Observational tests of eternal inflation: Bayesian model selection

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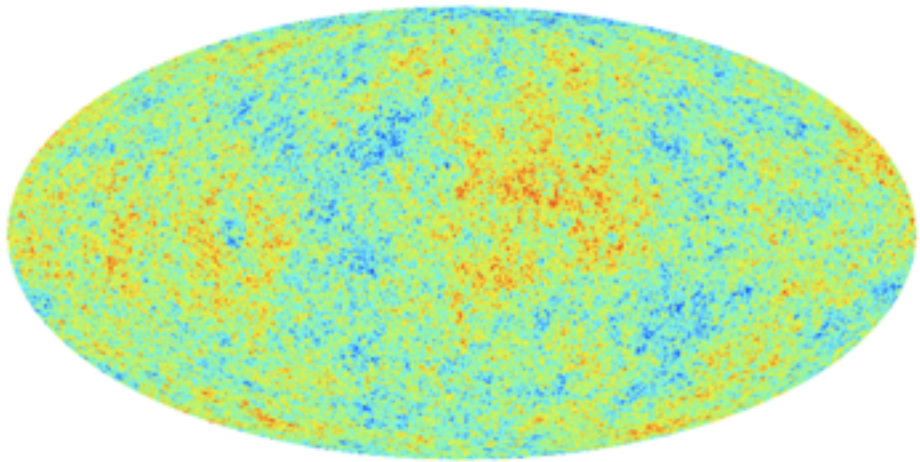
work in progress

Outline

- How do we decide if there are bubble collisions in CMB data?



  - Review of Bayesian statistics.
  - What are we testing? (model assumptions)
  - An analysis strategy.

Bayesian statistics

- The goal: $P(\text{Model}, \Theta \mid \text{data})$

How should I bet?

- Bayes' Theorem:

$$P(\text{Model}, \Theta \mid \text{data}) = \frac{P(\Theta)P(\text{data} \mid \text{Model}, \Theta)}{P(\text{data} \mid \text{Model})}$$

- Theory prior: $P(\Theta)$ $\quad \int P(\Theta) d\Theta = 1$
- Likelihood: $P(\text{data} \mid \text{Model}, \Theta)$
- Evidence (model averaged likelihood): $P(\text{data} \mid \text{Model})$

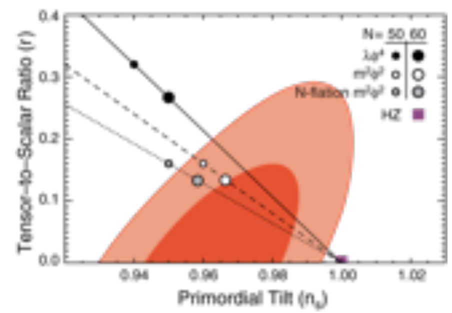
$$P(\text{data} \mid \text{Model}) = \int d\Theta P(\Theta) P(\text{data} \mid \text{Model}, \Theta)$$

Bayesian statistics

- The likelihood is used to quantify how consistent data is with a set of model parameters.

$$P(\text{data}\ |\text{Model}, \Theta)$$

exclusion plots



- This does NOT tell us how we should rank competing theories trying to describe the same data.

- To do so, we can apply Bayes' theorem at the level of Models:

$$P(\text{Model} \mid \text{data}) = \frac{P(\text{Model})P(\text{data}\ |\text{Model})}{P(\text{data})}$$

Bayesian model selection

- Let's say I have a model that fits the data fairly well, should I introduce a more complicated model that might fit it even better?

- We can decide by looking at the evidence ratio:

$$\frac{P(\text{Model 1} \mid \text{data})}{P(\text{Model 0} \mid \text{data})} = \frac{P(\text{Model 1})P(\text{data } |\text{Model 1})}{P(\text{Model 0})P(\text{data } |\text{Model 0})} = \frac{P(\text{data } |\text{Model 1})}{P(\text{data } |\text{Model 0})}$$

- The evidence naturally implements Occam's razor: the simpler model should be favored. Tension between volume of parameter space and goodness of fit.

$$P(\text{data } |\text{Model}) = \int d\Theta P(\Theta) P(\text{data } |\text{Model}, \Theta)$$

Bayesian model selection

- A model is specified both by:
  - The predictions for the data given a particular set of parameters Θ.
  - A prior $P(\Theta)$ that specifies what values these parameters can take.

Model 0: $\Lambda\mathrm{CDM}$

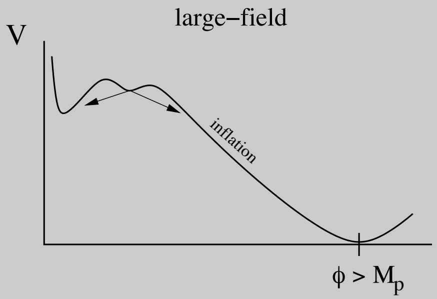
$$\Omega_m$$
$$\Omega_\Lambda$$
$$n_s$$
$$\Delta^2_{\mathcal{R}}$$
$$\vdots$$

Model 1: Bubble cosmology with collisions.

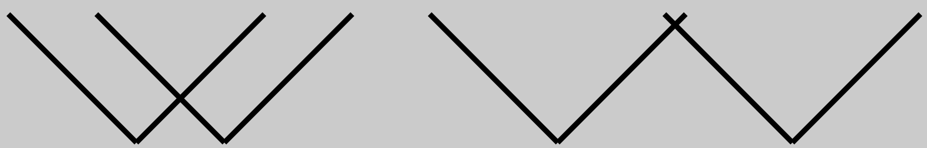
$$\Omega_m$$
$$\Omega_\Lambda$$
$$n_s \quad + \quad \Theta_{\mathrm{coll}} = ?$$
$$\Delta^2_{\mathcal{R}}$$
$$\vdots$$

Modeling bubble collisions

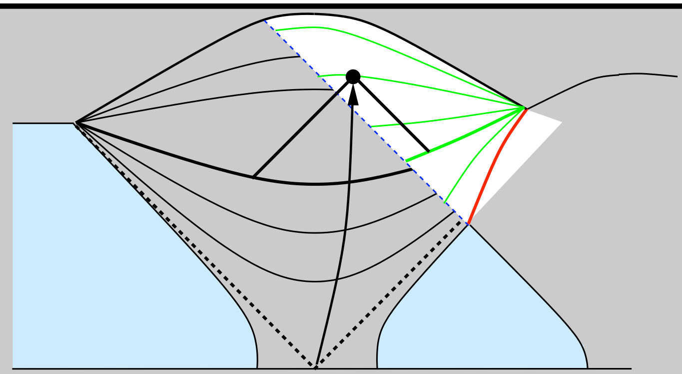
- What kinds of parameters specify the collision model?



- Nucleation rates.
- Cosmology in our bubble.
- Properties of colliding bubbles.



- Kinematics.



- Our position relative to collisions.

Modeling bubble collisions

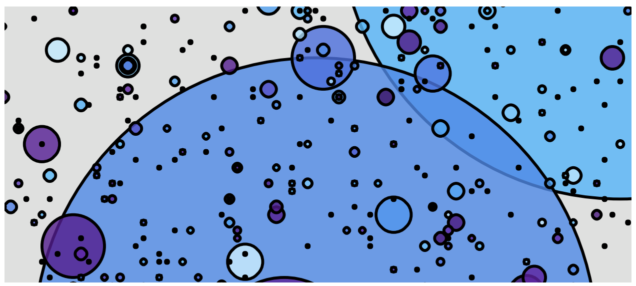
- What kinds of parameters specify the collision model?

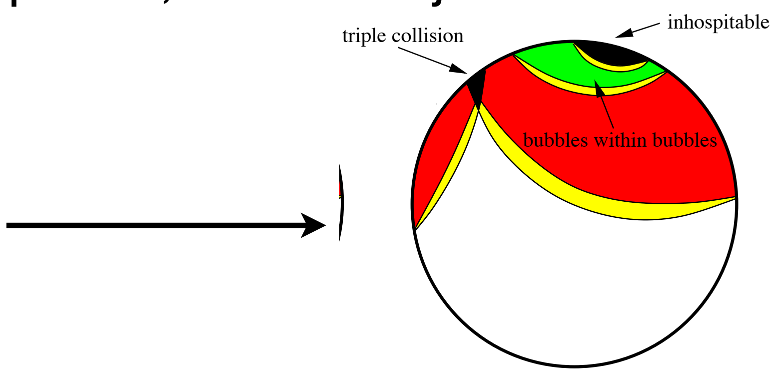
  - Global properties of the collision spacetime.

  - Observed properties of the collision spacetime.

Modeling bubble collisions

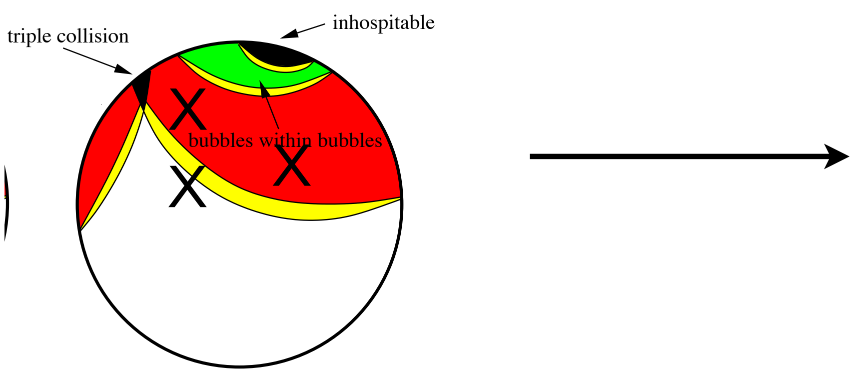
- Given unlimited computing power, we could just simulate eternal inflation.

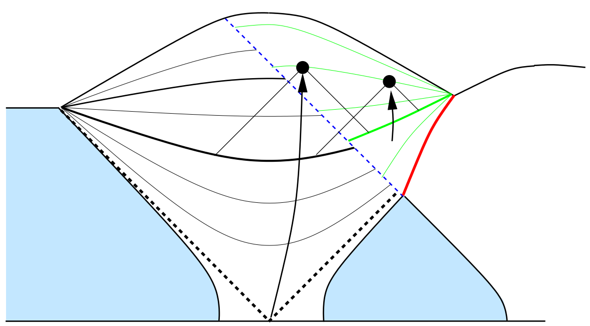




Constant time surface in our bubble.

- Putting observers in different places, we could then ask what they see.
- Counting various observers, we can also (perhaps) determine the prior.



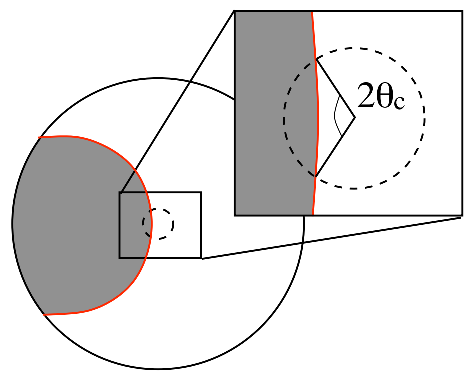


Modeling bubble collisions

- This is impossible.
- It is possibly also wasteful: no one observer can see all this structure.
- To confront data, we also need to determine some useful phenomenological parameters to map the fundamental parameters onto.

A first step:

- Assume the collision is a perturbation on top of inflation: $\Phi_{\rm coll}$
- Thanks to inflation, any observer sees a tiny piece at last scattering:



$$\Phi_{\rm coll} = \Phi(a)\left(\bar{c}_0 + \bar{c}_1(x - x_{\rm crit}) + \mathcal{O}((x - x_{\rm crit})^2)\right)\Theta(x - x_{\rm crit})$$

global properties are embedded in the $\bar{c}_i$

observables probe a subset of the $\bar{c}_i$

Modeling bubble collisions

- Projecting onto the past light cone of an observer:

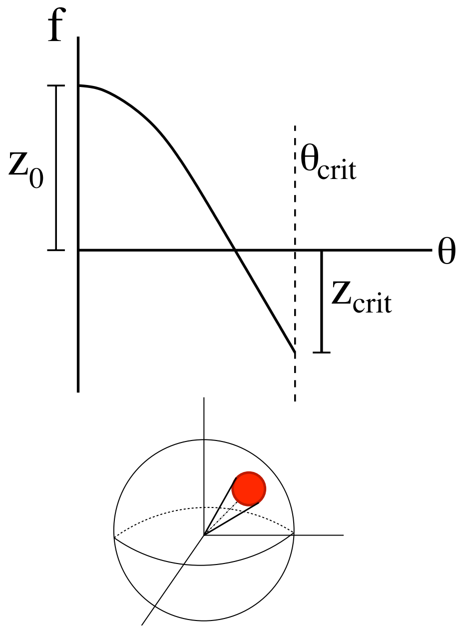
$$\frac{\delta T}{T} \simeq \frac{\Phi_{\rm coll}(a_{\rm ls})}{3} + 2\int_{a_{\rm ls}}^{1} da \frac{d\Phi_{\rm coll}}{da} + \left(\vec{v}\cdot\hat{\mathbf{n}} + \mathcal{O}(v^2)\right)$$

SW ISW doppler

- SW: depends only on the potential at last scattering.
- ISW: depends on how the potential evolves, and how the boundary propagates.
- Doppler: depends on where we formed.

Modeling bubble collisions

- Projecting onto the past light cone of an observer:



$$f(\hat{n}) = \left[\frac{z_{\rm crit} - z_0 \cos\theta_{\rm crit}}{1 - \cos\theta_{\rm crit}} + \frac{z_0 - z_{\rm crit}}{1 - \cos\theta_{\rm crit}} \cos\theta \right] \Theta(\theta_{\rm crit} - \theta)$$

$$\{\Phi(a), \bar{c}_0, \bar{c}_1, x_{\rm crit}\} \rightarrow \{z_0, z_{\rm crit}, \theta_{\rm crit}, \theta_0, \phi_0\}$$

- This form, with $z_{\rm crit} = 0$, first found by Chang, Kleban, and Levi.
- $z_{\rm crit} \neq 0$ depends on ISW and doppler contributions: how large?
- How is this template altered by the transfer function?

Model priors

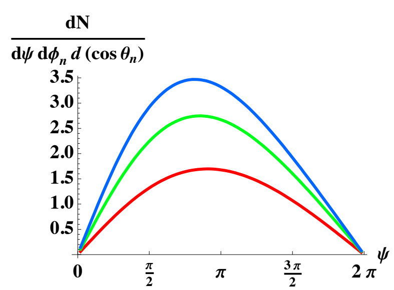
- Previous talks: observable collisions in our PLC are isotropic.

$$\mathrm{Pr}(\theta_0, \phi_0, \theta_{\rm crit}, z_0, z_{\rm crit}) = \mathrm{Pr}(\theta_0, \phi_0)\mathrm{Pr}(\theta_{\rm crit}, z_0, z_{\rm crit})$$

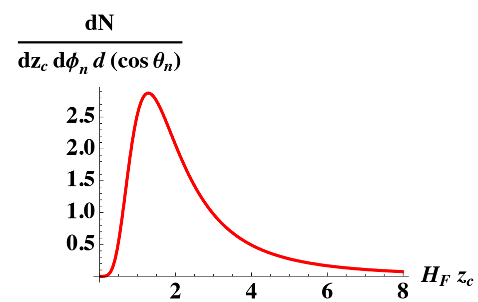
$$\mathrm{Pr}(\theta_0, \phi_0) = \frac{\sin\theta_0}{4\pi}$$

- Kinematics and observer position affects $\{z_0, z_{\rm crit}\}$ and $\theta_{\rm crit}$.

- Toy model tells us:



For all kinematical configurations.



For all angles.

- Until our understanding of the model improves, we choose flat priors:

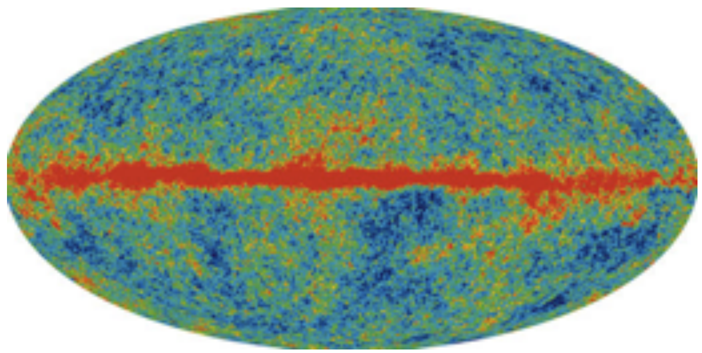
$$\mathrm{Pr}(\theta_{\rm crit}) = \frac{2}{\pi} \ , \quad 0 \leq \theta_{\rm crit} \leq \frac{\pi}{2}$$

$$\mathrm{Pr}(z_{\rm crit}) = \frac{1}{2\times 10^{-4}} \ , \quad -10^{-4} \leq z_{\rm crit} \leq 10^{-4}$$

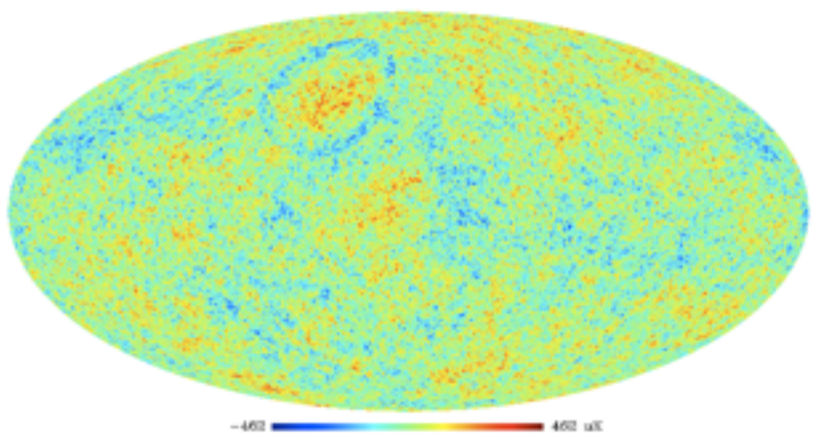
$$\mathrm{Pr}(z_0) = \frac{1}{2\times 10^{-4}} \ , \quad -10^{-4} \leq z_0 \leq 10^{-4}$$

The data

- The data under consideration: full-sky CMB maps.



- How do we model select?



Full-sky Bayesian analysis

- Assume collisions can be treated as independent sources on the sky.
- Assume that a theory is specified by the expected number of visible collisions on the full sky:

$$\Pr(\bar{N}_{\rm s}|\boldsymbol{d}) = \frac{\Pr(\bar{N}_{\rm s})\,\Pr(\boldsymbol{d}|\bar{N}_{\rm s})}{\Pr(\boldsymbol{d})}$$

- The evidence ratio we ultimately want to calculate is:

$$\frac{\Pr(\bar{N}_{\rm s}|\boldsymbol{d})}{\Pr(0|\boldsymbol{d})} = \frac{\Pr(\bar{N}_{\rm s})\ \Pr(\boldsymbol{d}|\bar{N}_{\rm s})}{\Pr(0)\ \Pr(\boldsymbol{d}|0)}$$

- Assume no theoretical prejudice:

$$\frac{\Pr(\bar{N}_{\rm s}|\boldsymbol{d})}{\Pr(0|\boldsymbol{d})} = \frac{\Pr(\boldsymbol{d}|\bar{N}_{\rm s})}{\Pr(\boldsymbol{d}|0)}$$

Full-sky Bayesian analysis

- The actual number of collisions is drawn from a Poisson distribution:

$$\Pr(N_\mathrm{s}|\bar{N}_\mathrm{s}) = \frac{\bar{N}_\mathrm{s}^{N_\mathrm{s}} e^{-\bar{N}_\mathrm{s}}}{N_\mathrm{s}!}$$

$$\Rightarrow \quad \Pr(\boldsymbol{d}|\bar{N}_\mathrm{s}) = \sum_{N_\mathrm{s}=0}^{\infty} \Pr(N_\mathrm{s}|\bar{N}_\mathrm{s}) \Pr(\boldsymbol{d}|N_\mathrm{s})$$

- The evidence is given by:

$$\Pr(\boldsymbol{d}|N_\mathrm{s}) = \int \mathrm{d}\boldsymbol{m}_1 \, \mathrm{d}\boldsymbol{m}_2 \ldots \mathrm{d}\boldsymbol{m}_{N_\mathrm{s}} \, \Pr(\boldsymbol{m}_1, \boldsymbol{m}_2, \ldots \boldsymbol{m}_{N_\mathrm{s}}) \Pr(\boldsymbol{d}|N_\mathrm{s}, \boldsymbol{m}_1, \boldsymbol{m}_2, \ldots \boldsymbol{m}_{N_\mathrm{s}})$$

$$\boldsymbol{m}_i = \{\theta_0^{(i)}, \phi_0^{(i)}, \theta_\mathrm{crit}^{(i)}, z_0^{(i)}, z_\mathrm{crit}^{(i)}\}$$

- From the independence of each collision:

$$\Pr(\boldsymbol{m}_1, \boldsymbol{m}_2, \ldots \boldsymbol{m}_{N_\mathrm{s}}) = \prod_{i=1}^{N_\mathrm{s}} \Pr(\boldsymbol{m}_i)$$

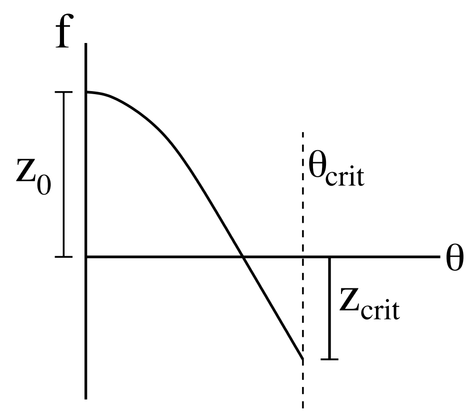
Full-sky Bayesian analysis

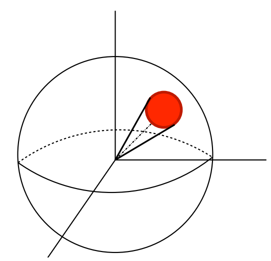
- The likelihoods are given by:

$$\Pr(\boldsymbol{d}|N_{\rm s},\boldsymbol{m}_1,\boldsymbol{m}_2,\ldots\boldsymbol{m}_{N_{\rm s}}) = \frac{1}{(2\pi)^{N_{\rm px}/2}|\mathsf{C}|}\exp\left(-\frac{1}{2}\left[\boldsymbol{d}-\sum_{i=1}^{N_{\rm s}}\boldsymbol{t}(\boldsymbol{m}_i)\right]\mathsf{C}^{-1}\left[\boldsymbol{d}-\sum_{i=1}^{N_{\rm s}}\boldsymbol{t}(\boldsymbol{m}_i)\right]^{\rm T}\right)$$

$$\Pr(\boldsymbol{d}|0) = \frac{1}{(2\pi)^{N_{\rm px}/2}|\mathsf{C}|}\exp\left(-\frac{1}{2}\boldsymbol{d}\;\mathsf{C}^{-1}\;\boldsymbol{d}^{\rm T}\right)$$

- Templates are defined as before:



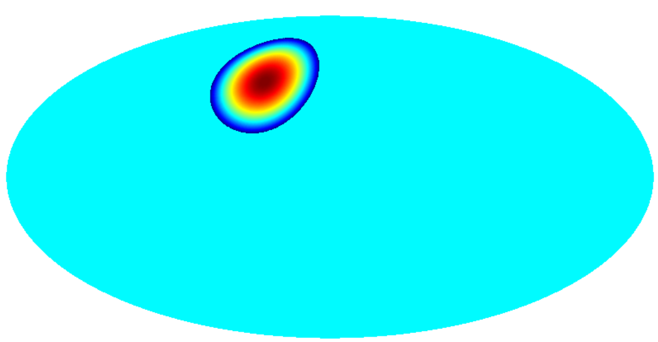


- In the absence of noise and finite instrumental resolution:

$$\mathsf{C}_{ij} = \sum_{\ell}\frac{2\ell+1}{4\pi}C_\ell\;P_\ell(\cos\theta_{ij})$$

Full-sky Bayesian analysis

- It is impossible to evaluate $\mathrm{Pr}(\boldsymbol{d}|N_\mathrm{s})$ (let alone $\mathrm{Pr}(\boldsymbol{d}|\bar{N}_\mathrm{s})$) directly:
  - Inverting C_{ij} at full WMAP resolution is impossible.
  - We must evaluate the likelihood over a $5\ N_\mathrm{s}$-dimensional parameter space to find $\mathrm{Pr}(\boldsymbol{d}|N_\mathrm{s})$; this is impossible for $N_\mathrm{s} \gg 1$.
- But, what if we knew something about the possible location and size of candidate collision events?



Full-sky Bayesian analysis

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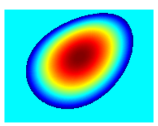
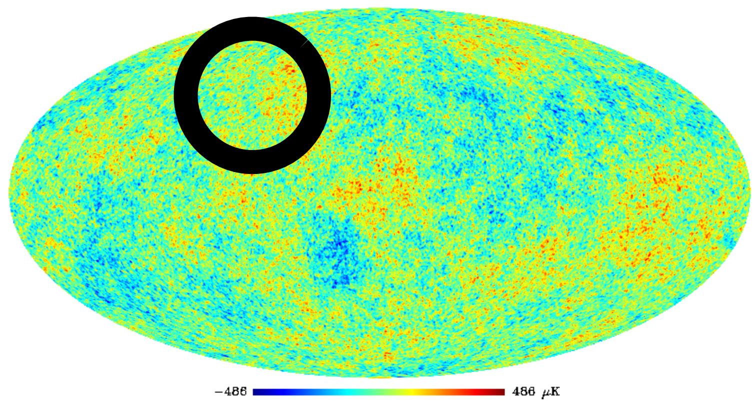
- But, what if we knew something about the possible location and size of candidate collision events?
  - The compact support for each template means we don't have to do a full-sky analysis, mitigating the above problems!



I'll present the strategy.
Hiranya will present its implementation.

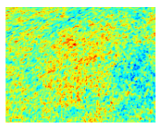
Full-sky Bayesian analysis

- How would this work? Lets say we have one candidate:

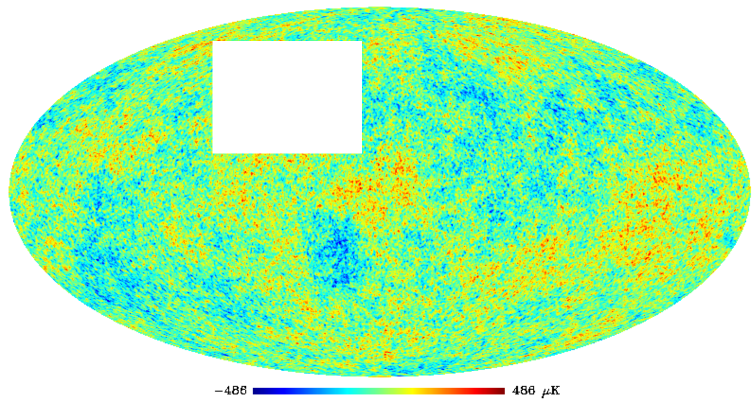


Full-sky Bayesian analysis

- How would this work? Lets say we have one candidate:
    - Split the sky into 2 regions: 1 ``blob” enclosing the candidate, and the rest of the sky.



r=1



r=2

- We need to evaluate:

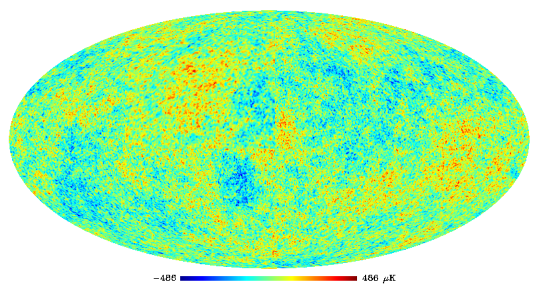
$$\frac{\Pr(\bar{N}_{\rm s}|\boldsymbol{d})}{\Pr(0|\boldsymbol{d})} = \Pr(0|\bar{N}_{\rm s}) + \Pr(1|\bar{N}_{\rm s})\frac{\Pr(\boldsymbol{d}|1)}{\Pr(\boldsymbol{d}|0)} + \Pr(2|\bar{N}_{\rm s})\frac{\Pr(\boldsymbol{d}|2)}{\Pr(\boldsymbol{d}|0)} + \ldots$$

- Start with the $N_{\rm s} = 0$ term: $\Pr(0|\bar{N}_{\rm s}) = e^{-\bar{N}_{\rm s}}$

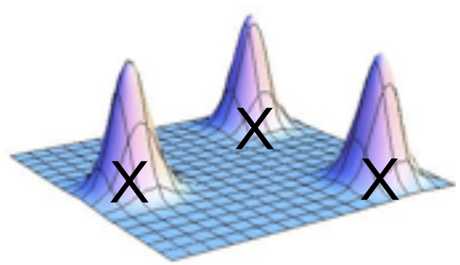
Full-sky Bayesian analysis

- The $N_{\rm s} = 1$ term.

$$\Pr(\boldsymbol{d}|1) = \int_{\rm region\ 1} {\rm d}\boldsymbol{m}\, \Pr(\boldsymbol{m})\Pr(\boldsymbol{d}|1,\boldsymbol{m}) + \int_{\rm region\ 2} {\rm d}\boldsymbol{m}\, \Pr(\boldsymbol{m})\Pr(\boldsymbol{d}|1,\boldsymbol{m})$$



- Assume we know the likelihood would be small outside of region 1:

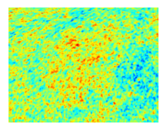


Full-sky Bayesian analysis

- The $N_\mathrm{s} = 1$ term.

likelihood small by assumption

$$\mathrm{Pr}(\boldsymbol{d}|1) = \int_{\mathrm{region}\ 1} \mathrm{d}\boldsymbol{m}\, \mathrm{Pr}(\boldsymbol{m})\mathrm{Pr}(\boldsymbol{d}|1,\boldsymbol{m}) + \int_{\mathrm{region}\ 2} \mathrm{d}\boldsymbol{m}\, \mathrm{Pr}(\boldsymbol{m})\mathrm{Pr}(\boldsymbol{d}|1,\boldsymbol{m})$$



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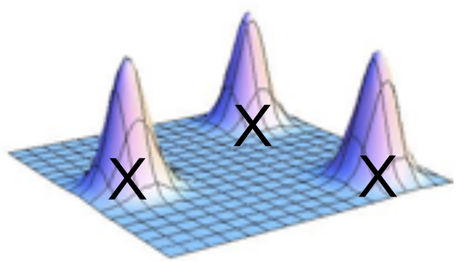
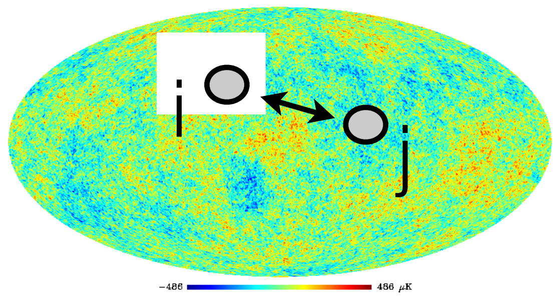


Full-sky Bayesian analysis

- Although we only integrate over region 1, the covariance still involves the whole sky:



$$\mathsf{C}_{ij} = \sum_\ell \frac{2\ell+1}{4\pi} C_\ell \; P_\ell(\cos\theta_{ij})$$

- If the blob fully encloses the template, we can approximate:

$$\Pr(\boldsymbol{d}|1) \propto e^{-[\boldsymbol{d}_1 - \boldsymbol{t}_1(\boldsymbol{m})]\mathsf{C}_1^{-1}[\boldsymbol{d}_1 - \boldsymbol{t}_1(\boldsymbol{m})]^\mathrm{T}/2} \;\times\; e^{-\boldsymbol{d}_2 \mathsf{C}_2^{-1} \boldsymbol{d}_2^\mathrm{T}/2}$$

$$\Pr(\boldsymbol{d}|0) \propto e^{-\boldsymbol{d}_1 \mathsf{C}_1^{-1} \boldsymbol{d}_1^\mathrm{T}/2} \;\times\; e^{-\boldsymbol{d}_2 \mathsf{C}_2^{-1} \boldsymbol{d}_2^\mathrm{T}/2}$$

- Under these approximations, we obtain:

$$\frac{\Pr(\boldsymbol{d}|1)}{\Pr(\boldsymbol{d}|0)} \simeq \frac{\int_{\mathrm{region}\ 1} \mathrm{d}\boldsymbol{m}\, \Pr(\boldsymbol{m})\; e^{-[\boldsymbol{d}_1 - \boldsymbol{t}_1(\boldsymbol{m})]\mathsf{C}_1^{-1}[\boldsymbol{d}_1 - \boldsymbol{t}_1(\boldsymbol{m})]^\mathrm{T}/2}}{e^{-\boldsymbol{d}_1 \mathsf{C}_1^{-1} \boldsymbol{d}_1^\mathrm{T}/2}}$$

Full-sky Bayesian analysis

- The $N_\mathrm{s} = 2$ term:

$$\Pr(\boldsymbol{d}|2) = \int\int d\boldsymbol{m}_1 d\boldsymbol{m}_2 \Pr(\boldsymbol{m}_1)\Pr(\boldsymbol{m}_2) \times \exp\left(-\frac{1}{2}\left[\boldsymbol{d} - \boldsymbol{t}(\boldsymbol{m}_1) - \boldsymbol{t}(\boldsymbol{m}_2)\right] \mathsf{C}^{-1} \left[\boldsymbol{d} - \boldsymbol{t}(\boldsymbol{m}_1) - \boldsymbol{t}(\boldsymbol{m}_2)\ \right]^\mathrm{T}\right)$$

- If we know the likelihood will be small in region 2, the highest likelihood occurs when both templates are in region 1.

$$\begin{aligned}\Pr(\boldsymbol{d}|2) &= \int_{\text{region 1}}\int_{\text{region 1}}() + 2\int_{\text{region 1}}\int_{\text{region 2}}() + \int_{\text{region 2}}\int_{\text{region 2}}() \\ &\simeq \int_{\text{region 1}}\int_{\text{region 1}}()\end{aligned}$$

- But, this is like having one template with twice the parameters!

$$\frac{\Pr(\boldsymbol{d}|1)}{\Pr(\boldsymbol{d}|2)} > \mathrm{Vol}(\boldsymbol{m}) \qquad \text{(for flat priors)}$$

- We can approximate the full sum by the $N_\mathrm{s} = 0$ and $N_\mathrm{s} = 1$ terms.

Full-sky Bayesian analysis

- For one blob:

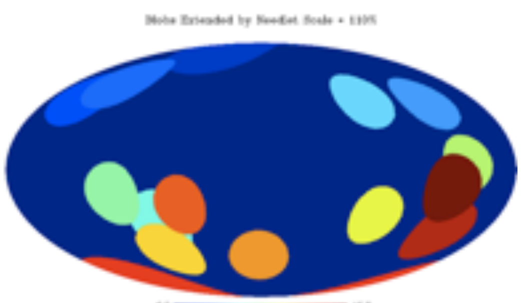
$$\frac{\Pr(\bar{N}_{\rm s}|\boldsymbol{d})}{\Pr(0|\boldsymbol{d})} \simeq e^{-\bar{N}_{\rm s}} + \bar{N}_{\rm s} e^{-\bar{N}_{\rm s}} \rho_1$$

- For two blobs, assuming they are well separated:

$$\frac{\Pr(\bar{N}_{\rm s}|\boldsymbol{d})}{\Pr(0|\boldsymbol{d})} \simeq e^{-\bar{N}_{\rm s}} + \bar{N}_{\rm s} e^{-\bar{N}_{\rm s}} (\rho_1 + \rho_2) + \bar{N}_{\rm s}^2 e^{-\bar{N}_{\rm s}} \rho_1 \rho_2$$

- For N_b blobs:

$$\frac{\Pr(\bar{N}_{\rm s}|\boldsymbol{d})}{\Pr(0|\boldsymbol{d})} \simeq \sum_{N_{\rm s}=0}^{N_b} \frac{\bar{N}_{\rm s}^{N_{\rm s}} e^{-\bar{N}_{\rm s}}}{N_{\rm s}!} \sum_{b_1, b_2, \ldots b_{N_{\rm s}}}^{N_b} \prod_{i=1}^{N_{\rm s}} \rho_{b_i} \prod_{i,j=1}^{N_{\rm s}} (1 - \delta_{b_i b_j})$$



Full-sky Bayesian analysis

- This method is a calculational trick: we use the full theory priors so there are no a posteriori choices.

- The accuracy of this method relies on how well we can identify candidate collisions. However, it is always a lower bound on the evidence ratio!

$$\frac{\Pr(\bar{N}_{\rm s}|\boldsymbol{d})}{\Pr(0|\boldsymbol{d})} \simeq \sum_{N_{\rm s}=0}^{N_b} \frac{\bar{N}_{\rm s}^{N_{\rm s}} e^{-\bar{N}_{\rm s}}}{N_{\rm s}!} \sum_{b_1, b_2, \ldots b_{N_{\rm s}}}^{N_b} \prod_{i=1}^{N_{\rm s}} \rho_{b_i} \prod_{i,j=1}^{N_{\rm s}} \left(1 - \delta_{b_i b_j}\right)$$

- Need to have both $\bar{N}_{\rm s} \simeq N_b$ and sizable ρ_{b_i} to favor the collision model.

- Work in progress trying to quantify the accuracy of our approximations.

Full-sky Bayesian analysis

- Easily generalized to include other data sets, i.e.polarization:

$$\left[\boldsymbol{d} - \sum_{i=1}^{N_\mathrm{s}} \boldsymbol{t}(\boldsymbol{m}_i)\right] \mathsf{C}^{-1} \left[\boldsymbol{d} - \sum_{i=1}^{N_\mathrm{s}} \boldsymbol{t}(\boldsymbol{m}_i)\right]^\mathrm{T} = \left[\boldsymbol{d}^{(T)} - \sum_{i=1}^{N_\mathrm{s}} \boldsymbol{t}^{(T)}(\boldsymbol{m}_i)\right] \mathsf{C}_{TT}^{-1} \left[\boldsymbol{d}^{(T)} - \sum_{i=1}^{N_\mathrm{s}} \boldsymbol{t}^{(T)}(\boldsymbol{m}_i)\right]^\mathrm{T}$$

$$+ \left[\boldsymbol{d}^{(T)} - \sum_{i=1}^{N_\mathrm{s}} \boldsymbol{t}^{(T)}(\boldsymbol{m}_i)\right] \mathsf{C}_{TE}^{-1} \left[\boldsymbol{d}^{(E)} - \sum_{i=1}^{N_\mathrm{s}} \boldsymbol{t}^{(E)}(\boldsymbol{m}_i)\right]^\mathrm{T}$$

$$+ \left[\boldsymbol{d}^{(E)} - \sum_{i=1}^{N_\mathrm{s}} \boldsymbol{t}^{(E)}(\boldsymbol{m}_i)\right] \mathsf{C}_{EE}^{-1} \left[\boldsymbol{d}^{(E)} - \sum_{i=1}^{N_\mathrm{s}} \boldsymbol{t}^{(E)}(\boldsymbol{m}_i)\right]^\mathrm{T}$$

$$\Rightarrow \quad \rho_{b_i} = \rho_{b_i}^{(TT)} \rho_{b_i}^{(TE)} \rho_{b_i}^{(EE)}$$

- Bayesian methods can be used to rank competing theories of spots: i.e. textures.
- Generalizable to study any features in a full-sky data set.

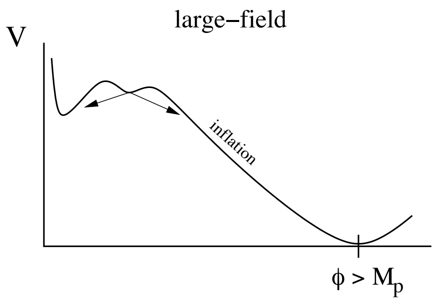
Conclusions

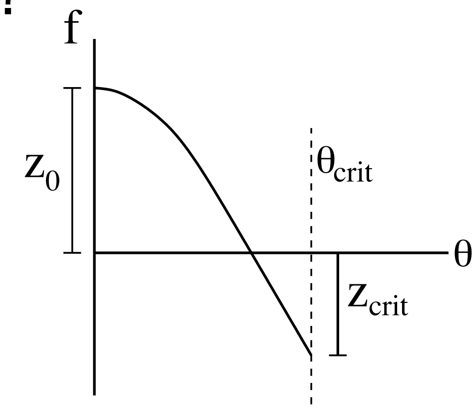
- Bayesian model selection is a consistent framework for determining if we should consider a theory with bubble collisions over one without.
- To do so, it is important to parameterize the theory of bubble collisions, and determine the priors for the parameters.
- A full sky Bayesian analysis can be approximated with a patch-wise analysis if we know something about the likelihood surface.

$$\frac{\Pr(\bar{N}_{\rm s}|\boldsymbol{d})}{\Pr(0|\boldsymbol{d})} \simeq \sum_{N_{\rm s}=0}^{N_b} \frac{\bar{N}_{\rm s}^{N_{\rm s}} e^{-\bar{N}_{\rm s}}}{N_{\rm s}!} \sum_{b_1, b_2, \ldots b_{N_{\rm s}}}^{N_b} \prod_{i=1}^{N_{\rm s}} \rho_{b_i} \prod_{i,j=1}^{N_{\rm s}} \left(1 - \delta_{b_i b_j}\right)$$

Open questions

- What is the mapping from a potential to a phenomenological model for the effects of collisions on the CMB?





$$+ \, \mathrm{Pr}(\boldsymbol{m})$$

- Are there any correlations between LCDM parameters and collisions?

$$\text{bubble collisions} \longrightarrow \text{detectable } r$$
$$\text{bubble collisions} \longrightarrow \text{undetectable } \Omega_k$$

$$\mathrm{Pr}(\boldsymbol{m}, \Lambda\mathrm{CDM}) \neq \mathrm{Pr}(\boldsymbol{m})\mathrm{Pr}(\Lambda\mathrm{CDM})$$

- What happens in a vast landscape?
  - Bayesian methods inherit the measure problem.
  - Question becomes academic, since the data will probably never be good enough to distinguish models in detail.