# Monopoles, Anomalies and Electroweak Symmetry Breaking


Csaba Csáki (Cornell)
with
Yuri Shirman (UC Irvine)
John Terning (UC Davis)


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# Outline

- Brief intro to monopoles
- A toy model for EWSB
- Detour on anomalies
- Monopole scattering and Rubakov-Callan effect
- Non-abelian magnetic charges
- A model with a heavy top
- Basic phenomenology

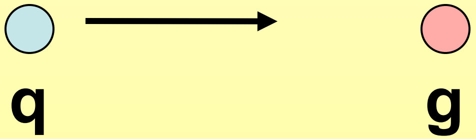
# A Brief History of Monopoles

- J.J. Thomson 1904: monopole + charge

$$\vec{J} = qg\vec{n}$$

- Implies Dirac quantization

- Implies the Rubakov-Callan effect




# A Brief History of Monopoles

- J.J. Thomson 1904: monopole + charge

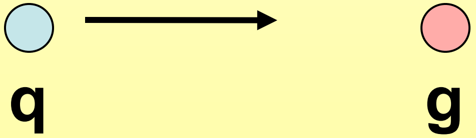
$$\vec{J} = \int d^3r \frac{1}{c} \vec{r} \times (\vec{E} \times \vec{B})$$

$$\vec{E} = \frac{q\vec{r}}{r^3}$$

$$\vec{B} = \frac{g(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

- Implies Dirac quantization

- Implies the Rubakov-Callan effect

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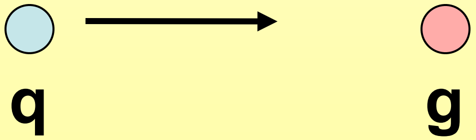



- Dirac 1930: Dirac string/monopole

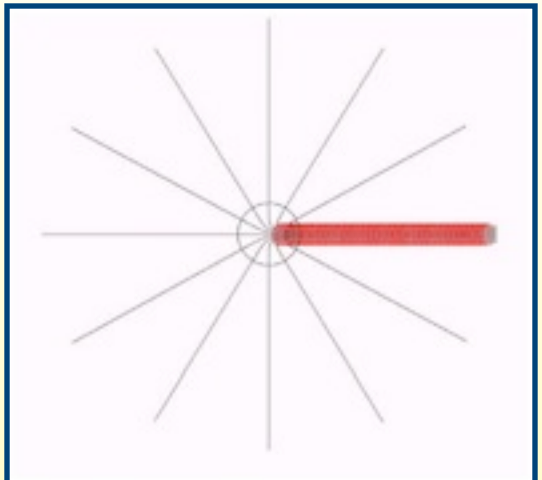


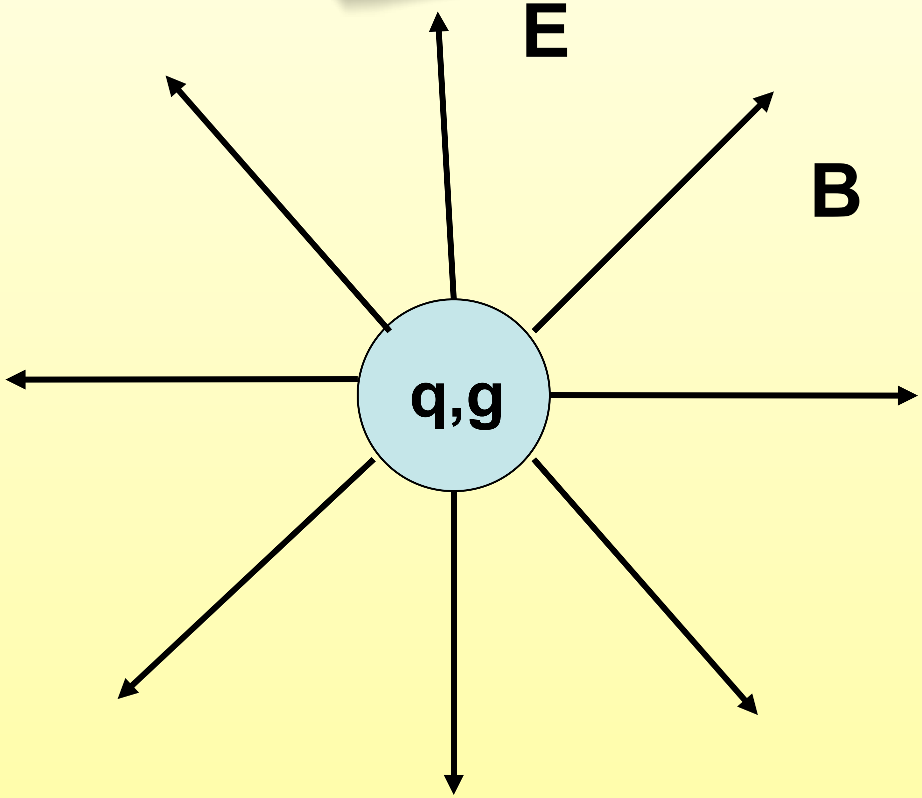
Figure 2: The vector potential of a monopole is singular on a string. When the string carries an integer number of flux quanta it can be gauged away.

- Dirac quantization:

$$qg = \frac{n}{2}$$

•Schwinger generalized quantization condition to dyons

$$q_1 g_2 - q_2 g_1 = \frac{n}{2}$$



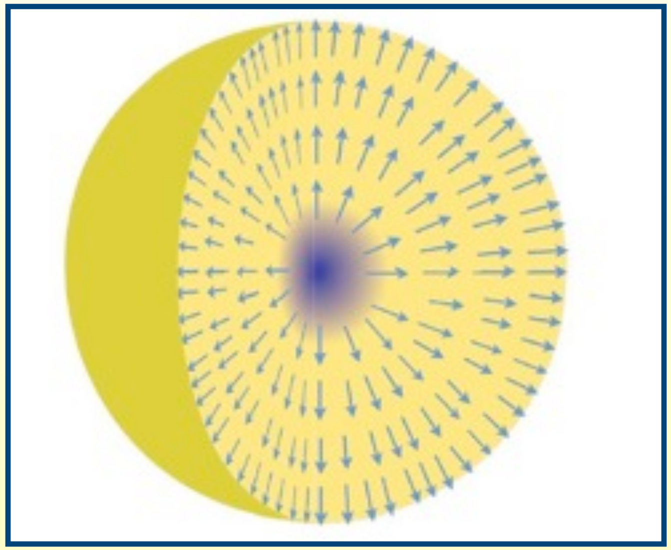

•Schwinger also tries to write theory of strong inter's using a model of hadrons with monopoles and dyons

•Our proposal in similar spirit, try to replace "technicolor-type" interactions with strong U(1) effects from dyons

•To our knowledge only known attempt to connect monopoles with "low-scale" particle pheno

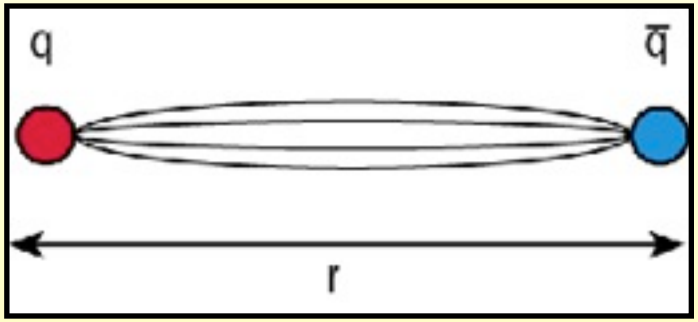
•1974: ‘t Hooft Polyakov monopole

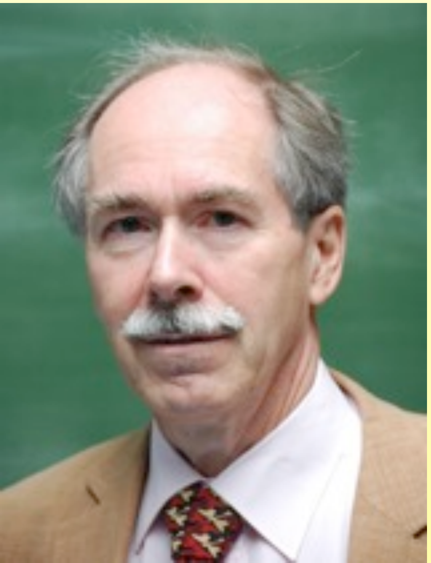
•Topological monopoles without singularity



• 1976: ‘t Hooft – Mandelstam: condensation of magnetic charges causes electric confinement

• Dual of Meißner effect where electric condensation confines magnetic fields



- Witten effect: magnetically charged objects pick up electric charge in the presence of q

$$q \to q + \frac{\theta}{2\pi} g$$

- θ can be physical in U(1) theories, if fermions massive

- Heuristic proof by Coleman

$$\mathcal{L}_\theta = \frac{\theta e^2}{8\pi^2}\vec{E}\cdot\vec{B}$$

- Monopole field plus arbitrary field:

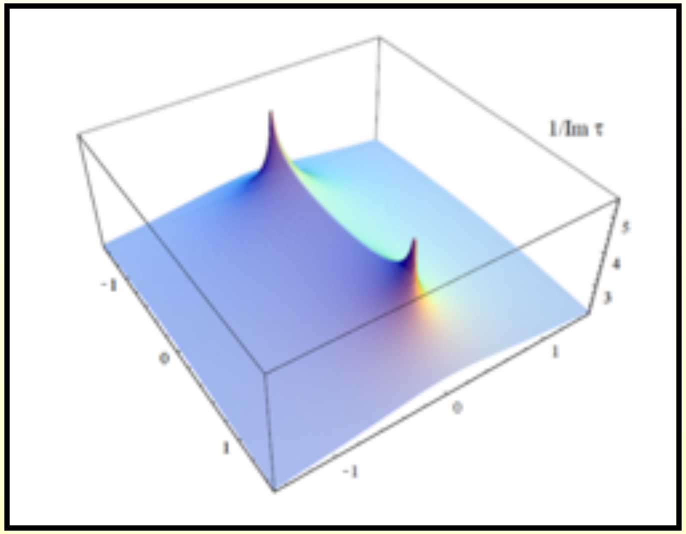
$$\vec{E} = -\nabla\phi$$
$$\vec{B} = \nabla\times\vec{A} + \frac{g}{4\pi}\frac{\vec{e}_r}{r^2}$$

- The Lagrangian, integrating by parts:

$$L_\theta = \frac{\theta e^2}{8\pi}\int dV(-\nabla\phi)\cdot(\nabla\times\vec{A} + \frac{g}{4\pi}\frac{\vec{e}_r}{r^2}) =$$
$$-\frac{\theta e^2 g}{32\pi^3}\int dV\,\phi\nabla(\frac{\vec{e}_r}{r^2}) = \frac{\theta e^2 g}{8\pi^2}\int dV\,\phi\delta(\vec{r})$$

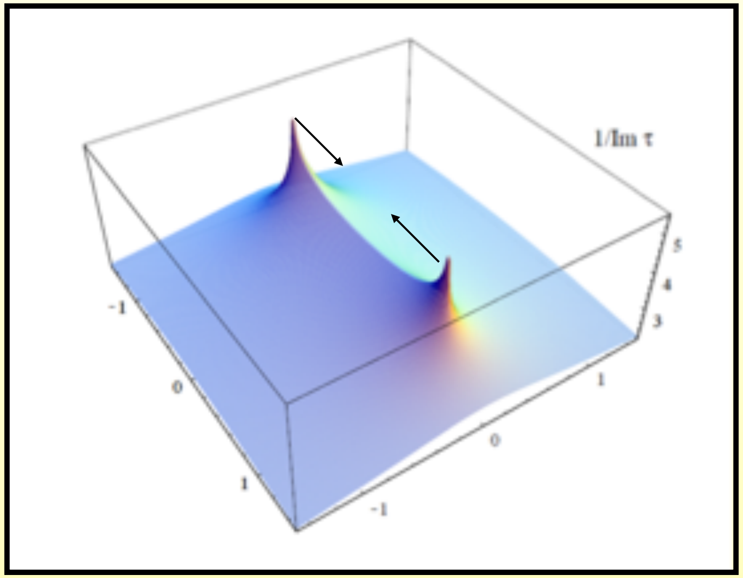
- Like a charge at the origin, q→q+q/(2p) g

- 1994: Seiberg, Witten: monopoles in N=2 SUSY theories can become massless (and condense if broken to N=1)




•Argyres Douglas (and also Intriligator and Seiberg):

•The points where monopoles and dyons are massless can coincide. Expect a fixed point (4D CFT)




**Idea:** use strong interactions between monopoles and electric charges to break electroweak symm.

**Similar to:** Schwinger 1960's theory of strong interactions using interactions of dyons (in the paper where he coined the term "dyon"

**Would be** like a technicolor-type theory built on U(1) dyons ("monocolor")

**Could have** some advantages wrt. technicolor

- Rubakov-Callan for top mass
- No new gauge group needed, just SM
- Different phenomenology…

# What kind of theory could be interesting?

- If only electric charges: U(1) IR free
- If only magnetic charges: dual U(1) IR free (free magnetic phase)
- Need electric and magnetic charges at the same time
- Argyres-Douglas: this is possible (in N=2 SUSY at very special points…)

# What we need for an interesting theory

- Want massless monopoles (relevant for IR dynamics)
- Should be fermionic (to avoid hierarchy problem)
- Should be chiral (to have quantum # of Higgs)
- All anomalies should cancel
- All Dirac quantization obeyed
- Magnetic charges should be vectorlike (to avoid confinement of electric charges)

# A toy model

- An extra generation with magnetic hypercharges

| | $SU(3)_c$ | $SU(2)_L$ | $U(1)_Y^{el}$ | $U(1)_Y^{mag}$ |
|---|---|---|---|---|
| $Q$ | $\square$ | $\square$ | $\frac{1}{6}$ | $3$ |
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| $\bar{U}$ | $\bar{\square}$ | $1$ | $-\frac{2}{3}$ | $-3$ |
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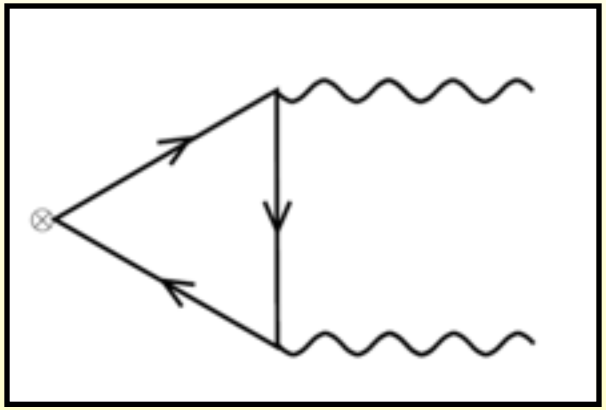
- All anomalies cancel, Dirac quantization OK

# A detour on anomalies with monopoles

- What is the chiral anomaly in the presence of dyons?



- Assume, can calculate anomalies for fields independently

- Then can do SL(2,Z) rotation where field is just an electron

# SL(2,Z)

- A set of field redefinitions that leaves physics unchanged (but Lagrangian NOT invariant, no sym)
- S-duality: has effect of $$g \to \frac{1}{g}$$
- Also exchanges electric and magnetic charges
- T-duality: shift of q: $$\theta \to \theta + 2\pi$$
- Together SL(2,Z). Can introduce "holomorphic" coupling parameter τ, under SL(2,Z) $$\tau \to \frac{a\tau + b}{c\tau + d}$$

$$\tau \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{e^2}$$

- Here a,b,c,d are integers and ad-bc=1
- The SL(2,Z) transformation of charges:

$$\begin{pmatrix} a & -b \\ -c & d \end{pmatrix} \begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} n \\ 0 \end{pmatrix}$$

- Where n=gcd(q,g) can always be achieved
- In this frame anomalies easy, just usual

$$\begin{aligned} \partial_\mu j^\mu_A(x) &= \frac{n^2}{16\pi^2} F'^{\mu\nu}\, {}^*F'_{\mu\nu} \\ &= \frac{n^2}{32\pi^2} \mathrm{Im}\left(F'^{\mu\nu} + i\, {}^*F'^{\mu\nu}\right)^2 \end{aligned}$$

- To transform back need SL(2,Z) for fields
- Maxwell equations:

$$\frac{\mathrm{Im}(\tau)}{4\pi}\partial_\mu\left(F^{\mu\nu}+i\,{}^*F^{\mu\nu}\right)=J^\nu+\tau K^\nu$$

- Will be SL(2,Z) covariant if fields transform (New?):

$$\left(F^{\mu\nu}+i\,{}^*F^{\mu\nu}\right)\to\frac{1}{c\tau^*+d}\left(F'^{\mu\nu}+i\,{}^*F'^{\mu\nu}\right)$$

- Chiral anomaly:

$$\begin{aligned}\partial_\mu j^\mu_A(x) &= \frac{1}{16\pi^2}\mathrm{Re}\left(q+\tau^* g\right)^2 F^{\mu\nu}\,{}^*F_{\mu\nu}+\frac{1}{16\pi^2}\mathrm{Im}\left(q+\tau^* g\right)^2 F^{\mu\nu}\,F_{\mu\nu}\\ &= \frac{1}{16\pi^2}\left\{\left[\left(q+\frac{\theta}{2\pi}g\right)^2-g^2\frac{16\pi^2}{e^4}\right]F^{\mu\nu}\,{}^*F_{\mu\nu}+\left[qg+\frac{\theta}{2\pi}g^2\right]F^{\mu\nu}\,F_{\mu\nu}\right\}\end{aligned}$$

- Need to cancel all terms separately!

$$\sum q_{X_i} q_i^2 = 0, \quad \sum q_{X_i} q_i g_i = 0, \quad \sum q_{X_i} g_i^2 = 0$$

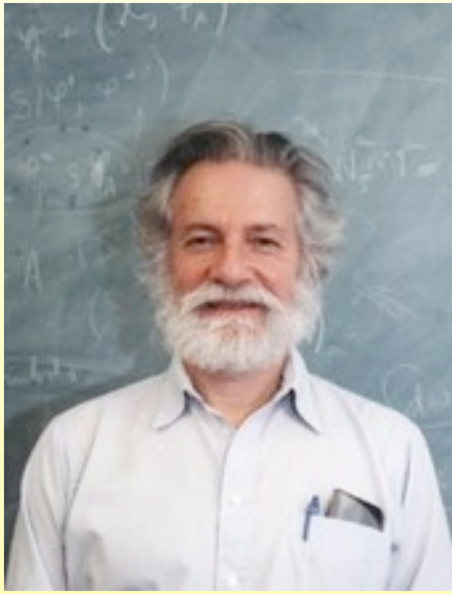
- Can argue similarly for gauge symmetries

- Need some Lagrangian formulation

- Use Zwanziger Lagrangian (local, gauge invariant but not Lorentz invariant)

- Two gauge fields, A electric, B magnetic

- Equations of motion Lorentz invariant



Daniel Zwanziger

- We found a trivial generalization including q term

$$
\begin{aligned}
\mathcal{L} = & -\text{Im}\frac{\tau}{8\pi n^2}\{[n\cdot\partial\wedge(A+iB)]\cdot[n\cdot\partial\wedge(A-iB)]\} \\
& -\text{Re}\frac{\tau}{8\pi n^2}\{[n\cdot\partial\wedge(A+iB)]\cdot[n\cdot^*\partial\wedge(A-iB)]\} \\
& -J\cdot A-\frac{4\pi}{e^2}K\cdot B
\end{aligned}
$$

- Using this we showed (similarly) that mixed gauge anomalies should cancel too:

$$
\begin{aligned}
\sum_j q_j^2\, g_j &= 0 \\
\sum_j q_j\, g_j^2 &= 0 \\
\sum_j g_j^3 &= 0
\end{aligned}
$$

# A toy model

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- All anomalies cancel, Dirac quantization OK

# What IR phase?

## 3 possibilities

- **Conformal fixed point** – if β- function like 1-loop: expect fixed point, not interesting for EWSB

- **IR-free** – electric charge outweighs magnetic charge, like in QED. Magnetic coupling becomes very large, forming of condensates and mass gap

- **Free magnetic** Magnetic charge outweighs electric

- **Assume:** not a fixed point. In this case plausible that it is IR free (more electric fields) - condensation

# Possible condensates

- Don't carry magnetic charge

- Have quantum number of Higgs

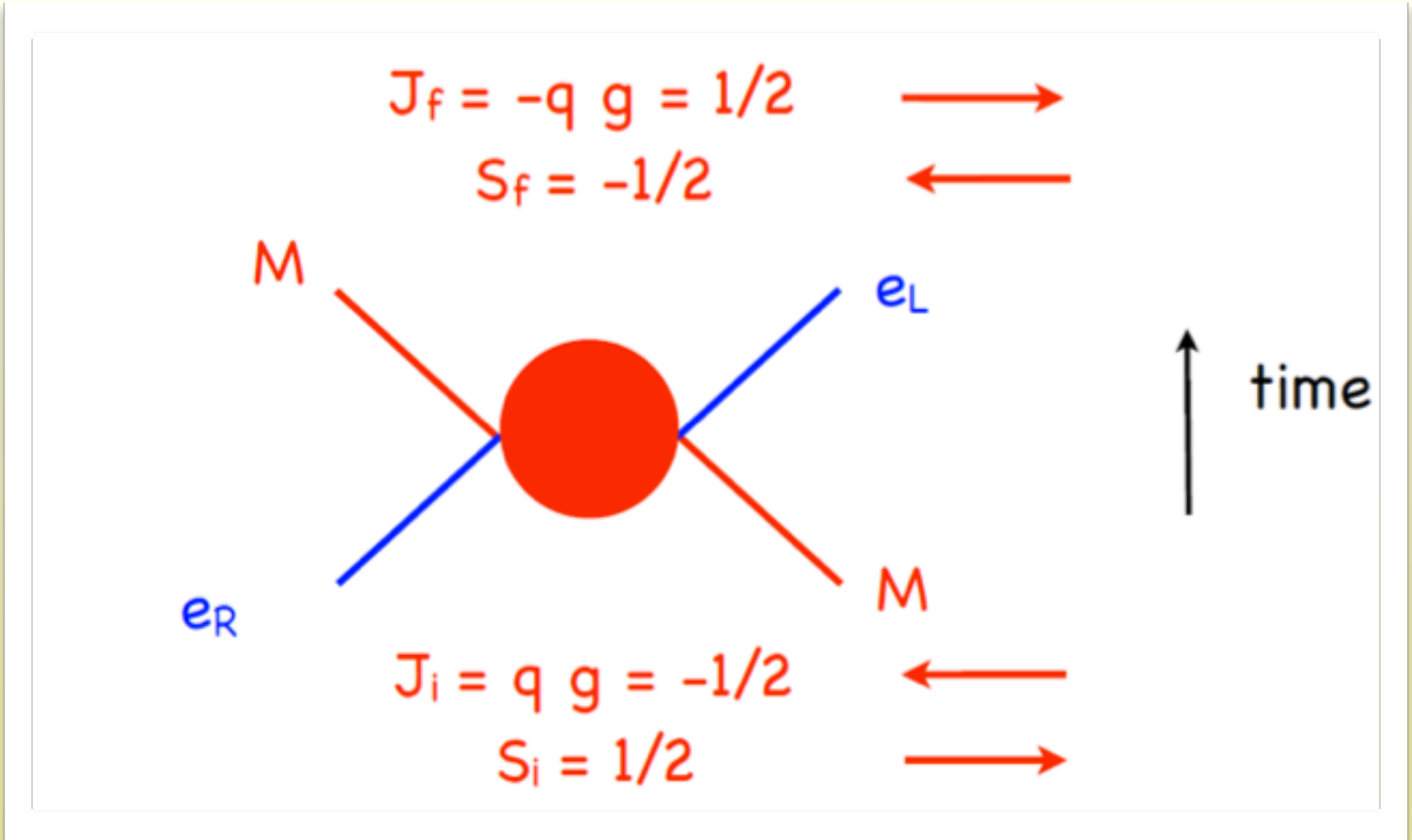
$$Q\bar{D} \sim (1, 2, \frac{1}{2}) \sim H, \quad Q\bar{U} \sim (1, 2, -\frac{1}{2}) \sim H^*,$$
$$L\bar{E} \sim (1, 2, \frac{1}{2}) \sim H, \quad L\bar{N} \sim (1, 2, -\frac{1}{2}) \sim H^*.$$

- Assume some of these condensates generated

$$\langle U_L \bar{U} \rangle \sim \langle D_L \bar{D} \rangle \sim \langle N_L \bar{N} \rangle \sim \langle E_L \bar{E} \rangle \sim \Lambda^d_{mag}$$

- $\Lambda_{mag}$ is a dynamical of order few x 100 GeV

# The Rubakov-Callan effect



# The Rubakov-Callan effect

- Even though no interaction between monopole and charge, angular momentum changes

- There has to be a contact interaction between monopoles and charges which is marginal

# The quantum picture

- Dirac equation in the presence of monopole peculiar for J=0

- For electron, positive helicity purely outgoing
  negative helicity purely incoming

- For positron just the opposite

- This is because $\vec{J}_{em} = -\frac{1}{2}\vec{n}$ and $\vec{J}_{tot} = \vec{J}_{em} + \vec{\sigma}$

- Need boundary condition at core of monopole – chirality should flip (or electric charge…)

# For spin 1/2

Squared Dirac eq.: $$\left[(\partial_\mu - ieA_\mu)^2 - \frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu} - m^2\right]\Psi = 0$$

In a monopole background:

$$\left[-\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) + \frac{1}{r^2}(\vec{L}^2 - q^2) - q\frac{\vec{\sigma}\cdot\hat{r}}{r^2} - (E^2 - m^2)\right]\Psi_\pm = 0.$$

Where

$$\vec{J} = \vec{L} + \frac{1}{2}\vec{\sigma} \qquad \vec{L} = \vec{r}\times(\vec{p} - e\vec{A}) + q\hat{r}$$

Eigenfunctions: “Monopole harmonics” (C.N. Yang and T.T. Wu)

$$Y_{q,l,m}(\theta,\varphi) = M_{q,l,m}(1-x)^{\frac{\alpha}{2}}(1+x)^{\frac{\beta}{2}}P_n^{\alpha,\beta}(x)e^{i(q+m)\varphi}$$

Need to diagonalize Dirac equation

$$\vec{L}^2 - q^2 - q\vec{\sigma}\cdot\hat{r} = \begin{bmatrix} (j+\frac{1}{2})(j+\frac{3}{2}) - q^2 - \frac{2q^2}{2j+1} & -q\frac{\left[(2j+1)^2-4q^2\right]^{\frac{1}{2}}}{2j+1} \\ -q\frac{\left[(2j+1)^2-4q^2\right]^{\frac{1}{2}}}{2j+1} & (j-\frac{1}{2})(j+\frac{1}{2}) - q^2 + \frac{2q^2}{2j+1} \end{bmatrix}$$

Eigenvalues: $\mu(\mu \pm 1)$ with $\mu = \sqrt{(j+\frac{1}{2})^2 - q^2}$

Wave function at origin: $\sim r^{\mu} \,\mathrm{or}\, r^{\mu-1}$

Since j=q±½ (for vanishing orbital) it is now possible that neither solution vanishes at core of monopole - need BC - leads to RC operators...

# But for toy model

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- **No** Rubakov-Callan generated
- Want something like $t_R U_L \rightarrow t_L U_R$
- $J_{in}$=3 x 2/3=2
- $J_{fin}$=-3 x 1/6 =-1/2
- Can not compensate with chirality flips…
- Need to modify model such that minimal Dirac charge is allowed

# Need for non-abelian magnetic charges

- Question similar to early 80's: can you have minimal Dirac charge with down quark e=-1/3?

- Naively contradicts Dirac quantization

- If monopole also carries color magnetic charge then possible

- This is what happens for GUT monopole

- Need to embed magnetic field into non-abelian groups as well – "non-abelian monopoles"

# GUT monopole:

$$Y = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -\frac{3}{2} & \\ & & & & -\frac{3}{2} \end{pmatrix}$$

- Specific U(1) transformations:

$$e^{\pm i \frac{2\pi}{3} Y} = \begin{pmatrix} \omega & & & & \\ & \omega & & & \\ & & \omega & & \\ & & & -1 & \\ & & & & -1 \end{pmatrix} \subset SU(3)\times SU(2)$$

- Monopole also carries **discrete** SU(3)xSU(2) magnetic charges

- Group really $SU(3)xSU(2)xU(1)/Z_6$

# Conserved quantity in presence of monopole

$$\begin{matrix} SU(3) \\ SU(2) \end{matrix} \left(\begin{array}{c|c} 1 & \\ \hline & -1 \end{array}\right) \propto \tfrac{1}{3}T_8 + (Y + T_3)$$

- The actual conserved quantity

$$J_z^{tot} = (\vec{L} + \vec{S})_z + Q + \tfrac{1}{3}T_8$$

- Leads to non-trivial Dirac quantization

# Non-abelian monopoles

- Magnetic field not aligned with $U(1)_Y$

$$\begin{aligned} \vec{B}_Y^a &= \frac{g}{g_Y}\frac{\hat{r}}{r^2}\,, \\ \vec{B}_L^a &= \delta_L^{a3}\frac{g\,\beta_L}{g_L}\,\frac{\hat{r}}{r^2}\,, \\ \vec{B}_c^a &= \delta_c^{a8}\frac{g\,\beta_c}{g_c}\,\frac{\hat{r}}{r^2}\,, \end{aligned}$$

- Dirac quantization loop

$$\int_{loop} e\,q\,A^\mu dx_\mu$$

- Now replaced by

$$\int_{loop} (g_c\,T_c^a G^{a\mu} + g_L\,T_L^a\,W^{a\mu} + g_Y Y\,B^\mu)\,dx_\mu$$

- The gauge field for Dirac calculation:

$$
\begin{aligned}
\vec{A}_Y &= \frac{g}{g_Y}\,\frac{1-\cos\theta}{r\sin\theta}\,\hat{e}_\phi\ . \\
\vec{A}^a_L &= \delta^{a3}_L\,\frac{g\,\beta_L}{g_L}\,\frac{1-\cos\theta}{r\sin\theta}\,\hat{e}_\phi\ , \\
\vec{A}^a_c &= \delta^{a8}_c\,\frac{g\,\beta_c}{g_c}\,\frac{1-\cos\theta}{r\sin\theta}\,\hat{e}_\phi\ ,
\end{aligned}
$$

- Dirac quantization: every component of matrix has to obey

$$
4\pi\left(T^8_c\,g\,\beta_c + T^3_L\,g\,\beta_L + Y g\right) = 2\pi n\ .
$$

# A model with a heavy top

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| $U_R$ | $\square^m$ | $1^m$ | $\frac{2}{3}$ | $\frac{1}{2}$ |
| $D_R$ | $\square^m$ | $1^m$ | $-\frac{1}{3}$ | $\frac{1}{2}$ |
| $N_R$ | $1$ | $1^m$ | $0$ | $-\frac{3}{2}$ |
| $E_R$ | $1$ | $1^m$ | $-1$ | $-\frac{3}{2}$ |

- We choose $b_L$=1 and $b_c$=1 for colored monopoles
- Dirac quantization now satisfied with minimal (1/2) Dirac charge

•Since $b_L$=1 magnetic field actually points always in direction of QED photon

•Can instead just look at QED electric and magnetic charges

| | $SU(3)_c$ | $U(1)^{el}_{em}$ | $U(1)^{mag}_{em}$ |
|---|---|---|---|
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| $N_L$ | 1 | 0 | $-\frac{3}{2}$ |
| $E_L$ | 1 | $-1$ | $-\frac{3}{2}$ |
| $U_R$ | $\square^m$ | $\frac{2}{3}$ | $\frac{1}{2}$ |
| $D_R$ | $\square^m$ | $-\frac{1}{3}$ | $\frac{1}{2}$ |
| $N_R$ | 1 | 0 | $-\frac{3}{2}$ |
| $E_R$ | 1 | $-1$ | $-\frac{3}{2}$ |

•Quantization condition now will be:

$$T^8_c g \beta_c + qg = \frac{n}{2}$$

•Dyons:

$$(q_1 g_2 - q_2 g_1) + (T^8_{c1} g_2 \beta_{c2} - T^8_{c2} g_1 \beta_{c1}) = \frac{n}{2}$$

- With this embedding:

$$\alpha^{mag} = \frac{\alpha^{-1}}{4} \sim 32$$

- Rubakov-Callan now generated:

- $u_R N_L \rightarrow u_L N_R$ satisfies the RC condition

- Initial spin +1, EM field J= 2/3 x (-3/2)=-1

- Final spin -1, EM field J= - 2/3 x (-3/2)=1

- Operator needs to be present:

$$\lambda_{ij}^{(u)} u_R^i N_L \left(u_L^j N_R\right)^\dagger$$

- Gauge invariant version: $\lambda^{(u)}_{ij} u^i_R L_L \left(q^j_L N_R\right)^\dagger$
- Some up-type quarks have to have large masses
- BUT: don't expect RC to break global symmetry
- Need to assume flavor physics at high scales breaks all flavor symmetries
- RC can be used to transmit flavor violation to low scales
- Can decouple flavor and EWSB scales via RC

- Down-type masses: 6-fermion RC operator

$$d_R + E_L + u_L + d_L^\dagger \rightarrow u_L + E_R$$

- After closing up up-quark leg get down mass

- $m_b \sim m_t/(16p^2)$

- Similarly for charged leptons. Neutrinos strongly suppressed

- PNGB's: RC can save us again, can transmit symmetry breaking:

$$Q_L E_R (L_L D_R)^\dagger$$

$$Q_L N_R (L_L U_R)^\dagger$$

# Basic Phenomenology

- After EWSB theory vectorlike, expect monopoles to pick up mass of order $L_{mag}$~500 GeV – TeV

- Since monopole points in QED direction, not confined, like “ordinary” QED monopole

- No magnetic coupling to Z

- Electric coupling is there, expect EWPO (S,T) like a heavy fourth generation – could be OK?

•**At LHC:** likely pair produced. Due to strong force strong attraction, will always annihilate at LHC. Large radiation, then annihilation. Lots of photons, some of them hard. Cross section? Not calculable. Naive estimate ~ few x pb **(A. Weiler)**

•**Cosmic ray** bounds? SLIM upper bound on monopole flux 1.3 $10^{-15}$ $cm^{-2}$ $sr^{-1}$ $s^{-1}$. Implies 1 mb bound on cross section, not strong.

•**Dark matter?** Monopole number conserved, baryon type monopole UUDE or UDDN could be stable

# Summary

•Use strong interactions from magnetic sector of U(1) to break EWS via condensation

•Monopoles can be aligned with QED, then no coupling to Z, not confined, minimal Dirac charge.

•Rubakov-Callan operators can transmit high scale flavor violation, separate flavor scale

•Should be visible at the LHC, lots of photons… CMS will trigger on it!