The EFT of inflation: new shapes of NG and consistency relation

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based on:
P. Creminelli, G. D’A., J. Noreña, M. Musso, E. Trincherini, ***JCAP*** **1102:006** [arXiv:**1011.3004**]
P. Creminelli, G. D’A., J. Noreña, M. Musso, *to appear*

Outline

- Effective theory of inflation
- Galilean symmetry and action for perturbations
- New shapes of non-Gaussianities
- Four-point function
- The not-so-squeezed limit of the bispectrum
- A better template for data analysis/simulations

Standard approach

Usual approach to inflation:

1) Build a Lagrangian for a scalar field: $\mathcal{L}(\phi, \partial_\mu \phi, \Box \phi, \dots)$

2) Solve EOM of scalar + FRW to find an inflating solution $\ddot{a} > 0$

$$\phi = \phi_0(t) \qquad \mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2$$

3) Expand in perturbations around this solution

$$\phi(t, \vec{x}) = \phi_0(t) + \delta\phi(t, \vec{x}) \qquad g_{\mu\nu} = g_{\mu\nu}^{\mathrm{FRW}} + \delta g_{\mu\nu}$$

4) Solve equations, work out predictions

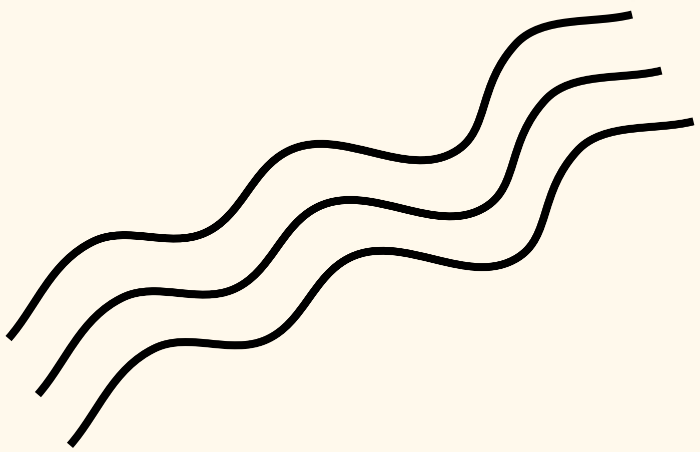
The EFT for inflation

Cheung et al. 2007

We can **focus directly on the theory of perturbations** around quasi de Sitter bkg

- Bkg solution (quasi-dS) spontaneously breaks time diffs

$$t \to t + \xi^0(t, \vec{x}) \quad \Rightarrow \quad \delta\phi \to \delta\phi + \dot{\phi}_0 \xi^0$$



- Can choose unitary gauge $\delta\phi = 0$

The graviton describes 3 degrees of freedom,

like in a broken gauge theory.

- The most general action in unitary gauge is constructed in terms of invariants of the 3D time slices:

$$S = \int \mathrm{d}t\mathrm{d}^3x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + M_{\rm Pl}^2\dot{H}g^{00} - M_{\rm Pl}^2(3H^2 + \dot{H}) + \frac{M_2^4(t)}{2}(g^{00}+1)^2 + \frac{M_3^4(t)}{2}(g^{00}+1)^3 + \cdots - \frac{\bar{M}_2^2(t)}{2}\delta K^2 + \dots\right]$$

Slow-roll

DBI, P(X)

Ghost infl.

Reintroducing the Goldstone

Stueckelberg trick: perform a broken time diff and promote the parameter to a field

$$t \to \tilde{t} = t + \xi^0(x) \qquad \xi^0(x) \to -\pi(x)$$

Simple example (slow-roll inflation):

$$\int \mathrm{d}^4x\sqrt{-g}\Big[A(t) + B(t)g^{00}\Big] \;\to\; \int \mathrm{d}^4x\sqrt{-g}\Big[A(t+\pi(x)) + B(t+\pi(x))\frac{\partial(t+\pi(x))}{\partial x^\mu}\frac{\partial(t+\pi(x))}{\partial x^\nu}g^{\mu\nu}(x)\Big]$$

Diff-invariant if π transforms non-linearly:

$$\pi(x) \to \pi(x) - \xi^0(x)$$

Decoupling limit: at high energy, no mixing with gravity!

$$S_\pi = \int \mathrm{d}^4x\left[\frac{1}{2}M_{\rm Pl}^2R - M_{\rm Pl}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3 + \dots\right]$$

$$c_s^{-2} = 1 - \frac{2M_2^4}{M_{\rm Pl}^2\dot{H}}$$

Large NG from small c_s

Validity of the EFT

Effective theory is valid for H << Λ

We probe small fluctuations $\phi_0(t+\pi(t,\vec{x}))$ $H\pi = -\zeta \simeq 10^{-5}$

Cosmological perturbations probe the theory at E ~ H

No need to solve for background to work out predictions!

We are interested in theories of the form

$$M_{\rm Pl}^2 \dot{H} (\partial\pi)^2 + M(\partial^2\pi)^3 + \ldots$$

We want cubic term to be of order ~ 10^{-3} the kinetic one.
In the H π >> 1 regime, we would have $10^5/\varepsilon$ boost -> outside the EFT!

Higher derivative terms *must* be small and *must* be evaluated on the lowest order e.o.m.
We cannot change the number of degrees of freedom.

Galilean symmetry

Nicolis, Rattazzi, Trincherini 2008

Shift symmetry on the gradient of a scalar

$$\phi \to \phi + b_\mu x^\mu + c$$

Lowest derivative galileons give 2nd order e.o.m!

$$\mathcal{L} \sim (\partial\phi)^2(\partial^2\phi)^n \,, \quad n \leq 3$$

Use these operators for an inflationary lagrangian (Burrage et al. 2010).
Bkg quite different from slow-roll, large non-Gaussianities given by cubic operators with 4 derivatives...

$$\ddot{\pi}\dot{\pi}^2 \,, \quad \dot{\pi}^2\nabla^2\pi \,, \quad \dot{\pi}\nabla\dot{\pi}\nabla\pi \,, \; \ddot{\pi}(\nabla\pi)^2 \,, \; \nabla^2\pi(\nabla\pi)^2$$

... but all these operators are equivalent to the ones with 3 derivatives arising in standard models

Non-minimal galileons, at least 2 derivatives per field.
Is the effective theory consistent? YES!
Do we have interesting predictions? YES!

Building up the action

Perturbations endowed with a Galilean symmetry, which non-linearly realize Lorentz symmetry

Difficult to use the geometrical language.
Useful to introduce a “fake” scalar which linearly realizes Lorentz symmetry

$$\psi(t, \vec{x}) \equiv t + \pi(t, \vec{x})$$

Starting from 2 derivatives per field, do we generate the minimal galileons in curved spacetime?

$R(\partial\psi)^2\partial^2\psi$ is not generated

We will have at least the suppressed $R^2(\partial\psi)^2\partial^2\psi$

Building up the action (2)

Lorentz invariant operators for ψ are products of traces of the matrix $\nabla_\mu \nabla_\nu \psi$

We need to subtract from each trace its bkg value.
So we need to worry about single traces, which can change the tadpole terms: $[\Psi^n]$

Consider the sum, which contains the single trace operators

$$\sum_p (-1)^p g^{\mu_1 p(\nu_1)} \ldots g^{\mu_n p(\nu_n)} \nabla_{\mu_1} \nabla_{\nu_1} \psi \ldots \nabla_{\mu_n} \nabla_{\nu_n} \psi$$

For n>3, we have too many indices and single traces are just products of shorter ones.
Otherwise, we have a total derivative in Minkowski, which in de Sitter gives
the minimal galileons:

$$H^2 \sum_p (-1)^p g^{\mu_1 p(\nu_1)} \ldots g^{\mu_n p(\nu_n)} \nabla_{\mu_1} \psi \nabla_{\nu_1} \psi \nabla_{\mu_2} \nabla_{\nu_2} \psi \ldots \nabla_{\mu_n} \nabla_{\nu_n} \psi$$

It is consistent to study the theory with all traces of Ψ, except the single traces,
plus the minimal cubic and quartic Galileons, suppressed by H^2

New operators

$$([\Psi\ldots\Psi]-c_1)([\Psi\ldots\Psi]-c_2) \longrightarrow (\delta^{ij}\nabla_i\nabla_j\pi)^3$$

$$([\Psi\ldots\Psi]-c_3)([\Psi\ldots\Psi]-c_4)([\Psi\ldots\Psi]-c_5) \longrightarrow \begin{array}{l}\nabla^2\pi(\nabla_i\nabla_j\pi)^2\\ \nabla^2\pi(\nabla_i\nabla_\mu\pi)^2\\ \nabla^2\pi(\nabla_\mu\nabla_\nu\pi)^2\end{array}$$

There is enough freedom to make these independent from quadratic operators.

We can check the mixing with gravity is subleading in slow-roll.

Final action has only 3 independent cubic operators:

$$S=\int \mathrm{d}^4x a^3\Big[-M_{\rm Pl}^2\dot{H}\left(\dot{\pi}^2-\frac{(\partial_i\pi)^2}{a^2}\right)+M_1\ddot{\pi}^3+M_2\,\ddot{\pi}\frac{(\partial_i\dot{\pi}-H\partial_i\pi)^2}{a^2}$$
$$+M_3\left(\ddot{\pi}\frac{(\partial_i\partial_j\pi)^2}{a^4}-2H\dot{\pi}\ddot{\pi}^2+3H^3\dot{\pi}^3\right)\Big]$$

Non Gaussianities

Almost free field in Bunch-Davies vacuum → almost Gaussian perturbations

Non Gaussianities of paramount importance to discriminate different models

With EFT, approach very similar to particle physics (EWPT):
measure observables, constrain operators

What is the best observable? Bispectrum in Fourier space of a conserved quantity

$$\left\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\right\rangle = (2\pi)^3\delta_D\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right)B(k_1,k_2,k_3)$$

The function B is approximately homogeneous of degree -6.
In this scale-invariant limit, it depends just on two ratios of lengths of 3-momenta:

$$B(k_1,k_2,k_3) = k_1^6 B(1,r_2,r_3)$$

The shape of non Gaussianities

Babich, Creminelli, Zaldarriaga 2004

In the scale-invariant limit, we need just 1 number to specify the PS.

Instead, the bispectrum is a 2-d function. Different operators → different shapes!

How do we measure the non Gaussianity?

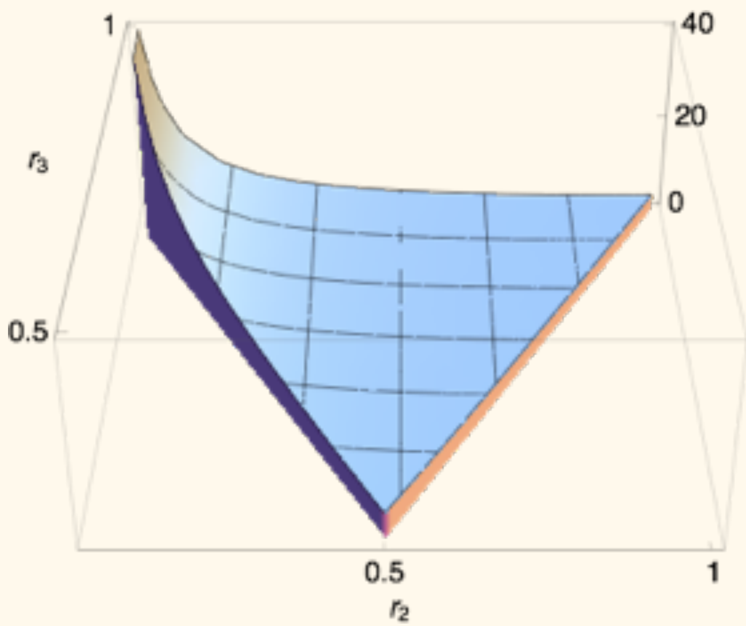
$$\hat{f}_{NL} = \frac{\sum_{\vec{k}_i} \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3} B(\vec{k}_1,\vec{k}_2,\vec{k}_3)/(\sigma^2_{k_1}\sigma^2_{k_2}\sigma^2_{k_3})}{\sum_{\vec{k}_i} B(\vec{k}_1,\vec{k}_2,\vec{k}_3)^2/(\sigma^2_{k_1}\sigma^2_{k_2}\sigma^2_{k_3})}$$

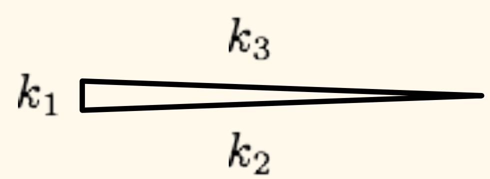
This suggests to quantify how similar are 2 shapes. Scalar product of bispectra:

$$B_1 \cdot B_2 = \int \mathrm{d}r_2 \mathrm{d}r_3\, r_2^4 r_3^4\, B_1(1,r_2,r_3) B_2(1,r_2,r_3)$$

Cosine of bispectra: $\cos(B_1,B_2) = \dfrac{B_1 \cdot B_2}{(B_1\cdot B_1)^{1/2}(B_2\cdot B_2)^{1/2}}$

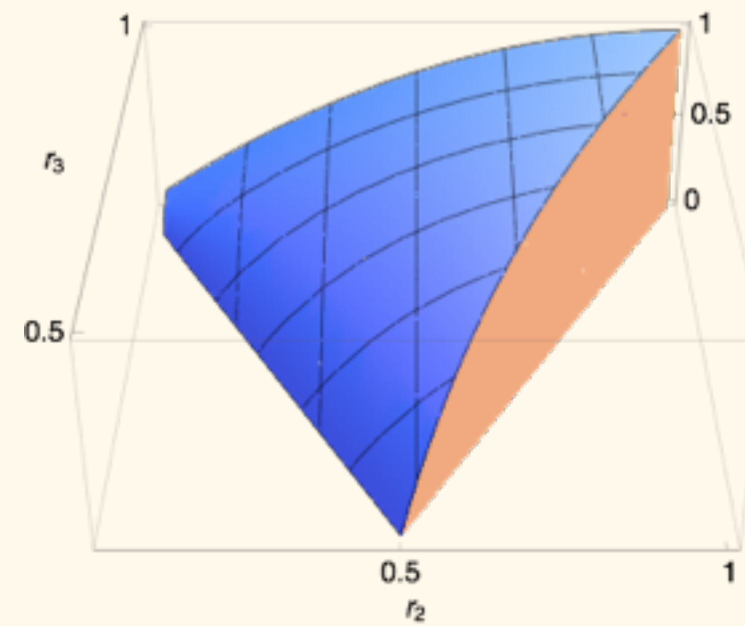
Shapes of non Gaussianities

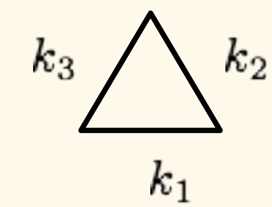




Local

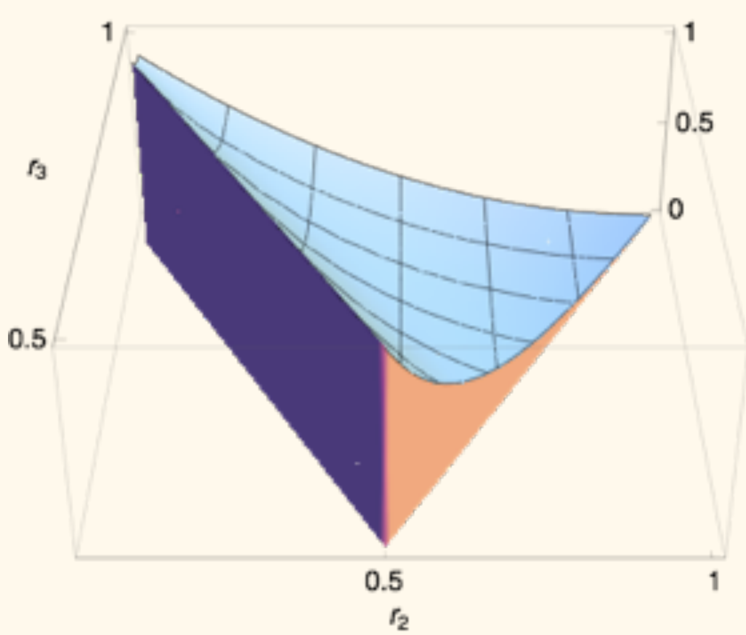
π^3

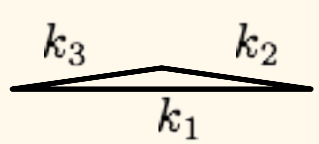




Equilateral

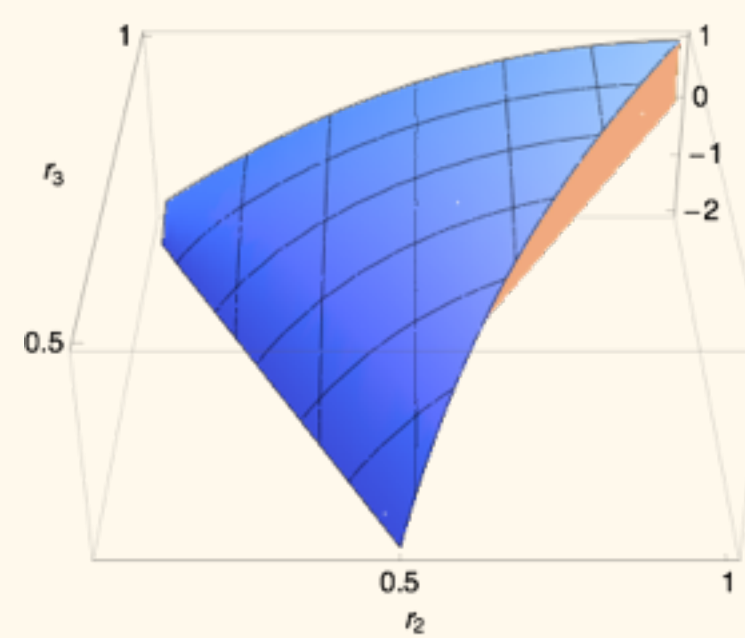
$(\partial\pi)^3$

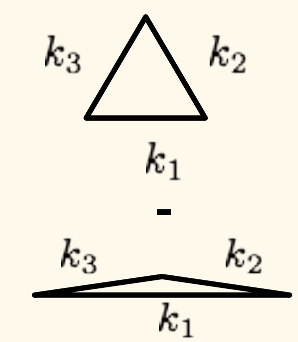




Enfolded

Modified vacuum





Orthogonal

$$\dot{\pi}\frac{(\partial_i\pi)^2}{a^2} + 5.4\frac{2}{3}\dot{\pi}^3$$

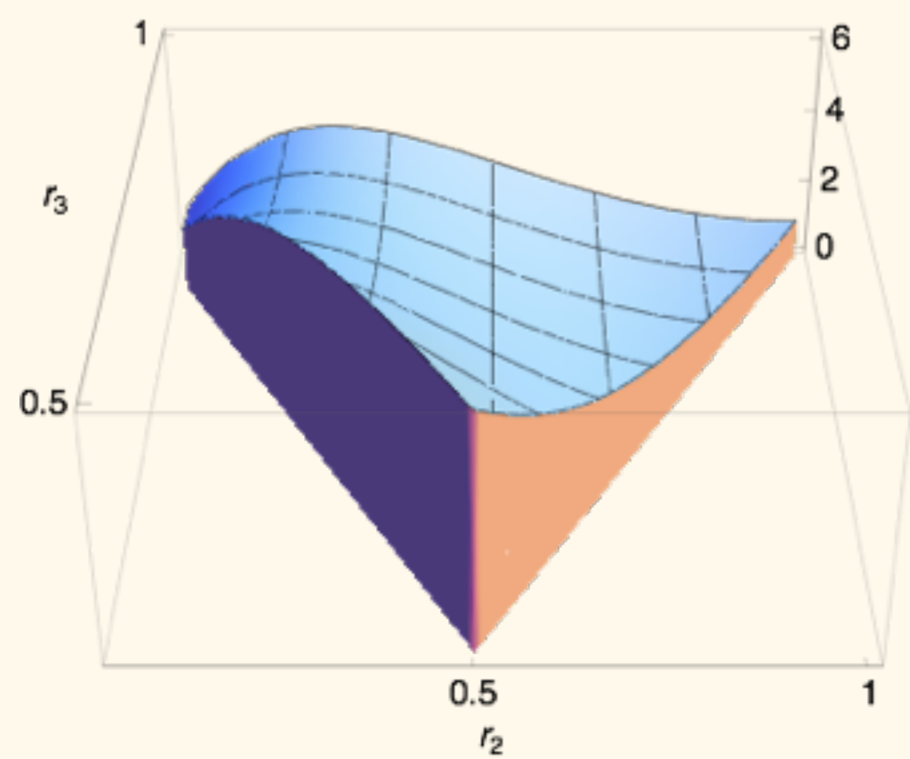
New shapes: M_3 operator

Standard EFT operator give equilateral and orthogonal shapes

$$S = \int \mathrm{d}^4 x a^3 \Big[- M_{\rm Pl}^2 \dot{H} \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) + M_1 \dot{\pi}^3 + M_2\, \ddot{\pi} \frac{(\partial_i \dot{\pi} - H \partial_i \pi)^2}{a^2} + M_3 \left(\ddot{\pi} \frac{(\partial_i \partial_j \pi)^2}{a^4} - 2H \dot{\pi} \ddot{\pi}^2 + 3H^3 \dot{\pi}^3 \right) \Big]$$

M_1 and M_2 operators give equilateral shape

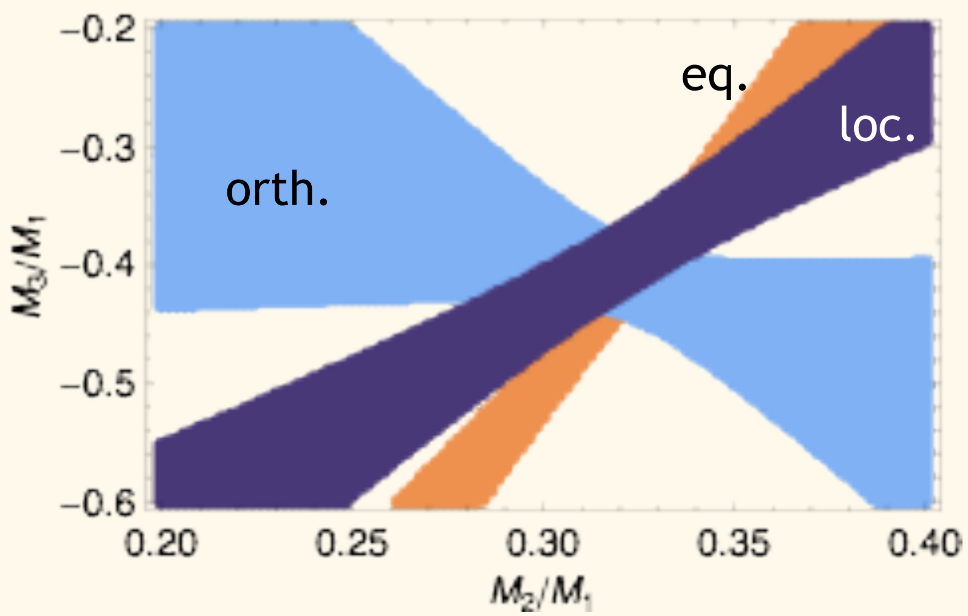
However, M_3 gives a "surfing" non Gaussianity



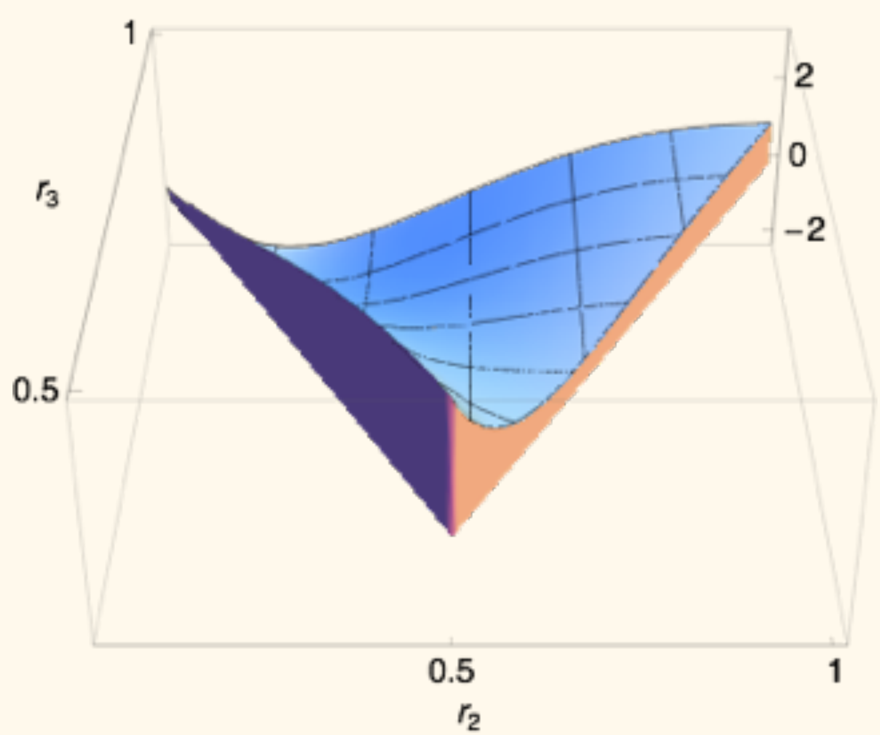
New shapes: orthogonal to standard ones

Orthogonal shape is found by tuning coefficients requiring small cosines with local and equilateral

Can we extend the space of shapes with our new operators? YES



Template	Cosine
Local	-0.15
Equilateral	0.03
Orthogonal	0.06
Enfolded	-0.03



Look where |cos| < 0.2

Intersection point at

$M_2 = 0.32\ M_1$, $M_3 = -0.42\ M_1$

This would require a dedicated template...

Constraints on parameters

Using the analysis of Smith et al. (2010) and WMAP7, we can put constraints on M_i

Choose equilateral template for M_1 and M_2: $f_{NL}^{eq} \equiv \frac{k^6}{6\Delta_\Phi^2} B(k,k,k)$

$$f_{\rm NL}^{\rm eq} = 26 \pm 140\,(68\%\,{\rm CL}) \longrightarrow \frac{M_1 H}{\varepsilon M_{\rm Pl}^2} = 240 \pm 1280 \quad \frac{M_2 H}{\varepsilon M_{\rm Pl}^2} = -80 \pm 470$$

For M_3 we can use enfolded template (cos = 0.94): $f_{NL}^{enf} \equiv \frac{k^6}{96\Delta_\Phi^2} B(k,k/2,k/2)$

$$f_{\rm NL}^{\rm enf} = 114 \pm 72\,(68\%\,{\rm CL}) \longrightarrow \frac{M_3 H}{\varepsilon M_{\rm Pl}^2} = 830 \pm 530$$

Four point function

Standard EFT: $\mathcal{L}_{1-\partial} = (\partial\pi_c)^2 + \frac{1}{\Lambda^2}(\partial\pi_c)^3 + \frac{1}{\Lambda^4}(\partial\pi_c)^4 + \dots$

$$\mathrm{NG}_3 \equiv \frac{\langle\zeta^3\rangle}{\langle\zeta^2\rangle^{3/2}} \simeq \frac{\mathcal{L}_3}{\mathcal{L}_2}\Big|_{E\sim H} \simeq \left(\frac{H}{\Lambda}\right)^2 \qquad \mathrm{NG}_4 \equiv \frac{\langle\zeta^4\rangle}{\langle\zeta^2\rangle^{2}} \simeq \frac{\mathcal{L}_4}{\mathcal{L}_2}\Big|_{E\sim H} \simeq \left(\frac{H}{\Lambda}\right)^4$$

$$\Longrightarrow \quad \mathrm{NG}_4 \sim \mathrm{NG}_3^2$$

Non-minimal galilean action: $\mathcal{L} = (\partial\pi_c)^2 + \frac{1}{\Lambda^2}(\partial^2\pi_c)^2 + \frac{1}{\Lambda^5}(\partial^2\pi_c)^3 + \frac{1}{\Lambda^8}(\partial^2\pi_c)^4 + \dots$

$$\mathrm{NG}_3 \simeq \left(\frac{H}{\Lambda}\right)^5 \qquad \mathrm{NG}_4 \simeq \left(\frac{H}{\Lambda}\right)^8$$

$$\Longrightarrow \quad \mathrm{NG}_4 \sim \mathrm{NG}_3^{8/5}$$

For a given cubic NG our model predicts a bigger 4 pt function

Usual parametrization: $\mathrm{NG}_3 \simeq f_{\mathrm{NL}}\Delta_\zeta^{1/2} \quad \mathrm{NG}_4 \simeq \tau_{\mathrm{NL}}\Delta_\zeta$

$f_{\mathrm{NL}} = 100$ implies $\tau_{\mathrm{NL}} \sim 10^4$ vs. $\tau_{\mathrm{NL}} \sim 10^5$

Consistency relation

Maldacena 2002
Creminelli, Zaldarriaga 2004
Cheung et al. 2007

Squeezed limit in single-field models: one of the modes is already a classical bkg when the other two exit the horizon

$$\langle \zeta_B(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \simeq \langle \zeta_B(\vec{k}_1)\langle \zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle_B\rangle \qquad k_1 \ll k_S$$

The long mode acts just as a rescaling of the coordinates

$$\langle \zeta(\vec{x}_2)\zeta(\vec{x}_3)\rangle_B = \xi(\tilde{\vec{x}}_2 - \tilde{\vec{x}}_3) \simeq \xi(\vec{x}_3 - \vec{x}_2) + \zeta_B(\vec{x}_+)(\vec{x}_3 - \vec{x}_2)\cdot \nabla \xi(\vec{x}_3 - \vec{x}_2)$$

Going back to Fourier space we get the consistency relation

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \simeq -(2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)P(k_1)P(k_S)\frac{\mathrm{d}\ln(k_S^3 P(k_S))}{\mathrm{d}\ln k_S}$$

The not-so-squeezed limit

Creminelli, G D'A, Noreña, Musso, *to appear*

At lowest order in derivatives

$$S_2 + S_3 = M_{\rm Pl}^2 \int d^4x \, \epsilon a^3 \left[(1 + 3\zeta_B)\dot{\zeta}^2 - (1+\zeta_B)\frac{(\partial_i \zeta)^2}{a^2} \right]$$

Long mode reabsorbed by coordinate rescaling $\vec{x} \to e^{\zeta_B} \vec{x}$

Corrections:

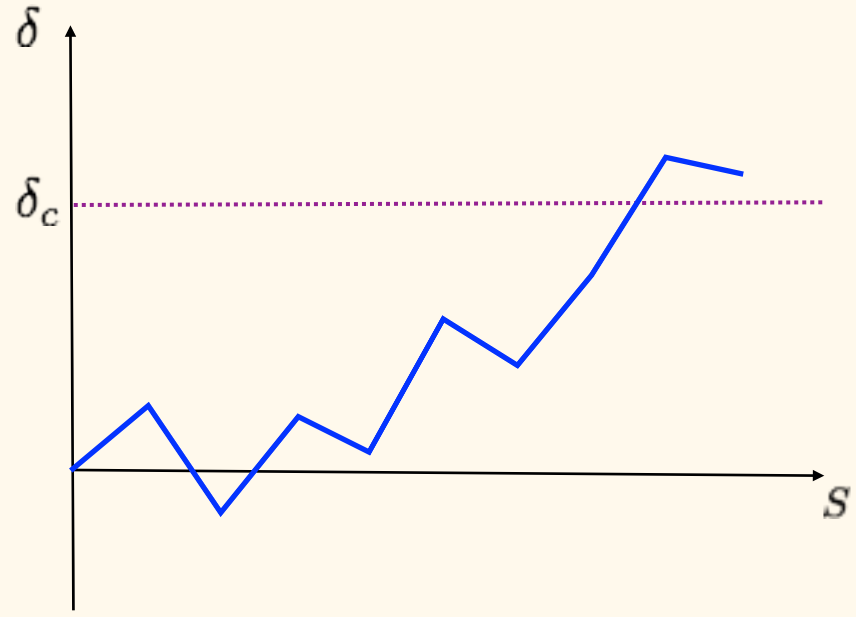
- Time evolution of ζ is of order k^2
- Spatial derivatives will be symmetrized with the short modes, giving k^2
- Constraint equations give order k^2 corrections

Final result: in the not-so-squeezed limit we have

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \simeq -(2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) P(k_1) P(k_S) \left[\frac{\mathrm{d}\ln(k_S^3 P(k_S))}{\mathrm{d}\ln k_S} + \mathcal{O}\left(\frac{k_L^2}{k_S^2}\right) \right]$$

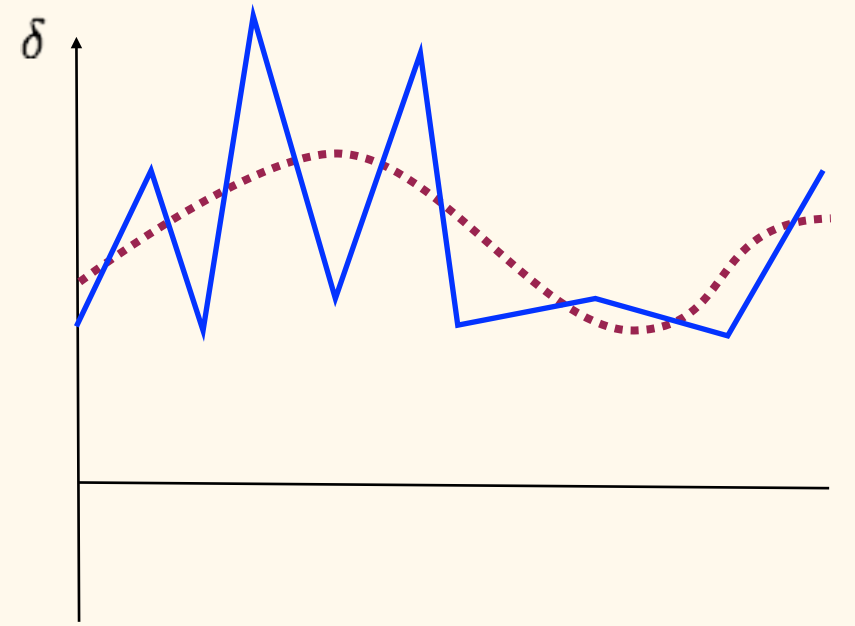
Why is this important? (1)

LSS is a powerful probe of NG.
One important observable: large scale bias (Dalal et al. 2007)



Galaxy formation in a nutshell:

When the overdensity in a certain region of space is larger than a threshold, the halo collapses and virializes.



Local NG induces a correlation between large scale and small scale perturbations, modifies the relation among halo and matter perturbations.

$$b_h(k) \sim \frac{\delta_h(k)}{\delta_m(k)}$$

Why is this important? (2)

Bias on large scales goes to a constant.
Corrections induced by NG (Matarrese & Verde 2008, Slosar et al. 2008):

$$\frac{\Delta b_h(k)}{b_h} \sim \frac{1}{\mathcal{M}_R(k)} \int \mathrm{d}k_1 k_1^2 \mathcal{M}_R(k_1) \int_{-1}^{1} \mathrm{d}\mu \mathcal{M}_R(|\vec{k}+\vec{k}_1|) \frac{B_\phi(k_1, k, |\vec{k}+\vec{k}_1|)}{P_\phi(k)}$$

$$\mathcal{M}_R(k) \sim W_R(k) T(k) k^2$$

Therefore, on large scales, for local NG, $\frac{\Delta b_h}{b_h} \sim \frac{f_{\rm NL}}{k^2}$

The large scale bias is very sensitive to the squeezed limit of the bispectrum.
A detection of bias going as k^{-1} would rule out all single field models!

A new template

Analysis of CMB is performed by using a sum of factorizable monomials in k's.

We choose the ones with a cosine close to unity w.r.t. the physical shape.

However, orthogonal and enfolded templates go to a constant in the squeezed limit, which is unphysical (Creminelli, G.D'A. Musso, Noreña, *to appear*).
For LSS observations, this gives wrong results! (e.g. bias at large scales)

Solution: we can introduce k^{-4} monomials and cancel divergences in the squeezed limit!

$$F_1(k_1,k_2,k_3) = \frac{16}{9\,k_1 k_2 k_3^4} + \frac{k_1^2}{9\,k_2^4 k_3^4} - \frac{1}{k_1^2 k_3^4} - \frac{1}{k_2^2 k_3^4} + \text{cycl.}$$

$$F_2(k_1,k_2,k_3) = \frac{1}{k_1^3 k_2^3} - \frac{1}{k_1 k_2^2 k_3^3} - \frac{1}{k_2 k_1^2 k_3^3} + \text{cycl.} \qquad F_3(k_1,k_2,k_3) = \frac{1}{k_1^2 k_2^2 k_3^2}$$

$$F_\alpha(k_1,k_2,k_3) = N f_{\rm NL} \Delta_\Phi^2 \left[\alpha F_1 + F_2 + 2(1+\alpha) F_3\right]$$

Model	α	|cos|
M3	0.71	0.95
orth.	0.55	0.98
enf.	0.60	0.98
eq.	0	1

Conclusions and future work

- Additional operators in the EFT Lagrangian
- New shapes for the 3-point function
- Potentially large 4-point function
- New (1-parameter) template which goes to 0 in the squeezed limit
- Shape orthogonal to everything: put constraints on this?
- Initial conditions for LSS simulations using the new template

Thank you!