\sigma} = dv^2 \, \mathrm{Tr} \left[\ln \left(\xi^{\dagger} \xi \right) \right]$$

is invariant under the linear transformation

$$\xi \longrightarrow g \, . \, \xi \, . \, h^{-1}(x)$$

for holomorphic functions h, i.e. theory has $\hat{H}$ gauge symmetry.

Now add vector superfield V for $\hat{H}$ and consider Kähler potential

$$K^V = v^2 \,\mathrm{Tr}\left[\xi^\dagger \xi e^{-V} + dV\right]$$

V is auxiliary superfield – no kinetic terms so integrate out.

Vector superfield EoM solution

$$de^V = \xi^\dagger \xi$$

Substitute vector superfield solution back into K^V to find

$$K^V = K^\sigma$$

- σ-model description recovered after integrating out V
- Two descriptions describe same low energy physics

HLS description

Most general Kähler potential is actually

$$K = K^\sigma + a(K^V - K^\sigma)$$

for real constant a.

Recall:

Vector superfield EoM solution

$$de^V = \xi^\dagger \xi$$

In Wess-Zumino gauge, scalar component is

$$d = \xi^\dagger \xi$$

Confirms that $d \neq 0$ is order parameter responsible for:

- breaking the flavour symmetry $G^c \to \hat{H}$
- completely breaking the $\hat{H}$ gauge symmetry

Is there a limit with $d \to 0$ such that symmetry is restored?

Suppose $\hat{T}^1 = \mathbb{1} \implies$ there is a broken $U(1)$ symmetry. Then

$$\xi = e^{\sum_a \Pi^a \hat{T}^a} = e^{\Pi^1} \left(e^{\sum_{a \neq 1} \Pi^a \hat{T}^a} \right)$$

and so

$$\xi^\dagger \xi = e^{2\bar{\kappa}} \left(e^{\sum_{a \neq 1} \Pi^{a\dagger} \hat{T}^a} e^{\sum_{a \neq 1} \Pi^a \hat{T}^a} \right)$$

where

$$\bar{\kappa} = \mathrm{Re}\left(\Pi^1\right)$$

$\bar{\kappa}$ is the scalar partner of the $U(1)$ GB $\implies$ $\bar{\kappa}$ is a quasi-GB.

$\Pi^{a\neq 1} = 0 \implies \xi^\dagger \xi = e^{2\bar{\kappa}} \implies d = e^{2\bar{\kappa}}.$

- $\bar{\kappa}$ rescales the VEV of $\xi^\dagger \xi$
- d is necessary to leave the value of $\bar{\kappa}$ undetermined
- $d \to 0$ can be taken by moving along the flat direction $\bar{\kappa}$
- Corresponds to restoring symmetry by scaling the order parameter to zero

The HLS description is dramatically simplified when $d \to 0$:

$$K^{\sigma} \longrightarrow 0 \qquad\qquad K^{V} \longrightarrow v^2\, \mathrm{Tr}\left[\xi^{\dagger}\xi e^{-V}\right]$$

Defining dimensionful chiral superfields by normalising them as

$$q = \sqrt{a} v \xi$$

it becomes $K = \mathrm{Tr}\left[q^{\dagger} q e^{-V}\right]$.

Result

Canonically normalised, unbroken gauge theory.

a and v act as normalisation constants.

This is the key feature that will lead to Seiberg duality.

Recap

Broken $U(1)$ flavour factors allow order parameters to be rescaled by moving along the associated quasi-GB direction.

Rescaling to zero restores gauge symmetry in the HLS description.

The result is a canonically normalised, unbroken gauge theory with flavour symmetry G^c and gauge group $\hat{H}$.

In SQCD R-symmetry provides the relevant $U(1)$.

What about the σ-model description?

- $d \to 0 \implies K^\sigma \to 0$ so σ-model description breaks down
- Result of flavour symmetry restoration
- At points of enhanced symmetry GBs become ex-GBs
- σ-model description flawed – revert to underlying theory

Note: 'new' massless DoF in both descriptions:

- Gauge fields in the HLS description
- Ex-GBs in the σ-model description

The two are related.

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Now apply to SQCD and relate these ideas to Seiberg duality.[1]

While the idea has been mentioned in the literature[2] it has not been explored fully.

[1] Good reviews of Seiberg duality in *Intriligator, Seiberg* - hep-th/9509066; *Terning* - hep-th/0306119

[2] First suggested in *Harada, Yamawaki* - hep-ph/9906445, hep-ph/0302103; rediscovered in *Komargodski* - 1010.4105; *Kitano* - 1109.6158

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SQCD with $N+n$ flavours and N colours:

	$SU(N)$	$SU(N+n)_L$	$SU(N+n)_R$	$U(1)_B$	$U(1)_R$
Q	$\square$	$\tilde{\square}$	$\mathbf{1}$	$+1/N$	$n/(N+n)$
$\tilde{Q}$	$\tilde{\square}$	$\mathbf{1}$	$\square$	$-1/N$	$n/(N+n)$

N colours $\implies$ quarks are $N \times N+n$ matrices.

At a generic point in moduli space scalar VEVs are rank N matrices

$$Q = \begin{pmatrix} \mathbf{v} & 0 \end{pmatrix} \qquad \tilde{Q} = \begin{pmatrix} \tilde{\mathbf{v}} \\ 0 \end{pmatrix}$$

Original flavour symmetry was

$$G = SU(N+n)_L \times SU(N+n)_R \times U(1)_B \times U(1)_R$$

Generic quark VEVs break it to

$$SU(n)_L \times SU(n)_R \times U(1)_{B'} \, [\times U(1)_{R'}]$$

Unbroken, complexified flavour transformations are

$$h_L = \begin{pmatrix} \mathbb{1} & 0 \\ h_{L,l} & h_{L,n} \end{pmatrix} \qquad h_R = \begin{pmatrix} \mathbb{1} & h_{R,u} \\ 0 & h_{R,n} \end{pmatrix}$$

where $\det\left(h_{L,n}h_{R,n}\right) = 1$.

GB superfields assembled into

$$\xi = e^{\kappa_R} \begin{pmatrix} e^{\kappa_B}\xi_N & \xi_u \\ 0 & \mathbb{1} \end{pmatrix} \qquad \tilde{\xi} = e^{\kappa_R} \begin{pmatrix} e^{-\kappa_B}\tilde{\xi}_N & 0 \\ \tilde{\xi}_l & \mathbb{1} \end{pmatrix}$$

where

- Representatives from $SU(N+n)_L$ and $SU(N+n)_R$ sectors
- κ_B comes from the broken $U(1)_B$
- κ_R comes from the broken $U(1)_R$

Need to find projection operators. Choose

- h_L to act on right
- h_R to act on left

Unique projection operator

$$\eta = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{1} \end{pmatrix}$$

η projects h_L, h_R to

$$SU(n)_L^c \times SU(n)_R^c \times U(1)_{B'}^c$$

transformations and ξ, $\tilde{\xi}$ to

$$\xi \,.\, \eta = \xi_\eta = e^{\kappa_R} \begin{pmatrix} \xi_u \\ \mathbb{1} \end{pmatrix} \qquad \longrightarrow \qquad g_L \cdot \xi_\eta \cdot h_{L,n}^{-1}$$

$$\eta \,.\, \tilde{\xi} = \tilde{\xi}_\eta = e^{\kappa_R} \begin{pmatrix} \tilde{\xi}_l & \mathbb{1} \end{pmatrix} \qquad \longrightarrow \qquad h_{R,n} \cdot \tilde{\xi}_\eta \cdot g_R^\dagger$$

κ_R appears as an overall scaling direction as before.

Allows for the symmetry restoring limit $\xi_\eta^\dagger \xi_\eta \to 0$.

First attempt: $SU(n)_L \times SU(n)_R \times U(1)_{B'}$ gauge theory with

	$SU(n)_L$	$SU(n)_R$	$U(1)_{B'}$	$SU(N+n)_L$	$SU(N+n)_R$
ξ_η	$\tilde{\square}$	$\mathbf{1}$	-1	$\square$	$\mathbf{1}$
$\tilde{\xi}_\eta$	$\mathbf{1}$	$\square$	$+1$	$\mathbf{1}$	$\tilde{\square}$

Problem: gauge group is anomalous.

Fine for a broken gauge group, but won't allow an unbroken limit.

Redefine gauge groups in terms of anomalous and anomaly free linear combinations

$$V = \frac{1}{2}(V_L + V_R) \qquad\qquad V' = \frac{1}{2}(V_L - V_R)\,.$$

$\mathrm{Tr}\,[V] = 0$ as $U(1)_{B'}$ transformations restricted to V'.

Leads to

	$SU(n)$	$U(n)'$	$SU(N+n)_L$	$SU(N+n)_R$
ξ_η	$\tilde{\square}$	$\tilde{\square}$	$\square$	$\mathbf{1}$
$\tilde{\xi}_\eta$	$\square$	$\tilde{\square}$	$\mathbf{1}$	$\tilde{\square}$

Kähler potential is

$$K = \mathrm{Tr}\left[(1-a)e^{2\bar{\kappa}_R}v^2\ln\left(\xi_\eta^\dagger\xi_\eta\right) + (1-\tilde{a})e^{2\bar{\kappa}_R}\tilde{v}^2\ln\left(\tilde{\xi}_\eta\tilde{\xi}_\eta^\dagger\right)\right] + \mathrm{Tr}\left[av^2\xi_\eta^\dagger\xi_\eta e^{-V-V'} + \tilde{a}\tilde{v}^2\tilde{\xi}_\eta\tilde{\xi}_\eta^\dagger e^{V-V'} + (av^2+\tilde{a}\tilde{v}^2)e^{2\bar{\kappa}_R}V'\right].$$

No trace term for V as $\mathrm{Tr}[V] = 0$.

Vector superfield EoM

$$\xi_\eta^\dagger\xi_\eta = e^{2\bar{\kappa}_R+V'}e^{V} \qquad \tilde{\xi}_\eta\tilde{\xi}_\eta^\dagger = e^{2\bar{\kappa}_R+V'}e^{-V}$$

V' appears as a scaling direction like $\bar{\kappa}_R$.

- Can absorb it into the chiral superfields
- Corresponds to fixing the gauge for $U(n)'^c$

Equivalently, could recast $\bar{\kappa}_R$ as part of a vector superfield (recovers ignored, spontaneously broken, gauged $U(1)_{R'}$).

Can generically exchange flat directions for vector superfields of complexified gauge symmetries.

Consequence of the huge amount of redundancy in the theory.

Absorb V' and define dimensionful DoF

$$V' = -\ln(\sigma\sigma^\dagger) \qquad q = \sqrt{a} v \xi_\eta \sigma \qquad \tilde{q} = \sqrt{\tilde{a}} \tilde{v} \sigma \tilde{\xi}_\eta$$

These transform under $SU(n) \times SU(N+n)_L \times SU(N+n)_R$ as

$$q \in (\tilde{\Box}, \Box, 1) \qquad \tilde{q} \in (\Box, 1, \tilde{\Box})$$

just like the dual quarks in Seiberg duality.

Kähler potential becomes

$$K = \mathrm{Tr}\left[q^\dagger q e^{-V} + \tilde{q}\tilde{q}^\dagger e^{V}\right] + e^{2\bar{\kappa}_R}[\ldots]$$

Result

Broken $SU(n)$ gauge theory with dual quarks.

Exactly what is expected from Seiberg duality!

Symmetry restored by taking $e^{2\bar{\kappa}_R} \to 0$ to recover unbroken dual.

Scaling permitted due to quasi-GB associated with R-symmetry.

Seiberg dual should also have a meson superfield $M \in (1, \tilde{\Box}, \Box)$.

In HLS description this is constructed from a flipped coset theory.

Choose

- h_L to act on left
- h_R to act on right

Leads to alternative projection operator

$$\eta' = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & 0 \end{pmatrix}$$

Projects h_L, h_R to identity – subspace invariant under $\hat{H}$.

$\implies$ flipped coset theory describes order parameters.

Meson superfield defined as

$$M = v\tilde{v}(\tilde{\xi}\,.\,\eta'\,.\,\xi) \longrightarrow g_R\,.\,M\,.\,g_L^\dagger$$

Problem: using standard and flipped coset descriptions leads to some DoF in ξ, $\tilde{\xi}$ being double counted.

Solution: include superpotential

$$W = \frac{1}{\mu} \mathrm{Tr}\left[M q \tilde{q}\right]$$

For arbitrary duality scale μ.

Double counted DoF projected out of effective theory.

Result

Full Seiberg dual recovered!

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All dual DoF defined in terms of GB superfields.

These are known explicitly for SQCD in the perturbative regime.

Check: write down explicit expressions for ξ, $\tilde{\xi}$ to find $M = \tilde{Q}Q$.

Expand original quarks as

$$Q = \begin{pmatrix} Q' & P \end{pmatrix} \qquad \tilde{Q} = \begin{pmatrix} \tilde{Q}' \\ \tilde{P} \end{pmatrix}$$

P, $\tilde{P}$ are pure GB superfields.

Now substitute expressions for ξ, $\tilde{\xi}$ into vector superfield EoM.

Result

$$V^\alpha \approx \mathrm{Tr}\left[S^\alpha\left(\frac{P^\dagger P}{v^2} - \frac{\tilde{P}\tilde{P}^\dagger}{\tilde{v}^2}\right)\right]$$

Dual gauge fields correspond to 'vector mesons'.

Valid for $v, \tilde{v} \gg \Lambda$.

What about duality scale μ and normalisation constants a, $\tilde{a}$?

- All contribute to order parameters of HLS description
- Must result in equivalent order parameter to original theory

For consistency with exact results from gaugino condensation:

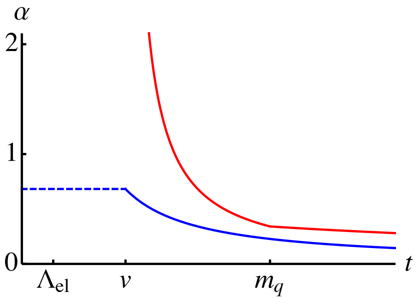
- $a = 2$ on baryonic branch $(M = 0,\ B \neq 0,\ \tilde{B} = 0)$
- $a = 1$ on mesonic branch $(M \neq 0,\ B = \tilde{B}$, i.e. $v = \tilde{v})$
- $\mu = \Lambda(v/\Lambda)^{2(n-N)/n}$ on mesonic branch (if $v \gg \Lambda$)

Result

HLS description suggests a particular choice of duality scale μ.

SQCD may help understand the origin of $a = 2$ – vector meson dominance in QCD.

RG flow on mesonic branch



Original theory higgsed at scale v.

Dual theory confines at same scale.

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As well as regular Seiberg duality the HLS interpretation accommodates:

- the derivation of the original theory from dual description
- massive quarks (both flip confining/higgsing behaviour)
- Seiberg duality for SO and Sp gauge groups
- Seiberg duality with a superpotential (e.g. adjoint SQCD)

Potential applications include:

- composite gauge boson scattering[1]
- lessons for QCD[2]
- systematic approach for finding non-supersymmetric dualities

[1] *Craig, Stolarski, Thaler* - 1106.2164; *Csaki, Shirman, Terning* - 1106.3074

[2] *Komargodski* - 1010.4105; *Kitano* - 1109.6158; *Kitano, Nakamura, Yokoi* - 1202.3260

Summary

- Gauge symmetry can be restored in supersymmetric HLS descriptions when the order parameters are quasi-GB VEVs
- Applying the HLS formalism to SQCD, this fact allows the full Seiberg dual to be recovered
- The dual gauge group is the HLS
- Dual gauge fields arise from 'vector mesons'
- The duality scale μ can be determined
- The approach extends to variants of Seiberg duality
- It can hopefully be applied to several interesting problems