Quantum causal dynamics without absolute space or time

by Aron C. Wall

based on “A discrete, unitary, causal theory of quantum gravity”
arXiv:1201.2489

with gratitude to William Donnelly, Laurent Freidel, Don Marolf, Lee Smolin, and Ted Jacobson. Supported by NSF grants PHY-0601800, PHY-0903572, the Maryland Center for Fundamental Physics, and the Simons Foundation.

A “DUCT”

is a

background-free DISCRETE model of spacetime

based on UNITARY replacement rules

which includes CAUSALity at a fundamental level

which is “just a THEORY” *(in the particle physics sense of a mathematical model that might or might not have anything to do with reality)*

Why study a discrete spacetime model?

1) Spacetime might be discrete in quantum gravity,
2) Regulates the ill-defined continuum path integral/canonical quantum gravity.

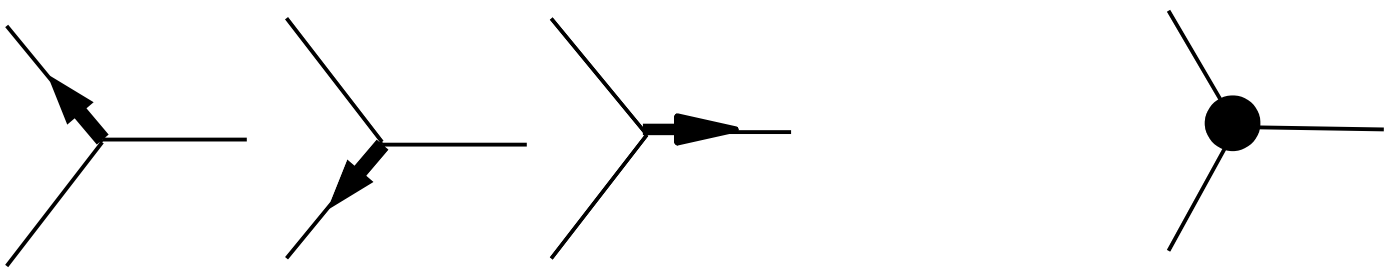
Kinematics

Kinematics of the DUCT

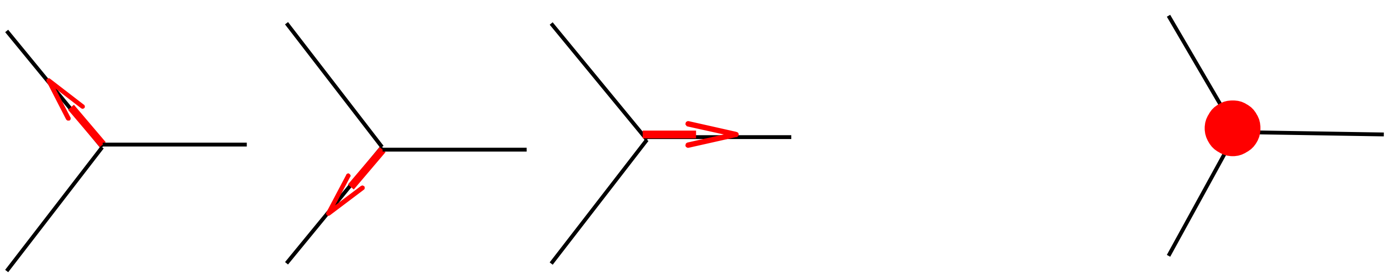
Inspired by loop quantum gravity, space at one time will be described by an *abstract labelled network* of some valence (e.g. trivalent)

Each vertex is labelled with a “future arrow” and a “past arrow”.

The future arrow governs evolution towards the future:
can point along one of the edges: or else to the vertex itself:



The past arrow governs past evolution, and has the same degrees of freedom:

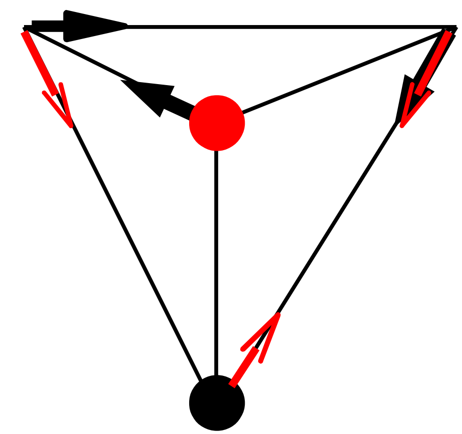


The two arrows are independent, so 4 x 4 = 16 dim. Hilbert Space per vertex (trivalent case). (Can generalize to case where arrows aren't independent.)

Kinematic Basis States

The vector space of kinematic states is defined by linearized superpositions of “basis states” A, B, C...

There is one basis state for every trivalent network labelled with past and future arrows, e.g:

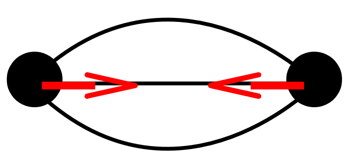


Analogue of spatial diffeomorphism invariance:
Any two graphs which differ by graph isomorphisms (any permutation of vertices and edges preserving the topology & arrows) are regarded as equivalent.

The natural inner product on this space is:

$$\langle A|B\rangle_{\mathrm{kin}} = |\mathrm{Aut}(A)|\delta_{AB}$$

where $|\mathrm{Aut}(A)|$ is the number of graph “automorphisms” (symmetries) of A

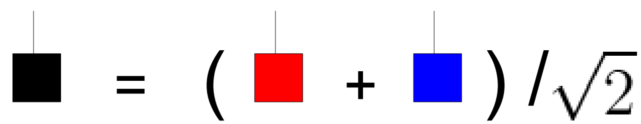
E.g.  has $|Z_2 \times Z_2| = 4$ symmetries

(If the network has a boundary, don't count symmetries which affect the boundary)

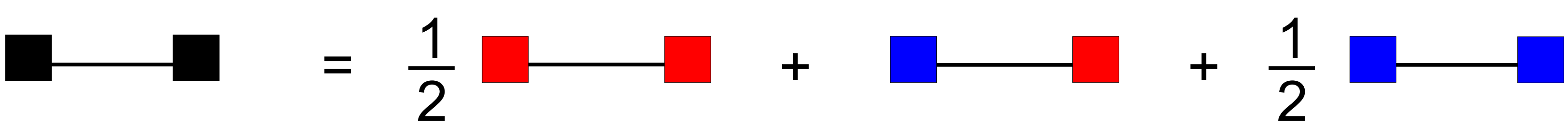
Why the Symmetry Factor is Necessary

Without the extra symmetry factor, inner product isn't invariant under local unitary transformations applied to the network

Important since dynamics will be defined using unitary replacement rules.

e.g. suppose that 

where the colored squares represent different possible states for networks with one external edge



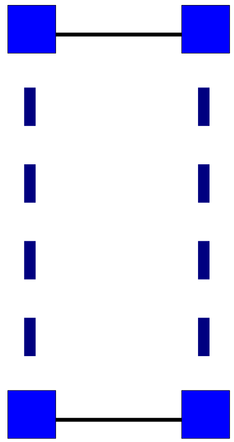
$$2 = 1/2 + 1 + 1/2$$

$$1 \neq 1/4 + 1 + 1/4$$

Twisted histories

Another way to think of the kinematic inner product, is that
it comes from a trivial “sum over histories” in which nothing happens:

The only “histories” that are allowed are graph isomorphisms,
and each history has unit amplitude:



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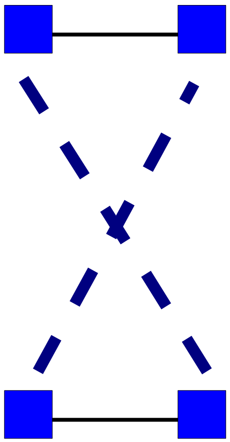

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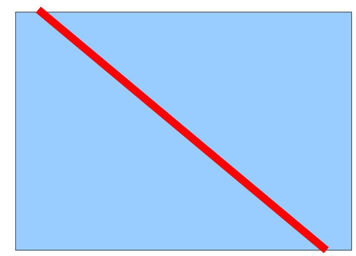
Why dynamics is hard

Reduction to physical state space

In a gravitational theory time translations are a gauge symmetry.
Thus dynamics should reduce the number of degrees of freedom.

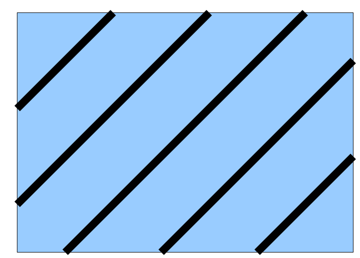
Two equivalent ways to do this:

1) Reduce to the physical subspace
by imposing a *constraint*.



physical states
are a subspace
of kinematic states

2) Reduce to the physical subspace
by modding out *pure gauge* states.



physical states
are orbits of the
kinematic state

These pictures are related by taking the dual of the state vector spaces.

I find picture #2 more convenient
every kinematic state corresponds to a physical state, so one can “mention”
a specific physical state before solving the dynamics (but it might be 0)

The problem of dynamics

Can define the physical state space by:

* imposing a Hamiltonian constraint $H(x)|\Psi\rangle_{\mathrm{phys}} = 0$
* or by defining a sum over histories (spin foam models).

Some of the problems that can potentially arise:

1) *Ultralocality.* The Hamiltonian only grows the network locally, but does not propagate degrees of freedom from vertex to vertex. (Smolin)

2a) *Diffeomorphism anomaly.*
In canonical approach, constraint algebra may not close, due to $[H(x), H(y)]$.

In covariant approach, must show that all states related by diffeomorphisms are null vectors.

3) *Not completely background-free.* Some proposed discrete dynamics presuppose a notion of "simultaneity".

"The classical limit and the form of the hamiltonian constraint in nonperturbative quantum gravity", arXiv:gr-qc/9609034v1.

Analogy to parallel processing

Problem is that time evolution has to work independently in different regions of space, without a “clock” to regulate the speeds of various processes.

Problem is already well-known in computer science!
In parallel processing, different processes are assigned to independent “threads”.

But not just any algorithm will do—consistency checks are required.
Problems that can arise:

1) *Race conditions:* two processes compete for the same data resource.
(e.g. Thread A toggles a bit while Thread B measures it)
BAD in computing because computation must give unique answer.
BAD in quantum gravity because time evolution should be unambiguous.

2) *Deadlock:* processes “waiting” for each other to finish get stuck forever
(e.g. A is waiting for B to finish, B for C and C for A)
BAD in computing because computation shouldn't halt before outputting the answer.
BAD in quantum gravity because time should not just come to an end somewhere
(except maybe at singularities).

The universe “computer” must be cleverly programmed to avoid these problems.

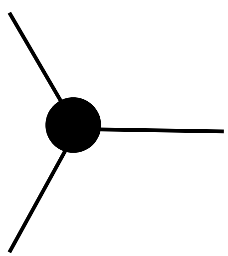
How the DUCT solves this problem

Initial and Final Evolution Regions

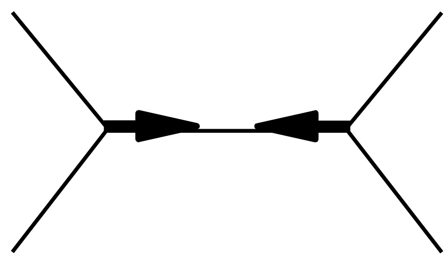
Forwards time evolution takes “initial evolution regions” (IER's), and replaces that piece of the network with “final evolution regions” (FER's) that have the same number of external edges.

An IER is a *minimal* set of vertices whose future arrows point to each other, plus the edges which the arrows point along. All other edges are “external”.

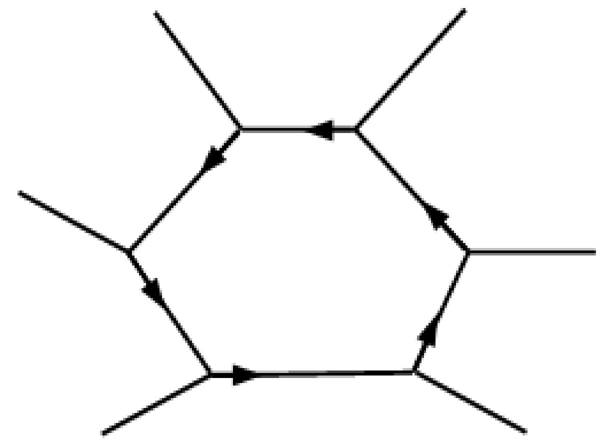
a)



b)



c)

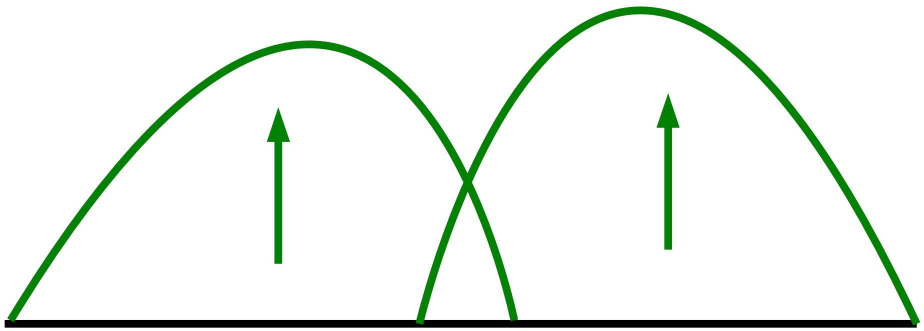


This is the simplest rule I can think of that:

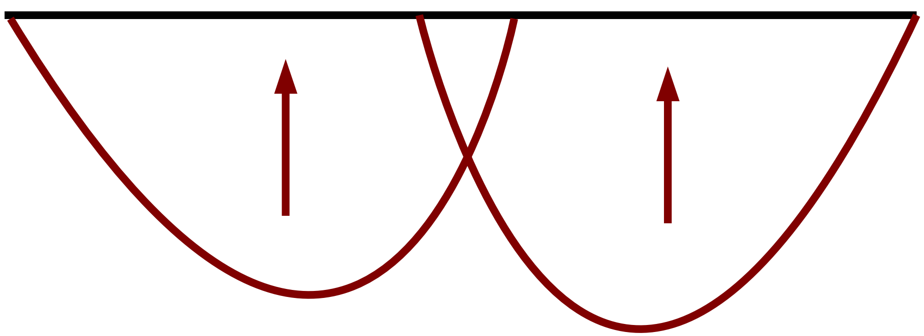
1) avoids deadlock. In any closed network there is at least one IER.
2) avoids race conditions. No two IER's can overlap with each other.

The same rule defines “final evolution regions” (FER's) using past arrows.

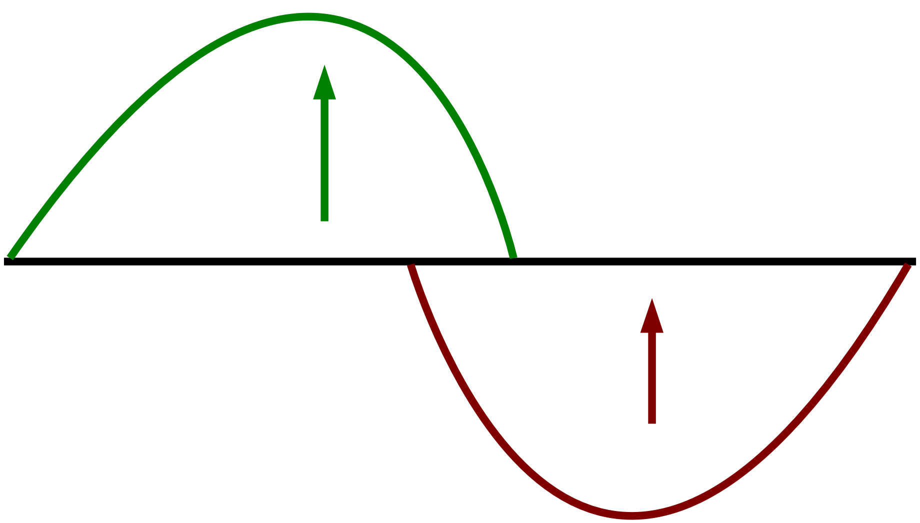
Overlapping IER's are inconsistent if histories are made of indivisible time steps:



As are overlapping FER's:



But an IER and a FER can overlap:



Dynamics of the DUCT

Based on unitary transition amplitudes:

Sort the IER's and FER's into classes based on the number of external edges.

For each class Q, identify a unitary operator $I(Q)$ which maps from the vector space of IER's to the vector space of FER's:

* I is unitary with respect to the *kinematic inner product* of the IER/FER.
* I must be symmetric w.r.t. permuting the boundary edges.

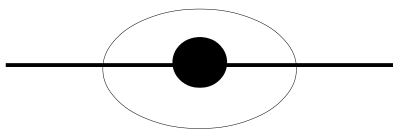
The elements of the I matrices are the "constants of nature" of the DUCT.

Since there is no "absolute time", I implements a gauge symmetry:
if you start with a network A and act on it using I to get a (superposition of) networks B, then A and B are *identified* as the same "physical state".

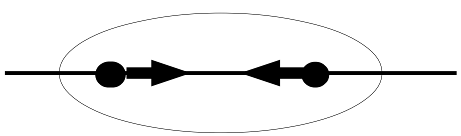
Example of a Bivalent DUCT

EVOLUTION REGIONS WITH TWO EXTERNAL EDGES:

12 possible IER's



(3 past arrow states)

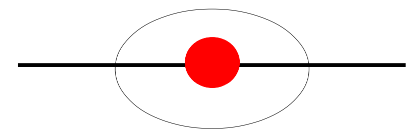


(9 past arrow states)

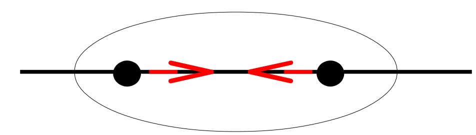
$$\xrightarrow{I}$$

12 x 12
unitary matrix

12 possible FER's

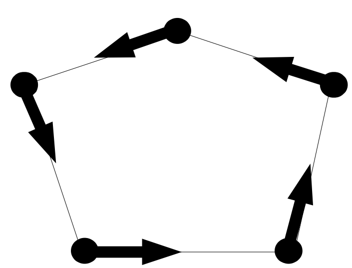


(3 future arrow states)

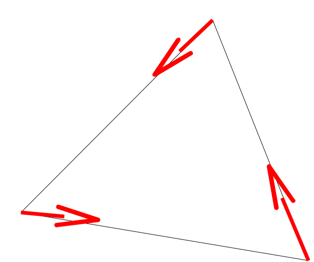


(9 future arrow states)

EVOLUTION REGIONS WITH ZERO EXTERNAL EDGES:



infinitely many—
need restrictions
to get finite
# of parameters

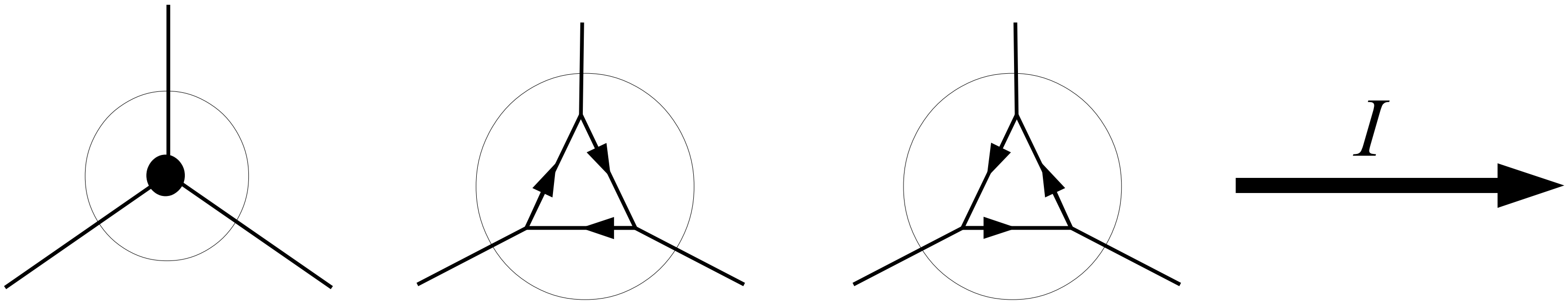


e.g. evolution rule acts independently on each vertex.

Trivalent DUCT

Similar, but more complicated. Can restrict to Pachner moves for simplicity:

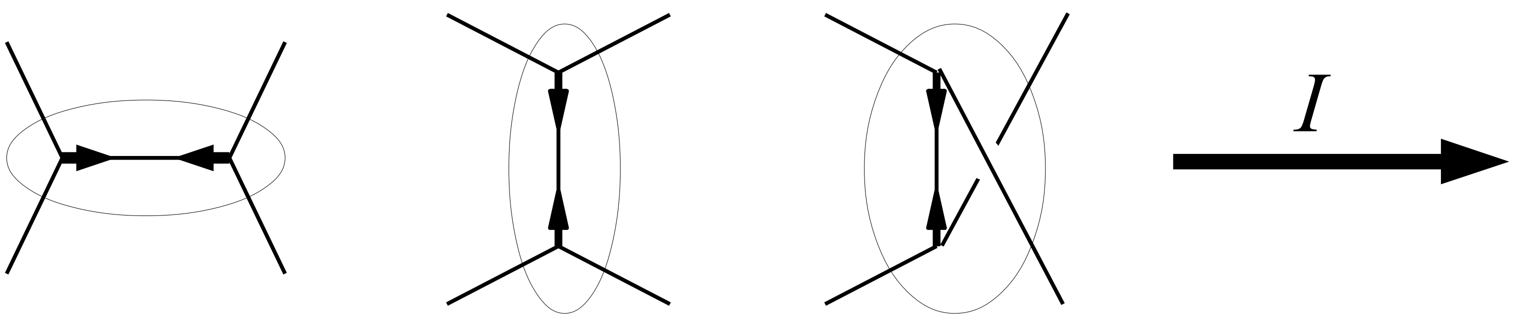
IER'S WITH 3 EXTRNAL EDGES



corresponding FER's

4 states   64 states   64 states

IER'S WITH 4 EXTRNAL EDGES (PACHNER GRAPHS ONLY)

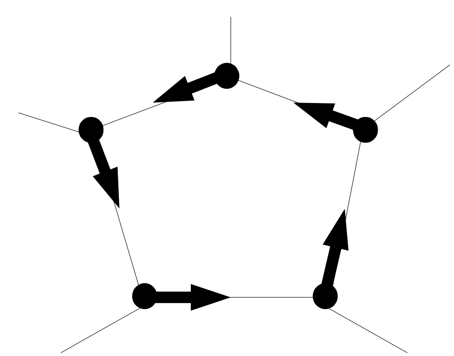


corresponding FER's

16 states   16 states   16 states

IER'S WITH LOOPS OF N VERTICES (BESIDES N = 3)

One U per possible value of N.
Requires ansatz to keep finite # of parameters



Defining the Sum over histories

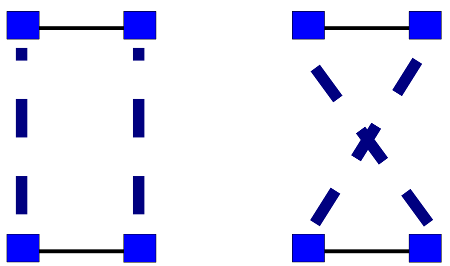
Definition of a history

The inner product on the physical state space of a DUCT is defined using a sum over histories:

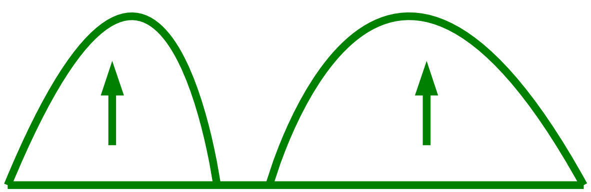
A *history* consists of any sequence of transitions from a basis state A to a basis state B in which IER's are replaced with FER's (forwards evolution), or else FER's are replaced with IER's (backwards evolution).

<u>Fine Print:</u>

1) **Shift Diffeos:** Two histories that differ by graph isomorphisms of intermediate network states are to be regarded as equivalent, so long as the permutations the full (vertex, edge, arrow, transitions) structure, and A and B are *not* affected.

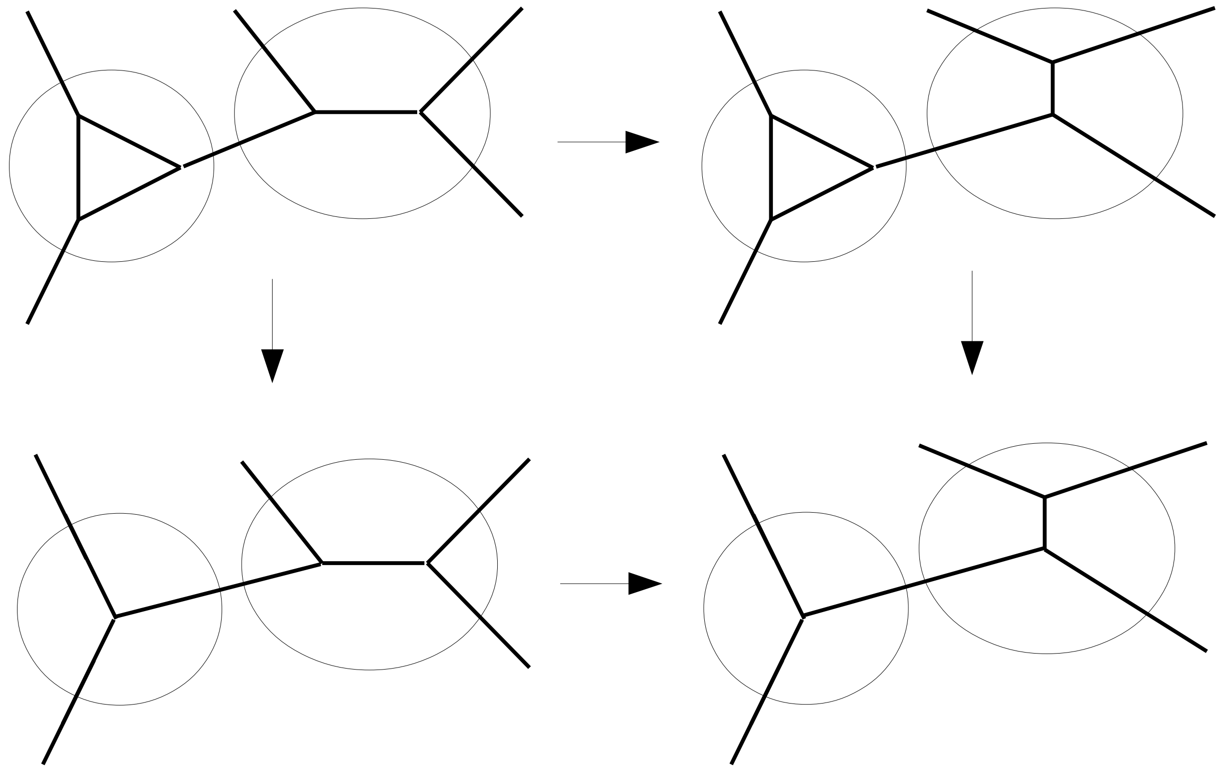


2) **Lapse Diffeos:** When two transitions occur in disjoint regions of the network, it does not matter which order they occur in.



3) **No Backtracking:** It is not possible to evolve forwards and backwards in the same spatial location.

Many-Fingered Time



Replacement rules occuring in disjoint regions
can be evolved forwards in time independently of each other.

The two paths from the start to the finish are the *same* history.

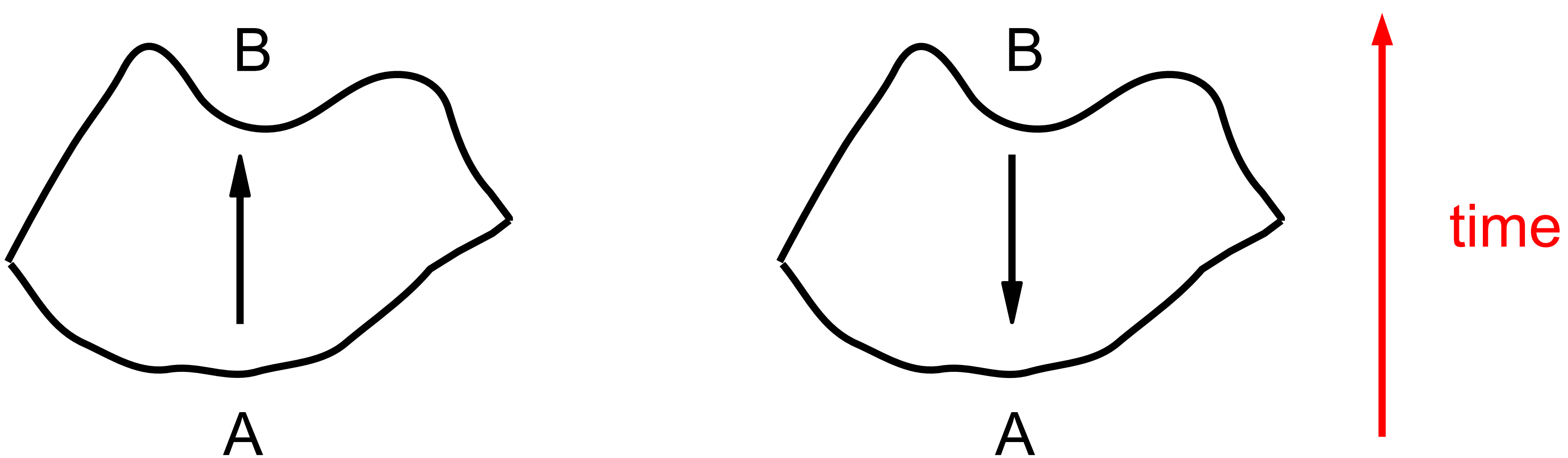
Necessary to avoid “absolute time”. Analogous to lapse diffeos in GR.

Forwards and Backwards Evolution

Histories are defined by applying I operators in sequence to replace IER's with FER's. Can also use I^{-1} to replace FER's with IER's.

In order to preserve covariance, must include histories with backwards evolution in the sum over histories. (Teitelboim "Causality Versus Gauge Invariance in Quantum Gravity and Supergravity", Phys. Rev. Lett. 50, 705 (1983).)

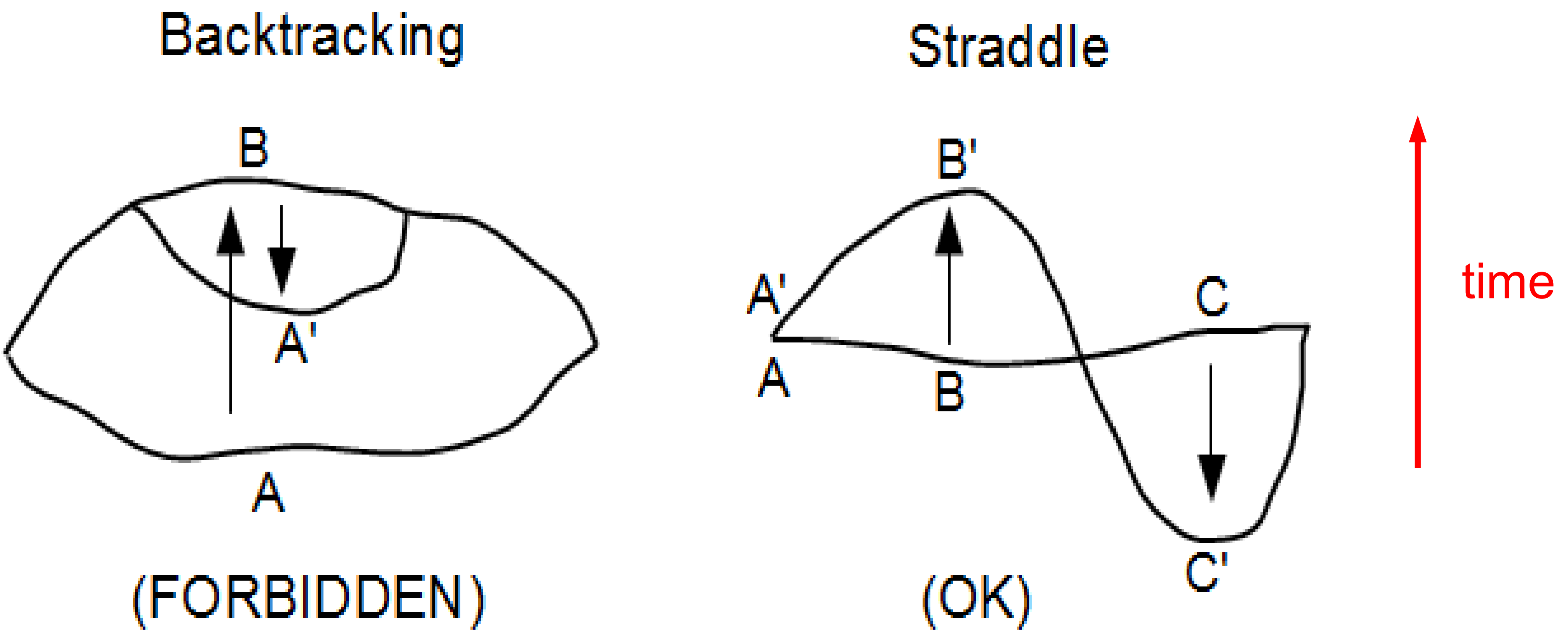
If time evolution leads to a gauge identification of two states A = B, this must hold regardless is to the future and which is to the past.



Forwards evolving history
(replacing IER's with FER's)

Backwards evolving history
(replacing FER's with IER's)

Combining forwards and backwards evolution



Regardless of the relative time positioning, there is a causal structure (partial ordering), defined relative to the red time arrow.

Feynman Rules

Physical inner product between two states calculated by sum over histories:

$$\langle B|A\rangle_{\text{phys}} = \sum_{H(A,B)} \frac{1}{\text{Aut}(h)} \prod_{n=0}^{t-1} T(h_n \to h_{n+1})$$

* H(A,B) is the class of histories beginning with network A and ending with B,
  t is the total number of transitions in the history h.
  (subject to equivalence relations and "no backtracking rule")

* $T(t_n \to t_{n+1})$ $\begin{cases} T_{ai} = |\text{Aut}(i)| I_{ai} & \text{to replace IER } i \text{ with FER } a \\ T^\dagger_{ia} = |\text{Aut}(a)| I^{-1}_{ia} & \text{to replace FER } a \text{ with IER } i \end{cases}$

* Symmetry factors:
Aut(*h*), Aut(*i*), Aut(*a*) = automorphism groups of h, IER, FER respectively.

I "derived" these rules by trial and error—leave anything out and you'll lose the "lapse" diffeomorphism gauge symmetry of the inner product!

Physical Features

Properties of the Inner Product

Assuming the sum over histories converges, the inner product is:

1) Hermitian: $\langle A|B\rangle = \langle B|A\rangle^*$ because the time-reverse of any history has a conjugate amplitude.

2) Gauge degenerate: if *i* labels an IER and *a* labels the FER's it can evolve to, then the difference between two states related by time evolution is a "null" vector:

$$\langle B| \left(|C \smile i\rangle - \sum_a I_{ai} |C \smile a\rangle \right) = 0$$

$C \smile i$ means the network formed by joining the IER *i* to some *C*. No diff anomaly!

3) Positivity: $\langle A|A\rangle \geq 0$

I believe this holds for all states, but can only prove it so far in situations where each state has a finite past. (Requires either states w/ boundary, or modified rules)

Given these properties, one gets a Hilbert Space of "physical states" after modding out null vectors. Dual to usual picture in canonical quantum gravity, where physical states are obtained by imposing a constraint.

Convergence issues

No guarantee that the sum over histories is absolutely convergent.
Since the sum over histories is “Lorentzian” there may be histories with large absolute amplitudes whose phases cancel to nearly zero.

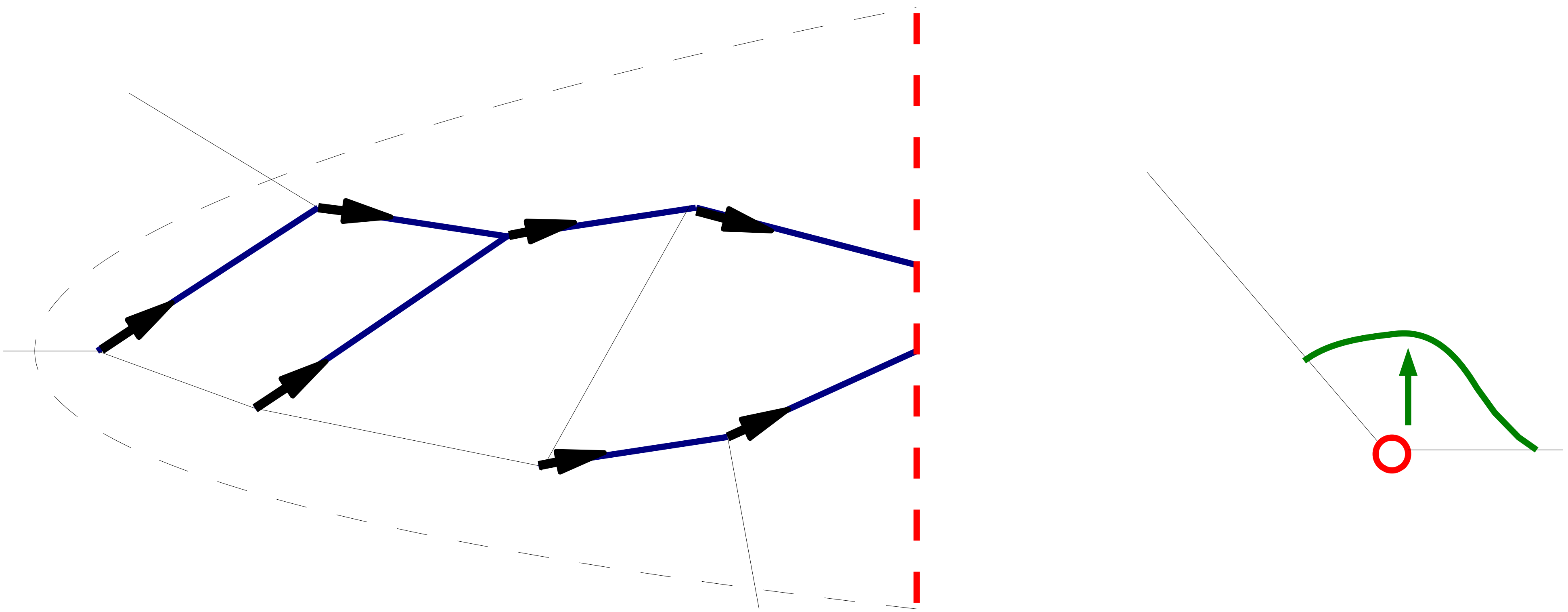
At worst, number of histories diverges exponentially with transitions t.
(Combinatorics good because histories must be causal.)

Suggests that if inner product does not converge, it can be “Abel resummed” by adding a damping factor $0 \leq c \leq 1$ to each transition element:

$$\langle B|A\rangle = \sum_{H(A,B)} \frac{c^t}{|\mathrm{Aut}(h)|} \prod_{n=0}^{t-1} T(h_n \to h_{n+1}),$$

Analytically continue to $c = 1$. Assuming no obstructions, resulting inner product should still be hermitian and gauge-invariant.

Analogue of Null Surfaces



If all the arrows in a region line up, it is impossible to evolve that region any farther without first evolving what is behind the red line.
Similar to a null surface in GR. Blue lines are like “horizon generators”.

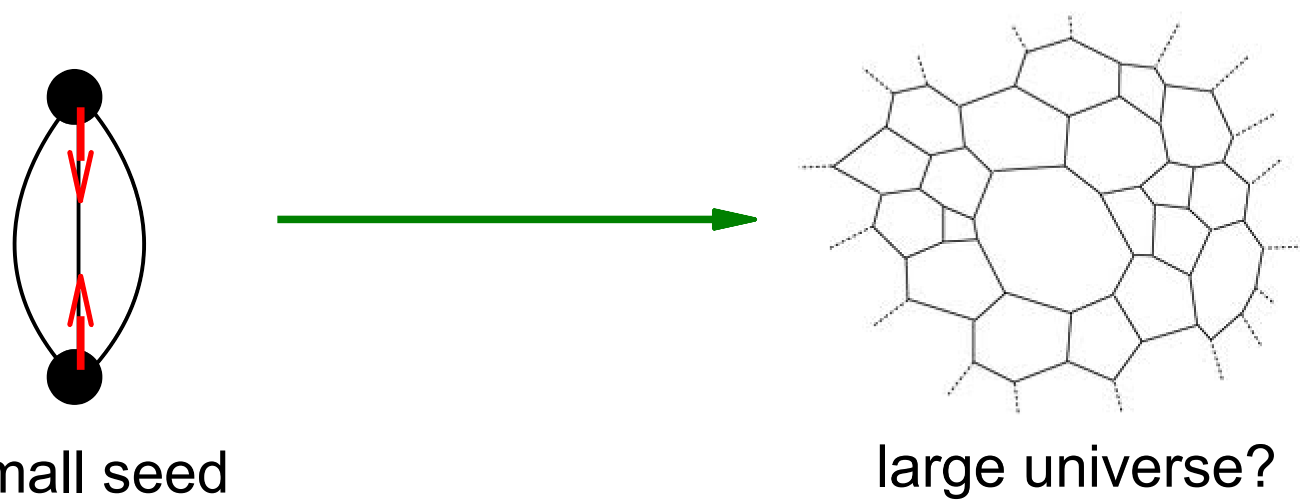
Can ask questions about horizon thermodynamics.

Universe from a small seed?

No direct analogue of the Hartle-Hawking state since no processes which change the number of connected components are allowed.

But one can choose a “physical state” that has a small seed configuration lying on it's gauge orbit. Presumably it “expands” to increase its entropy.

Physical predictions require selecting gauge-invariant observables.



Status of the model

* Assuming the issues of convergence and positivity work out okay, a DUCT is a “background-free quantum theory with local degrees of freedom propagating causally” (Baez definition of quantum gravity, in “Higher-Dimensional Algebra and Planck-Scale Physics” http://math.ucr.edu/home/baez/planck/)

* can generalize to models in which past and future arrows can't be measured simultaneously due to QM uncertainty relations.
(These models can have single-digit numbers of free parameters.)

* not clear whether Lorentz symmetry can emerge from a discrete theory of this sort. (Perhaps continuum limit would be an aether theory?)

* not clear whether an approximate semiclassical spacetime emerges (probably related to the existence of a stable vacuum state)

Further Research Possibilities

Computer simulations—1) Does sum over histories converge?
2) Is the inner product always positive?
3) Evidence for a continuum limit?

But simulations probably have to be "small" due to Lorentzian signature:
No Monte Carlo algorithm, number of histories goes exponentially

Can also try simulating classical analogue of model (deterministic rules)

Couple model to a fixed boundary to test holographic ideas
No "holographic principle" built in explicitly, since the model is based on local degrees of freedom—but it is subject to diffeo invariance.

Test argument that diffeo invariance implies holography?
Marolf, "Unitarity and Holography in Gravitational Physics" (arXiv:0808.2842)
id. "Holographic Thought Experiments" (arXiv:0808.2845)

The DUCT may still be missing important physical ingredients.
Even if it fails, it might help diagnose what else is needed.

THE END

Quantum Gravity

Goal to construct theory incorporating lessons from GR & QFT, e.g.

* *Background-free:* there should be no *a priori* fixed space or time background used to define the the dynamics. Instead, the dynamics of matter should affect the nature of space and time themselves. (GR)

* *Quantum mechanical:* states are vectors in Hilbert spaces, while processes are described by unitary operators. (QM)

* *Causal:* local degrees of freedom propagate causally with respect to the "causal structure" of spacetime.
(Both GR and QFT, but in GR the causal structure is itself dynamical)

i.e. one wants a "background-free quantum theory with local degrees of freedom propagating causally" (Baez)

hope is to obtain GR and QFT in appropriate limits.
(may require additional properties not listed above)