New particle mass spectrometry at the LHC using M_{CT2} variable

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Ref) arXiv:0912.2354
arXiv:1008.0391

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Contents

1. **Comparison of** M_T **&** M_{CT} **/** M_{T2} **&** M_{CT2}
   - General event topology at hadron collider

2. **Amplification of singular structures in endpoint region by** $M_{CT/CT2}$
   - Reducing systematic uncertainties in endpoint extraction

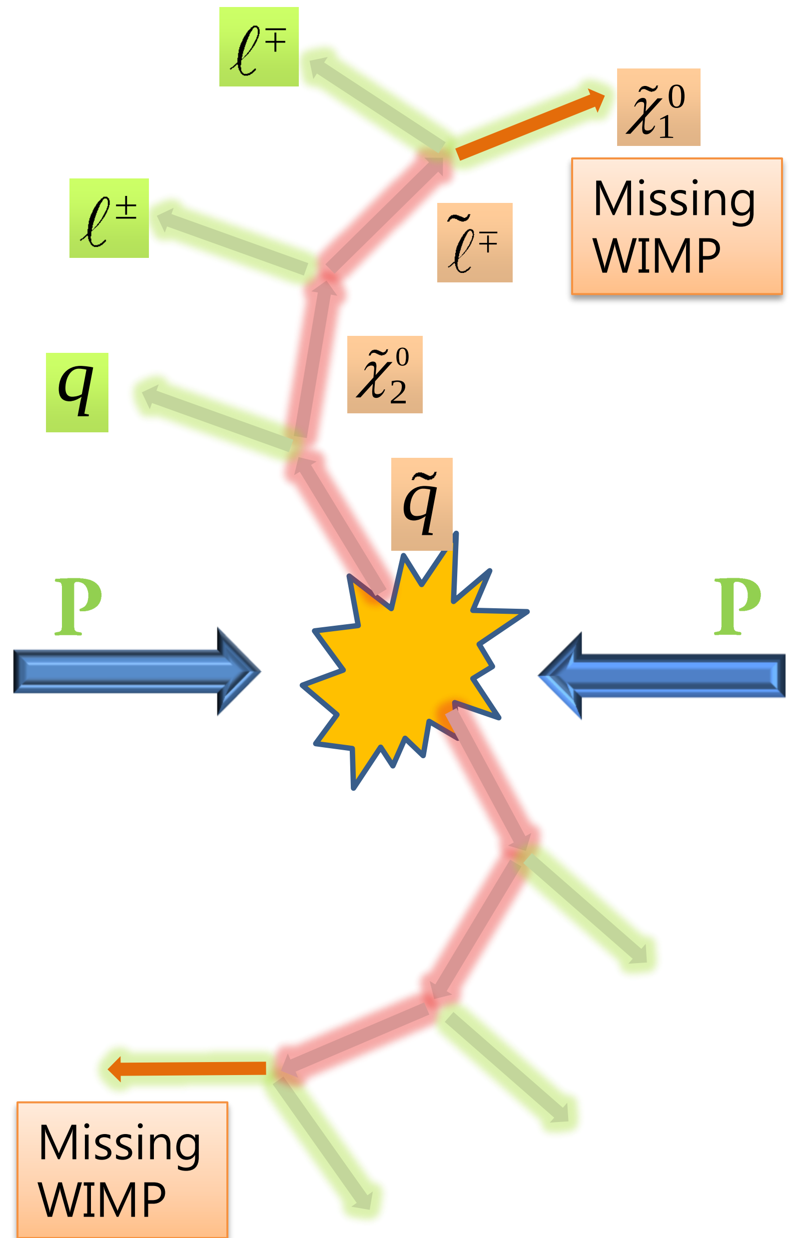
3. **A variety in transverse boost for** $M_{CT/CT2}$
   - Transversification
   VS
   Magnifying/Utilizing the boost effect

Dark matter production at the LHC

- **BSM models with WIMP DM candidate**
  - **Supersymmetry(R-parity)**
  - **Universal Extra Dimension (KK-parity)**
  - **Little Higgs(T-parity)**

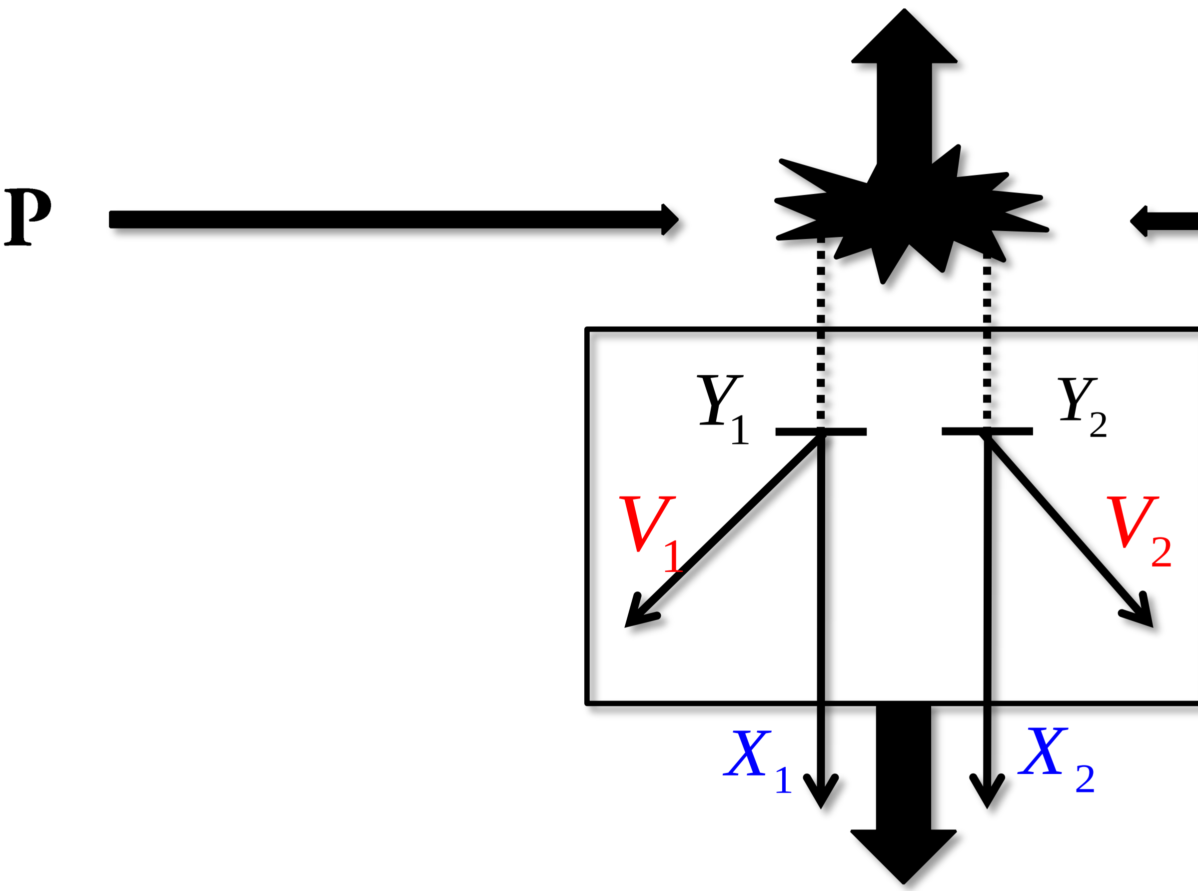
- **Measurement of new particle masses is not an easy task.**

  ✓ **Several missing particles (N ≥ 2)**
  ✓ **Partonic CM frame ambiguity**
  ✓ **Complex event topologies**



Mass Measurement with Missing Particle(s)

$\vec{\delta}_T$: Transverse momentum from ISR or initial decays of $Y_{1,2}$



$Y_{1/2}$: Mother particles

V_1 , V_2 : Visible particles

$X_{1/2}$: Missing particles

$-\vec{\delta}_T$ of boosted $Y_{1,2}$ system with single step decay

In the Standard Model …

- M_T **(Transverse mass)** V. Barger, A. Martin and R. Phillips Z. Phys. C 21,99 (1983), J. Smith, W. van Neerven and J. Vermaseren Phys. Rev. Lett 50, 1738 (1983)

  for M_W **measurement in** $p\bar{p} \to \delta_T + W(\to \ell + \nu)$ **: Single Y/X**
- $M_T^2 = m_\ell^2 + m_\nu^2 + 2\,(E_T^\ell E_T^\nu - p_T^\ell p_T^\nu) = 2p_T^\ell p_T^\nu\,(1 - \cos\varphi) \leq M_W^2$

J. Smith et. al. Phys. Rev. Lett 50, 1738 (1983)

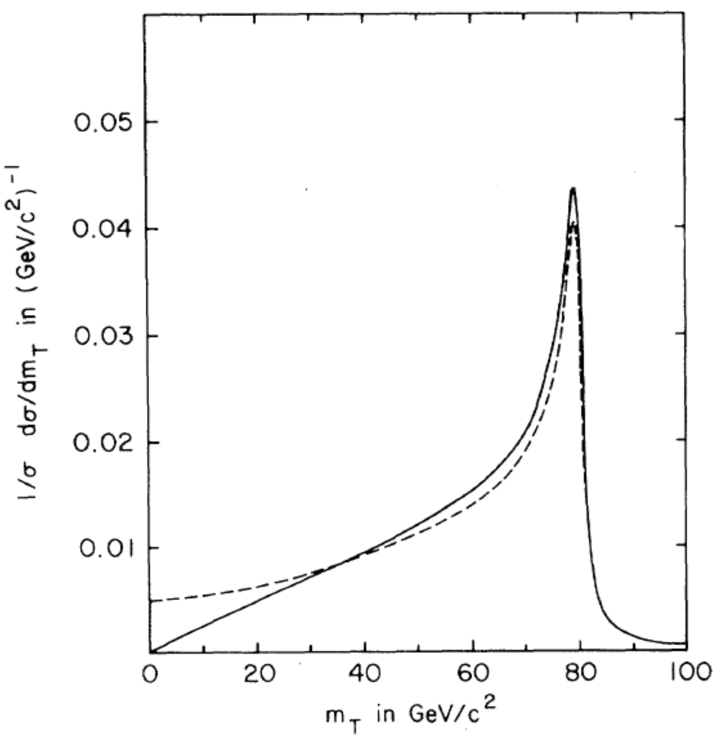
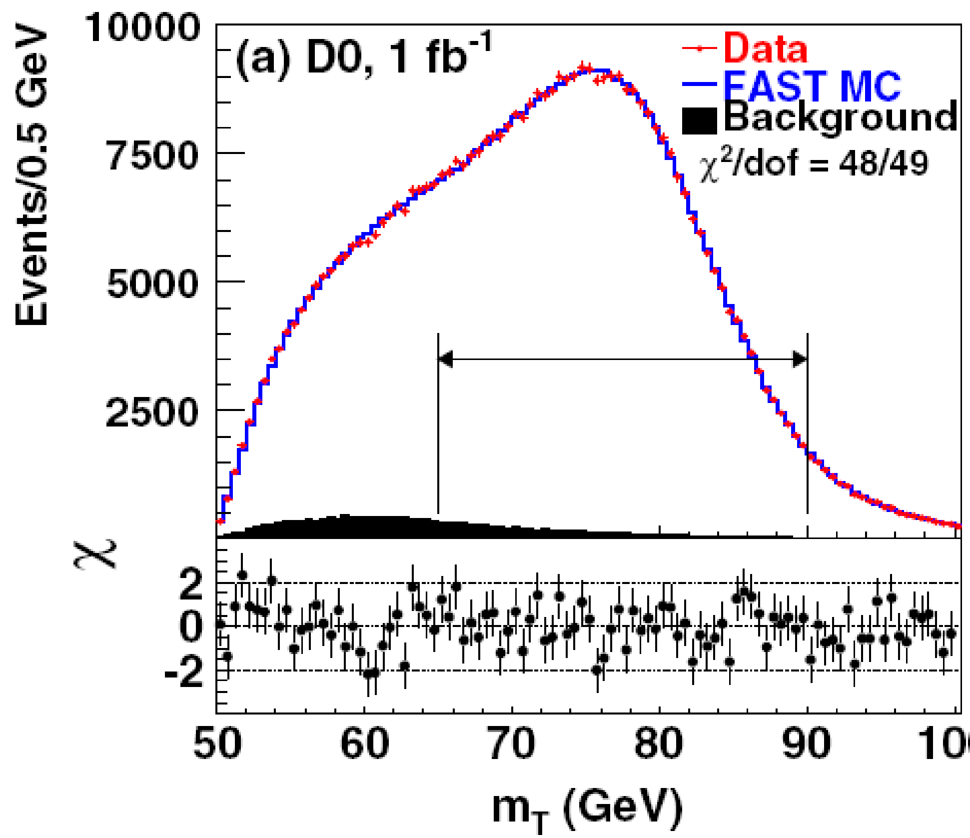


FIG. 1. $\sigma^{-1} d\sigma/dm_T$ for $M = 80$ GeV/c^2 and $\Gamma = 2.5$ GeV/c^2. The solid line is for $p_T^W = 0$ GeV/c, while the dashed line is for $p_T^W = 50$ GeV/c.

D0 Collaboration, Phys. Rev. Lett 103, 141801 (2009)



$\Rightarrow M_W = 80.401 \pm 0.021(\mathrm{stat}) \pm 0.038(\mathrm{sys})$ GeV

Questions for New Physics at the LHC

1) What if we don't know M_X as well as M_Y?

   ⇒ The endpoint relation $M_T^{max} = M_Y$ is not conserved anymore for trial missing particle mass, $\chi \neq M_X$.

2) What if there exist multiple missing particles in the boosted decay system ?? (DM candidates in Z_2-parity conserving NP models)

   ⇒ Only the sum of Tr. momenta is known.

3) Can we determine both of the masses, simultaneously, in such a non-reconstructable event with short decay chains ???

•For reconstructable events, see Refs) M. Nojiri, et al 2006,
H.Cheng, J. Gunion, Z. Han and B. McElrath et al, 2007-2009

Basic properties of M_T and M_{CT}(Contransverse mass)

$$pp \to \delta_T + Y(\to V + X)$$

- **Let's consider a system *Y*, where its $\sqrt{S}$ is resonant / non-resonant.**

- M_T **of** V **and** X **(for resonant Y with** $\sqrt{S} = M_Y$ **) :**

$$M_T(Y)^2 \equiv m_X{}^2 + m_V{}^2 + 2\sqrt{m_V{}^2 + |V_T|^2}\sqrt{m_X{}^2 + |X_T|^2} - 2V_T \cdot X_T$$
$$\leq S(= M_Y{}^2)$$

$\Rightarrow$ Colinear boost (or Frame) invariant endpoint as $\sqrt{S} = M_Y$!

Basic properties of M_T and M_{CT}(Contransverse mass)

- M_{CT} **for non-resonant Y :**

$$M_{CT}(Y)^2 \equiv m_X{}^2 + m_V{}^2 + 2\sqrt{m_V{}^2 + |V_T|^2}\sqrt{m_X{}^2 + |X_T|^2} + 2V_T \cdot X_T$$

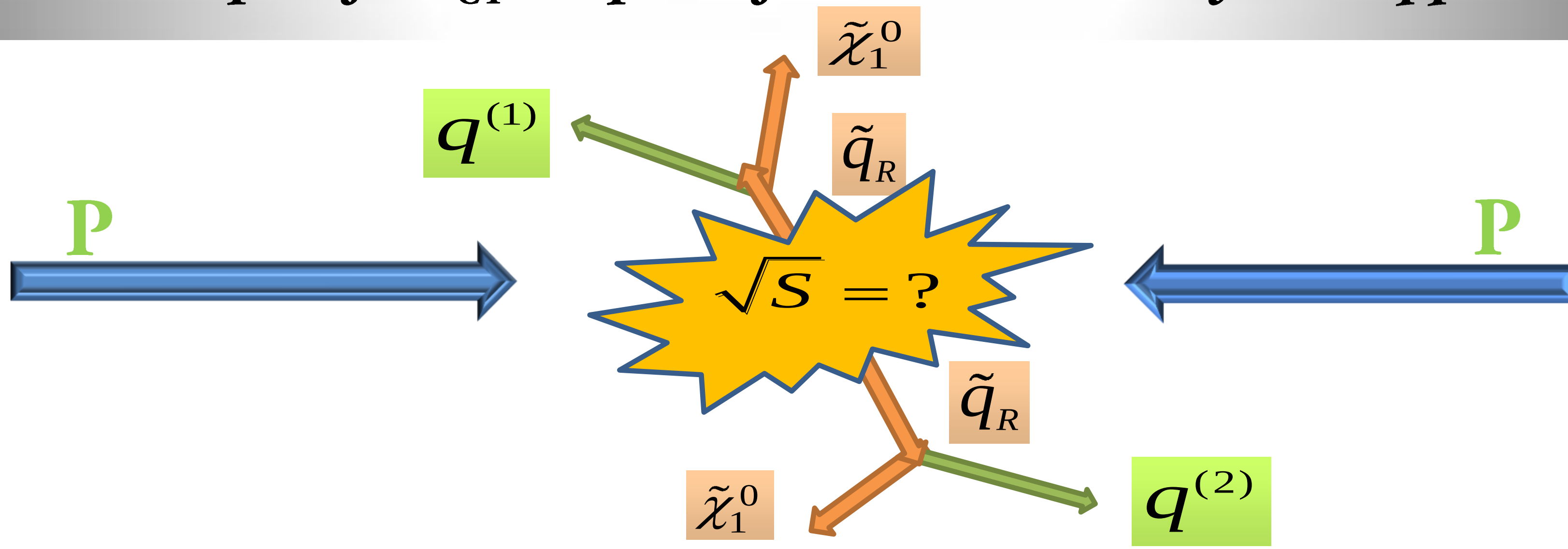
$$\leq$$

$$M_C(Y)^2 \equiv m_X{}^2 + m_V{}^2 + 2\sqrt{m_V{}^2 + |V|^2}\sqrt{m_X{}^2 + |X|^2} + 2V \cdot X$$

$\Rightarrow$ Contra-linear boost (Back to back boosts of V and X) invariant endpoint!

$\Rightarrow$ $\sqrt{S}$ invariant endpoint in a fixed frame.

Example of M_{CT} endpoint for non-resonant system (qq)



$$M_{CT}(qq,\sqrt{S} > 2M_{\tilde{q}})^2 \equiv 2|q_T^{(1)}||q_T^{(2)}| + 2q_T^{(1)} \cdot q_T^{(2)}$$

$$\leq$$

$$M_C(qq,\sqrt{S} > 2M_{\tilde{q}})^2 \equiv 2|q^{(1)}||q^{(2)}| + 2q^{(1)} \cdot q^{(2)}$$

= In CM frame,

$$M_C(qq,\sqrt{S} = 2M_{\tilde{q}})^2 = 2|q_0^{(1)}||q_0^{(2)}| + 2q_0^{(1)} \cdot q_0^{(2)}$$

$$\leq 4|q_0^{(1)}||q_0^{(2)}| = 4\left(\frac{m_{\tilde{q}}^2 - m_{\tilde{\chi}_1^0}^2}{2m_{\tilde{q}}}\right)^2$$

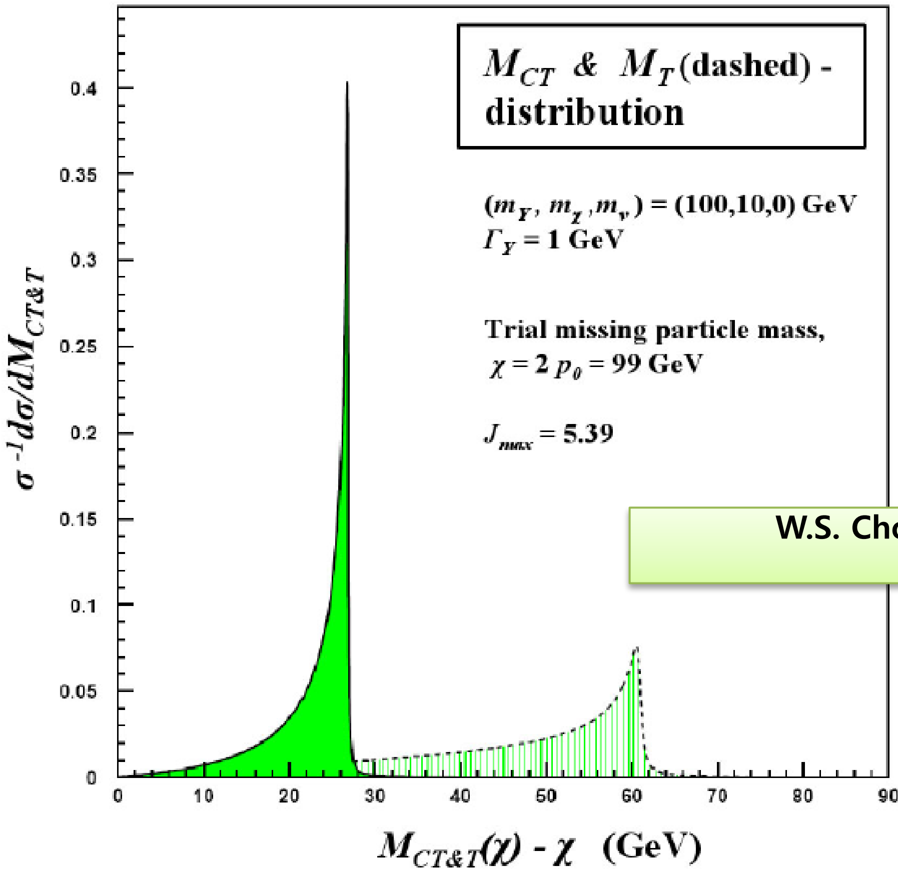
→The inequality holds as long as the RH-squark pair system is Tr. rest.

M_{CT} for a resonance decay of Y with test mass of X, χ

$$pp \to \delta_T + Y(\to V + X)$$

$$M_{CT/T}(Y)^2 \equiv \chi^2 + m_V{}^2 + 2\sqrt{m_V{}^2 + |V_T|^2}\sqrt{\chi^2 + |X_T|^2} \pm 2V_T \cdot X_T$$

$$(\delta_T = 0,\ m_V = 0) \le \chi^2 + 2\left[|p_0|\sqrt{\chi^2 + |p_0|^2} \mp |p_0|^2\right], \quad |p_0| = \frac{m_Y{}^2 - m_X{}^2}{2m_Y}$$

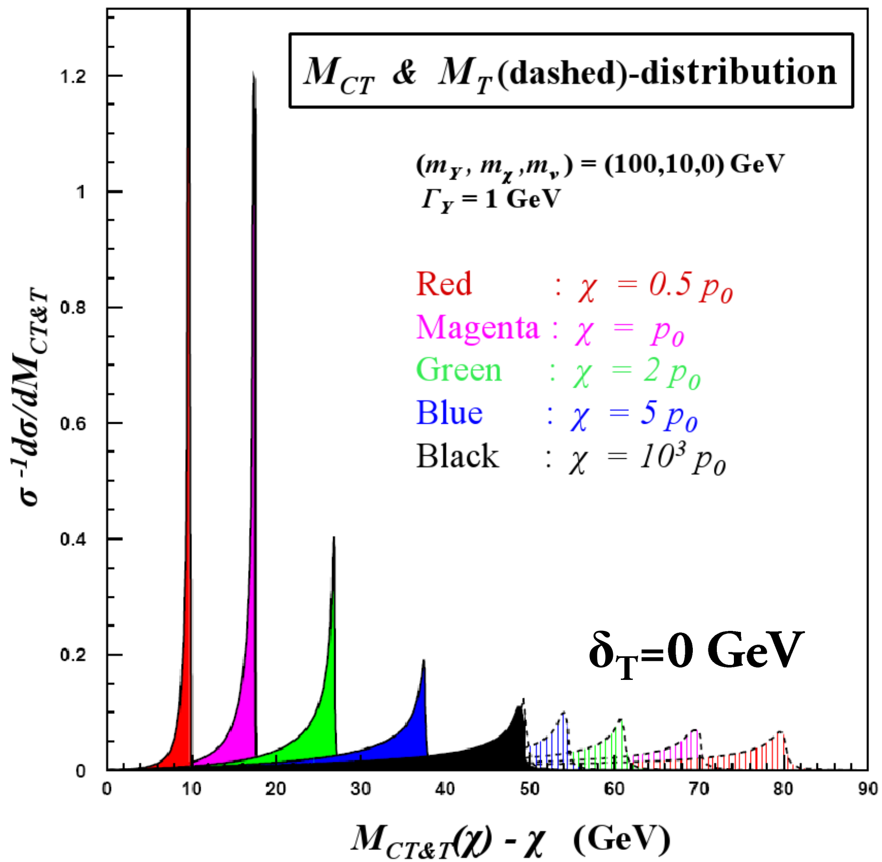


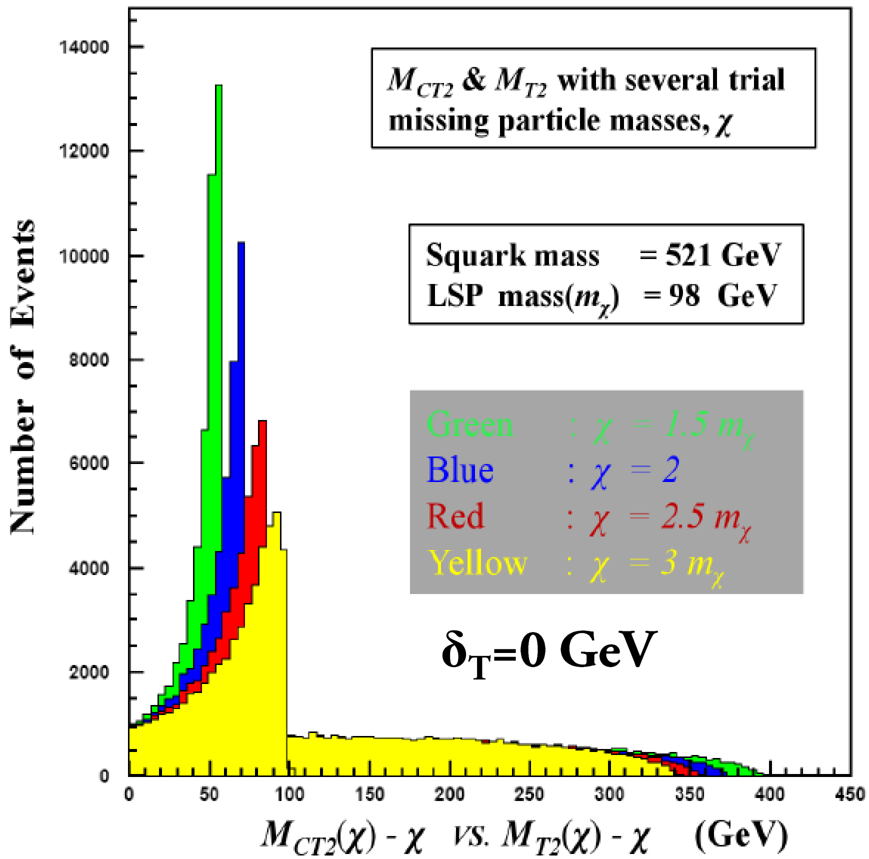
W.S. Cho, J.E.Kim and J.H.Kim, arXiv:0912.2354,
Phys.Rev.D81,095010(2010)

Large Jacobi Factor in the Endpoint Region of $M_{CT}/M_{CT2} \Leftrightarrow M_T/M_{T2}$:

- Compact distribution
  for the internal momentum scale from the decay in system.
- Controlled by trial missing particle mass, χ
- Accentuation of singular structure in the endpoint region
- Reduction of systematic error in endpoint extraction

W.S. Cho, J.E.Kim and J.H.Kim, arXiv:0912.2354, Phys.Rev.D81,095010(2010)





M_{CT2} (ConStransverse mass)

W.S.Cho, J.E.Kim and J.H.Kim, Phys.Rev.D81,095010(2010)

$$M_{CT2/T2} \text{ for } pp \to \delta_T + Y_1 Y_2 (\to V_1 X_1 + V_2 X_2)$$

$$M_{CT2/T2} \equiv \min[\max\{M_{CT/T}(Y_1), M_{CT/T}(Y_2)\}],$$

$$M_{CT/T}(Y_i)^2 \equiv \chi^2 + 2|V_{iT}|\sqrt{\chi^2 + |X_{iT}|^2} +- 2V_{iT} \cdot X_{iT}$$

- χ = Trial missing particle mass, massless visible assumed.
- min & max over all possible missing Tr. Momentum,

$$X_{1T} + X_{2T} = \not{E}_T$$

- M_{CT2}=Mixture of M_{T2} [C. Lester and D. Summers (1999)] and M_{CT} [Tovey (2008), Cho et al, Serna(2008), M_{CT} as a part of M_{T2} sol. (χ=0) for non-resonant massless visibles]

- IF $m_\nu \sim 0$, M_{CT2} (χ) projection can have significantly amplified endpoint structure (χ = Trial missing particle mass)

$$J_{max}(\chi) \Rightarrow \infty \ \text{ as } \chi \Rightarrow 0$$

- One can control $J_{max}(\chi)$ by judicious value of χ

$$\sigma^{-1}\frac{d\sigma}{dM_{CT}(\chi)} \sim J\sigma^{-1}\frac{d\sigma}{dM_T(\chi)}$$

$$J = \frac{M_{CT}(\chi)}{M_T(\chi)}\frac{(e_X + |\mathbf{p}_{0T}|)^2}{(e_X - |\mathbf{p}_{0T}|)^2}$$

$$\rightarrow \begin{cases} \frac{M_C(\chi)}{M(\chi)}\frac{(E_X+|\mathbf{p}_0|)^2}{(E_X-|\mathbf{p}_0|)^2} & \text{Endpoint region, } J_{max} \\ 1 & \text{Minimum region} \end{cases}$$

- A faint Break Point (~ signal endpoint with irred. BGs) with small slope difference is amplified by $J_{max}^2(\chi)$:

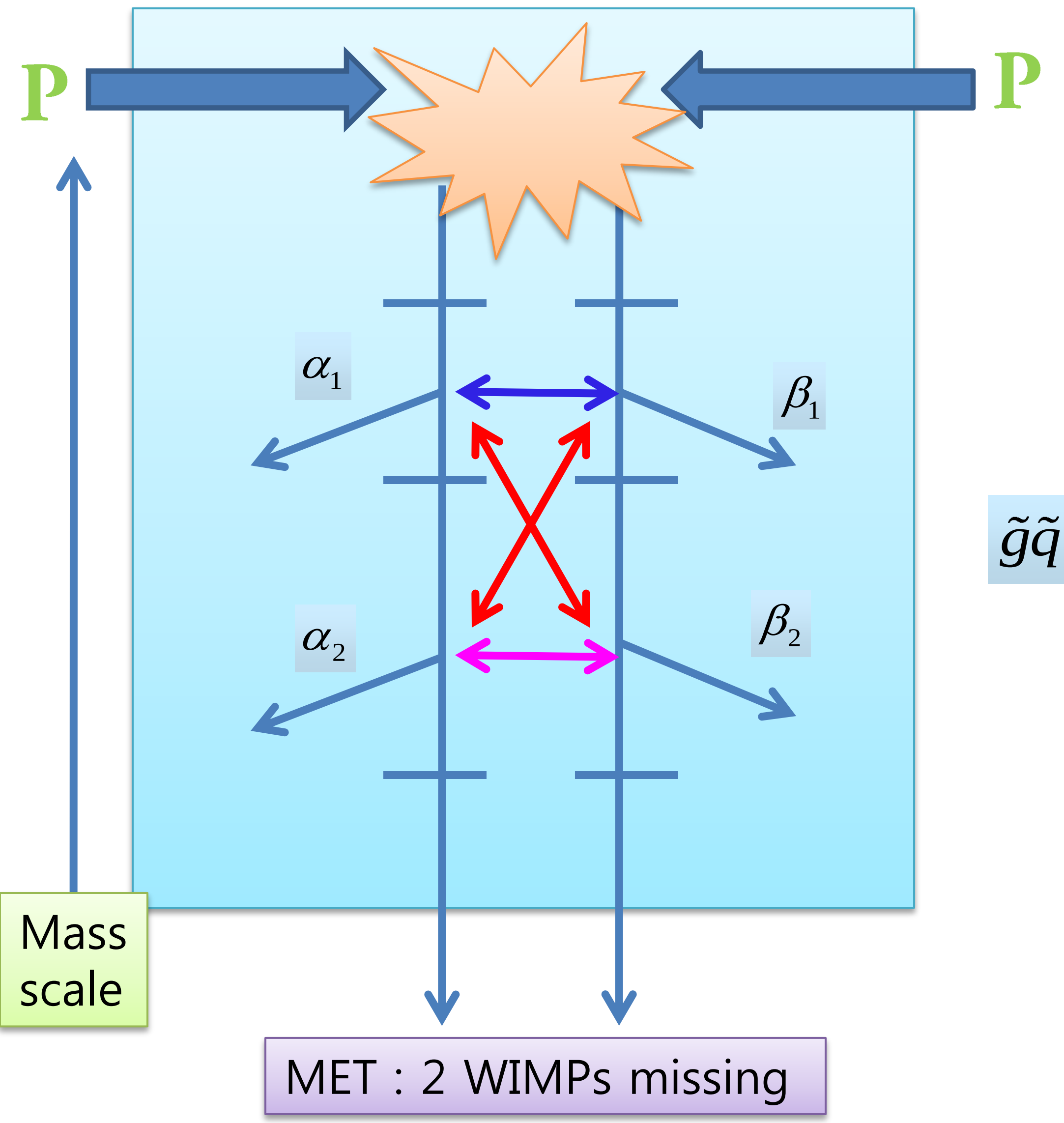
$$\Delta a \rightarrow \Delta a` = J_{max}^2(\chi)\, \Delta a \,, \qquad \delta_{BP}^{\;2} \sim \frac{\sigma^2}{\Delta a^2}$$

With the accentuated BP structure, the fitting scheme (function/range) can be elaborated, and it can significantly reduces the systematic uncertainties in extracting the position of the BPs !

➔ The most reliable range for local fitting for BP.

Simple Example :

$$\tilde{g}\tilde{g} \rightarrow (q+\tilde{q})+(q+\tilde{q}) \rightarrow (qq\tilde{\chi}_1^0)+(qq\tilde{\chi}_1^0)$$



4jets➔ 6 possible pairs of jets / 3 Independent decay crossing pairs exist

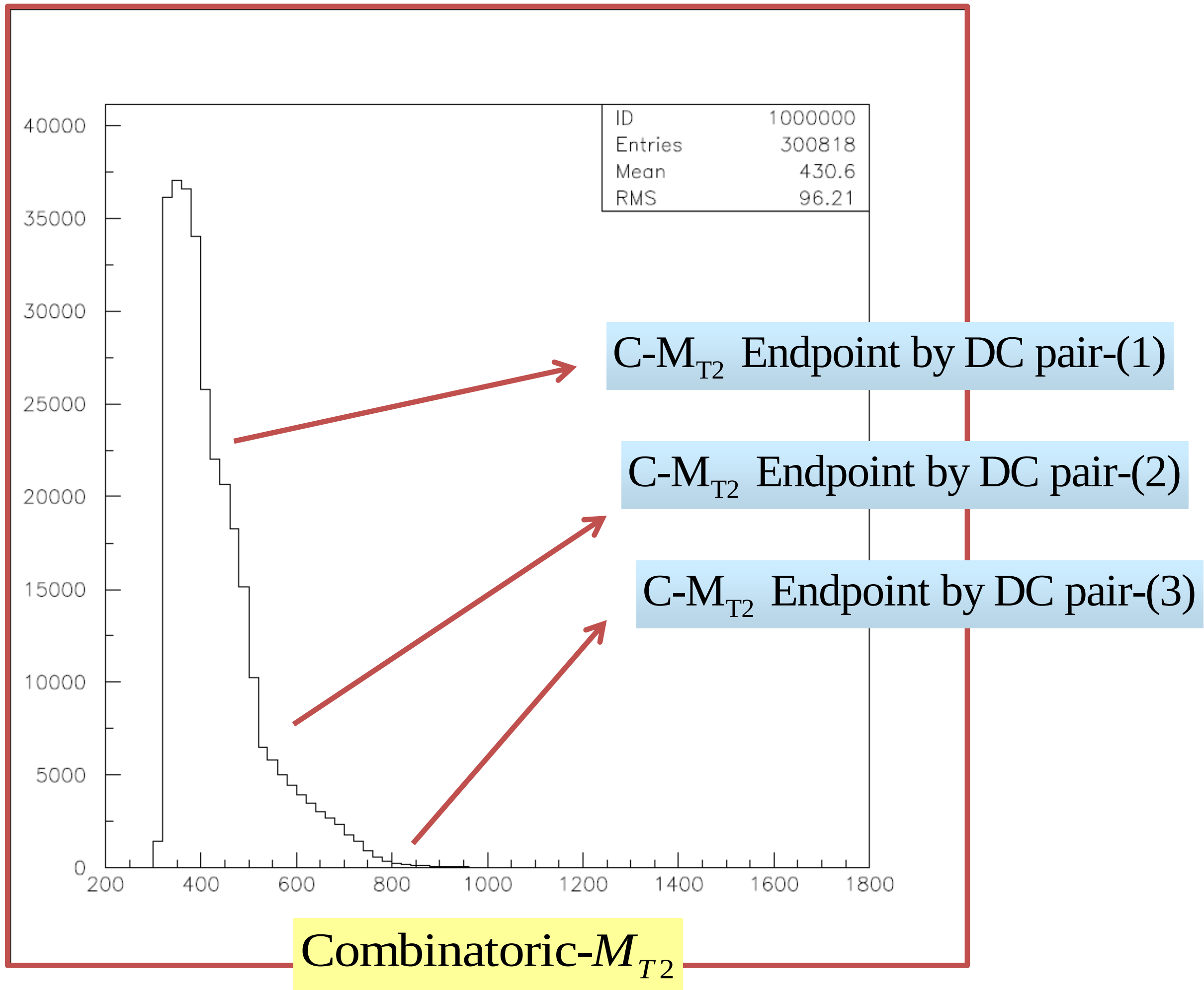
1) α(1)-β(1)
2) α(1)-β(2)/α(2)-β(1)
3) α(2)-β(2)

$$\tilde{g}\tilde{q} \rightarrow (qq\tilde{\chi}_1^0)+(q\tilde{\chi}_1^0)$$

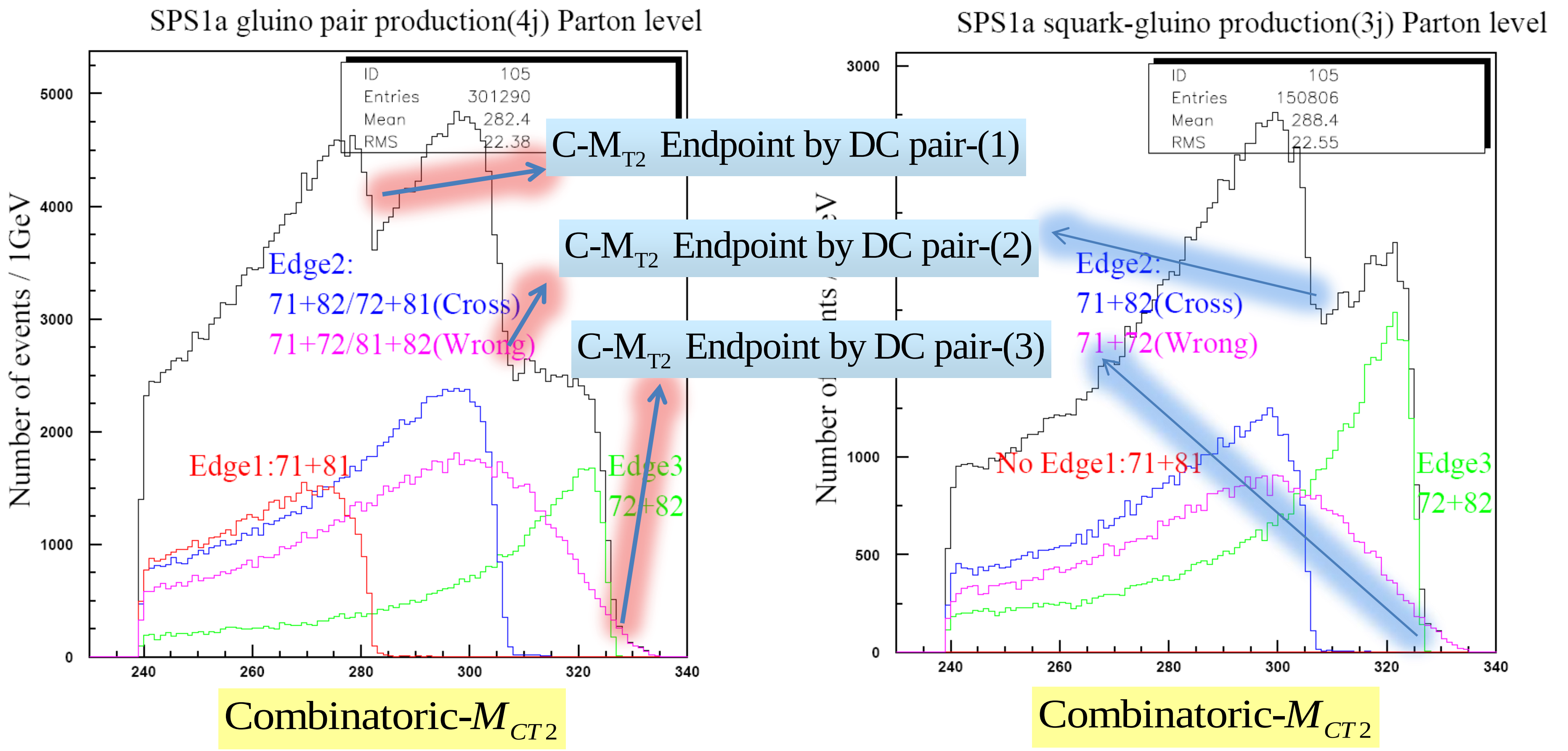
3 jets ➔ 3 pairs / 2 Independent decay crossing pairs exist

2) α(1)-β(2)/α(2)-β(1)
3) α(2)-β(2)

Partonic level results : C-M_{T2}



Partonic level results : C-M_{CT2}



→ Systematic errors for physical constraints reduced by $O(1/J_{max})$ in local fitting of break points.

J_{max} : Jacobian factor near the endpoint region

This enhances our observability for several endpoints.

(Previously)
Impose hard cut, and remove the BG events near the endpoint.

(Now)
Well, moderate cut & irreducible BGs are okay, if there exist dim BPs from signal endpoints. We can magnify it !

- ***M_{CT2} for $\delta_T \neq 0$??***

W.S. Cho, W. Klemm and M. M. Nojiri, [arXiv:1008.0391]

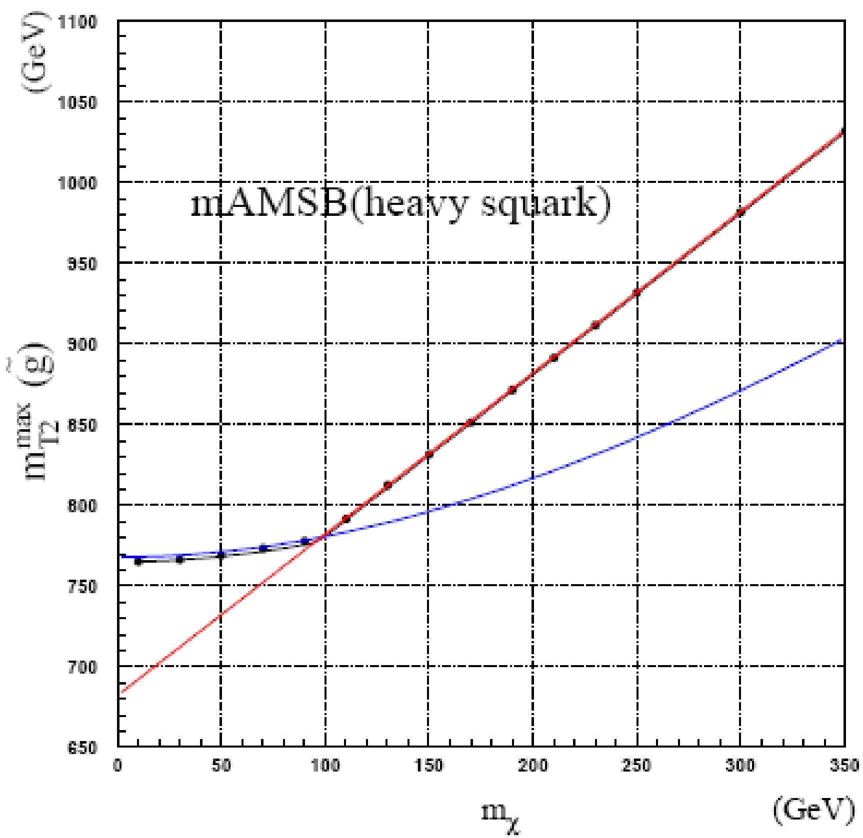
➔ ***Resolving power to determine both the mother and missing particle masses.***

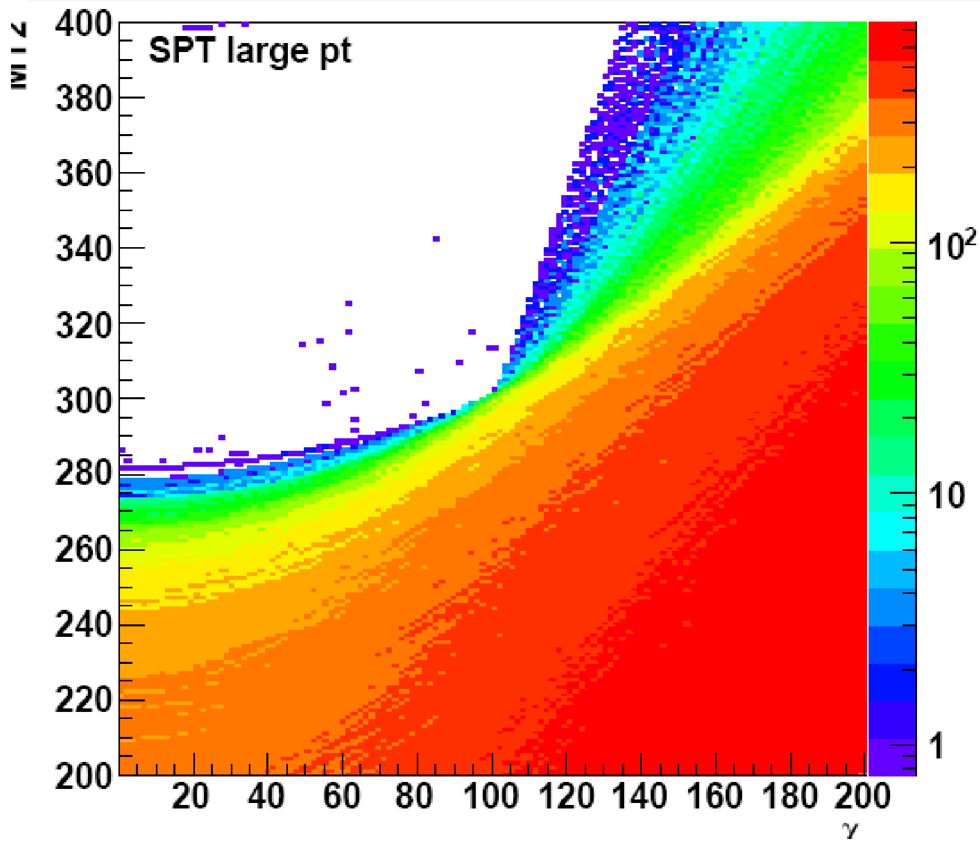
Can we measure both the M_Y and M_X? → M_{T2} - kink Methods

- Using $M_{T2}^{max}(\chi)$ / kink position at true masses, (M_Y, M_X)
- The kink is from the variety of kinematic configurations for $M_{T2}^{max}(\chi)$

1. ``Mass Kink" from M_{Vis} variation (i.e. only for $N_{V1\&V2} \geq 2$) :
Cho et al, 0709.0288, 0711.4526

2. ``Boosted Kink" from various recoiling configurations by δ_T:
: A. Barr et al, 0709.2740, 0711.4009;
M. Burns et al, 0810.5576





● ***However, the BK structure may not be easy to identify as it requires very large δ_T .***

→ ``M_{T2}-bowl" (Statistical approach to pinpoint BK)
P. Konar, et al, 0910.3679;
T. Cohen, et al, 0905.1201

$M_{CT2}^{max}(\chi)$ with Non-zero Tr. Boost : Magnifying the boost effect

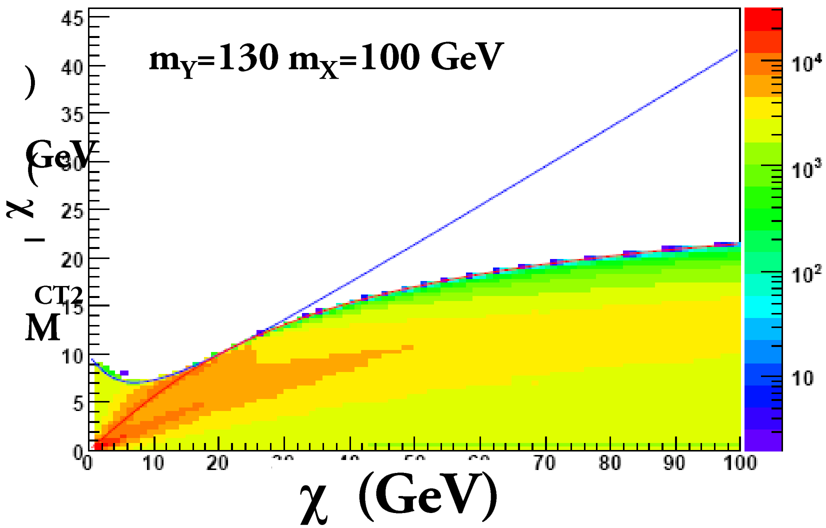
W.S. Cho, W. Klemm and M. M. Nojiri, [arXiv:1008.0391]

$$M_{CT2} \text{ for } pp \to \delta_T + Y_1 \, Y_2 (\to V_1 X_1 \; + \; V_2 X_2)$$

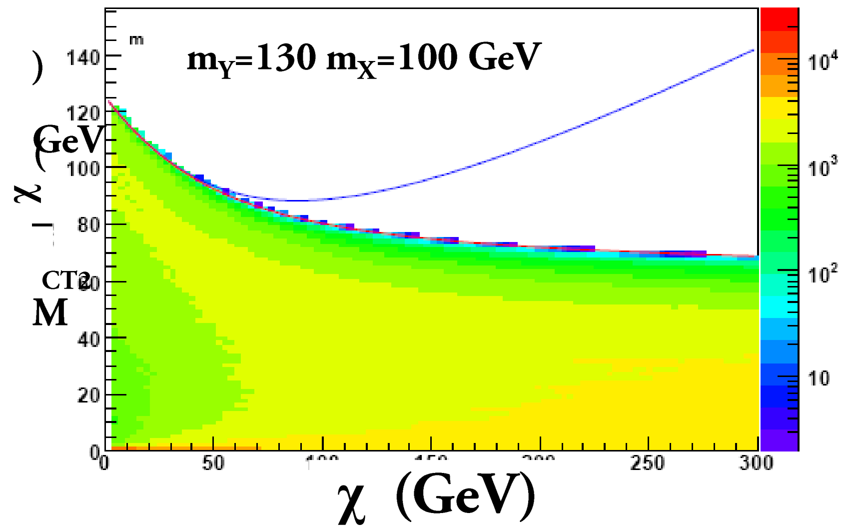
$$M_{CT2}^{\max} \equiv \begin{pmatrix} 2\chi^2 + \dfrac{|\delta_T|^2}{4} & \text{for } \chi \le \chi_* \\ \chi^2 + 2\alpha\left(\dfrac{|\delta_T|}{2} - \alpha\right) + 2\alpha\sqrt{\chi^2 + \left(\dfrac{|\delta_T|}{2} - \alpha\right)^2} & \text{for } \chi \ge \chi_* \end{pmatrix}$$

$$\alpha \equiv \left(\frac{m_Y^2 - m_X^2}{2m_Y}\right)\left[\frac{|\delta_T|}{2m_Y} + \sqrt{1 + \left(\frac{|\delta_T|}{2m_Y}\right)^2}\right], \qquad \chi_*^2 = \frac{|\delta_T|}{2}\left(2\alpha - \frac{|\delta_T|}{2}\right)$$

δ_T=20 GeV

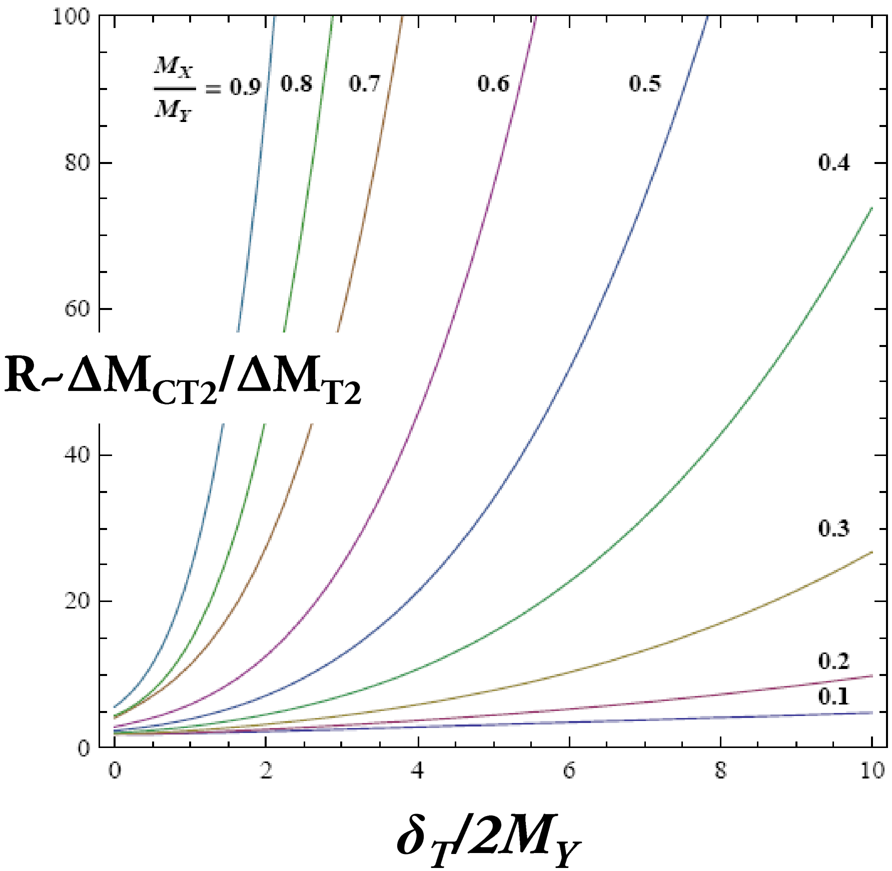


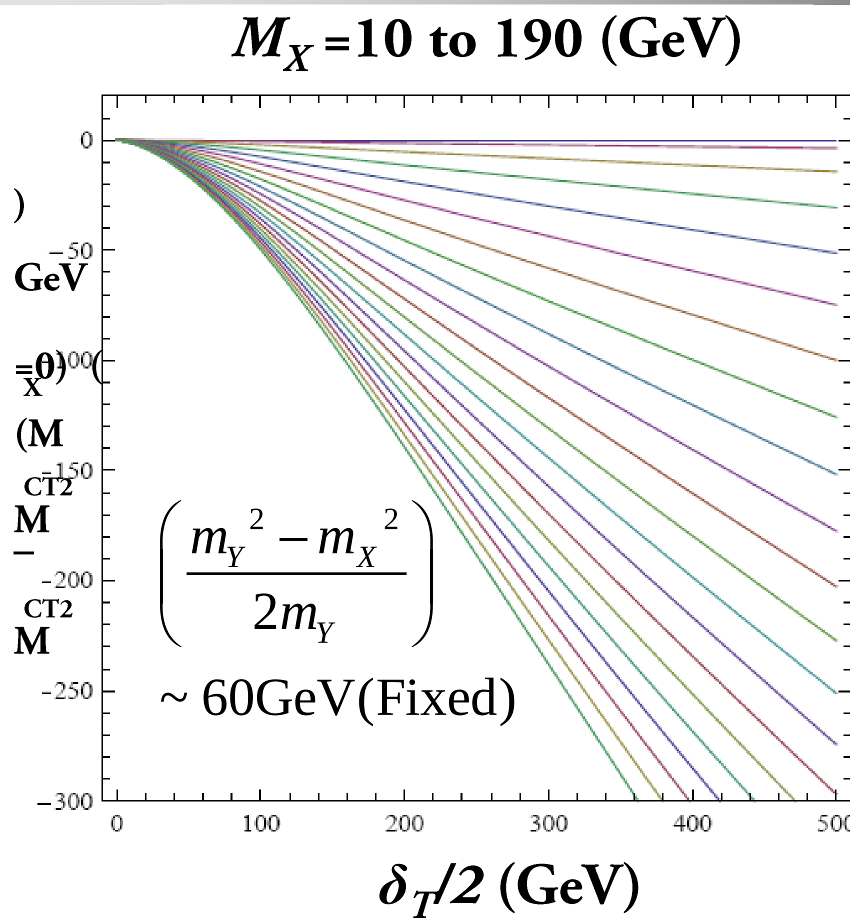
δ_T=250 GeV



Sensitive and elastic recoiling of the endpoint with respect to external boost momentum

- Especially for near degenerate mass spectrum
- Provides enhanced experimental resolution for M_Y and M_X with moderate value of boost momentum

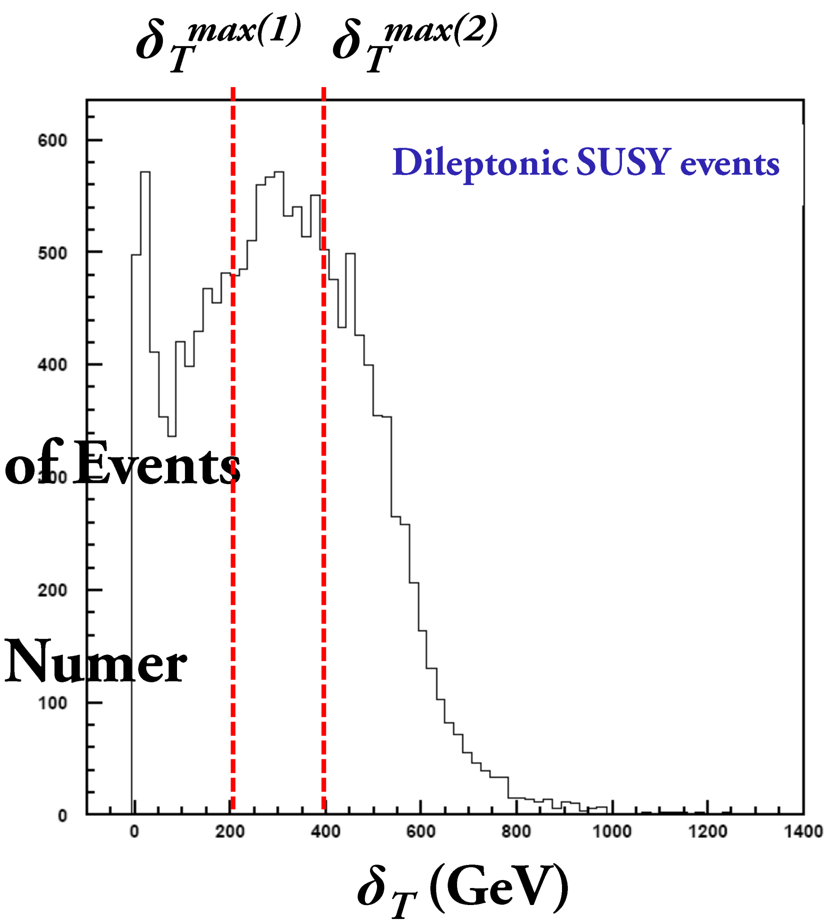




SUSY example : $m_{\tilde{\chi}_1^\pm}$ & $m_{\tilde{\nu}}$ measurement using same sign dileptonic events

More ref) K. Matchev, et al. arXiv:0909.4300,0910.1584; P. Konar, et al. Phys. Rev. Lett 105,051802(2010)

$$M_{CT2} \text{ for } pp \rightarrow \delta_T(\text{ISR/initial decays}) + \tilde{\chi}_1^\pm \tilde{\chi}_1^\pm (\rightarrow \ell^\pm \tilde{\nu}_\ell \; + \; \ell^\pm \tilde{\nu}_\ell)$$



- **SUSY Point**

$$m_{\tilde{g}},\; m_{\tilde{q}},\; m_{\tilde{\chi}_1^\pm/\tilde{\chi}_2^0},\; m_{\tilde{\nu}},\; m_{\tilde{\chi}_1^0}$$

$$(\sim 720, \sim 640, 231, 157, 123)\ \text{GeV}$$

- **Event selection cuts**

[The ATLAS Collaboration, CERN-OPEN-2008-020]

1. SS dileptonic event with $N_{jet} \geq 2$
2. $P_T(\ell_1, \ell_2) \geq 20$ GeV
3. $P_T(j_1, j_2) \geq (100,80)$ GeV
4. *Missing Tr. P* ≥ 100 GeV

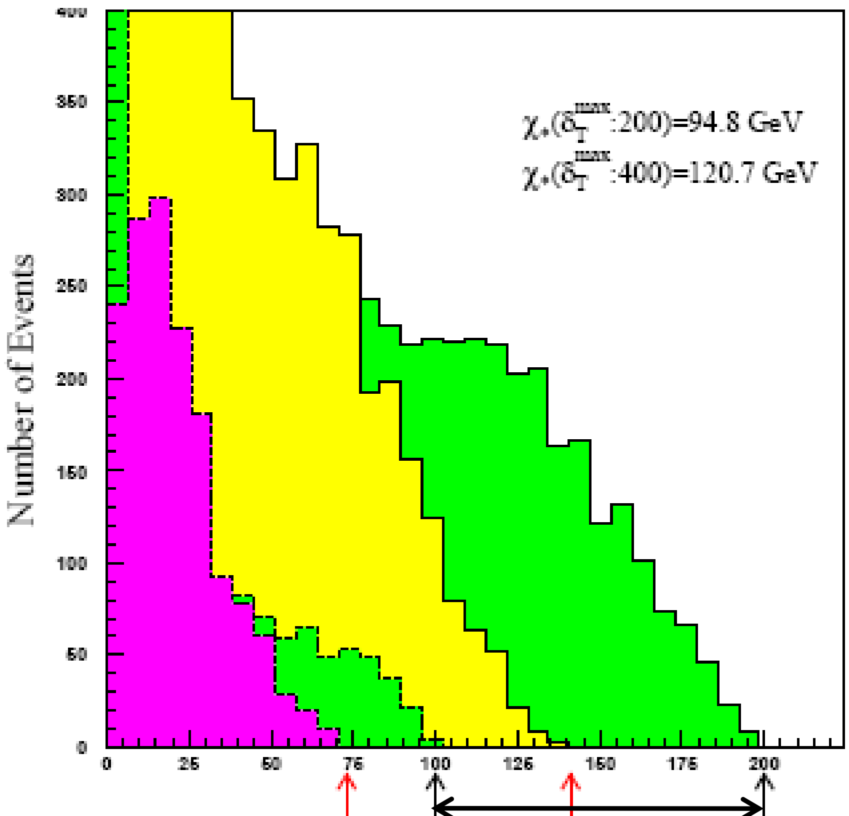
Additionally,

5. $\delta_T \leq \delta_T^{max}$= 200 / 400 GeV

Result)

$M_{CT2}(\chi)$-χ distribution of SS Dileptonic events with $\delta_T \leq 200,400$ GeV

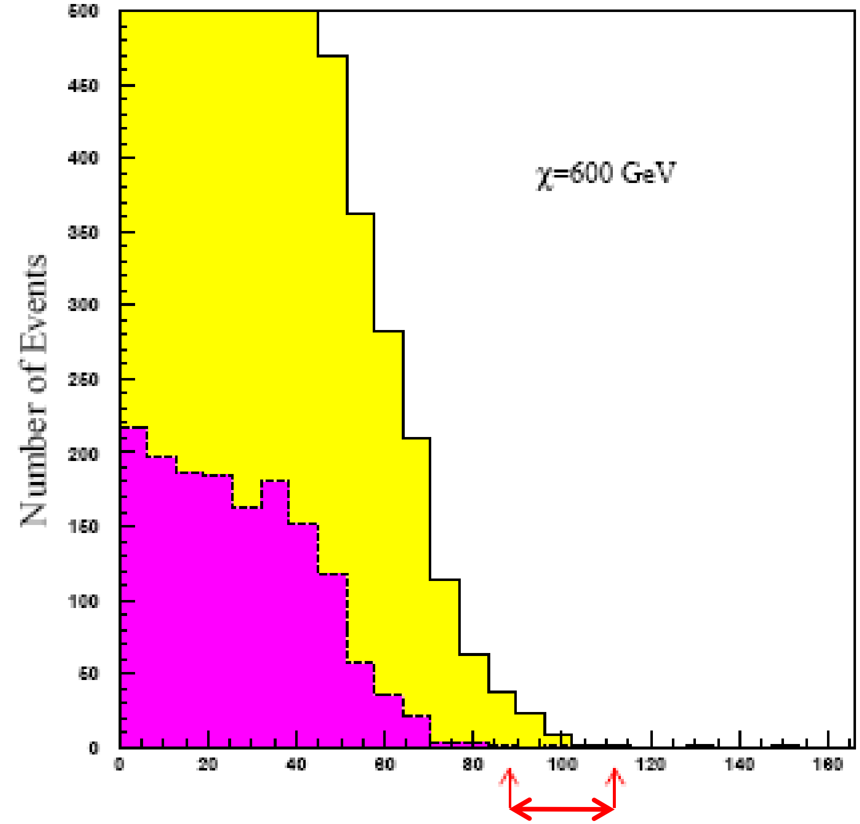
$M_{CT2}(\chi=0|\chi_*+10,\delta_T^{max}=200|400)$ - χ



ΔM_{CT2}^{max}=100 GeV

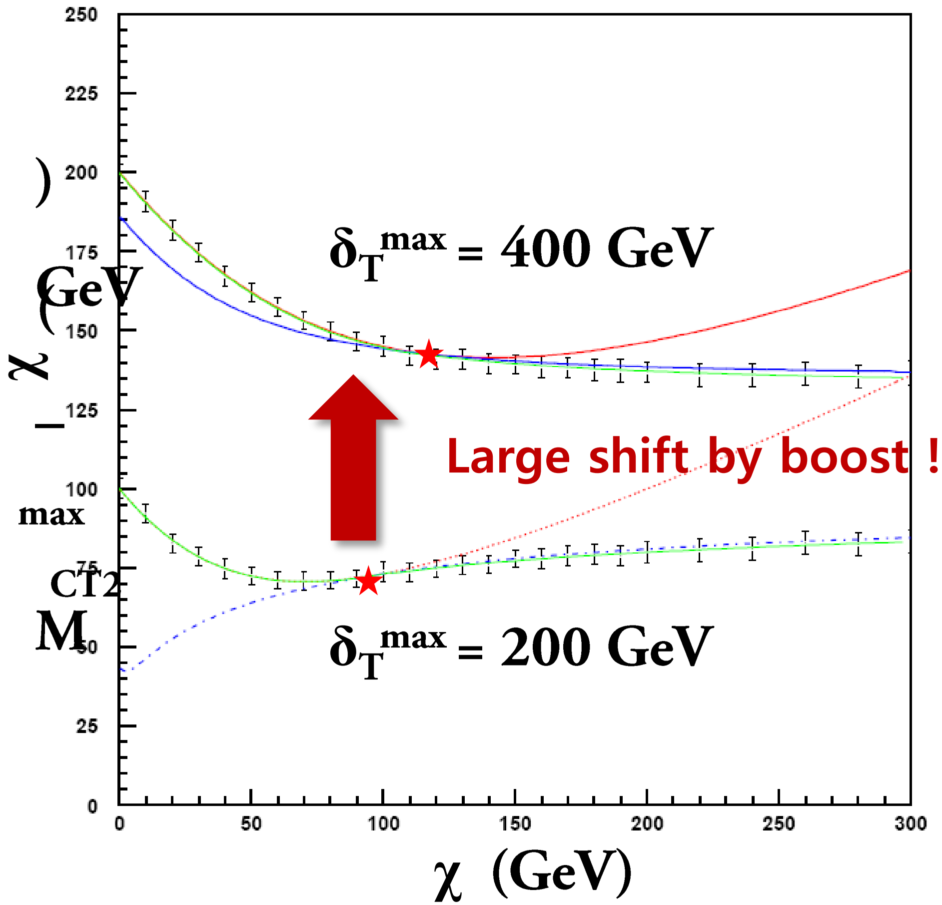
ΔM_{CT2}^{max}~ 65 GeV

$M_{T2}(\chi=600,\delta_T^{max}=200|400)$-$\chi$



ΔM_{T2}^{max}~ 20 GeV

Result) True/Reconstructed $M_{CT2}^{max}(\chi)$, Reconstructed Masses



★ : **Position of the kink,**

$$\chi_*^{\ 2} = \frac{|\delta_T^{\ max}|}{2}\left(2\alpha - \frac{|\delta_T^{\ max}|}{2}\right)$$

$$= (94.8{,}120.7)\ \text{GeV}$$

$$\alpha \equiv \left(\frac{m_Y^{\ 2} - m_X^{\ 2}}{2m_Y}\right)\left[\frac{|\delta_T^{\ max}|}{2m_Y} + \sqrt{1 + (\frac{|\delta_T^{\ max}|}{2m_Y})^2}\right]$$

Segmented fitting with $M_{CT2}^{max}(\chi,\delta_T^{max})$ provides two α values.

$$\alpha(\delta_T^{\ max} = 200\,|\,400)\ (GeV)$$

$$= (92.4 \pm 2.5\,|\,132.6 \pm 3.4)$$

$$\Rightarrow (M_{\tilde{\chi}_1^{\pm}}, M_{\tilde{\nu}_\ell})$$

$$= (231.2 \pm 9.9\,|\,159.3 \pm 5.9)$$

Summary

1. M_{CT2} distribution can have compact endpoint structure with respect to the internal momentum scale from decay system.

→ Small slope discontinuities are amplified by $J(x)^2$, accentuating the breakpoint structures clearly, reducing sys. uncertainties.

2. It shows very sensitive endpoint recoiling by the external boost momentum of the decay system like a flubber ball.

→ Can have signficant resolving powers by the boost effect.

3. It can be utilized for the mass measurement in boosted decay systems which must be the most fundamental and general element of complex event topologies at future hadron colliders.

Backup slides

Error analysis with histogram : $(x_i, y_i \pm \sigma_i)$

$\sigma_i =$ statistical error of the i-th bin

Statistical error for breakpoint(BP)

(using Least Square methods)

$$\delta_{BP}{}^2 \sim \frac{\sigma^2}{\Delta a^2} \rightarrow \frac{J^2 \sigma^2}{J^4 \Delta a^2} \sim \frac{1}{J^2} \delta_{BP}{}^2$$

$$\therefore \delta_{BP}{}^{(stat)}(M_{\pi T2}) \sim \frac{1}{J} \delta_{BP}{}^{(stat)}(M_{T2})$$

However, the error propagation factor ~ J for getting p^0,

$\delta_{p^0}{}^{(stat)}(M_{\pi T2}) \sim \delta_{p^0}{}^{(stat)}(M_{T2})$: No advantage for statistical errors.

Systematic error for BP using Segmented Linear Regression:

$(\mathrm{x_i}, \mathrm{y_i}) \rightarrow$ Find the BP with maximal "Coefficient of Explanation"

$$\delta_{BP}{}^{2} \sim \frac{\sum \varepsilon^{2}}{\Delta a^{2}} \rightarrow \frac{\sum \varepsilon'^{2}}{J^{4} \Delta a^{2}} \sim \frac{1}{J^{4}} \delta_{BP}{}^{2}$$

$(\sum \varepsilon'^{2} \sim \sum \varepsilon^{2},$ similar square sum of residuals after maximization *with* elaborated fitting functions)

$$\therefore \delta_{BP}{}^{(sys)}(M_{\pi T2}) \sim \frac{1}{J^{2}} \delta_{BP}{}^{(sys)}(M_{T2})$$

Taking into account the error propagation factor,

$$\delta_{p^{0}}{}^{(sys)}(M_{\pi T2}) \sim \frac{1}{J} \delta_{p^{0}}{}^{(sys)}(M_{T2}) : \text{ O(1/J) reduction is expected!}$$

Combinatoric - $M_{CT2}(\alpha_i - \beta_j)$ [work in progress]

Let's take a system of interest with transverse momentum, **- δ_T**.

$$\mathbf{pp} \rightarrow (\delta_T) + (\alpha_1, \ldots \alpha_\mathrm{i} \ldots \alpha_\mathrm{N} / \beta_1 \ldots \beta_\mathrm{j} \ldots \beta_\mathrm{N} / \text{New physics missing PTLs})$$

$$\rightarrow (\delta_T) + (\alpha_\mathrm{i} \quad \beta_\mathrm{j} + \text{(assumed to be) missing particles}(\not{E}_\mathrm{T}'))$$

(i=1..N, j=1..M)

$$C - M_{CT2}{}^2(\alpha_\mathrm{i}\text{-}\beta_\mathrm{j}) \equiv \min[\max\{M_{CT}(A_i), M_{CT}(B_j)\}]$$

$$M_{CT} \equiv \chi^2 + 2|\mathbf{p_T}|\sqrt{\chi^2 + |\mathbf{k_T}|^2} + 2\mathbf{p_T} \cdot \mathbf{k_T},$$

- $\mathbf{p_T}$ = visible transverse momenta
- ***χ* = universal test mass for** A_{i+1} & B_{j+1}(in general $M_{A_{i+1}} \neq M_{B_{j+1}}$)
- $\mathbf{k_T}(\alpha) + \mathbf{k_T}(\beta) = -(\alpha_{iT} + \beta_{jT}) - \delta_T = \not{E}_T'$
- min&max over all possible invisible missing momentum $\mathbf{k_T}$