Cosmology of multi-scalar-singlet extensions of the Standard Model

Aleksandra Drozd
Institute of Theoretical Physics, University of Warsaw

University of California Davis, April 10th, 2012

The project "International PhD Studies in Fundamental Problems of Quantum Gravity and Quantum Field Theory" is realized within the MPD programme of Foundation for Polish Science, cofinanced from European Union, Regional Development Fund

Multi-scalar-singlet extension of Standard Model

- Model
- Theoretical Constraints
- Cosmological Constraints

In collaboration with B. Grządkowski and J. Wudka, arXiv:1112.2582 [hep-ph], JHEP04(2012)006

Model

Standard Model + N-component singlet scalar $\vec{\varphi}$
O(N) symmetry and Z_2 symmetry: $\vec{\varphi} \to -\vec{\varphi}$

$$L_{\mathrm{scalar}} = D_\mu H^\dagger D^\mu H + \frac{1}{2}\partial_\mu\vec{\varphi}\,\partial^\mu\vec{\varphi} - V(\varphi)$$

Scalar Potential $V(\varphi)$

$$-\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + \frac{1}{2}\mu_\varphi^2\vec{\varphi}^2 + \frac{1}{4!}\lambda_\varphi\left(\vec{\varphi}^2\right)^2 + \lambda_x H^\dagger H\vec{\varphi}^2$$

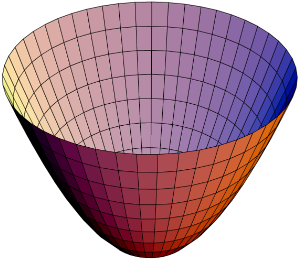
H - Higgs doublet, $\langle H\rangle = (0, v/\sqrt{2})$, $v = 246$ GeV, $\langle\vec{\varphi}\rangle = 0$

after EWSB: $m_h^2 = 2\lambda_H v^2$, $m_\varphi^2 = \mu_\varphi^2 + \lambda_x v^2$

Theoretical Constraints - Tree Level Unitarity

- $m_H^2 \leqslant \frac{8\pi}{3} v^2$
- $|\lambda_\varphi| \leqslant 8\pi$
- $|\lambda_x| \leqslant 4\pi$

Theoretical Constraints - Tree Level Vacuum Stability



- quartic couplings are all positive ($\lambda_{\mathrm{H}}, \lambda_{\varphi}, \lambda_{\mathrm{x}} > 0$)

or

- λ_{x} is negative and $\lambda_{\mathrm{x}}^2 < \lambda_{\mathrm{H}}\lambda_{\varphi}/6$

Triviality - constraining couplings and cut-off

$$m_h^2 = 2\lambda_H v^2$$

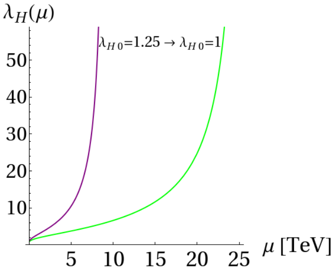


Fig. 1: Running of λ_H for $N = 1$, $\lambda_{\varphi 0} = 1$, $\lambda_{x0} = 1$, $\lambda_{H0} = 1$ ($m_h = 348\text{GeV}$) and $\lambda_{H0} = 1.25$ ($m_h = 389\text{GeV}$).

1-loop beta functions for the scalar sector

$$16\pi^2\mu\frac{d\lambda_H}{d\mu} = 24\lambda_H^2 - 6y_t^4 + 12y_t^2\lambda_H + 2N\lambda_x^2 + \dots$$

$$16\pi^2\mu\frac{d\lambda_x}{d\mu} = \lambda_x\left(12\lambda_H + \lambda_\varphi + 8\lambda_x + 6y_t^2 + \dots\right)$$

$$16\pi^2\mu\frac{d\lambda_\varphi}{d\mu} = 48\lambda_x^2 + \frac{1}{3}(8+N)\lambda_\varphi^2$$

y_t - top quark Yukawa coupling

with initial conditions: $\lambda_H(\mu = M_W) = \lambda_{H0}$, $\lambda_x(\mu = M_W) = \lambda_{x\,0}$, $\lambda_\varphi(\mu = M_W) = \lambda_{\varphi\,0}$

Triviality - constraining couplings and cut-off

$$m_h^2 = 2\lambda_H v^2$$

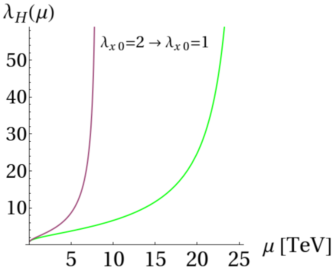


Fig. 1: Running of λ_H for $N = 1$, $\lambda_{\varphi 0} = 1$, $\lambda_{x0} = 1, 2$, $\lambda_{H0} = 1$ ($m_h = 348$GeV).

1-loop beta functions for the scalar sector

$$16\pi^2 \mu \frac{d\lambda_H}{d\mu} = 24\lambda_H^2 - 6y_t^4 + 12y_t^2\lambda_H + 2N\lambda_x^2 + \ldots$$

$$16\pi^2 \mu \frac{d\lambda_x}{d\mu} = \lambda_x \left(12\lambda_H + \lambda_\varphi + 8\lambda_x + 6y_t^2 + \ldots\right)$$

$$16\pi^2 \mu \frac{d\lambda_\varphi}{d\mu} = 48\lambda_x^2 + \frac{1}{3}(8 + N)\lambda_\varphi^2$$

y_t - top quark Yukawa coupling

with initial conditions: $\lambda_H(\mu = M_W) = \lambda_{H0}$, $\lambda_x(\mu = M_W) = \lambda_{x\,0}$, $\lambda_\varphi(\mu = M_W) = \lambda_{\varphi\,0}$

Triviality

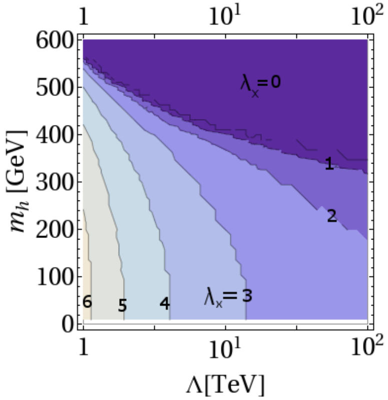


Fig. 2: Maximum λ_x contour lines for $N = 1$, $\lambda_\varphi(M_W) = 0.1$.

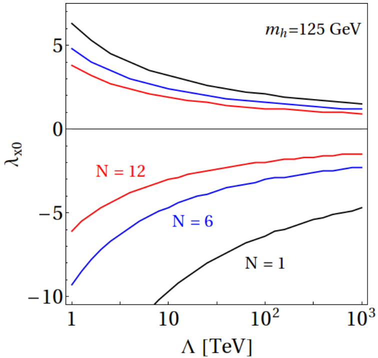


Fig. 3: Maximum and minimum λ_x for $m_h = 125$ GeV, $\lambda_\varphi(M_W) = 0.1$ for $N = 1, 6, 12$.

$$\lambda_{x\mathrm{MAX}}(m_h \sim 125\,\mathrm{GeV}) = \begin{cases} 2 & \Lambda \sim 100\,\mathrm{TeV} \\ 6 & \Lambda \sim 1\,\mathrm{TeV} \end{cases}$$

Theoretical Constraints

- Consistency Condition
- Unitarity
- Vacuum Stability
- Triviality

Dark Matter Candidate

- Z_2 implies $\vec{\varphi}$ stable

Dark Matter Candidate

Thermal Production of Dark Matter

the Boltzmann equation ($f = n/T^3$):

$$\frac{df}{dT} = \frac{\langle \sigma v \rangle}{K}(f^2 - f_{EQ}^2), \quad K = \sqrt{\frac{4\pi^3 g(T)}{45 m_{Pl}^2}}$$

thermally averaged cross section for DM + DM → SM + SM:

$$\langle \sigma_{DM+DM \to X+X'} v \rangle = \frac{1}{n_{EQ}^2} \int d\Gamma_{X,X'} e^{-(E_X + E_{X'})/T} \times |M_{DM+DM \to X+X'}|^2 (2\pi)^4 \delta^4(p_{DM} + p_{DM'} - p_X - p_{X'})$$

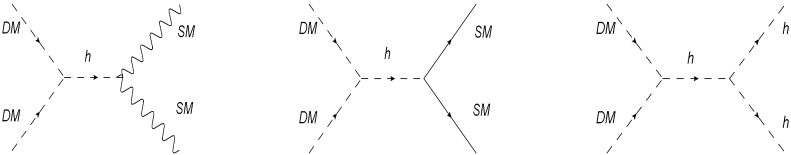
$$\Omega_{DM} = \frac{\rho_{DM}}{\rho_{critical}} = \frac{m_{DM} n}{\rho_{critical}}$$

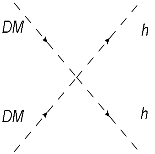
Dark Matter Candidate

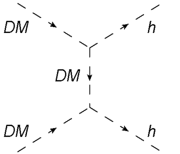
Thermal Production of Scalars

the Boltzmann equation ($f_i = n_i/T^3$):

$$\frac{df_i}{dT} = \frac{\langle \sigma v \rangle_i}{K}\left(f_i^2 - f_{EQ}^2\right), \quad K = \sqrt{\frac{4\pi^3 g_i(T)}{45 m_{Pl}^2}}$$







$$\Omega_{DM} = \sum_i \Omega_{DM}^i = N\Omega_{DM}^i$$

Dark Matter Abundance - Cold Dark Matter (CDM)

$$\Omega^{i}_{DM}h^2(\lambda_x, m_\varphi, N, m_h, \ldots) = 1.06 \times 10^9 \frac{m_\varphi/T_f}{\sqrt{g_*}\, m_{Pl}\langle\sigma v\rangle}\frac{1}{GeV} = 0.110 \pm 0.018$$

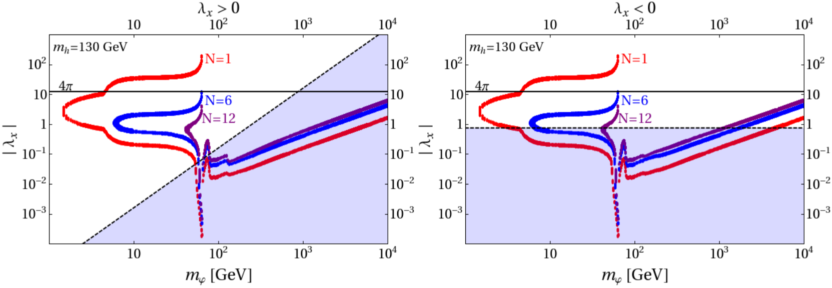


Fig. 4: $\lambda_x(m_\varphi)$ for $N = 1, 6, 12$, $m_h = 130$ GeV. Blue regions are allowed. Thick black line is a unitarity constraint $|\lambda_x| < 4\pi$.

RIGHT: dashed line is a stability constraint $\lambda_x^2 < \lambda_H\lambda_\varphi/6$ for $\lambda_\varphi = 8\pi$.

LEFT: dashed line is for consistency condition $\lambda_x < m_\varphi^2/v^2$.

Dark Matter Candidate
- Feebly Interacting Dark Matter (FIDM)

$$n\langle\sigma v\rangle = \Gamma \ll H$$

Dark Matter Abundance - FIDM

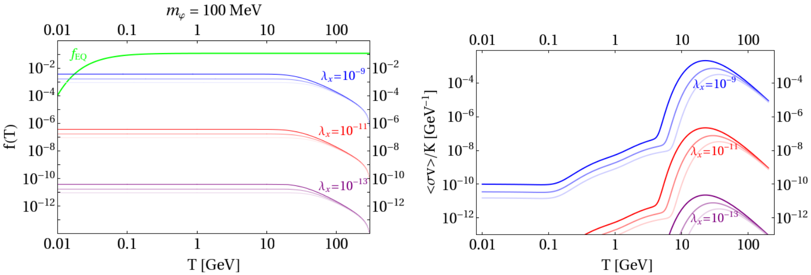


Fig. 5: RIGHT: Thermally averaged cross section and
LEFT: solutions to Boltzman equation with initial condition $f(T = \infty) = 0$ for $\lambda_x = 10^{-13}, 10^{-11}, 10^{-9}$. Splitting comes from $m_h = 100, 130, 160$ GeV. Green curve is the equilibrium density f_{EQ}.

Dark Matter Abundance - FIDM

Thermal Production of Dark Matter

the Boltzmann equation ($f = n/T^3$):

$$\frac{df}{dT} = \frac{\langle \sigma v \rangle}{K}(f^2 - f_{EQ}^2), \quad K = \sqrt{\frac{4\pi^3 g(T)}{45 m_{Pl}^2}}$$

for $f^2 \ll f_{EQ}^2$:

$$\frac{df}{dT} \sim -\frac{\langle \sigma v \rangle}{K} f_{EQ}^2, \quad \Omega = \frac{m_\varphi f_\varphi(T_\gamma) T_\gamma^3}{\rho_{crit}}$$

Dark Matter Abundance - FIDM

$$\Omega h^2 = \frac{m_\varphi f_\varphi(T_\gamma) T_\gamma^3}{\rho_{crit}} = 0.11 \pm 0.018$$

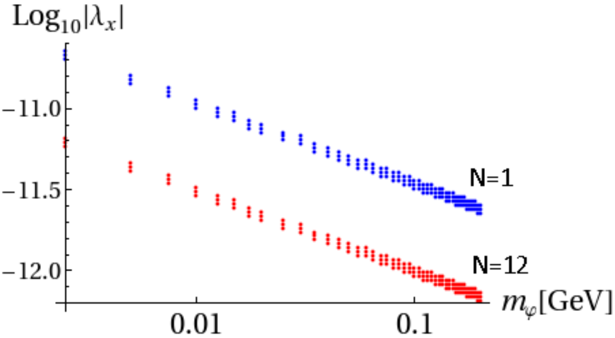
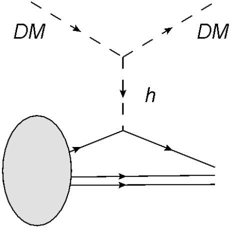


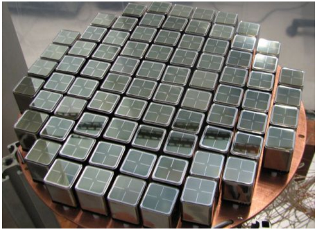
Fig. 6: Solutions to Boltzman equation in FIDM case, $m_h = 130$ GeV, $N = 1, 12$.

Direct Detection

$$\mathrm{DM} + \mathrm{N} \to \mathrm{DM} + \mathrm{N}$$



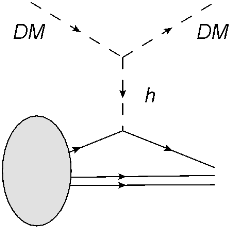
CRESST-II



XENON 100

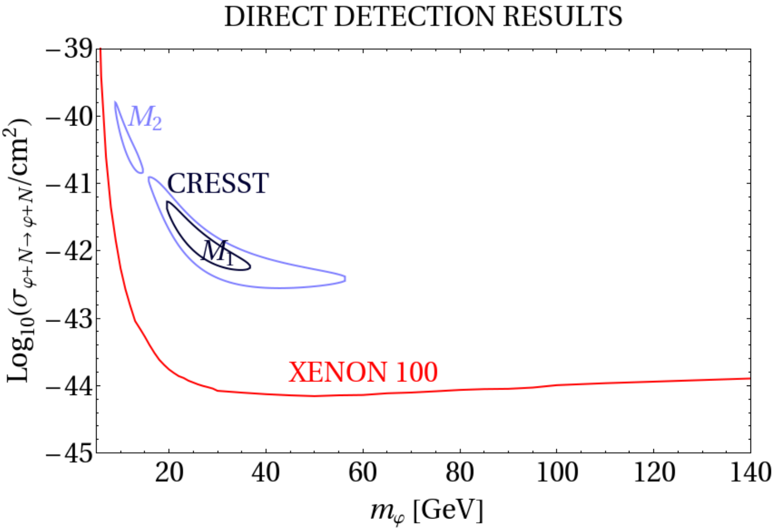
Direct Detection

$$\mathrm{DM} + \mathrm{N} \rightarrow \mathrm{DM} + \mathrm{N}$$



$$\sigma_{\mathrm{DM+N\rightarrow DM+N}} = \frac{1}{\pi}\frac{\lambda_x^2 m_N^2 \left(\sum_q f_q^N\right)^2}{m_h^4 m_\varphi^2}$$

Direct Detection



Direct Detection - XENON 100

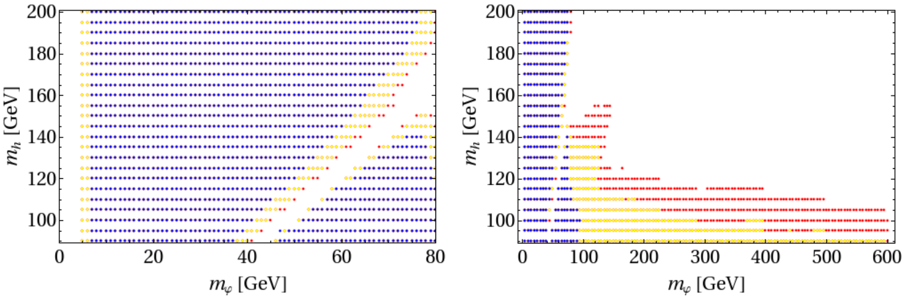
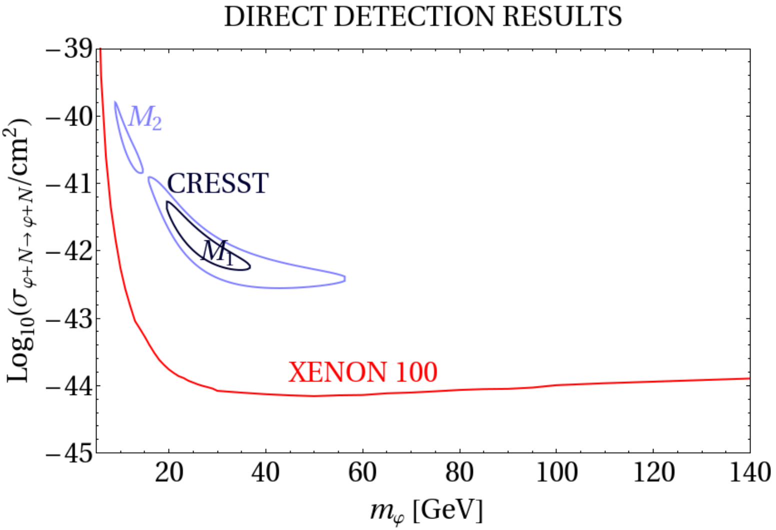
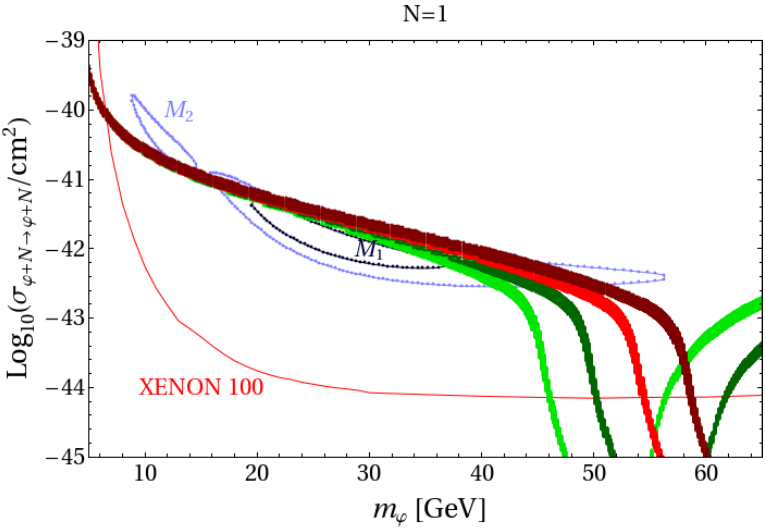


Fig. 7: Forbidden parameters (m_φ, m_h) for $N = 1, 6, 12$ (blue, yellow, red) in CDM case.

Direct Detection

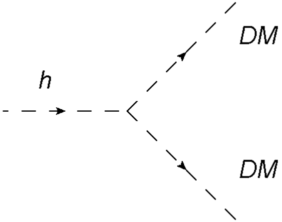


Direct Detection - CRESST



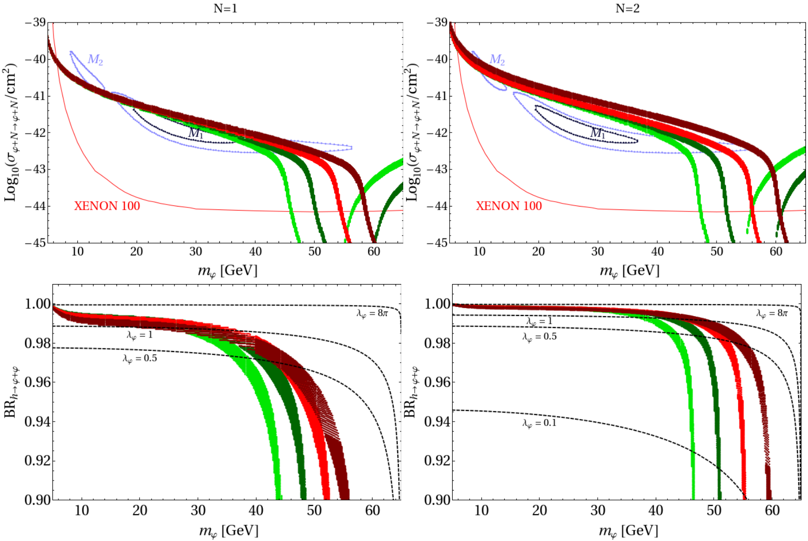
Branching Ratio $h \rightarrow \varphi\varphi$

$$BR_{h\rightarrow\varphi\varphi} = \frac{\Gamma_{h\rightarrow\varphi\varphi}}{\Gamma_{h\rightarrow SM}+\Gamma_{h\rightarrow\varphi\varphi}}$$



$$\Gamma_{h\rightarrow\varphi\varphi} \propto N\lambda_x^2$$

Direct Detection - CRESST



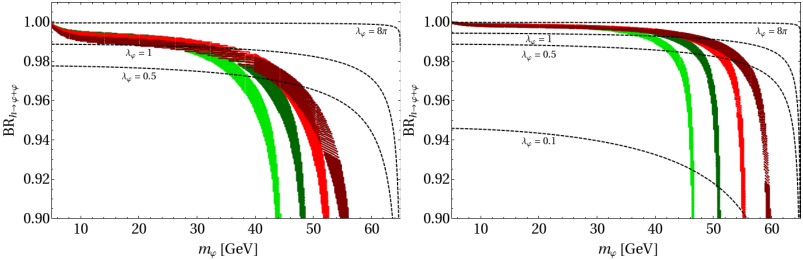
Direct Detection - CRESST

consistency constraint:

$$\lambda_x < m_\varphi^2 / v^2$$

stability constraint for negative λ_x:

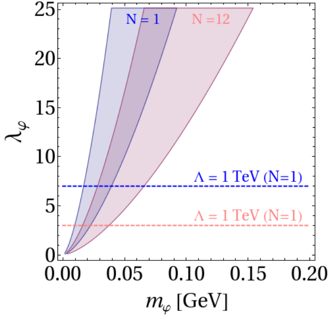
$$\lambda_x^2 < \lambda_\varphi \lambda_H / 6$$



Self-Interaction Rate

Steinhardt & Spergel: Phys.Rev.Lett.84:3760-3763, 2000

$$\frac{\sigma_{\varphi\varphi\to\varphi\varphi}}{m_\varphi} = (2.05 \times 10^3 - 2.57 \times 10^4)\mathrm{GeV}^{-3}$$



Governato et.al., arXiv:1202.0554 [astro-ph.CO]

Conclusions

1. Strongest theoretical constraints on $|\lambda_x|$:
   - vacuum stability $\Rightarrow \lambda_x^2 < \lambda_H \lambda_\varphi / 6$
   - $\mu_\varphi^2 > 0 \Rightarrow \lambda_x < m_\varphi^2 / v^2$

2. XENON 100 constraints on CDM:
   - $m_h \sim 2m_\varphi$ & $|\lambda_x| < 0.1$ for large number of scalars.

3. CRESST constraints on CDM:
   - agree with predictions within singlet-scalar model
   - Higgs not observable in LHC

4. Steinhardt and Spergel bound implies very light (below 50 MeV) and feebly coupled singlet scalars.

Thank you for your attention

International PhD Projects Programme (MPD) -
Grants for Innovations

FACULTY OF PHYSICS WARSAW UNIVERSITY
INNOVATIVE ECONOMY NATIONAL COHESION STRATEGY
FNP Foundation for Polish Science
EUROPEAN UNION EUROPEAN REGIONAL DEVELOPMENT FUND

Extra Slides

Triviality

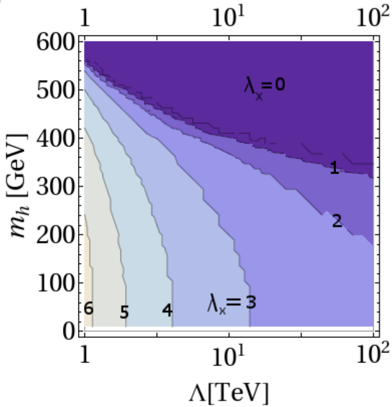


Fig. 2: Maximum λ_x contour lines for $N = 1$, $\lambda_\varphi(M_W) = 0.1$.

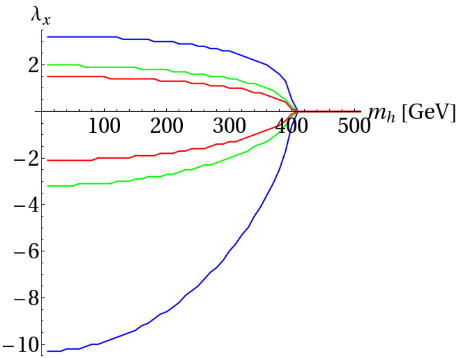


Fig. 3: Maximum and minimum λ_x for $\Lambda = 10$ TeV, $\lambda_\varphi(M_W) = 0.1$ for $N = 1, 6, 12$.

$$\lambda_{xMAX}(m_h \sim 125\,\mathrm{GeV}) = \begin{cases} 2 & \Lambda \sim 100\,\mathrm{TeV} \\ 6 & \Lambda \sim 1\,\mathrm{TeV} \end{cases}$$

DIRECT DETECTION

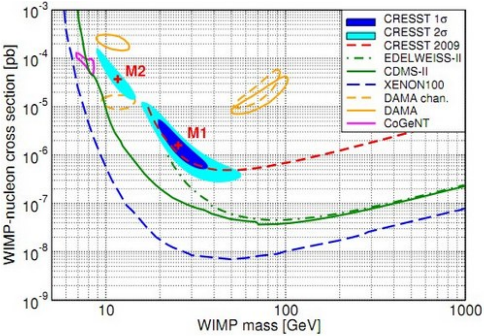


Fig. 13. The WIMP parameter space compatible with the CRESST results discussed here, using the background model described in the text, together with the exclusion limits from CDMS-II [12], XENON100 [13], and EDELWEISS-II [14], as well as the CRESST limit obtained in an earlier run [1]. Additionally, we show the 90% confidence regions favored by CoGeNT [15] and DAMA/LIBRA [16] (without and with ion channeling). The CRESST contours have been calculated with respect to the global likelihood maximum M1.