Constructing a Dynamics for Causal Set Quantum Gravity

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Outline

- An Introduction to the Causal Set Approach to Quantum Gravity
  – L.Bombelli, J.Lee, D. Meyer and R. Sorkin, 1987
- Dynamics from First Principles
  - Classical Stochastic Dynamics
    – D. Rideout and R. Sorkin, 2001
  - The quantum measure and quantum dynamics.
    – R. Sorkin
    – with F. Dowker and S. Johnston
- Continuum Inspired Dynamics
  - The Benincasa-Dowker Action for causal sets.
    – D. Benincasa and F. Dowker
  - Markov Chain Monte Carlo – First Steps.
    – with J. Henson D. Rideout and R. Sorkin

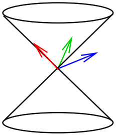
The Causal Set Hypothesis

The causal set approach is based on two fundamental building blocks:

The Causal Structure Poset

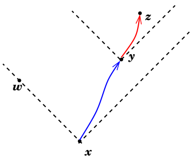
Spacetime Discreteness

The Causal Structure Poset $(M, \prec)$ Associated with (M, g)



Timelike: $g_{ab}v^a v^b < 0$
Null: $g_{ab}v^a v^b = 0$
Spacelike: $g_{ab}v^a v^b > 0$

The Causal Structure Poset $(M, \prec)$ Associated with (M, g)



- $x \prec y$ if there is a future directed causal curve from x to y
- If (M, g) has no closed causal curves, then $(M, \prec)$ is a partially ordered set
  - M is the *set* of events.
  - $\prec$ is:
    - Acyclic: $x \prec y$ and $y \prec x \Rightarrow x = y$
    - Reflexive: $x \prec x$
    - Transitive: $x \prec y$, $y \prec z \Rightarrow x \prec z$

How primitive is $(M, \prec)$?

– Zeeman, Penrose, Kronheimer, Hawking, Geroch, Ellis, Malament, etc..

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$(M, \prec)$ determines the conformal class of the metric.

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If $f : (M_1, g_1) \to (M_2, g_2)$ is a causal bijection between two future and past distinguishing spacetimes, then f is a smooth conformal isometry. Namely, f and f^{-1} are smooth and $f_* g_1 = \Omega^2 g_2$.

S. W. Hawking, A.R. King, P.J. McCarthy, J. Math. Phys. (1976); D. Malament, J. Math. Phys. (1977); O. Parrikar, S. Surya (2011).

- Causal structure $= 9/10^{\text{th}}$ of the spacetime geometry.
- Volume element $= 1/10^{\text{th}}$ of the spacetime geometry.

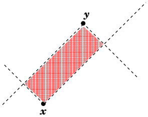
Spacetime geometry $=$ Causal Structure + Volume

Be Wise, Discretise! —- Mark Kac

- Planck scale physics: $l_p = \sqrt{G\hbar/c^3}$

  Black Hole Entropy, Resolution of Singularities, Regularisation of QFTs, etc.

- Discreteness can give the spacetime volume element:



Discreteness

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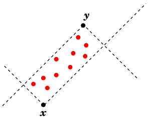
- Discreteness can give the spacetime volume element:

  A spacetime region of volume V has $n \sim V/V_p$ Planck volumes



The Causal Set Hypothesis

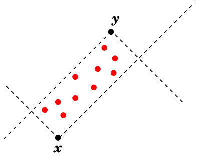
The Causal Structure Poset

Spacetime Discreteness

$\Downarrow$

The underlying structure of spacetime is a *causal set* or locally finite poset $(C, \prec)$

- Discreteness implemented via *local finiteness*: $|Fut(x) \cap Past(y)| < \infty$



Reconstructing Spacetime From a Causal Set

Spacetime geometry = Causal Structure + Volume

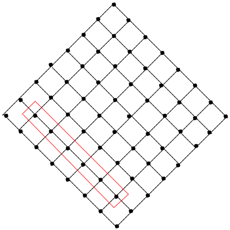
Causal Structure $\rightarrow$ Partially Ordered Set

Spacetime Volume $\rightarrow$ Number

Order + Number $\sim$ Spacetime geometry

Reconstructing Spacetime From a Causal Set

- Regular lattice does not preserve Number-Volume correspondence



- Random lattice generated via a Poisson process:
  $P_V(n) \equiv \frac{1}{n!} e^{-\rho V} (\rho V)^n, \quad < N >= \rho V$

Reconstructing Spacetime From a Causal Set

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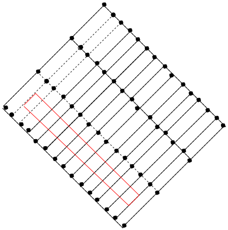

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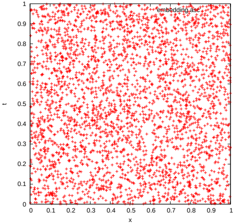
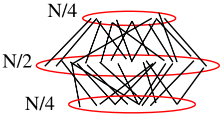


- Local Lorentz invariance: there are no preferred directions – L.Bombelli, J.Henson, R. Sorkin, Mod.Phys.Lett. 2009

Spacetime-like causal sets

- A generic causal set looks nothing like spacetime



- C is *"approximated"* by (M, g) if it admits a *"faithful embedding"* $\Phi : C \to (M, g)$
  - Order relation in $C \leftrightarrow$ induced causal order in $\Phi(C)$
  - $\Phi(C) \subset (M, g)$ is a high probability Poisson sprinkling in (M, g)

The Inverse Problem: Reconstructing continuum geometry and topology from the causal set

Timelike Distance, Dimension, Homology, D'Alembertian, Scalar Curvature..

Dynamics for Causal sets

- From first principles
  - Classical Sequential Growth and Observables
  - Quantum Sequential Growth (Quantum Measure formulation and the construction of Observables)
  - Biggest Challenge: Emergence of Einstein gravity, continuum spacetime
- Continuum inspired Dynamics: $Z = \sum_{c \in \Omega} e^{iS[c]/\hbar}$
  - A Non-local Action
  - Wick Rotation without changing the sample space.
  - Markov Chain Monte Carlo methods
    - Local moves and KR posets.
    - A 2D model of causal set quantum gravity – some interesting leads.
  - Biggest Challenge: What are the covariant observables?

Causal Set Cosmology

- Classical Sequential Growth – D.P. Rideout, R.D. Sorkin, Phys. Rev D (2000)

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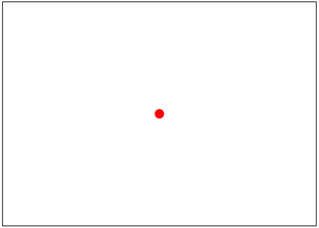

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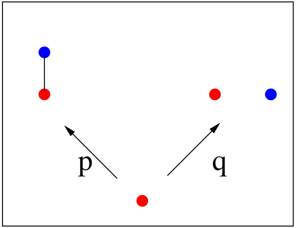

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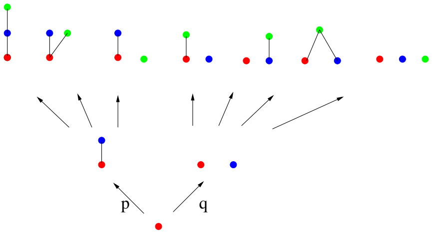

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- Transitive percolation: p: probability of adding in a link and $q = 1 - p$: probability for no relation.

- Principles:
  - General Covariance or Label Independence,
  - Bell-causality condition

Classical Stochastic Theory

Classical Stochastic Dynamics is a Probability Measure Space: $(\Omega, \mathcal{A}, \mu)$

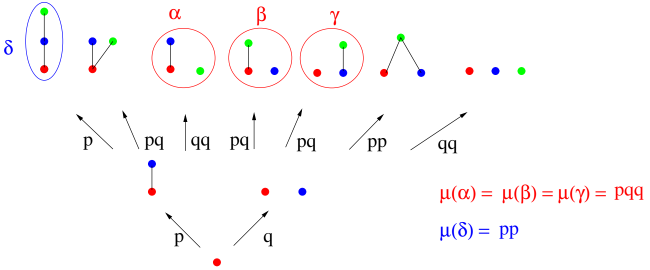
- Sample Space Ω: space of histories.
- Event Algebra $\mathcal{A}$: set of all propositions about the system.
  - $\mathcal{A}$ is closed under finite set union, intersection and complementation.
  - $\Omega \in \mathcal{A}$.
- Probability Measure $\mu : \mathcal{A} \to [0, 1]$: finitely additive

  Kolmogorov Sum Rule: $\mu(\alpha_1 \sqcup \alpha_2) = \mu(\alpha_1) + \mu(\alpha_2)$

Lesson: An observable is a measurable set

- Sequential growth generates causal sets that are labelled.
  - Ω : space of all "completed" labelled, *past finite* causal sets
  - $\mathcal{A}$ is generated by the cylinder sets $\{\mathrm{cyl}(c_i)\}$ where c_i are labelled causal sets of size $n < \infty$.

    Example: cyl(..)=set of all causal sets whose first two elements form a 2-antichain.
  - μ in terms of p:



Covariant Observables

- Sequential growth generates causal sets that are labelled.
- Finite time events are not covariant.
  - Complete $\mathcal{A}$ to include infinite time events: $\mathcal{S}_{\mathcal{A}}$ is the sigma algebra generated from $\mathcal{A}$
    - $\mathcal{S}$ is an algebra
    - $\mathcal{S}$ is closed under countable unions and intersections

    Example from classical random walk: Walker eventually returns to the origin.
  - $(\Omega, \mathcal{A}, \mu) \rightarrow (\Omega, \mathcal{S}_{\mathcal{A}}, \mu^*)$

    Caratheodary-Hahn theorem: The extension exists and is unique.

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- Take the quotient of $(\Omega, \mathcal{S}_{\mathcal{A}}, \mu^*)$ with respect to relabellings: $(\Omega', \mathcal{S}', \mu')$

  Physical characterisation of this space in terms of past sets.

  G. Brightwell, H.F. Dowker, R.S. Garcia, J. Henson, R.D. Sorkin, Phys. Rev. D(2003)

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Is there an analog of this for the quantum case?

Quantum Theory

Quantum theory can be thought of as a generalisation of classical stochastic theories.

Quantum Dynamics is a Quantum Measure Space $(\Omega, \mathcal{A}, \mu)$

R. D. Sorkin, Mod. Phys. Lett. A 9, 3119 (1994), R. D. Sorkin, J. Phys. Conf. Ser. (2007), Fay Dowker, Yousef Ghazi-Tabatabai, J.Phys.A(2008)

Quantum Theory

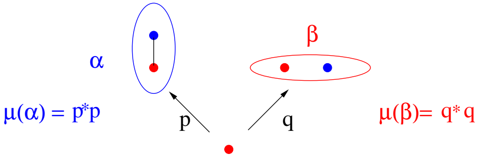
- Ω and $\mathcal{A}$ same as in classical stochastic theories.
- μ is non-additive: $\mu(\alpha \sqcup \beta) = \mu(\alpha) + \mu(\beta) + I(\alpha, \beta)$

Quantum Sum Rule

$$\mu(\alpha_1 \sqcup \alpha_2 \sqcup \alpha_3) = \mu(\alpha_1 \sqcup \alpha_2) + \mu(\alpha_1 \sqcup \alpha_3) + \mu(\alpha_2 \sqcup \alpha_3) - \mu(\alpha_1) - \mu(\alpha_2) - \mu(\alpha_3).$$

- What is the interpretation of μ?
  - Principle of Preclusion: If $\mu(\alpha) = 0$, then α doesn't happen or is *precluded*
  - The Anhomomorphic Logic/Coevent/Piombino Interpretation.
    R. D. Sorkin, J. Phys. Conf. Ser. (2007),
    Fay Dowker, Yousef Ghazi-Tabatabai, J.Phys.A(2008)

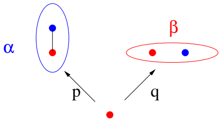
- $p, q \in \mathbb{C}$.



What is $\mu(\alpha \sqcup \beta)$?

- $p, q \in \mathbb{C}$.
- Decoherence functional $D : \Omega \times \Omega \to \mathbb{C}$.
  - Properties:
    - Hermetian: $D(\alpha, \beta) = D^*(\beta, \alpha)$
    - Finitely Biadditive: $D(\sqcup_{i=1}^{n} \alpha_i, \beta) = \sum_{i=1}^{n} D(\alpha_i, \beta)$
    - Normalised: $D(\Omega, \Omega) = 1$
    - Strongly Positive: For any finite collection $\{\alpha_i\}$, $M_{ij} = D(\alpha_i, \alpha_j)$ is positive semi-definite.
  - Example: For a unitary system, $D(\gamma, \gamma') = A^*(\gamma)A(\gamma')\delta(\gamma(T) - \gamma'(T))$

- $p, q \in \mathbb{C}$.



- Complex Percolation: $D(\alpha, \beta) \equiv A^*(\alpha)A(\beta)$

$\mu(\alpha \sqcup \beta) = |p|^2 + |q|^2 + 2Re(p^* q)$

$p + q = 1 \Rightarrow |p| + |q| = 1 + \zeta \quad \zeta \geq 0$

What is the analogue of the Caratheodary-Hahn extension theorem for the quantum measure?

Quantum Measure as a Vector Measure

- The Quantum Measure as a Quantum Vector Measure: $(\Omega, \mathcal{A}, \mu_{\mathrm{v}})$

  μ_{v} takes values in the *histories Hilbert Space*.

- Caratheodary-Hahn-Kluvanek theorem:
  An extension $(\Omega, \mathcal{A}, \mu_{\mathrm{v}}) \rightarrow (\Omega, \mathcal{S}_{\mathcal{A}}, \mu_{\mathrm{v}}^{*})$ exists and is unique
  provided μ_{v} satisfies certain convergence conditions

- Quantum Measure only extends for "Real-Complex" Percolation: $p \in [0, 1]$
  - Real amplitudes, but D is still non-additive.

$$D(\alpha \sqcup \beta) = \mu_{\mathrm{v}}(\alpha)^2 + \mu_{\mathrm{v}}(\beta)^2 + 2\mu_{\mathrm{v}}(\alpha)\mu_{\mathrm{v}}(\beta)$$

  - Observables identical to those of classical transitive percolation.

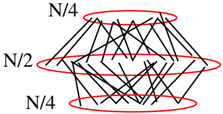
- Open questions:
  - Is it enough to get some if not all observables?
  - What fundamental principles should we choose?

Continuum-Inspired Approach

- Covariant sum-over-histories formulation:

$$Z = \sum_{C \in \Omega} A(C), \quad \text{eg}: \quad A(C) = \exp^{iS(C)}$$

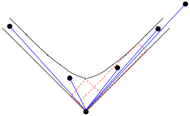
- In the $N \to \infty$ limit, Ω is dominated ($\sim e^{N^2/4}$) by the 3-level Kleitman-Rothschild causal sets.



- Entropy v/s action: Can spacetime emerge from the theory?

The Benincasa-Dowker Action

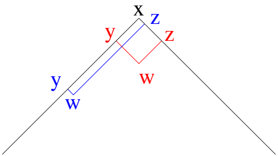
- What is the nearest neighbour of an event?



Eg: Minkowski causal set has an infinite valency.

- Scalar field in a slowly varying frame:

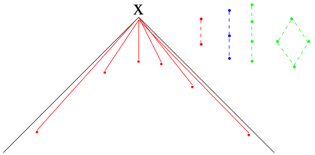
$$\Box\phi(x) \sim \phi(x) - \phi(y) - \phi(z) + \phi(w)$$



The Benincasa-Dowker Action

- $$B\phi(x) = \frac{4}{l_p^2}[-\frac{1}{2}\phi(x) + (\sum_{y\in N_1^-(x)} -2\sum_{y\in N_2^-(x)} + \sum_{y\in N_3^-(x)})\phi(y)]$$
  $$= \frac{4}{\sqrt{6}l_p^2}[-\phi(x) + (\sum_{y\in N_1^-(x)} -9\sum_{y\in N_2^-(x)} +16\sum_{y\in N_3^-(x)} -8\sum_{y\in N_4^-(x)})\phi(y)],$$

– R. Sorkin, gr-qc/0703099, D. Benincasa and F.Dowker PRL, 2010



- For curved spacetime: $\lim_{l\to 0} B\phi(x) = (\Box - \frac{1}{2}R(x))\phi(x)$
- $\frac{1}{\hbar}S^{(2)}[C] = N - 2N_1 + 4N_2 - 2N_3$

  $\frac{1}{\hbar}S^{(4)}[C] = N - N_1 + 9N_2 - 16N_3 + 8N_4$

The Mesoscale

- Need to Introduce an intermediate scale $l_k >> l_p$ to dampen fluctuations.

- Gives rise to a family of actions:

$$S(\epsilon)/\hbar = \epsilon^2 \times \left(N - 2\epsilon^2 \sum_{n=0}^{N-2} f(n,\epsilon) \right)$$

$\epsilon = l_p/l_k \in [0, 1]$

$$f(n,\epsilon) = (1-\epsilon)^n - 2\epsilon n(1-\epsilon)^{n-1} + \frac{1}{2}\epsilon^2 n(n-1)(1-\epsilon)^{n-2} \quad (1)$$

- As $\epsilon \to 1$, recover the Benincasa-Dowker Action.

Wick Rotation and the Thermodynamic Partition Function

- Wick Rotation:
  - $S_1[\zeta, C] = \zeta N - \zeta^{-1} 2N_1 / + \zeta 4N_2 - \zeta^{-1} 2N_3$

    $$\zeta \to i\zeta \quad \Rightarrow \quad iS_1[\zeta, C] \to S_1^E[\zeta, C] = -(\zeta N + \zeta^{-1} 2N_1 / + \zeta 4N_2 + \zeta^{-1} 2N_3)$$
  - $\zeta \times S_2[C] = \zeta(N - 2N_1 / + 4N_2 - 2N_3)$

    $$\zeta \to i\zeta \quad \Rightarrow \quad i\zeta S_2[C] \to -\zeta S_2^E[C]$$
- Space of Configurations Ω is still “Lorentzian”.

$$Z_1 = \sum_{C \in \Omega} e^{S_1^E[C,\zeta]} \qquad Z_2 = \sum_{C \in \Omega} e^{-\zeta S_2^E[C]}$$

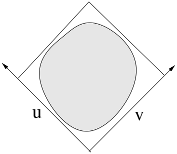
– with J. Henson, D. Rideout, R. Sorkin

- Link Move:
  - Pick a pair $x, y \in C$
  - If $x \prec y$
    - if $x \prec_L y$, then remove the link
    - Else do nothing
  - If x, y are unrelated, and "suitable", then add a link.

    If $z \prec w$ such that $z \prec x$ and $y \prec w$, then they are unsuitable. (Eg. of a kinematical rejection)
- This move equilibriates and can reproduces the Uniform Distribution
- Problem with KR posets: the moves are not efficient enough for these.

2D Causal set QG – A Simpler Problem

- Interval spacetime: $g_{ab}dx^a dx^b = -\Omega^2(u,v)\, dudv.$



- The Causal Set Analogue: $U = \{u_1, u_2, \ldots, u_N\}$ and $V = \{v_1, v_2, \ldots, v_N\}$

$$x \prec y \quad \Leftrightarrow \quad u(x) < u(y) \quad \text{and} \quad v(x) < v(y)$$

$\Phi(C) = U \cap V$ is a 2D ORDER

All causal sets that faithfully embed into interval spacetimes are 2D orders.

Minkowski spacetime is a prediction of a Unimodular, Continuum Inspired Dynamics
– with G. Brightwell and J. Henson

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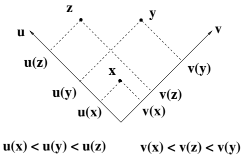

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MonteCarlo for 2D orders

- $Z = \sum_{2Dorders} \exp(-\zeta S^{E}_{2D}(\epsilon))$.
- The Move:
  - $U = (u_1, u_2, \ldots u_i, \ldots u_j, \ldots u_N)$, $V = (v_1, v_2, \ldots v_i, \ldots v_j, \ldots v_N)$
  - Pick a pair (u_i, v_i) and (u_j, v_j) at random and exhchange: $u_i \leftrightarrow u_j$
  - $U' = (u_1, u_2, \ldots u_j, \ldots u_i, \ldots u_N)$, $V' = (v_1, v_2, \ldots v_i, \ldots v_j, \ldots v_N)$

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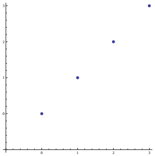
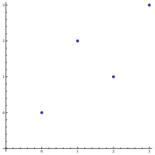
  - $U' = (u_1, u_2, \ldots u_j, \ldots u_i, \ldots u_N)$, $V' = (v_1, v_2, \ldots v_i, \ldots v_j, \ldots v_N)$
- (1) $U = (1, 2, 3, 4)$, $V = (1, 2, 3, 4)$ – The 4-Chain



(2) Exchange: $u_2 \leftrightarrow u_3$

MonteCarlo for 2D orders

- $Z = \sum_{2Dorders} \exp(-\zeta S_{2D}^{E}(\epsilon))$.
- The Move:
  - $U = (u_1, u_2, \ldots u_i, \ldots u_j, \ldots u_N)$, $V = (v_1, v_2, \ldots v_i, \ldots v_j, \ldots v_N)$
  - Pick a pair (u_i, v_i) and (u_j, v_j) at random and exhchange: $u_i \leftrightarrow u_j$
  - $U' = (u_1, u_2, \ldots u_j, \ldots u_i, \ldots u_N)$, $V' = (v_1, v_2, \ldots v_i, \ldots v_j, \ldots v_N)$
- (3) $U' = (1, 3, 2, 4)$, $V' = (1, 2, 3, 4)$ – The Diamond

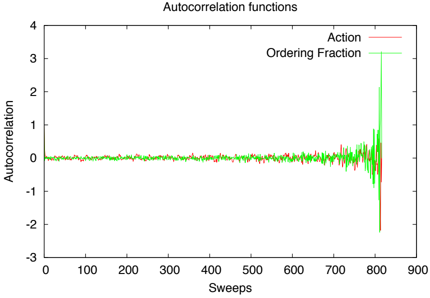


Progress with Simulations

- $N = 50$ with about 800 sweeps.

Autocorrelation Function: $\chi(t) = \int dt' (\mathcal{O}(t') - <\mathcal{O}>)(\mathcal{O}(t'+t) - <\mathcal{O}>)$

MonteCarlo Methods in Statistical Physics, Newman and Barkema



Progress with Simulations

- $N = 50$ with about 800 sweeps.

  Autocorrelation Function: $\chi(t) = \int dt' (\mathcal{O}(t') - <\mathcal{O}>)(\mathcal{O}(t'+t) - <\mathcal{O}>)$

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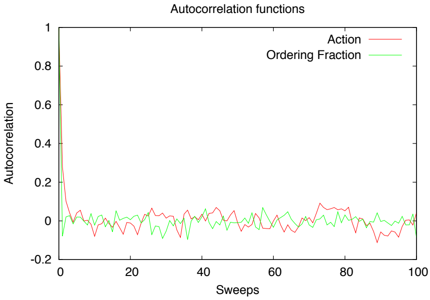
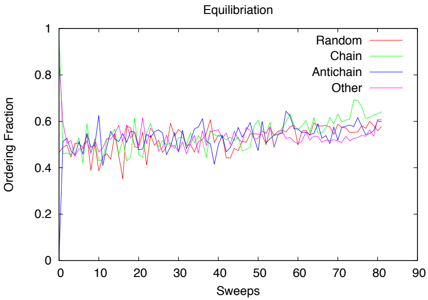


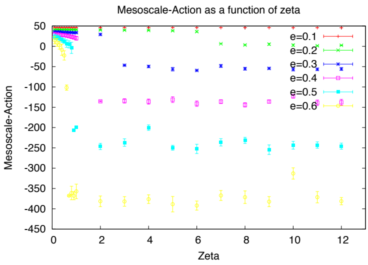
Progress with Simulations

- $N = 50$ with about 800 sweeps.



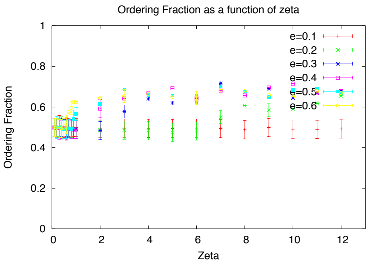
Progress with Simulations

- $N = 50$ with about 800 sweeps.
- Signs of a cross-over?



Progress with Simulations

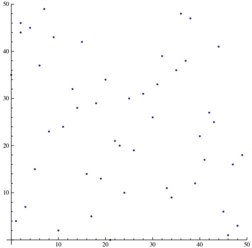
- $N = 50$ with about 800 sweeps.
- Signs of a cross-over?



Progress with Simulations

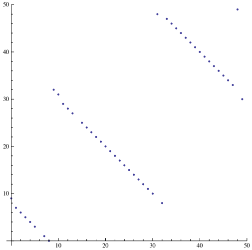
- $N = 50$ with about 800 sweeps.
- Signs of a cross-over?
- Small zeta Phase

Out[1]=



Progress with Simulations

- $N = 50$ with about 800 sweeps.
- Signs of a cross-over?
- Small zeta Phase
- Indications of a "crystalline" phase?



Open Directions

- What do we really get when we analytically continue back?
- Behaviour of cross over as a function of N.
- Other observables?