Taus and theory

Patrick Fox

Fermilab

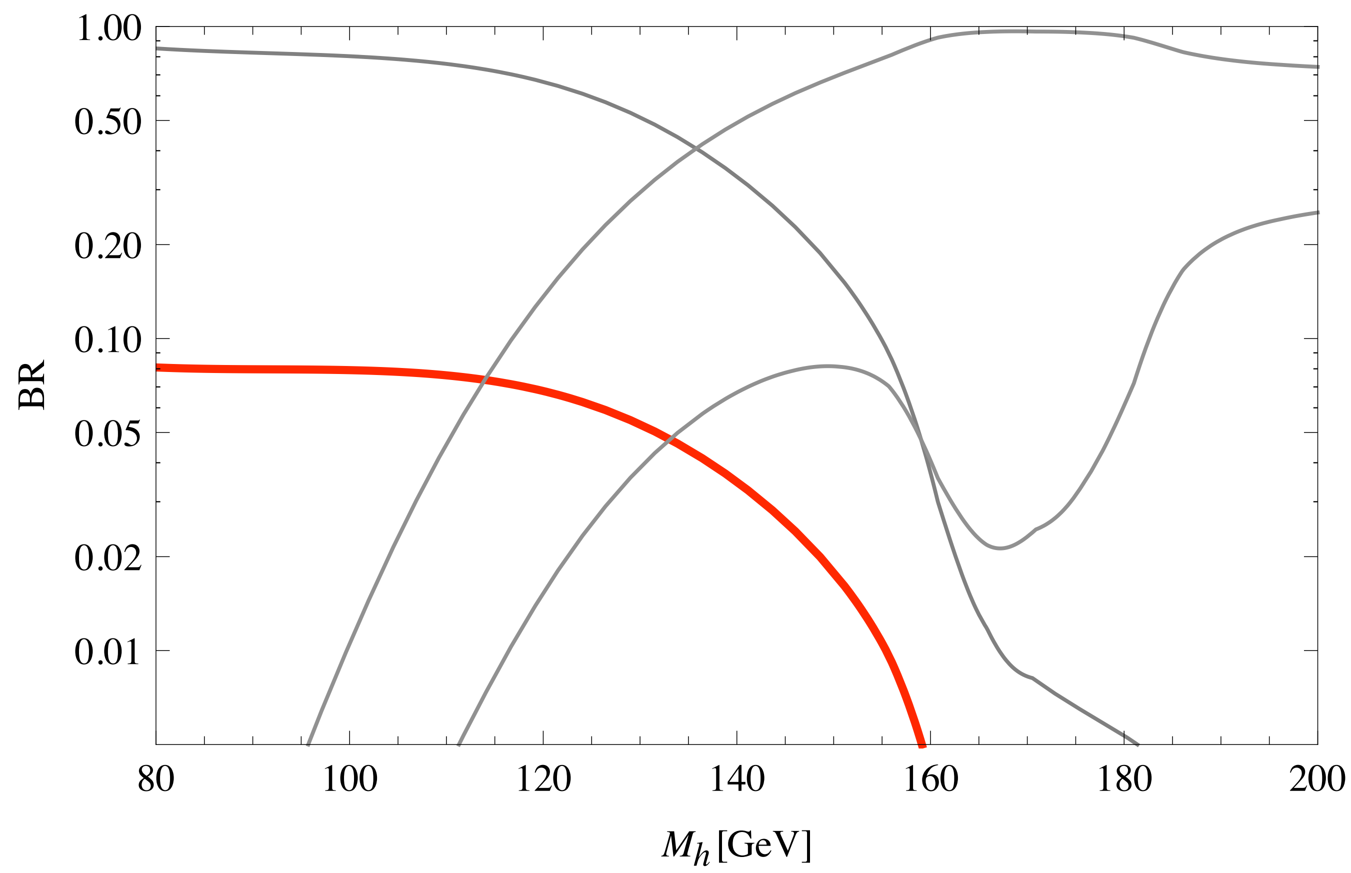
with Bogdan Dobrescu
(arXiv:1001.3147)

with Bogdan Dobrescu and
Adam Martin
(arXiv:1005.4238)

Why study taus?

Tests of SM

- 3rd generation lepton, largest lepton Yukawa
- Couples more strongly to electroweak symmetry breaking
- Sizable branching ratio of Higgs
- VBF with H➔tau tau
- Lepton universality



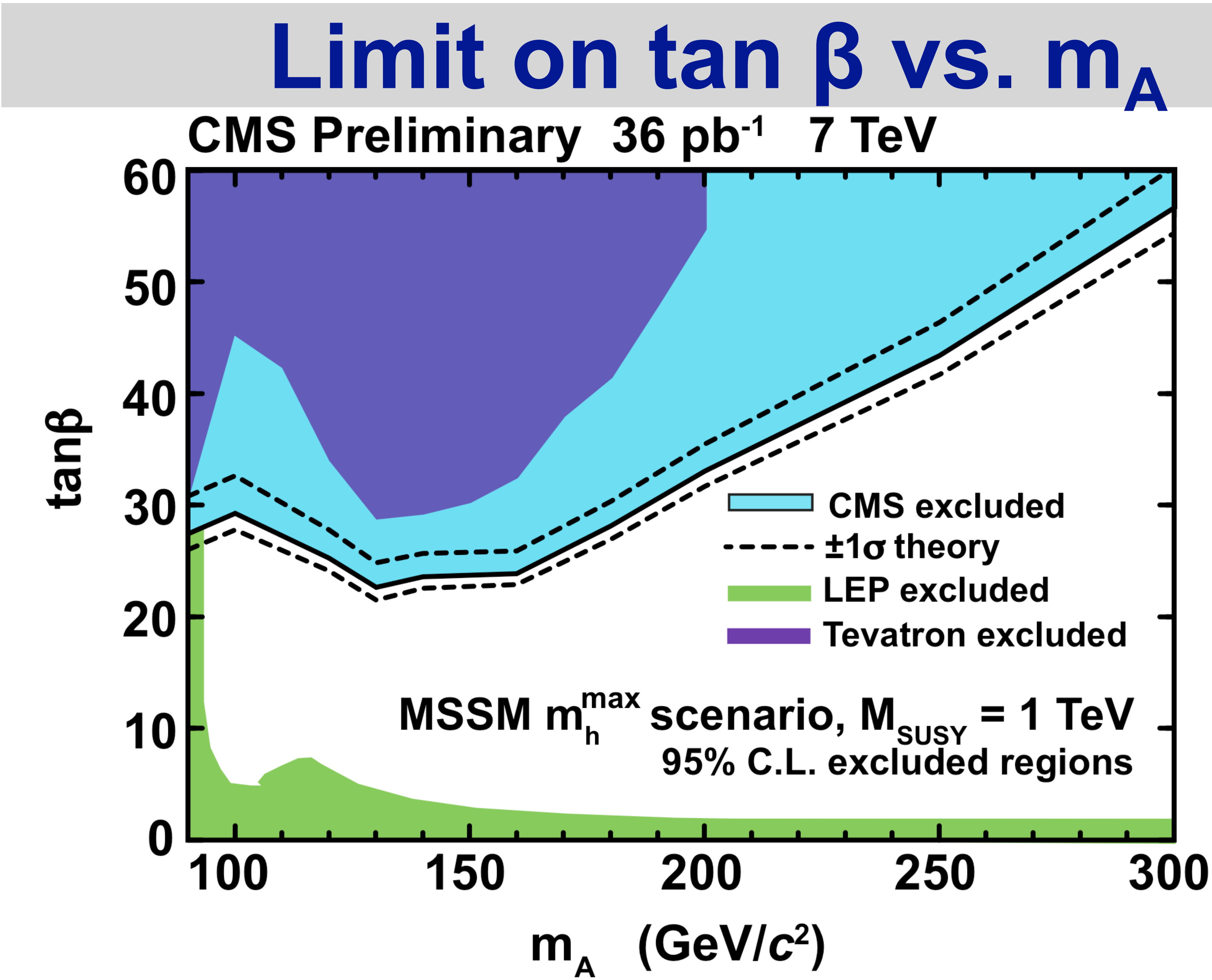
Why study taus?

Search for BSM physics

- In 2HDM, at large tan beta, can have enhanced couplings to down quark and leptons
- $H^{\pm} \rightarrow \tau^{\pm}\nu$
- 3rd generation leptoquarks $LQ \rightarrow \tau b$
- Insight into flavour puzzle
- Stau NLSP in some regions of SUSY, tau rich events

Why study taus?

Because we can!



Christian Veelken,
@ Moriond, 2011

A model where taus are <u>even more</u> important

A new phase of an old model?

- MSSM review
- The MSSM at and near $\tan\beta = \infty$
- Loop generated masses
- Collider phenomenology
- A taste of flavour
- Conclusions

Fermion masses in the SM

- SM fermions are chiral
- Higgs couplings responsible for all fermion masses

$$y_t t_R t_L h + y_b b_R b_L h^* + \ldots$$

- Yukawa's have large hierarchies, and strange patterns
- At the weak scale

$$y_f = \frac{m_f}{v}$$

$$y_t \sim 1 \qquad\qquad y_b \sim \frac{1}{60}$$

MSSM

- Anomalies require two Higgs (Higgsino) doublets

$$H_u : \left(1, 2, \frac{1}{2}\right) \qquad\qquad H_d : \left(1, 2, -\frac{1}{2}\right)$$

SM fermions and MSSM sfermions:

$$Q : \left(3, 2, \frac{1}{6}\right) \quad U^c : \left(\bar{3}, 1, -\frac{2}{3}\right) \quad D^c : \left(\bar{3}, 1, \frac{1}{3}\right) \quad L : \left(1, 2, -\frac{1}{2}\right) \quad E^c : (1, 1, 1)$$

- Holomorphy forces a Type-II 2HDM i.e. one Higgs (H_u) couples only to up-type quarks and one (H_d) only couples to down-type quarks and leptons

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$$\boxed{W = y_u\, \hat{u}^c \hat{H}_u \hat{Q} - y_d\, \hat{d}^c \hat{H}_d \hat{Q} - y_\ell\, \hat{e}^c \hat{H}_d \hat{L} + \mu\, \hat{H}_u \hat{H}_d}$$

2HDM

At tree level can define $\tan\beta \equiv \frac{v_u}{v_d}$

The MSSM Yukawa couplings

$$y_u^{MSSM} = \frac{y_u^{SM}}{\sin\beta} \qquad y_d^{MSSM} = \frac{y_d^{SM}}{\cos\beta}$$

<u>Ratios of Yukawas (in each sector) in MSSM same as in SM</u>

Usually perturbativity [$y_b \leq \mathcal{O}(1)$] places a constraint on yb:

$$\tan\beta \lesssim 50 - 60$$

[except Hamzaoui and Pospelov]

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I wish to consider the case of $\tan\beta \approx \infty$

2HDM in MSSM

Assign R-charges

$$R[\hat{H}_d, \hat{Q}, \hat{u}^c, \hat{e}^c] = 0 \text{ and } R[\hat{H}_u, \hat{d}^c, \hat{L}] = 2$$

Tree-level Higgs potential is (no B_μ term)

$$\underbrace{\left(|\mu|^2 + m_{H_u}^2\right)}_{<0} |H_u|^2 + \underbrace{\left(|\mu|^2 + m_{H_d}^2\right)}_{>0} |H_d|^2 + \frac{g'^2}{8}\left(|H_u|^2 - |H_d|^2\right)^2 + \frac{g^2}{2}\left|H_u^\dagger T^a H_u + H_d^\dagger T^a H_d\right|^2$$

$$M_{h^0}^2 = -2\left(|\mu|^2 + m_{H_u}^2\right) = M_Z^2$$

$$M_{H^0}^2 = M_{A^0}^2 = 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2$$

$$M_{H^\pm}^2 = M_{A^0}^2 + M_W^2$$

Only H_u gets a vev: $\tan\beta = \infty$

Conclusions

My model predicts that all fermions other than up, charm and top are massless

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Furthermore.......

Down-type fermion masses

$$W = y_u\, \hat{u}^c \hat{H}_u \hat{Q} - y_d\, \hat{d}^c \hat{H}_d \hat{Q} - y_\ell\, \hat{e}^c \hat{H}_d \hat{L} + \mu\, \hat{H}_u \hat{H}_d$$

All chiral symmetries explicitly broken by superpotential

$$U(3)^5 \to U(1)_B \times U(1)_L$$

Once SUSY is broken can generate new "wrong-type" Yukawas

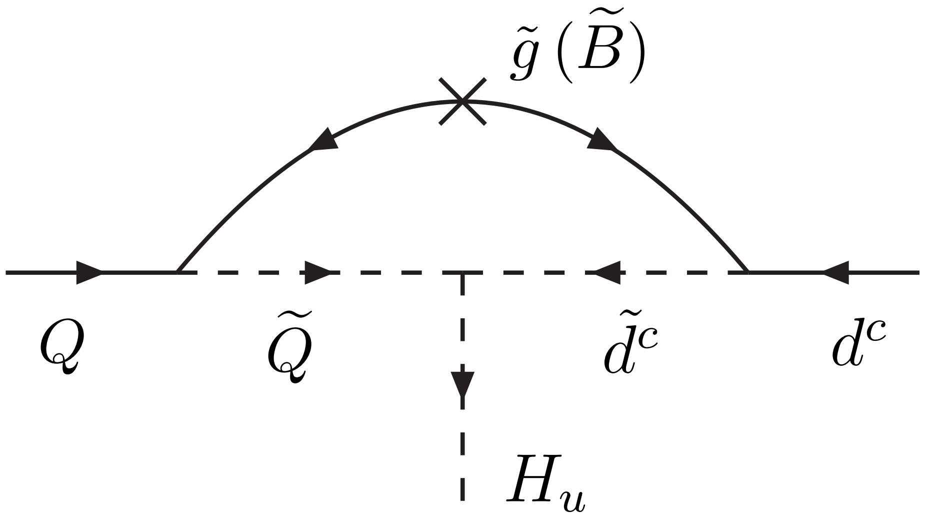
[Hall, Rattazzi, Sarid; Haber and Mason;.....]

$$-y'_d\, d^c H_u^\dagger Q - y'_\ell\, e^c H_u^\dagger L + \mathrm{H.c.}$$

Loop generation of masses (a short domino)

[Dobrescu and PJF; Graham and Rajendran]

Uplifted Higgs couplings

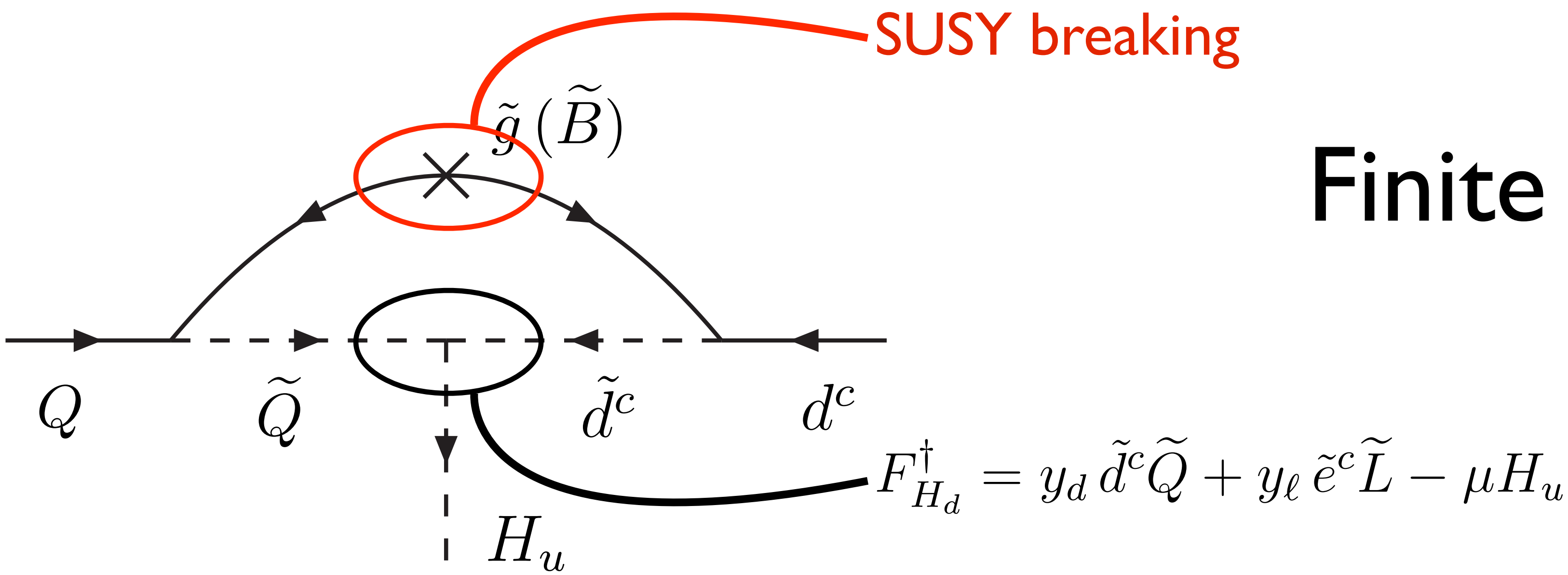


Finite loop

$$y'_d = -y_d \frac{2\alpha_s}{3\pi} \frac{|\mu|}{M_{\tilde{d}}} F\left(\frac{M_{\tilde{g}}}{M_{\tilde{Q}}}, \frac{M_{\tilde{d}}}{M_{\tilde{Q}}} \right)$$

$$F(x,y) = \frac{2xy}{x^2 - y^2} \left(\frac{y^2 \ln y}{1 - y^2} - \frac{x^2 \ln x}{1 - x^2} \right) \qquad 0 < F(x,y) < 1$$

Uplifted Higgs couplings



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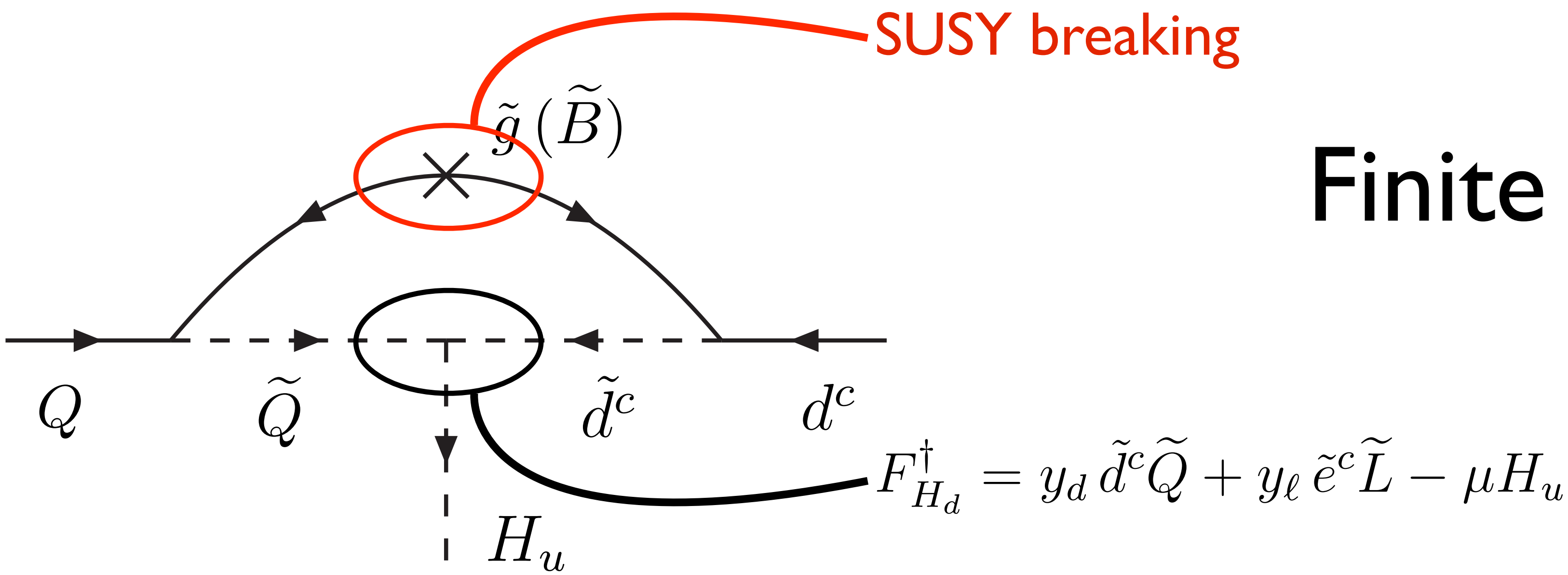

Finite loop

Lagrangian parameter

Effective Yukawa

$$y'_d = -y_d \frac{2\alpha_s}{3\pi} \frac{|\mu|}{M_{\tilde{d}}} F\left(\frac{M_{\tilde{g}}}{M_{\tilde{Q}}}, \frac{M_{\tilde{d}}}{M_{\tilde{Q}}}\right)$$

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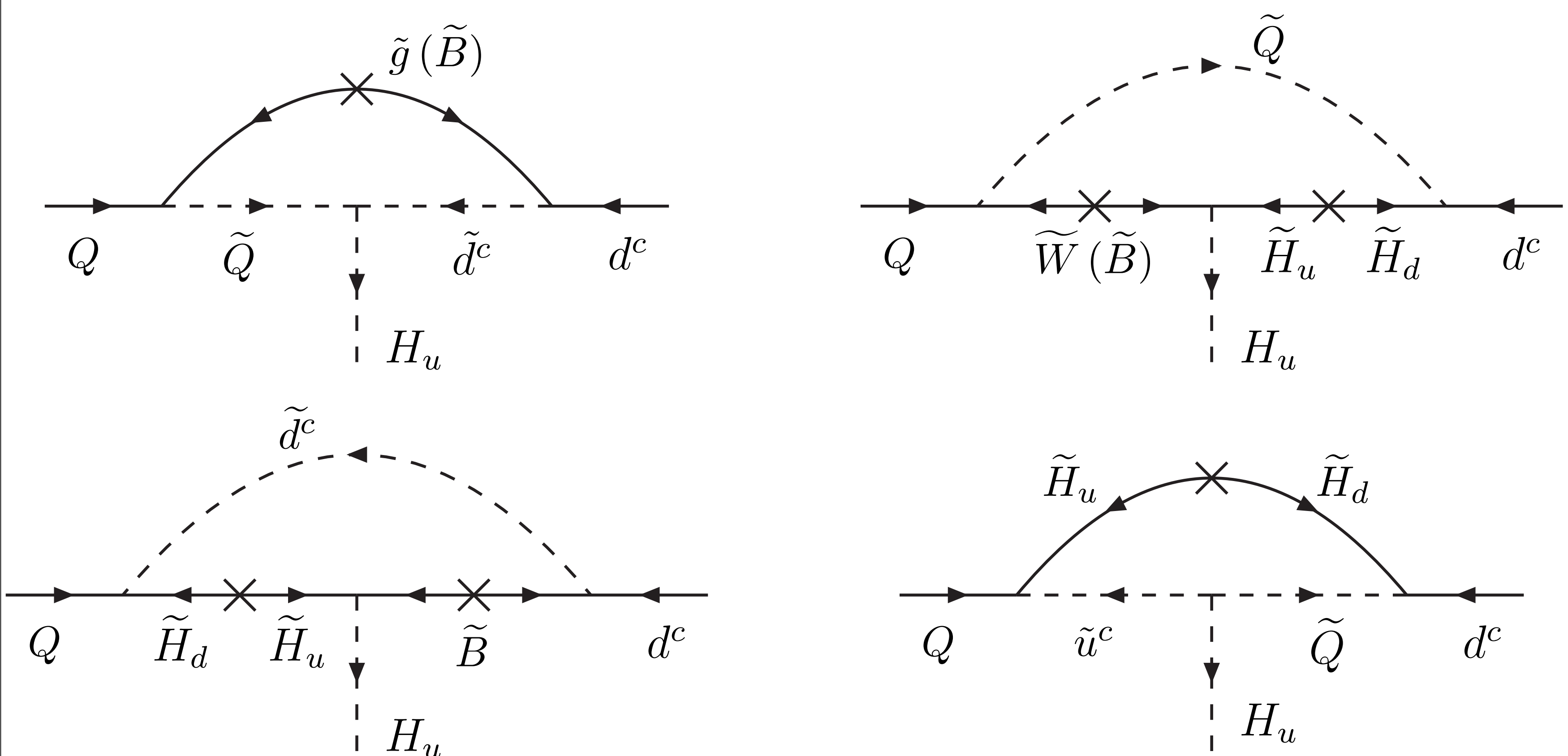
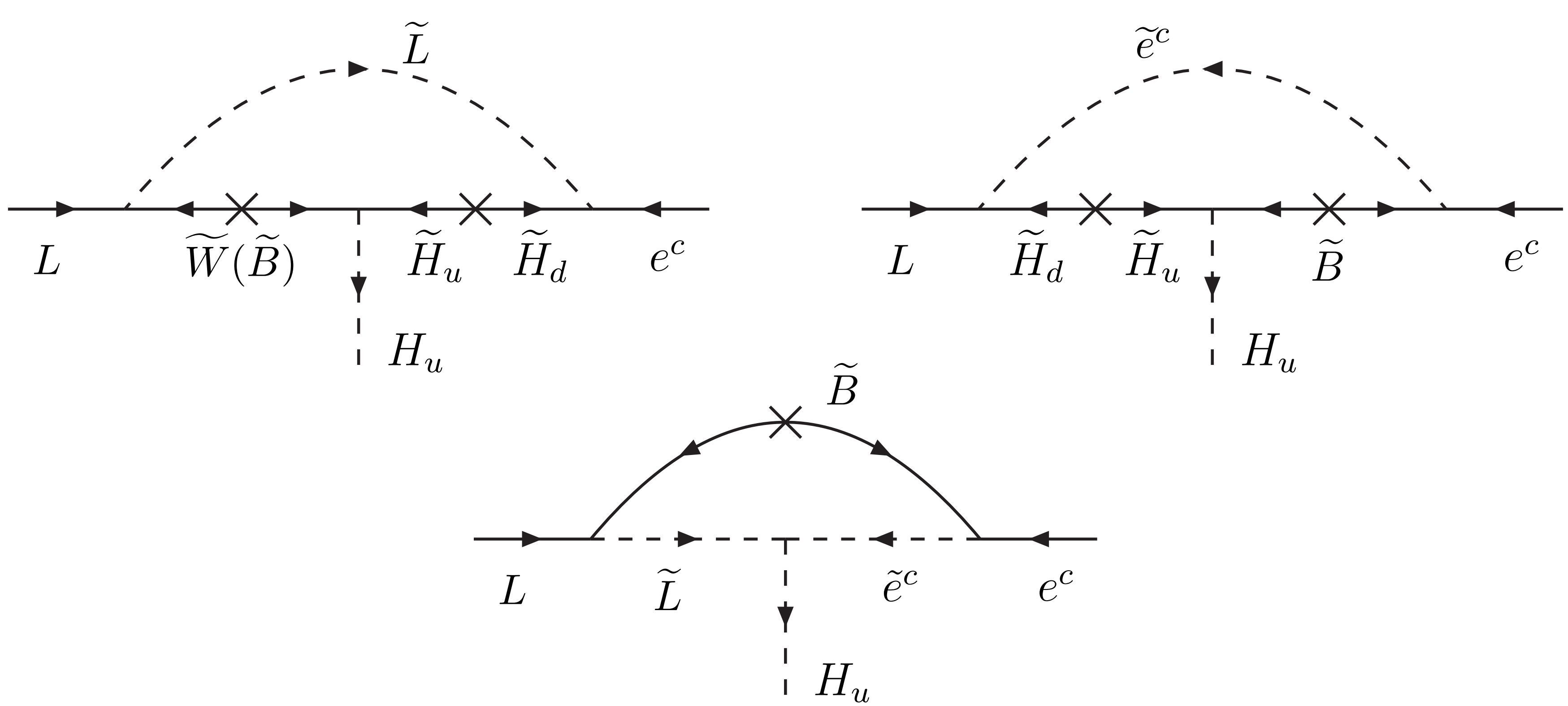


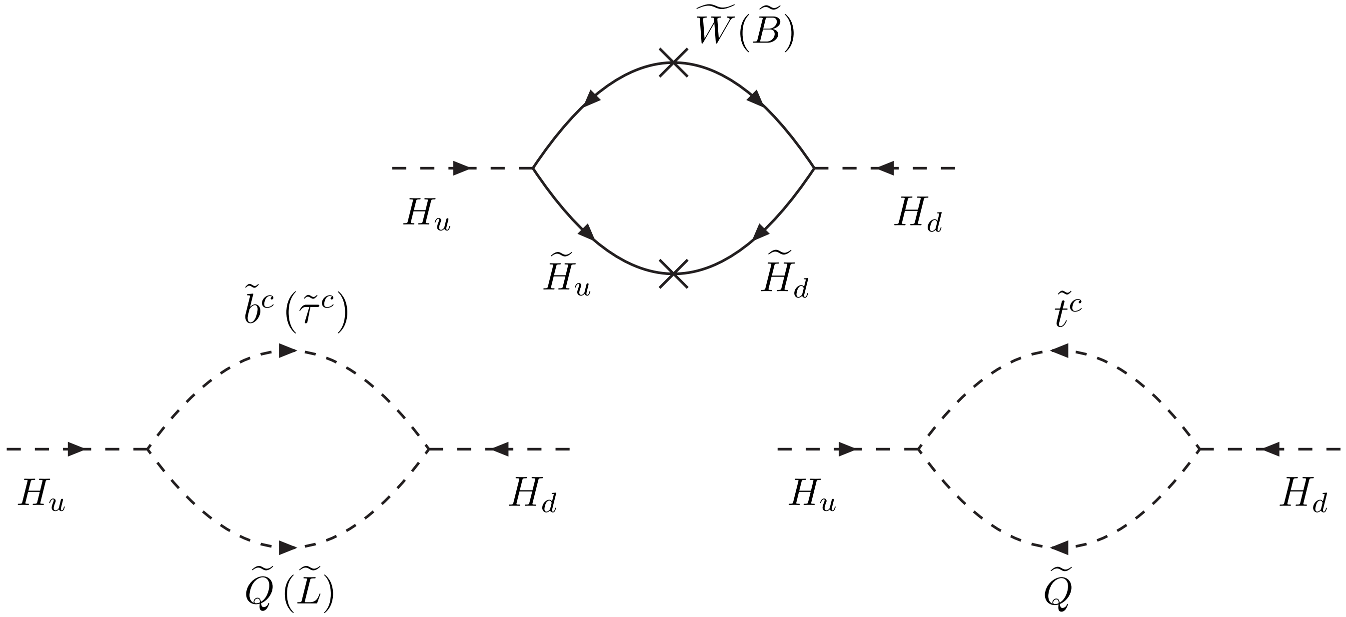
Down-type quarks



Leptons

Loop corrections to $\tan\beta$

Once SUSY is broken B_μ generated at one loop



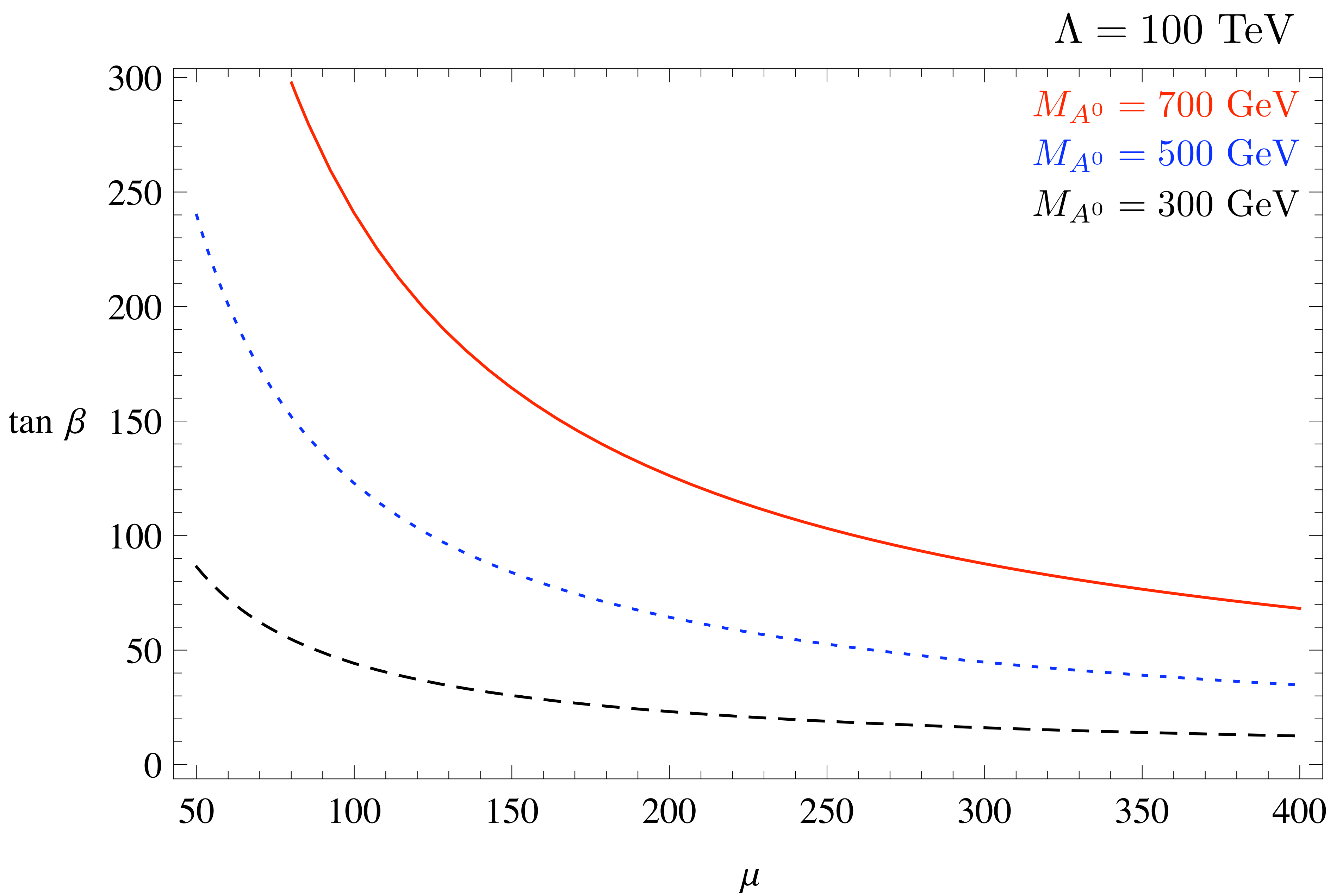
Log divergent

$$b = -\frac{\alpha\mu}{2\pi}\left[\frac{3}{s_W^2}M_{\tilde{W}}G(|\mu|,M_{\tilde{W}})e^{-2i\theta_W} + \frac{1}{c_W^2}M_{\tilde{B}}G(|\mu|,M_{\tilde{B}})e^{-2i\theta_B}\right]$$
$$-\frac{\mu}{8\pi^2}\left[3y_b^* A_b G(M_{\tilde{Q}},M_{\tilde{b}}) + y_\tau^* A_\tau G(M_{\tilde{L}},M_{\tilde{\tau}}) + 3y_t^* A_t G(M_{\tilde{Q}},M_{\tilde{t}})\right]$$

$$G(m_1,m_2) = \frac{1}{m_2^2 - m_1^2}\left(m_2^2\ln\frac{\Lambda}{m_2} - m_1^2\ln\frac{\Lambda}{m_1}\right)$$

Loop corrections to $\tan\beta$

$$\frac{v_u}{v_d} \equiv \tan\beta \approx \frac{1}{|b|} M_{A^0}^2 \left[1 + O(1/\tan^2\beta)\right] \gg 1$$

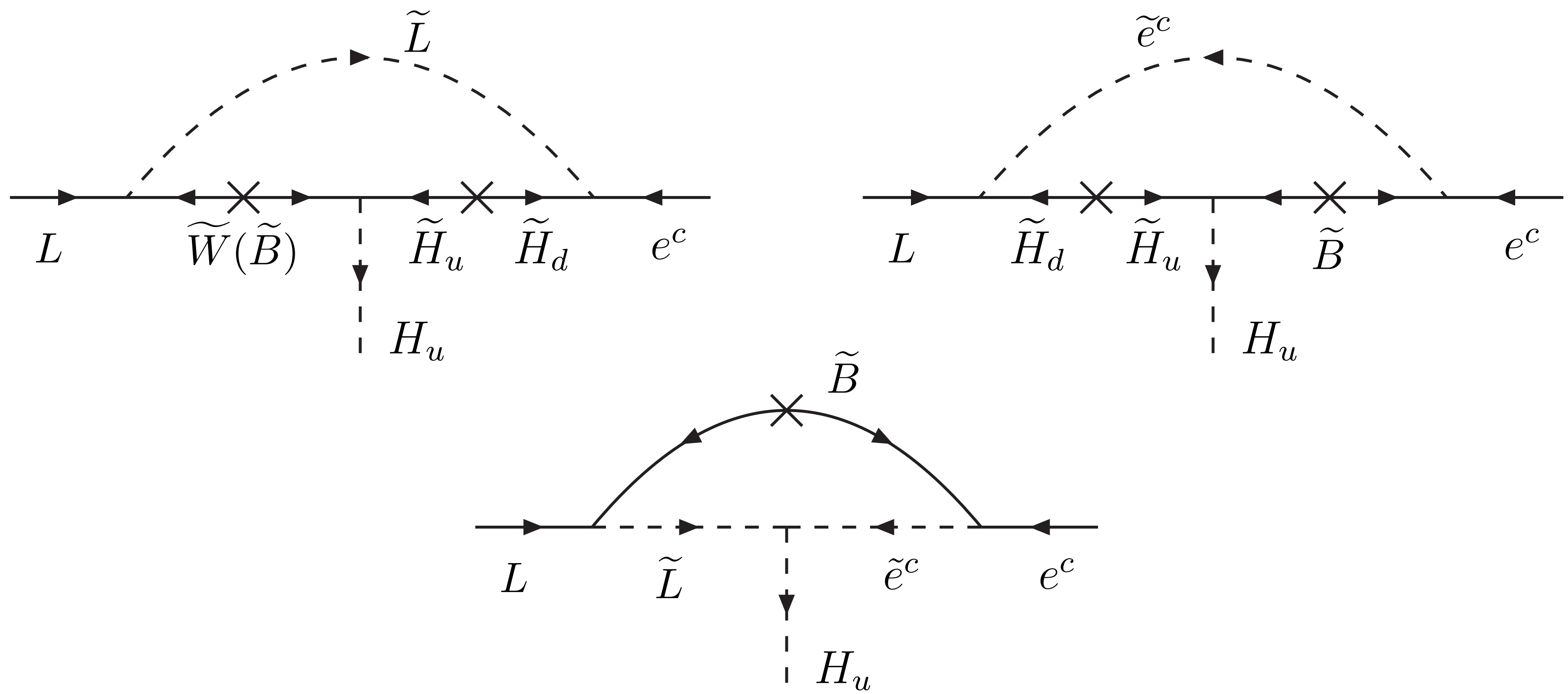


$M_{\tilde{B}} = 100$ GeV,

$$\left(\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2} \right)$$

(Ignoring A-terms)

Tau mass

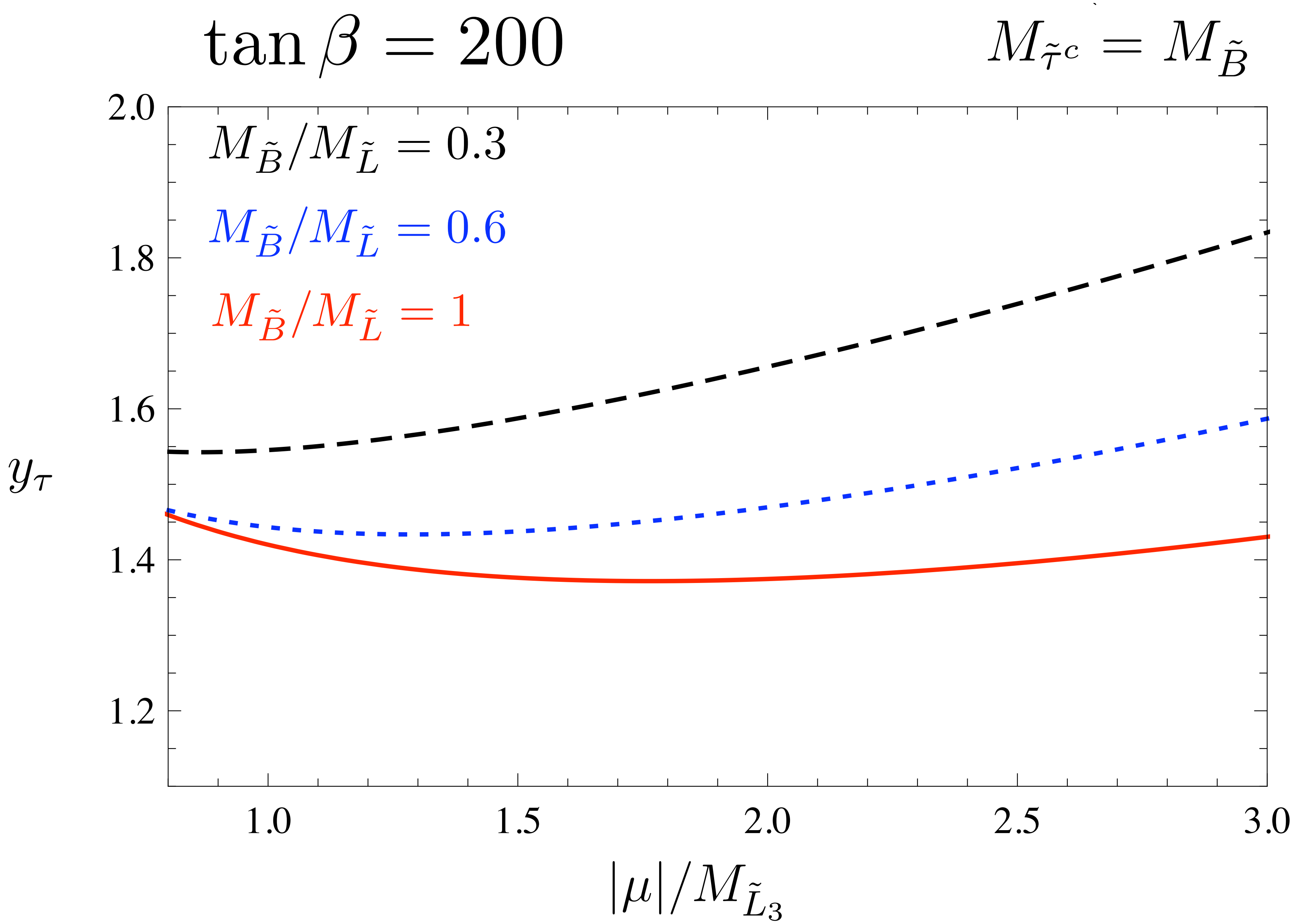


$$y'_\ell = \frac{y_\ell\,\alpha}{8\pi} e^{i(\theta_W - \theta_\mu)} \left\{ \frac{3}{s_W^2} F\left(\frac{M_{\tilde{W}}}{M_{\tilde{L}}}, \frac{|\mu|}{M_{\tilde{L}}}\right) + \frac{e^{i(\theta_B - \theta_W)}}{c_W^2} \left[-F\left(\frac{M_{\tilde{B}}}{M_{\tilde{L}}}, \frac{|\mu|}{M_{\tilde{L}}}\right) \right.\right.$$
$$\left.\left. + 2F\left(\frac{M_{\tilde{B}}}{M_{\tilde{e}}}, \frac{|\mu|}{M_{\tilde{e}}}\right) + \frac{2|\mu|}{M_{\tilde{e}}} F\left(\frac{M_{\tilde{B}}}{M_{\tilde{L}}}, \frac{M_{\tilde{e}}}{M_{\tilde{L}}}\right) \right] \right\}$$

Tau mass

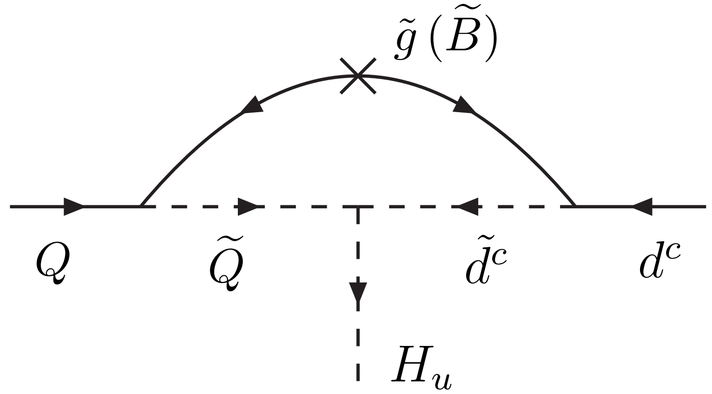
With $\tan\beta \neq \infty$

$$m_\ell = y_\ell v_d + y'_\ell v_u$$



$$\left(\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2} \right)$$

Bottom quark mass



$$(y'_d)_F = -\frac{y_d}{3\pi} e^{i(\theta_g - \theta_\mu)} \frac{2|\mu|}{M_{\tilde{d}}} \left[\alpha_s F\left(\frac{M_{\tilde{g}}}{M_{\tilde{Q}}}, \frac{M_{\tilde{d}}}{M_{\tilde{Q}}} \right) + \frac{\alpha e^{i(\theta_B - \theta_g)}}{24 c_W^2} F\left(\frac{M_{\tilde{B}}}{M_{\tilde{Q}}}, \frac{M_{\tilde{d}}}{M_{\tilde{Q}}} \right) \right]$$

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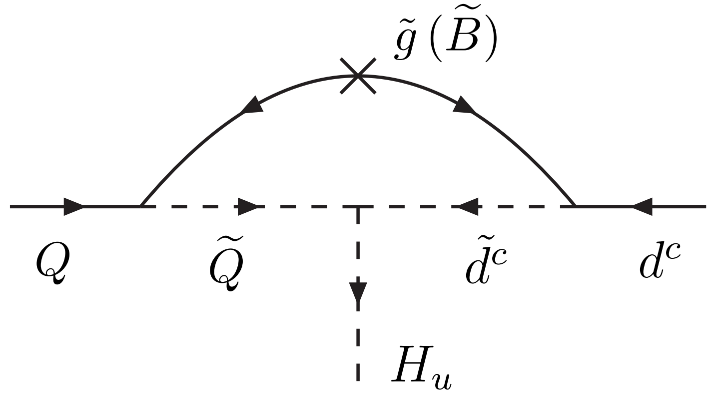

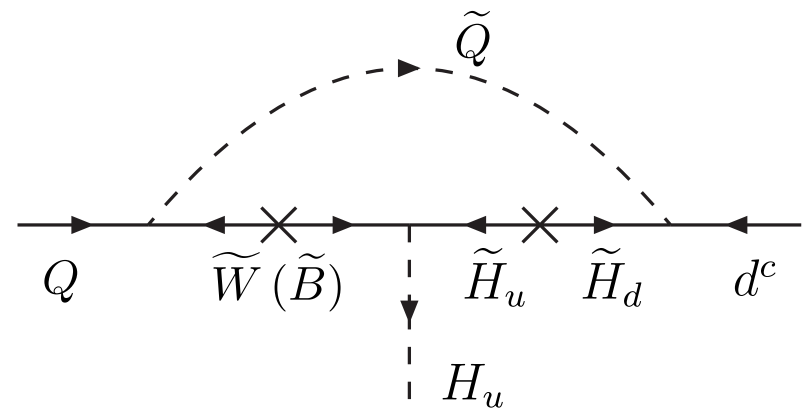

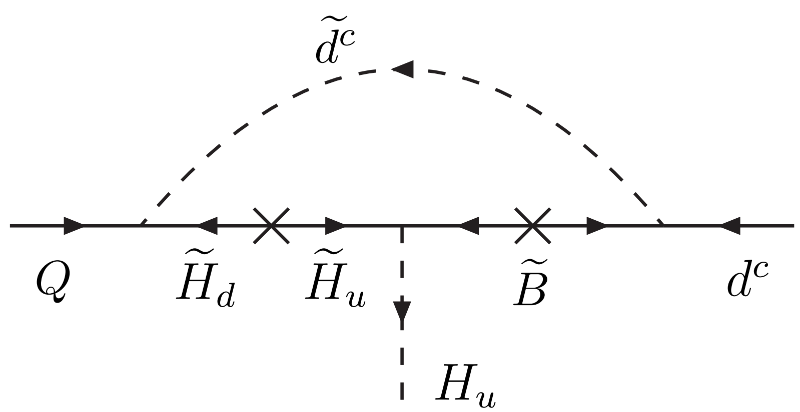

$$(y_d')_F = -\frac{y_d}{3\pi} e^{i(\theta_g-\theta_\mu)} \frac{2|\mu|}{M_{\tilde{d}}} \left[\alpha_s F\left(\frac{M_{\tilde{g}}}{M_{\tilde{Q}}}, \frac{M_{\tilde{d}}}{M_{\tilde{Q}}}\right) + \frac{\alpha e^{i(\theta_B-\theta_g)}}{24 c_W^2} F\left(\frac{M_{\tilde{B}}}{M_{\tilde{Q}}}, \frac{M_{\tilde{d}}}{M_{\tilde{Q}}}\right)\right]$$

$$(y_d')_{\tilde{H}} = \frac{y_d \alpha}{8\pi} e^{i(\theta_W-\theta_\mu)} \left\{ \frac{3}{s_W^2} F\left(\frac{M_{\tilde{W}}}{M_{\tilde{Q}}}, \frac{|\mu|}{M_{\tilde{Q}}}\right) + \frac{e^{i(\theta_B-\theta_W)}}{3c_W^2}\left[F\left(\frac{M_{\tilde{B}}}{M_{\tilde{Q}}}, \frac{|\mu|}{M_{\tilde{Q}}}\right) + 2F\left(\frac{M_{\tilde{B}}}{M_{\tilde{d}}}, \frac{|\mu|}{M_{\tilde{d}}}\right)\right]\right\}$$

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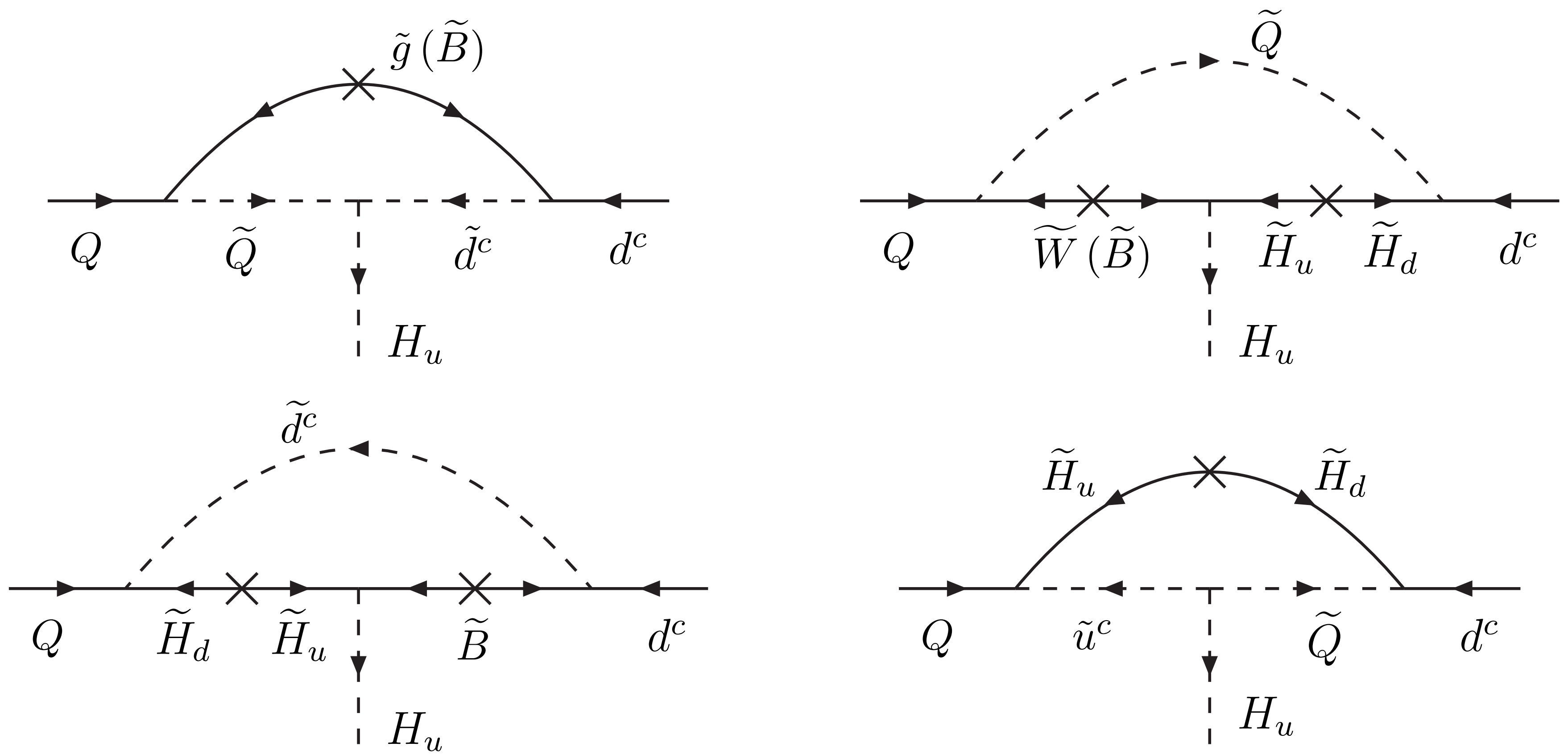

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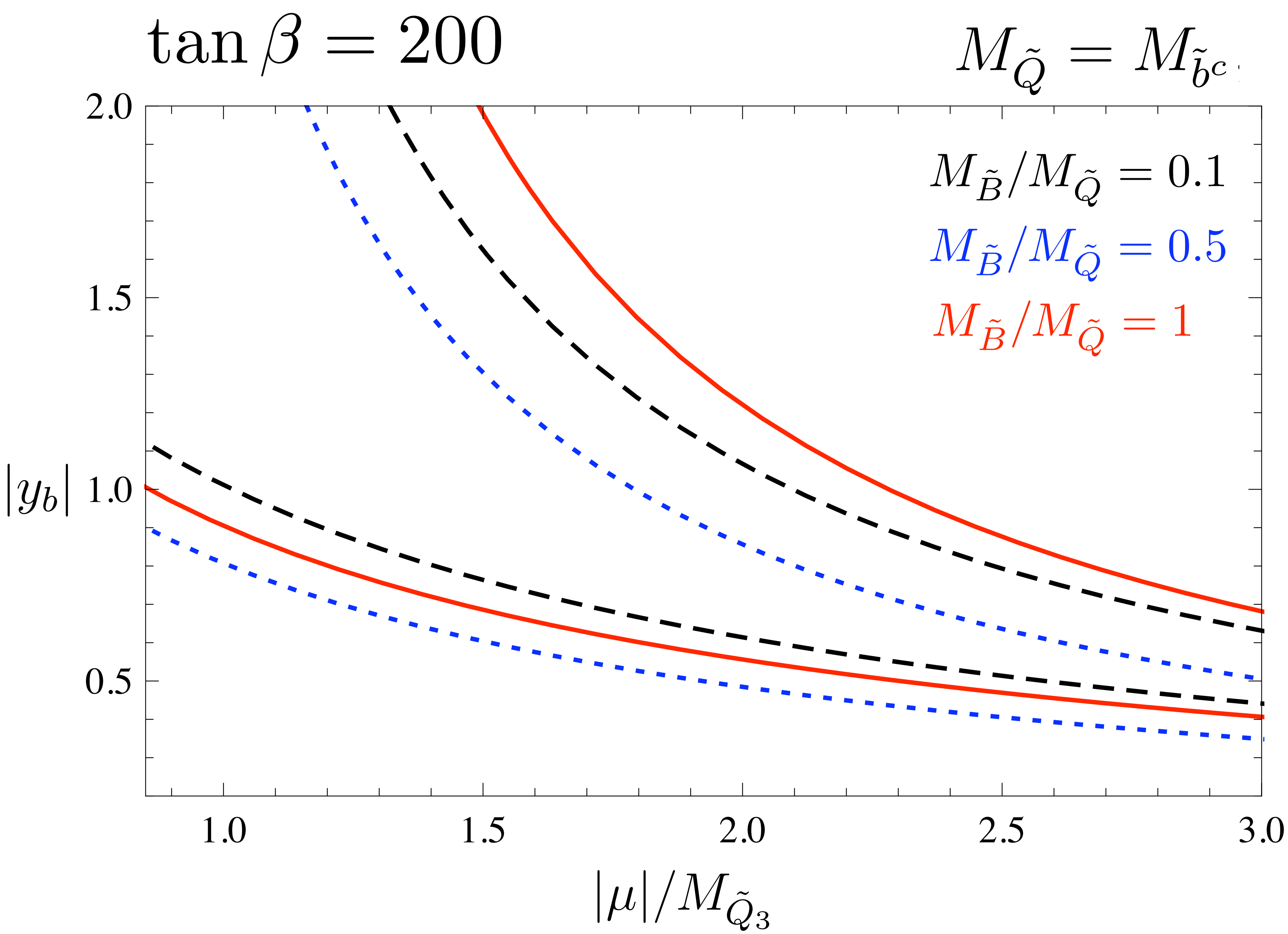
$$(y_d')_A = \frac{y_u y_d}{16\pi^2} e^{-i\theta_\mu} \frac{A_u^*}{M_{\tilde{u}}} F\left(\frac{M_{\tilde{u}}}{M_{\tilde{Q}}}, \frac{|\mu|}{M_{\tilde{Q}}}\right)$$

Bottom quark mass



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$$\theta_\mu = \theta_B = \theta_W$$

(Ignoring A-terms)

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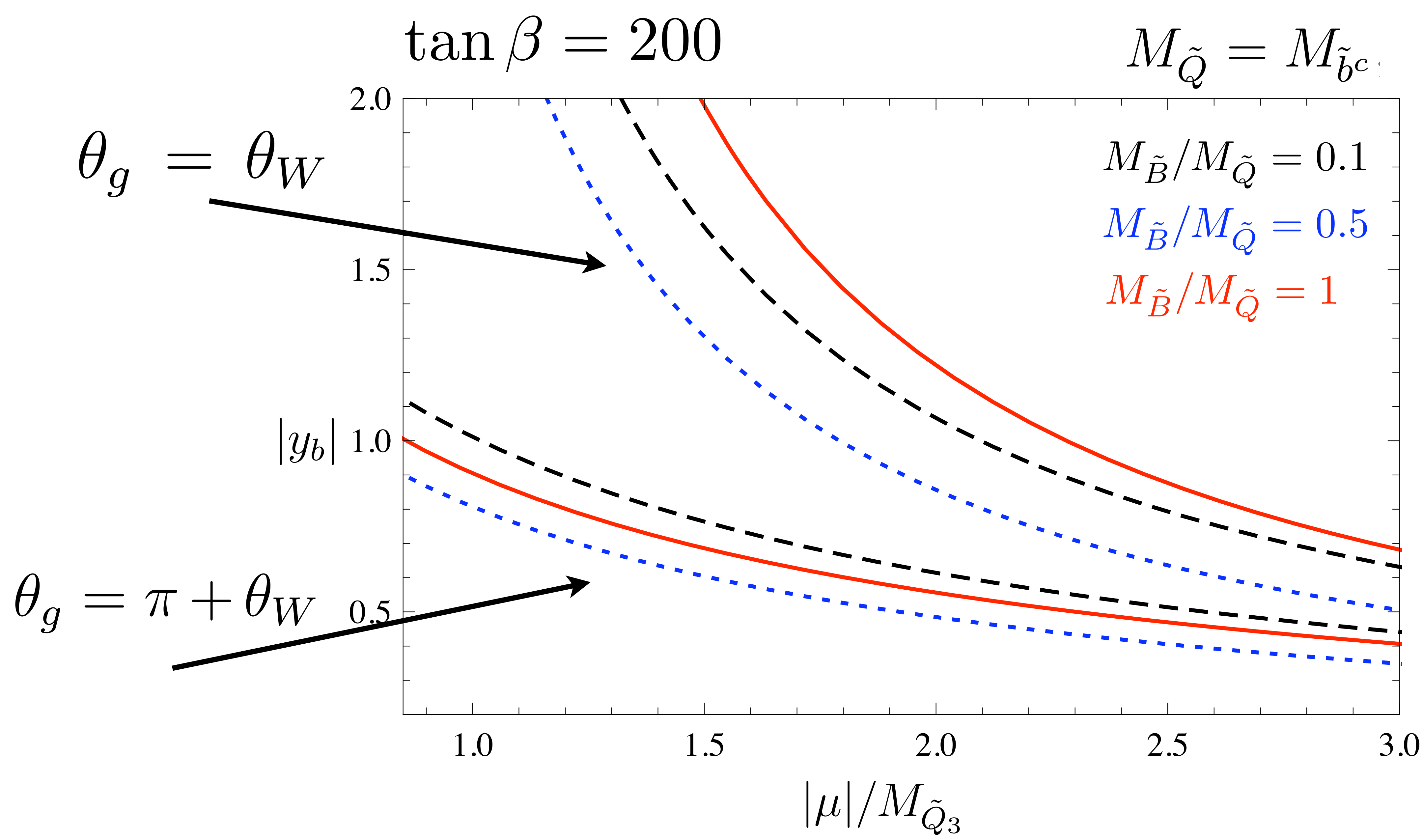

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Uplifted Higgses

$$\tan\alpha = -\left(\frac{M_{A^0}^2 + M_Z^2}{M_{A^0}^2 - M_Z^2}\right)\frac{1}{\tan\beta}\left[1 + O(1/\tan^2\beta)\right]$$

Higgs (h^0) that couples to WW mainly in H_u

$$M_{h^0}^2 \simeq M_Z^2\left(1 - \frac{4M_A^2}{\left(M_{A^0}^2 - M_Z^2\right)\tan^2\beta}\right) + \Delta(M_{h^0}^2)$$

Heavy Higgses (A^0, H^0,$H^\pm$) in H_d

$$M_{H^0}^2 \simeq M_{A^0}^2\left(1 + \frac{4M_Z^2}{\left(M_{A^0}^2 - M_Z^2\right)\tan^2\beta}\right)$$

$$M_{H^\pm}^2 \; = \; M_{A^0}^2 + M_W^2$$

Uplifted Higgses

- Couplings of heavy Higgses larger than in MSSM
- Width of heavy Higgses go up
- Branching ratios and production altered

$$y^b_{H^0} = -\frac{1}{\sqrt{2}}\left(y_b \cos\alpha + y'_b \sin\alpha\right) \approx -\frac{y_b}{\sqrt{2}} \quad ,$$

$$y^b_{A^0} = y^b_{H^-} = \frac{1}{\sqrt{2}}\left(y_b \sin\beta - y'_b \cos\beta\right) \approx \frac{y_b}{\sqrt{2}}$$

$$y^b_{h^0} = \frac{1}{\sqrt{2}}\left(y_b \sin\alpha - y'_b \cos\alpha\right) \approx -\frac{1}{\sqrt{2}}\left[\frac{y_b}{\tan\beta}\left(\frac{M^2_{A^0}+M^2_Z}{M^2_{A^0}-M^2_Z}\right) + y'_b\right]\left[1 + O(1/\tan^2\beta)\right]$$

$$B(H^0, A^0 \to \tau^+\tau^-) \approx \frac{y^2_\tau}{y^2_\tau + 3y^2_b} \approx 30\% - 80\%$$

cf. usual MSSM/2HDM ~10%

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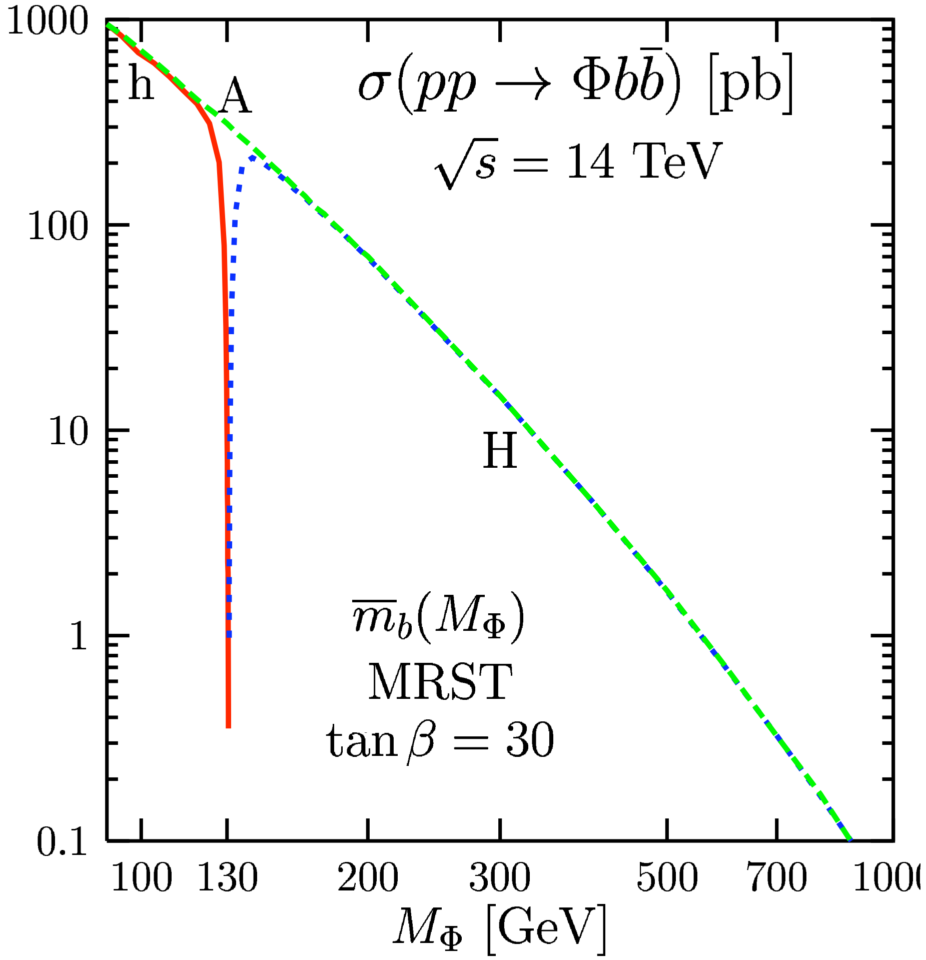
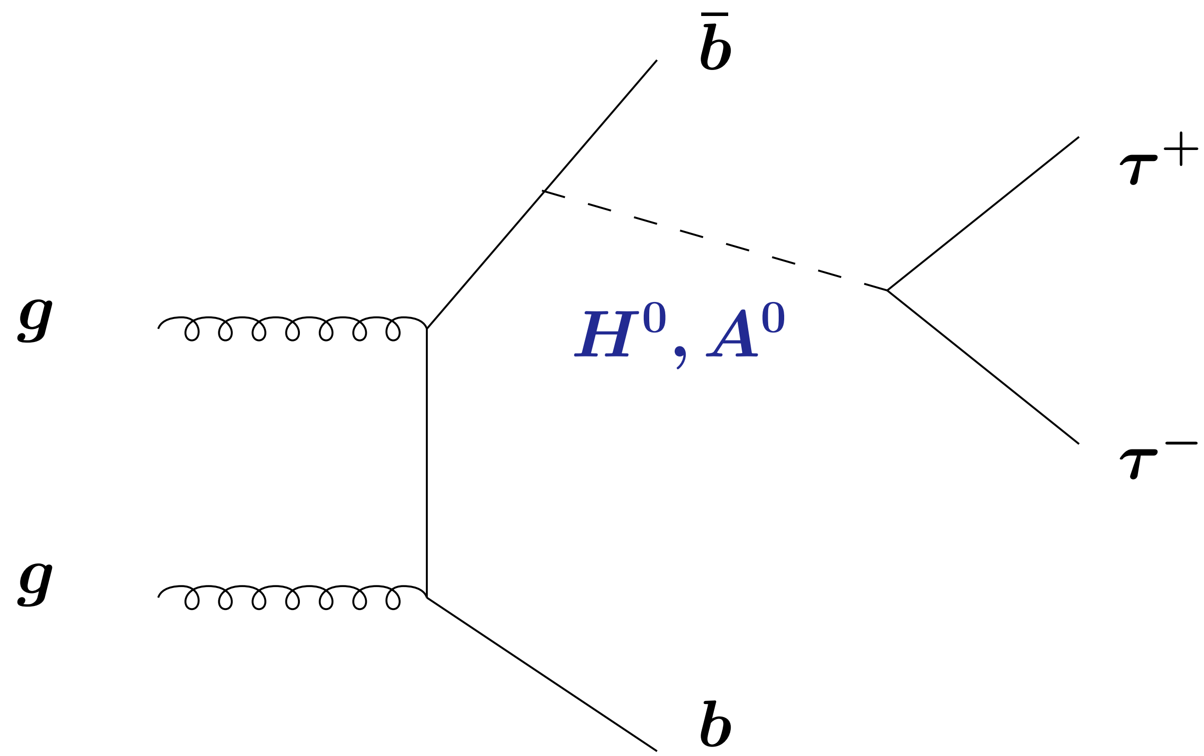
Uplifted Higgses at hadronic machines

- Production of heavy Higgses through gluon fusion with b loops and in association with b's increases.
- Decays to taus can dominate



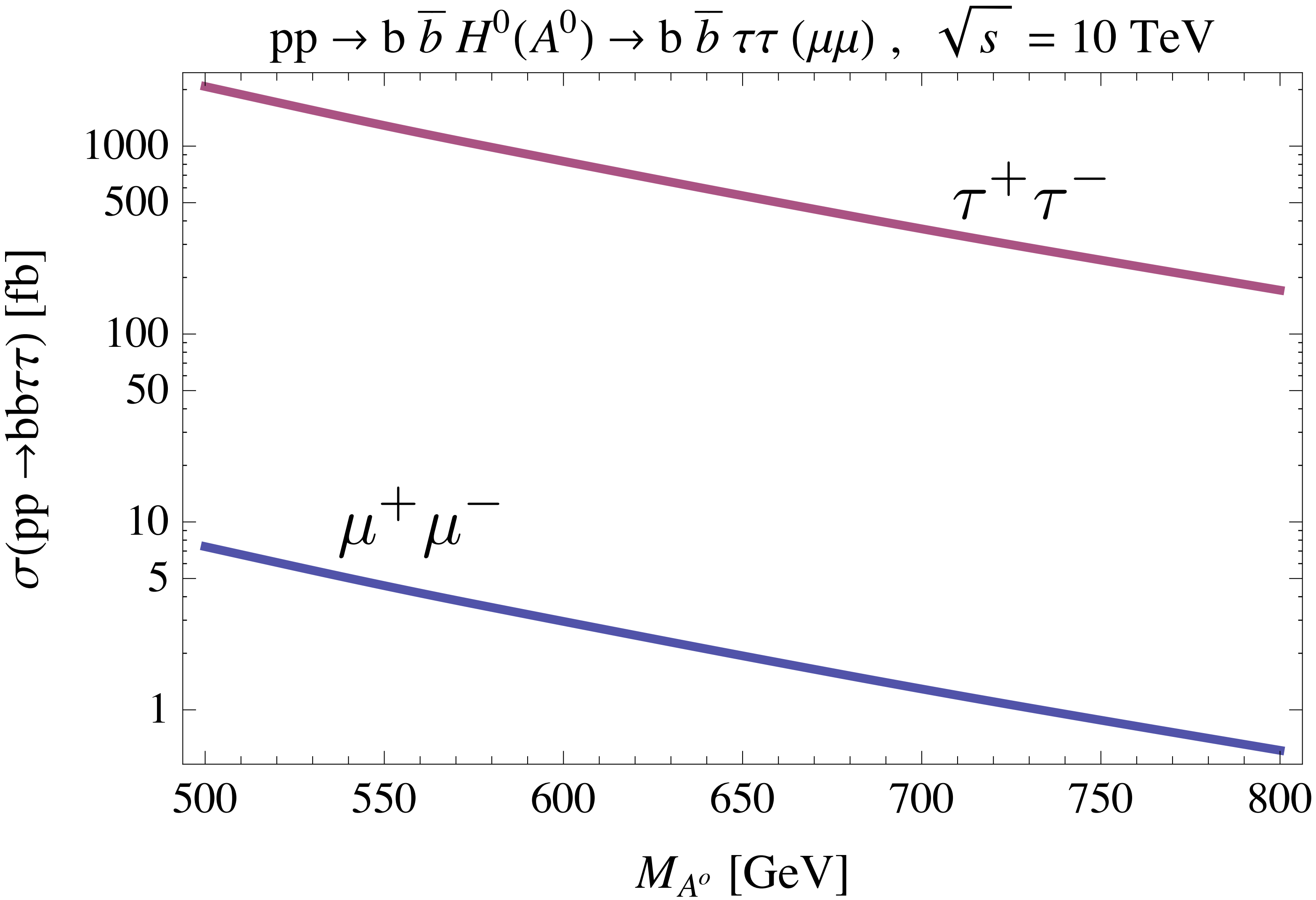
usual MSSM

A. Djouadi, hep-ph/0503173



Uplifted Higgses at hadronic machines

$$B(A^0 \to \mu^+\mu^-) \approx \frac{m_\mu^2}{m_\tau^2} B(A^0 \to \tau^+\tau^-) \approx 0.1 - 0.3\%$$

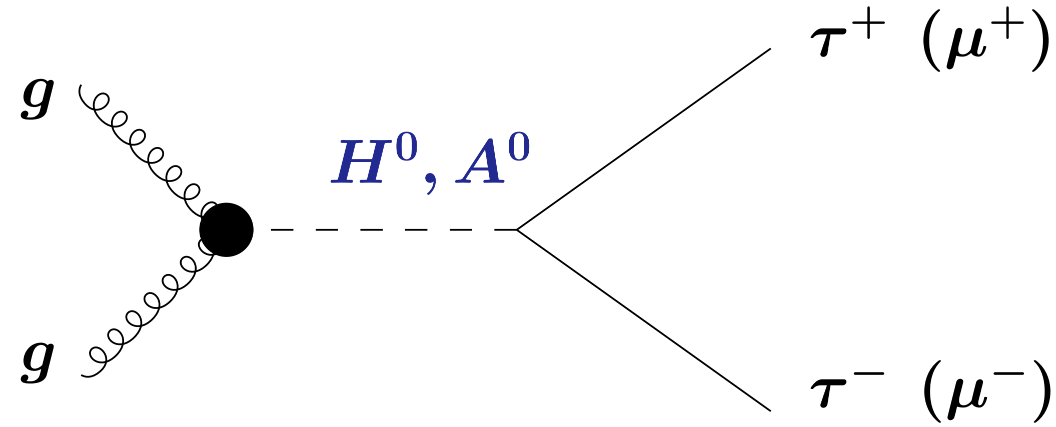


$$K \approx 2, \; y_b = 1, \; y_\tau = 1.5$$

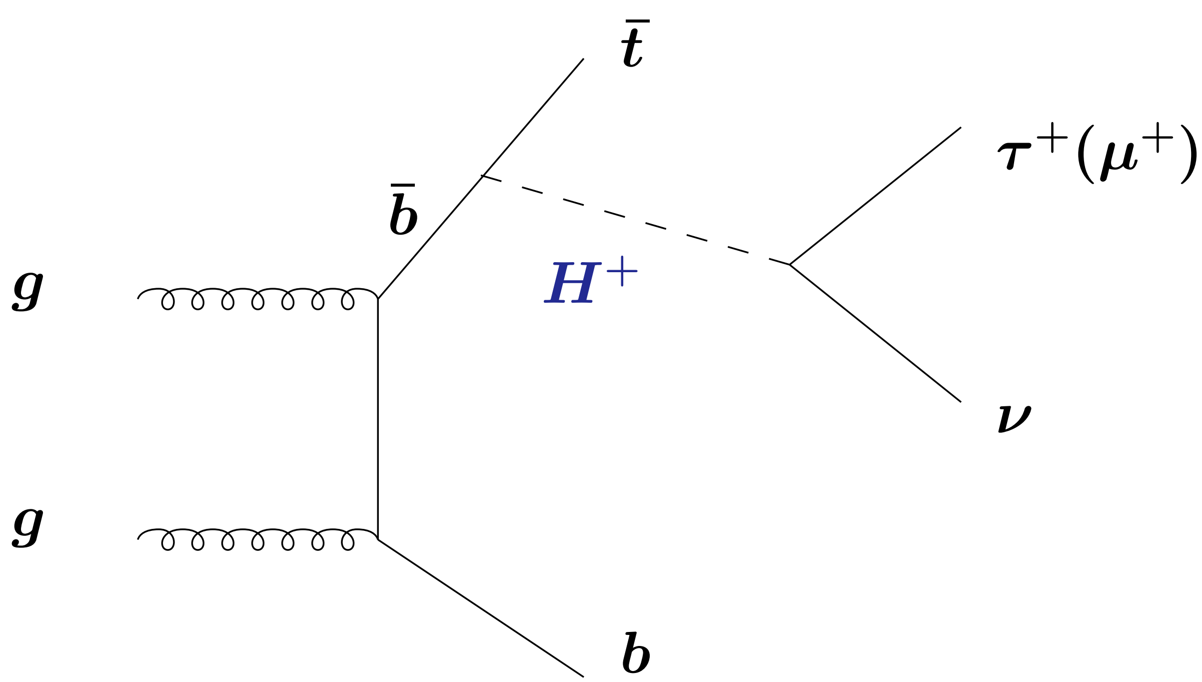
Uplifted Higgses at hadronic machines

Gluon fusion with tau/muon final state

b and $\tilde{b}$ loops $\Rightarrow$



Charged Higgs



A taste of uplifted flavour

$$R = \frac{BR(B^+ \to \tau^+ \nu)}{BR(B^+ \to \tau^+ \nu)_{SM}}$$

$$R_{\rm 2HDM} = \left(1 - \tan^2\beta \frac{m_B^2}{m_{H^+}^2}\right)^2$$

Rate in MSSM is reduced relative to SM
Observation is above SM expectation
Strong bounds on charged Higgs

$$\mathrm{BR}(B^+ \to \tau^+ \nu)_{\rm SM} = (0.80 \pm 0.12) \times 10^{-4}$$

$$\mathrm{BR}(B^+ \to \tau^+ \nu)_{\rm exp} = (1.73 \pm 0.35) \times 10^{-4}$$

A taste of uplifted flavour

$$R = \frac{BR(B^+ \to \tau^+ \nu)}{BR(B^+ \to \tau^+ \nu)_{SM}}$$

$y_b \times y_\tau$

$$R_{\rm 2HDM} = \left(1 - \tan^2\beta \frac{m_B^2}{m_{H^+}^2}\right)^2$$

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Observation is above SM expectation
Strong bounds on charged Higgs

$$\mathrm{BR}(B^+ \to \tau^+ \nu)_{\rm SM} = (0.80 \pm 0.12) \times 10^{-4}$$

$$\mathrm{BR}(B^+ \to \tau^+ \nu)_{\rm exp} = (1.73 \pm 0.35) \times 10^{-4}$$

Uplifted "separates" Yukawa couplings from masses, extra phases that enter. Has potential to allow NP to constructively interfere

Conclusions

$\tan\beta$

$1 \qquad \sim 50 \qquad \infty$

Usual MSSM — Uplifted MSSM

- Down-type fermion masses generated at one loop by fields of MSSM
- Ratios of Yukawas not as in MSSM
- $\tan\beta$ a potentially confusing parameter
- Higgs production at hadronic machines increased
- Decays to taus dominate
- Easier to find the heavy Higgses

Flavour violation in processes involving the third gen.

Explanation of PAMELA excess? [Kadota, Freese, Gondolo]

Formulae

Uplifted lepton coupling

$$y'_\ell = \frac{y_\ell\,\alpha}{8\pi} e^{i(\theta_W-\theta_\mu)} \left\{ \frac{3}{s_W^2} F\left(\frac{M_{\tilde{W}}}{M_{\tilde{L}}}, \frac{|\mu|}{M_{\tilde{L}}}\right) + \frac{e^{i(\theta_B-\theta_W)}}{c_W^2} \left[-F\left(\frac{M_{\tilde{B}}}{M_{\tilde{L}}}, \frac{|\mu|}{M_{\tilde{L}}}\right) \right.\right.$$

$$\left.\left. + 2F\left(\frac{M_{\tilde{B}}}{M_{\tilde{e}}}, \frac{|\mu|}{M_{\tilde{e}}}\right) + \frac{2|\mu|}{M_{\tilde{e}}} F\left(\frac{M_{\tilde{B}}}{M_{\tilde{L}}}, \frac{M_{\tilde{e}}}{M_{\tilde{L}}}\right) \right] \right\}$$

Uplifted down-quark coupling

$$(y'_d)_F = -\frac{y_d}{3\pi} e^{i(\theta_g-\theta_\mu)} \frac{2|\mu|}{M_{\tilde{d}}} \left[\alpha_s F\left(\frac{M_{\tilde{g}}}{M_{\tilde{Q}}}, \frac{M_{\tilde{d}}}{M_{\tilde{Q}}}\right) + \frac{\alpha e^{i(\theta_B-\theta_g)}}{24 c_W^2} F\left(\frac{M_{\tilde{B}}}{M_{\tilde{Q}}}, \frac{M_{\tilde{d}}}{M_{\tilde{Q}}}\right) \right]$$

$$(y'_d)_{\tilde{H}} = \frac{y_d \alpha}{8\pi} e^{i(\theta_W-\theta_\mu)} \left\{ \frac{3}{s_W^2} F\left(\frac{M_{\tilde{W}}}{M_{\tilde{Q}}}, \frac{|\mu|}{M_{\tilde{Q}}}\right) + \frac{e^{i(\theta_B-\theta_W)}}{3 c_W^2} \left[F\left(\frac{M_{\tilde{B}}}{M_{\tilde{Q}}}, \frac{|\mu|}{M_{\tilde{Q}}}\right) + 2F\left(\frac{M_{\tilde{B}}}{M_{\tilde{d}}}, \frac{|\mu|}{M_{\tilde{d}}}\right) \right] \right\}$$

$$(y'_d)_A = \frac{y_u y_d}{16\pi^2} e^{-i\theta_\mu} \frac{A_u^*}{M_{\tilde{u}}} F\left(\frac{M_{\tilde{u}}}{M_{\tilde{Q}}}, \frac{|\mu|}{M_{\tilde{Q}}}\right)$$

$$y'_d = (y'_d)_F + (y'_d)_{\tilde{H}} + (y'_d)_A$$