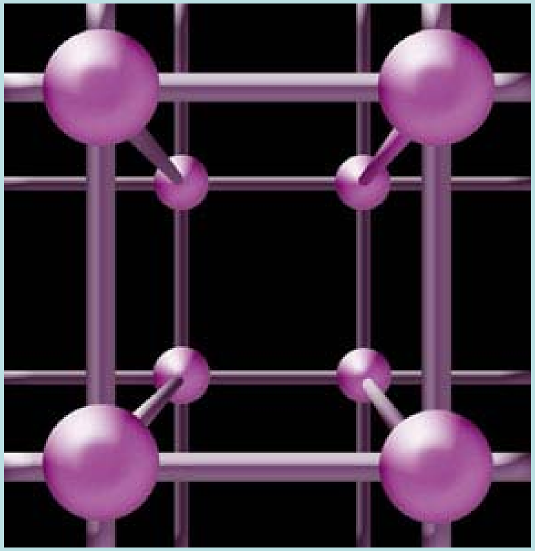
TeV Physics and Conformality



UC Davis

September 27, 2010

Beyond QCD on the Lattice

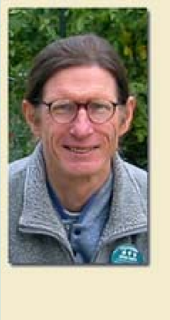
1. Collaboration with George Fleming and Ethan Neil
   1) arXiv:0712.0609, PRL 100, 171607 , 2008

   2) arXiv:0901.3766 PR D79, 076010, 2009

2. LSD collaboration **J. C. Osborn, R. Babich, R. C. Brower, M. A. Clark, C. Rebbi, D. Schaich, M. Cheng, J. Kiskis, R. Soltz, P. M. Vranas, T. Appelquist, G. T. Fleming, E. T. Neil, M. Lin**

   1) arXiv:0910.2224 PRL 104, 071601 (2010)

   2) arXiv: this week (David Schaich)



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USQCD clusters, LLNL BG/L, NSF Teragrid

New Strong Dynamics at TeV Energies?

New, SM-singlet Sector
 Conformal behavior

Electroweak breaking
 Near-conformal infrared behavior: walking technicolor, conformal technicolor (M. Luty et al)

Other possible uses of (near) conformal-symmetry
 Flavor hierarchies (Nelson&Strassler), SUSY flavor problem

Conformal Symmetry / Scale Invariance / Dilatation Symmetry

No fixed scales (masses)

Might expect at high energies or temperatures

Examples $(h/2\pi = c = 1)$:

$\sigma\ (e+e- \rightarrow \text{Hadrons}) \sim 1/\ E_{cm}^2$

$F(T) \sim T^4$

Not Quite

Quantum field theories *typically* depend on a renormalization scale Λ.

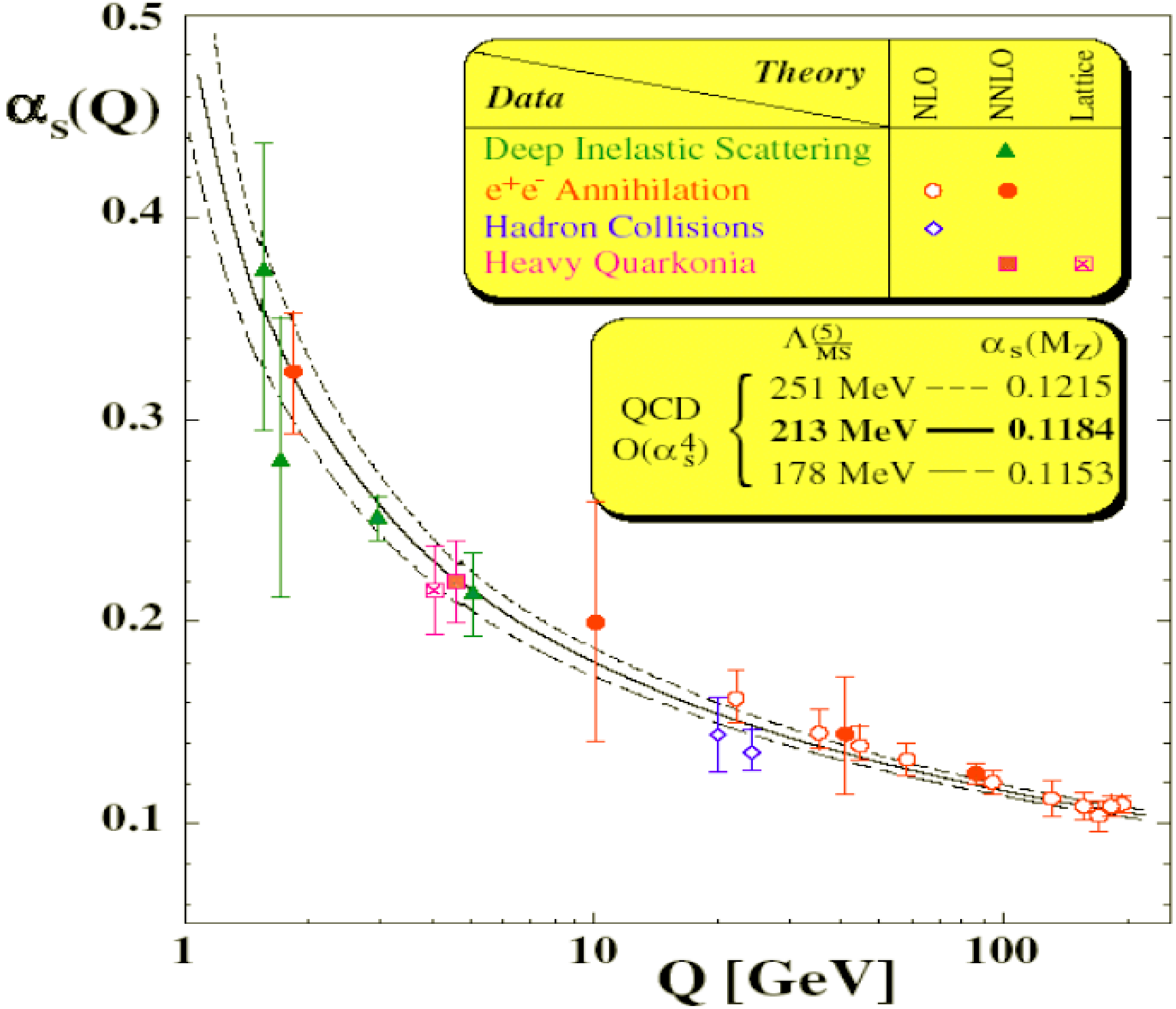
$$\alpha \equiv g^2/4\pi = \alpha(q^2 / \Lambda^2)$$

For a Yang-Mills theory

$$\underset{q^2\to\infty}{\sim} 1/\log(q^2 / \Lambda^2)$$

Asymptotic freedom

QCD



$\alpha_s(Q)$
0.5
0.4
0.3
0.2
0.1
1
10
100
Q [GeV]
Theory
Data
NLO
NNLO
Lattice
Deep Inelastic Scattering
e+e- Annihilation
Hadron Collisions
Heavy Quarkonia
$\Lambda^{(5)}_{\overline{MS}}$
$\alpha_s(M_Z)$
QCD
$O(\alpha_s^4)$
251 MeV 0.1215
213 MeV 0.1184
178 MeV 0.1153

QCD Infrared Features

1. Quark and gluon confinement

   $\Lambda \sim 200$ MeV

2. Spontaneous chiral symmetry breaking $SU(2)_L x SU(2)_R \rightarrow SU(2)$

3 Pions = PNGB's

F = 93 MeV

$<\bar{\psi}\psi> \sim 4\,\pi\,F^3$

TeV-Scale Theories

- SU(N) Gauge Theories with N_f Massless Fermions (fundamental and other representations)

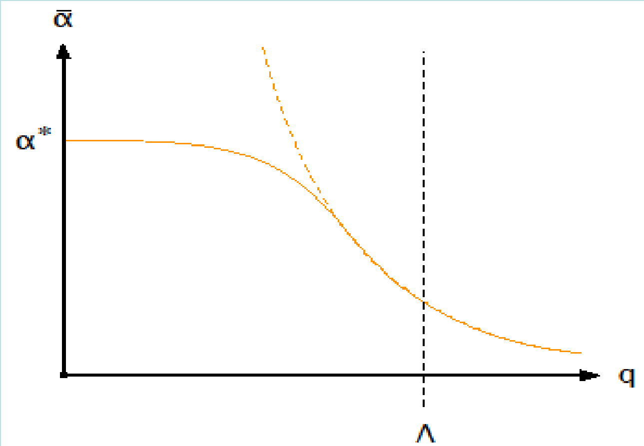
- Asymptotically-free (can take lattice spacing to zero) $N_f < N_{af}$

- Vary N_f and study how the infrared behavior changes (Chiral symmetry breaking and confinement versus *infrared* conformal behavior.)

SUSY SU(N) Theories: Strange and interesting possibilities

Conformal Window

$N_{af} > N_f > N_{fc}$ (Fermion screening)



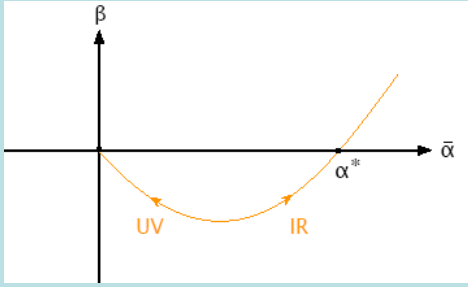
Perturbative for $N_f \rightarrow N_{af}$

Gross and Wilczek, antiquity

William Caswell, 1974 --→

Banks and Zaks, 1982

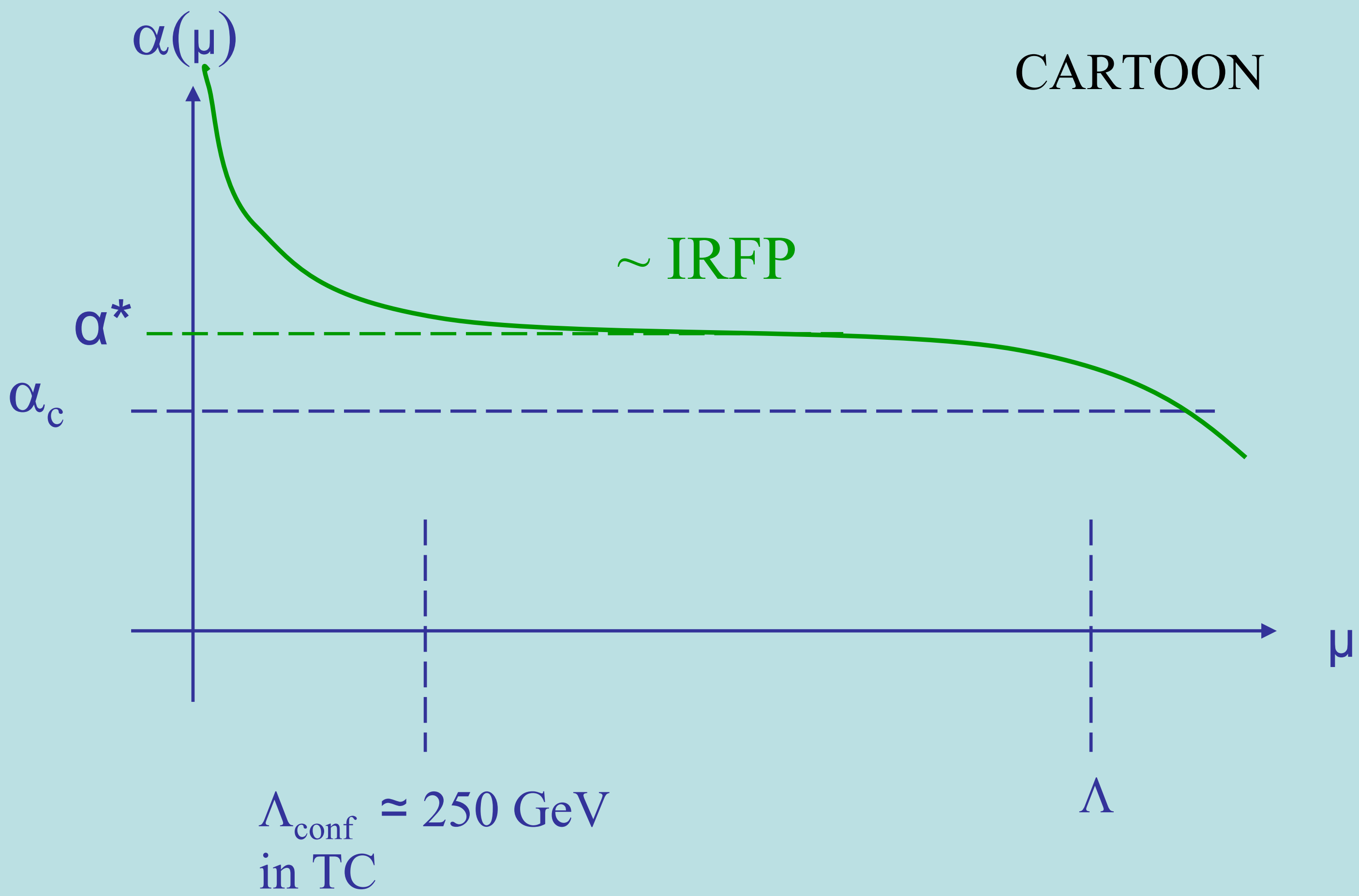
……….



α^* increases as N_f decreases.

N_{fc}: α^* becomes large enough to trigger chiral symmetry breaking and confinement.
(Probably not accessible in perturbation theory)

NEAR-CONFORMAL BEHAVIOR
For N_f just below N_{fc} (Walking)



Questions

1. Value of N_{fc}?

2. Order of the phase transition?

2. Correlation functions and anomalous dimensions inside the conformal window ($N_f > N_{fc}$)

3. Below and near the transition ($N_f < N_{fc}$)

   Approximate IRFP (Walking)? Condensate enhancement?
   Parity Doubling? EW precision studies ? Dilaton ?
   Implications for the LHC?

Topics

(1) N_{fc} in SU(N) QCD

→Walking

(2) Dilaton (1 Slide)

(3) Chiral Symmetry Breaking and Condensate Enhancement

(4) Parity Doubling and EW Precision Studies

(1) N_{fc} in SU(N) QCD

- Degree-of-Freedom Inequality (Cohen, Schmaltz, TA 1999). Fundamental rep:

  ← Conjecture

  $N_{fc} < \ 4\,N[1 - 1/18N^2 + \ldots]$

- Gap-Equation Studies, Instantons (1996 ….):

  $N_{fc} \cong 4\,N$

  Pioneers:
  Maskawa & Nakajima 1974
  Kugo & Fukuda 1975

  ←Suspect methods

Lattice-Simulation Study of the Extent of the Conformal window in an SU(3) Gauge Theory with N_f Fermions in the Fundamental Representation

Conformality violated by a, L !!

Fleming, Neil, TA
2007 …….

Earlier, inconclusive lattice work

Iwasaki et al (2004) SU(3): $N_{fc} \sim 7$

Focus: Gauge Invariant and Non-Perturbative Definition of the Running Coupling from the Schroedinger Functional of the Gauge Theory

ALPHA Collaboration: Luscher, Sommer, Weisz, Wolff, Bode, Heitger, Simma, …

Transition amplitude from a prescribed state at t=0 to one at t=T= L± a (Dirichlet BC).(m = 0)

Using Staggered Fermions as in

U. Heller, Nucl. Phys. B504, 435 (1997)
Miyazaki & Kikukawa

Focus on N_f = multiples of 4:

16: Perturbative IRFP

12: IRFP "expected", Simulate

8 : IRFP uncertain , Simulate

4 : Confinement, ChSB

SF Running Coupling

$$\frac{1}{\bar{g}^2(L)} = \frac{1}{2}\left[\frac{1}{\bar{g}^2(L, L-a)} + \frac{1}{\bar{g}^2(L, L+a)}\right]$$

One Large Scale!

In perturbation theory:

$N_f = 16$ IRFP at $g^{*2}_{SF} = 0.47$ $(g^{*2}_{SF}/4\pi \approx .04)$

$N_f = 12$ IRFP at $g^{*2}_{SF} = 5.18$ $(g^{*2}_{SF}/4\pi \approx 0.4)$

$N_f \leq 8$ No perturbative IRFP

Lattice Simulations

MILC Code (Urs Heller)

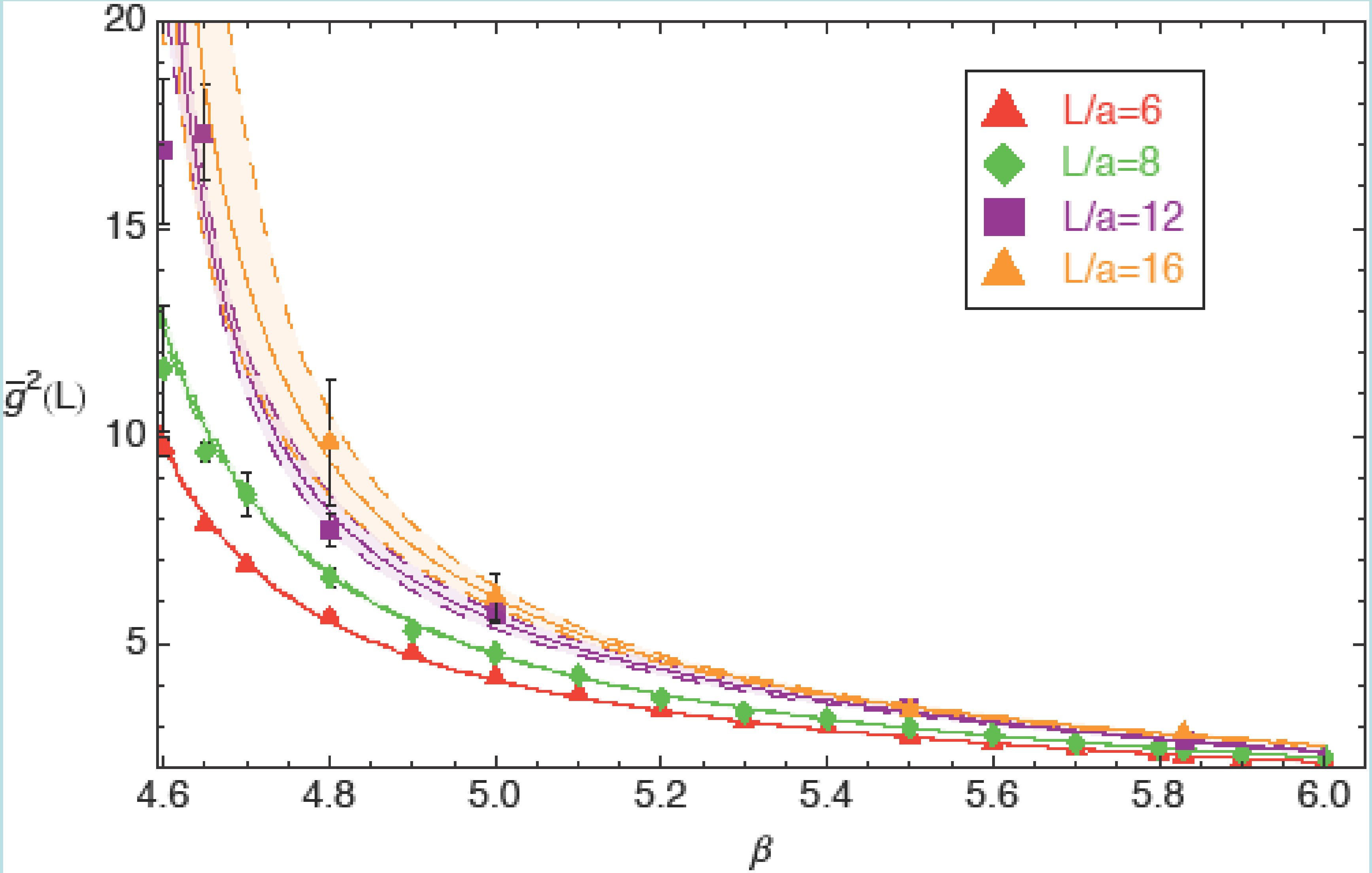
Staggered Fermions

$$N_f = 8, 12$$

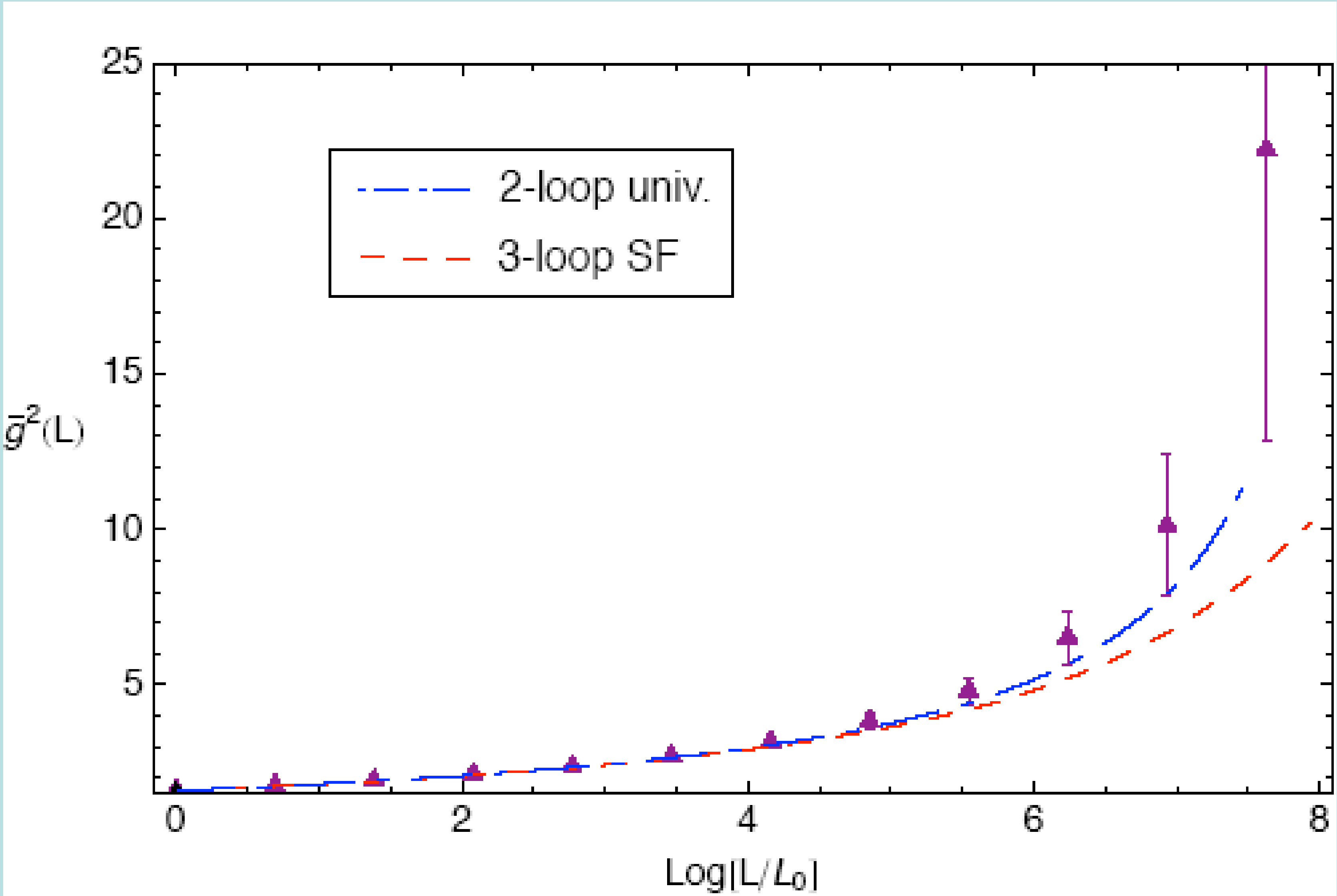
Range of Lattice Couplings g_0^2 (= 6/β) and Lattice Sizes L/a → 20

O(a) Lattice Artifacts due to Dirichlet Boundary Conditions

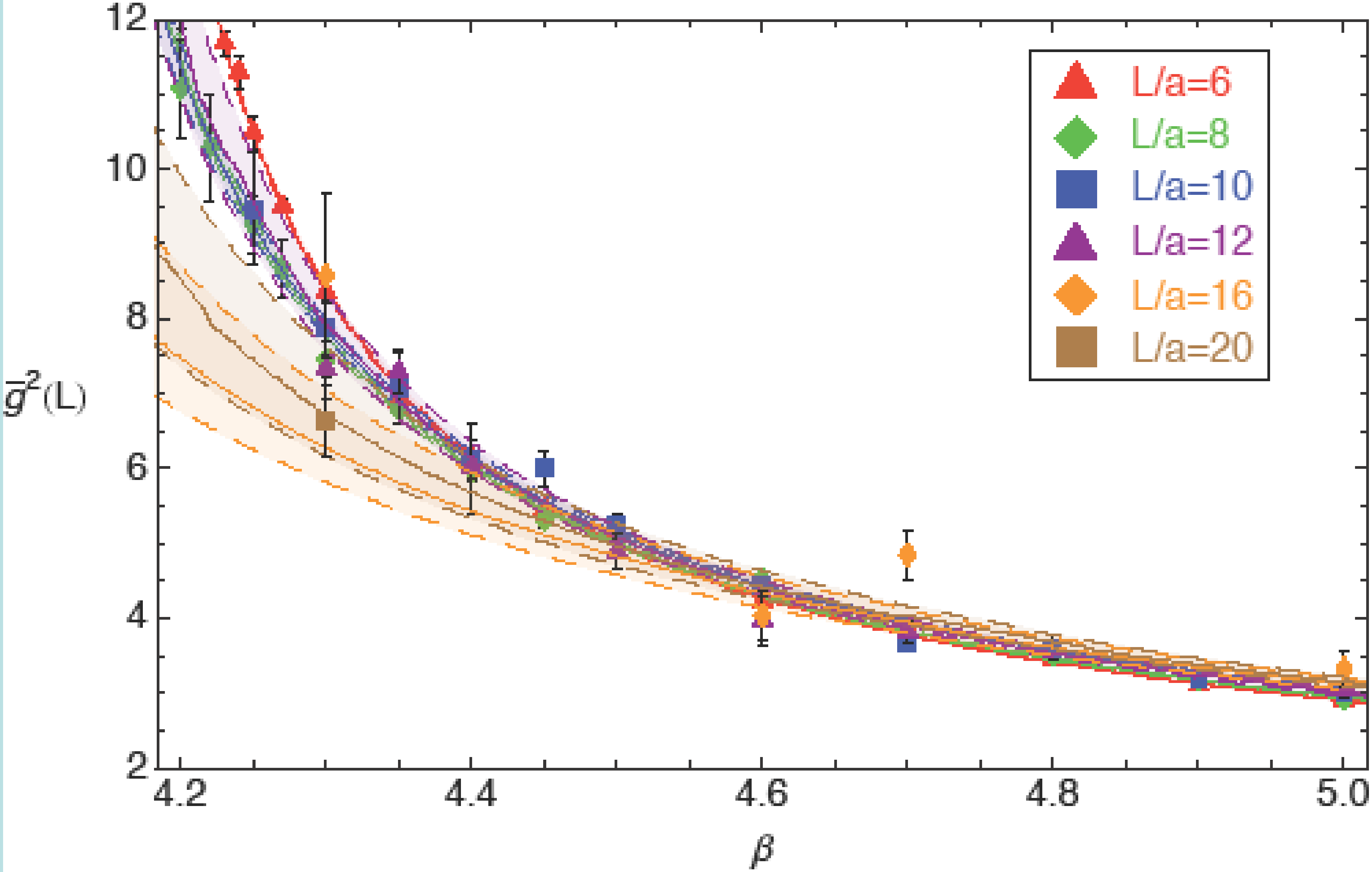
$N_f = 8$ Data with Fits



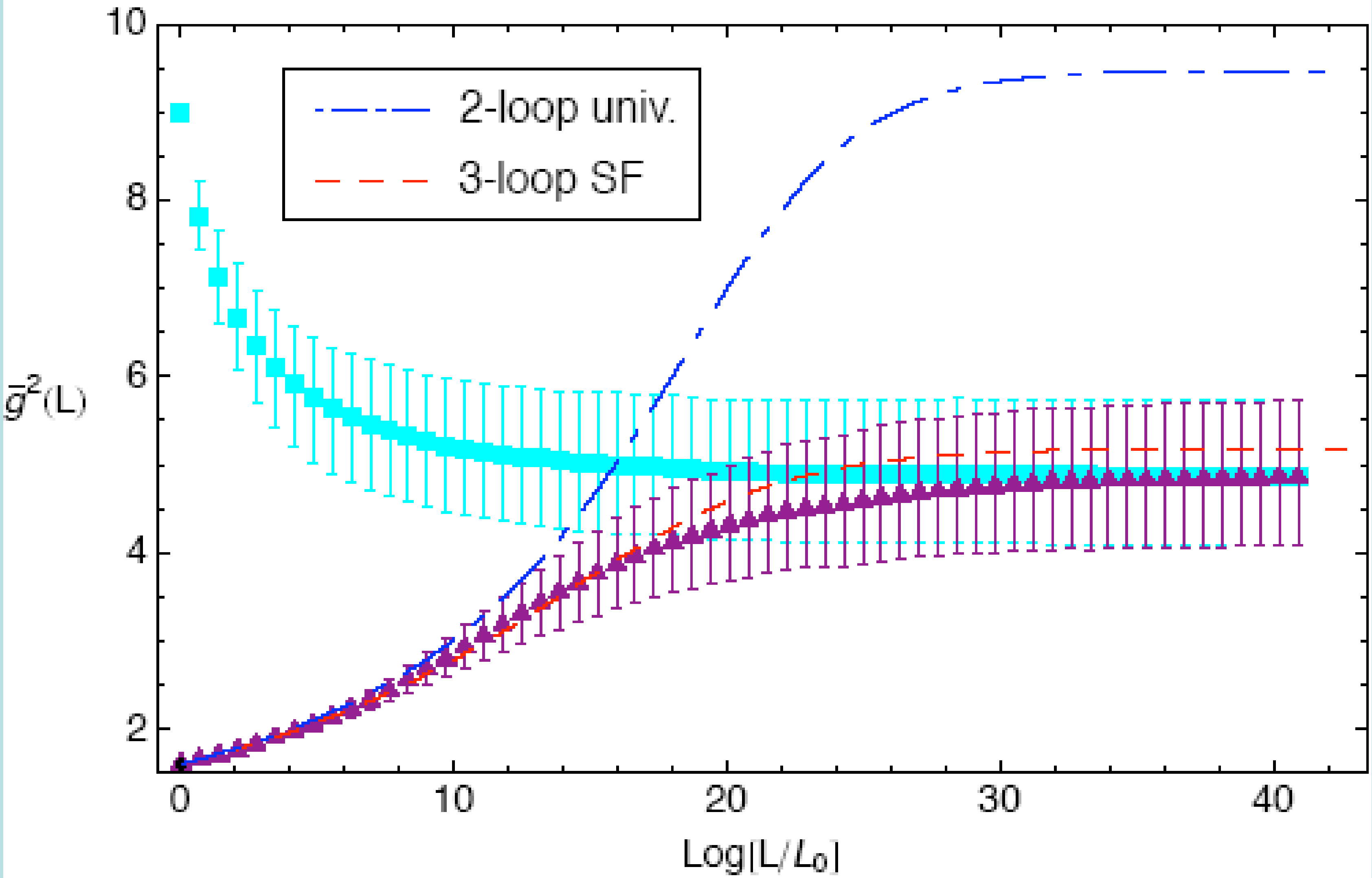
N_f = 8 Continuum Running



$N_f = 12$ Data with Fits



N_f = 12 Continuum Running



Conclusions

1. Lattice evidence that for an SU(3) gauge theory with N_f Dirac fermions in the fundamental representation $8 < N_{fc} < 12$

2. N_f=12: Relatively weak IRFP

3. N_f=8: Confinement and chiral symmetry breaking.

Employing the Schroedinger-functional running coupling defined <u>at</u> the box boundary L

Current $N_f = 12$ Activity

Summary of lattice results at $N_f = 12$ (3-color fundamental)

Group	Fermion action	Gauge action	Method	Conformal?	Refs (arXiv)
Fleming, Neil, TA	staggered	unimproved (plaquette)	SF coupling	Yes	0712.0609 0901.3766
Jin & Mawhinney	staggered	improved (DBW2)	Spectrum	No	0910.3216
Hasenfratz	improved staggered (stout)	unimproved (plaquette)	MCRG	Yes	0911.0646 priv. comm.
Kuti, Holland, Fodor, et al.	improved staggered (stout)	improved (Symanzik)	Spectrum	No	0911.2463
Deuzeman, Lombardo, Pallante	improved staggered (Naik)	improved (Symanzik)	Thermal trans.	Yes	0904.4662

Lots of Related Activity

1. Other N_f values such as N_f=10.

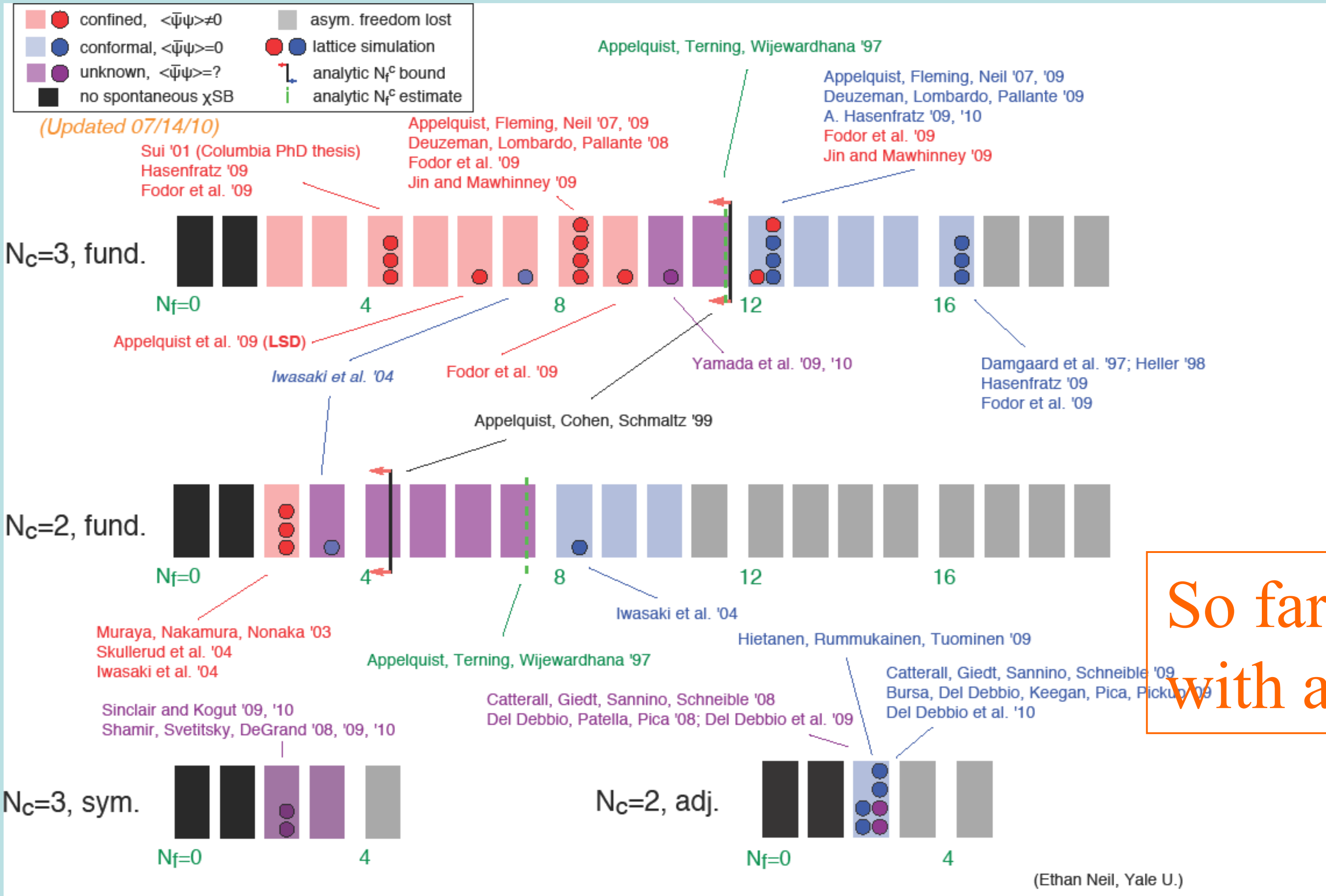
   Yamada et al

2. Other gauge groups and representation assignments for the fermions

   Sannino, del Debbio, DeGrand, Shamir, Svetitsky; Hietenan et al, Sinclair, Kogut, Catterall. ……..

3. Examine physical quantities such as the static potential (Wilson loop) and correlation functions.

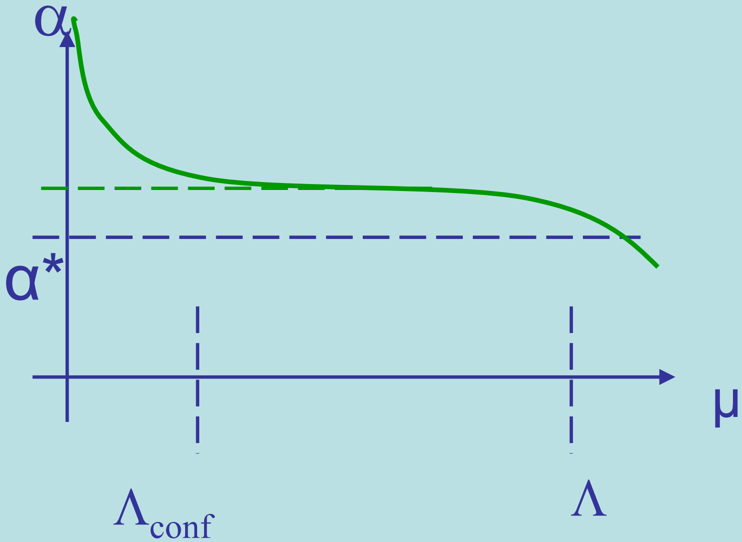
   Bilgici et al arXiv:0902.3768



So far, consistent with a single $N_{fc}(N)$

Lattice is now a tool for studying strongly coupled gauge theories.

(2) Dilaton



An (approximate) NGB (a PNGB) associated with the spontaneous breaking of (approximate) scale symmetry

Yang Bai and TA
arXiv:1006.4375 : $M_d^2 \sim (N_{fc} - N_f)\Lambda^2_{conf}$

Dilaton Phenomenology:
Goldberger, Grinstein, Skiba PRL 2008

(3) Chiral Symmetry Breaking and Condensate Enhancement (LSD)

arXiv:0910.2224
PRL 104, 071601 (2010)

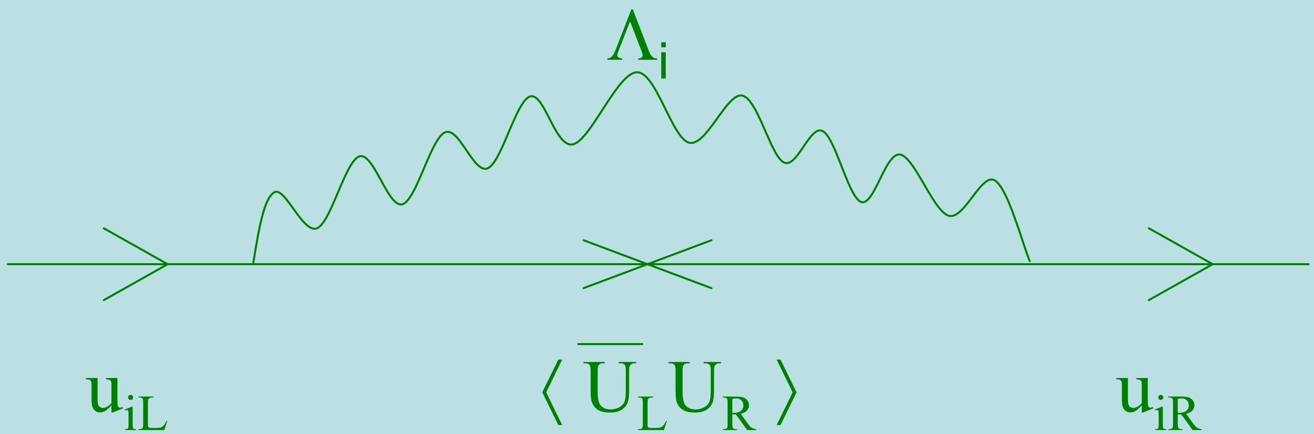
Walking:

As the conformal window is approached ($N_f \rightarrow N_{fc}$),
$<\bar{\psi}\,\psi>$ is enhanced relative to its nominal value $4\pi\, F^3$.

LSD Program:

Search for enhancement of $<\bar{\psi}\,\psi> / F^3$ by starting at $N_f = 2$,
and then $N_f = 6$. (Creeping Toward the Conformal Window)
($\Lambda = a^{-1}$)

WHY?



$$M_{ii}^{(u)} \cong 4\pi \frac{F^3}{\Lambda_i^2}$$

$$\exp\left\{\int \frac{d\mu}{\mu} \gamma[\alpha_{TC}(\mu)]\right\}$$

$d=3-\gamma$

$\gamma \rightarrow 1$?

$\gamma \rightarrow 2$?

M. Luty

Some Details

- Domain-wall fermions, Iwasaki improved action
- USQCD: Chroma, CPS

- $32^3 \times 64$ lattice ($L_s = 16$)
- $m_f = .005, .01, .015, .02, .025$, $m = m_f + m_{res}$

- $N_f^2 - 1$ PNGB's

- Simulate: M_P , F, $<\bar{\psi}\ \psi>$, M_V $M_P L > 4$

Extrapolate to m=0 with Chiral Perturbation Theory

- $M^2_{Pm} = 2m \langle \psi\, \psi \rangle / F^2 \{1 + zm\, [\alpha_{M1} + (1/N_f)\log(zm)] + \ldots\}$

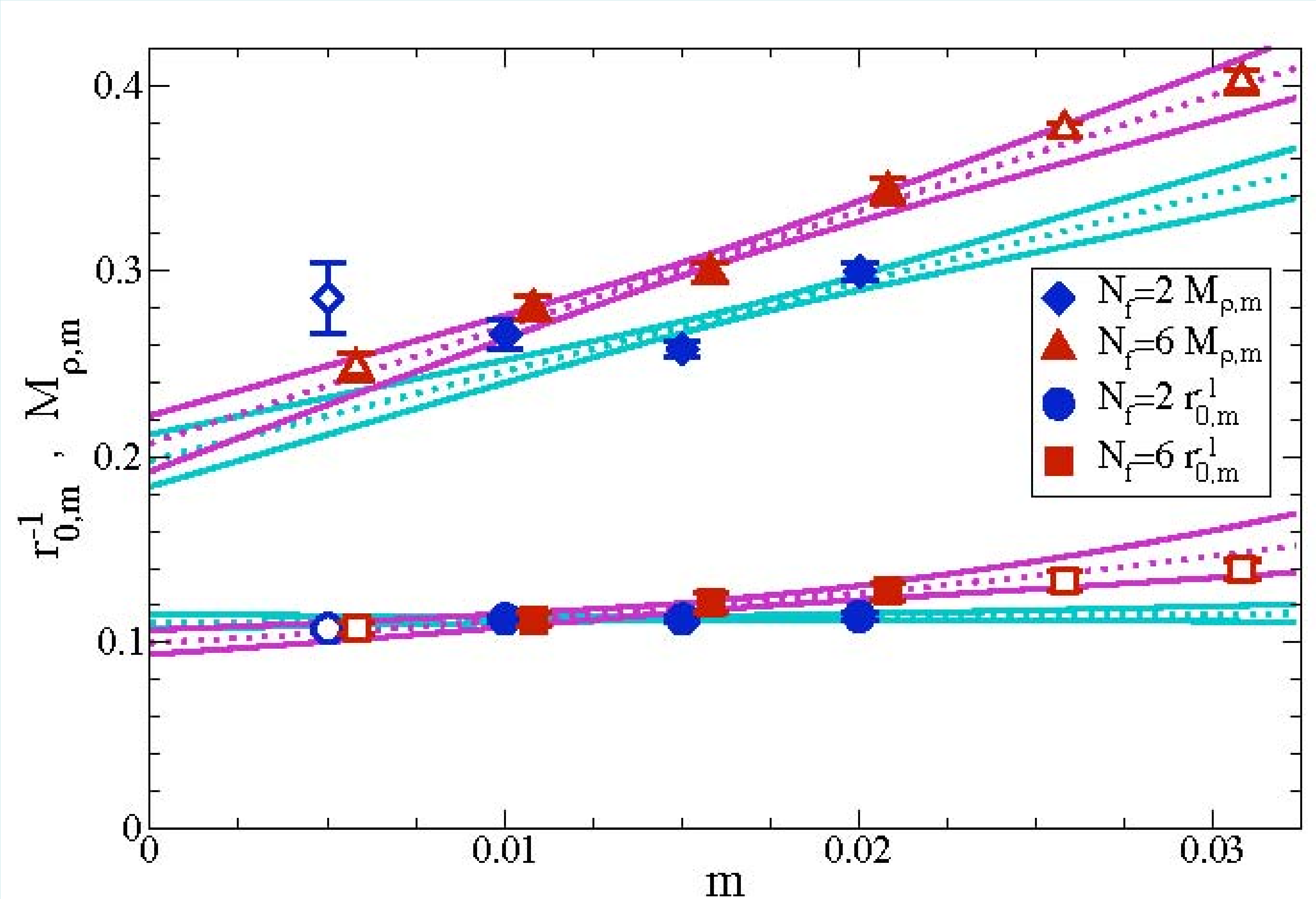
$$z \equiv 2 \langle \bar{\psi}\, \psi \rangle / (4\pi)^2 F^4$$

- $F_m = F\{ 1 + zm\, [\alpha_{F1} - (N_f/2)\log(zm)] + \ldots\}$

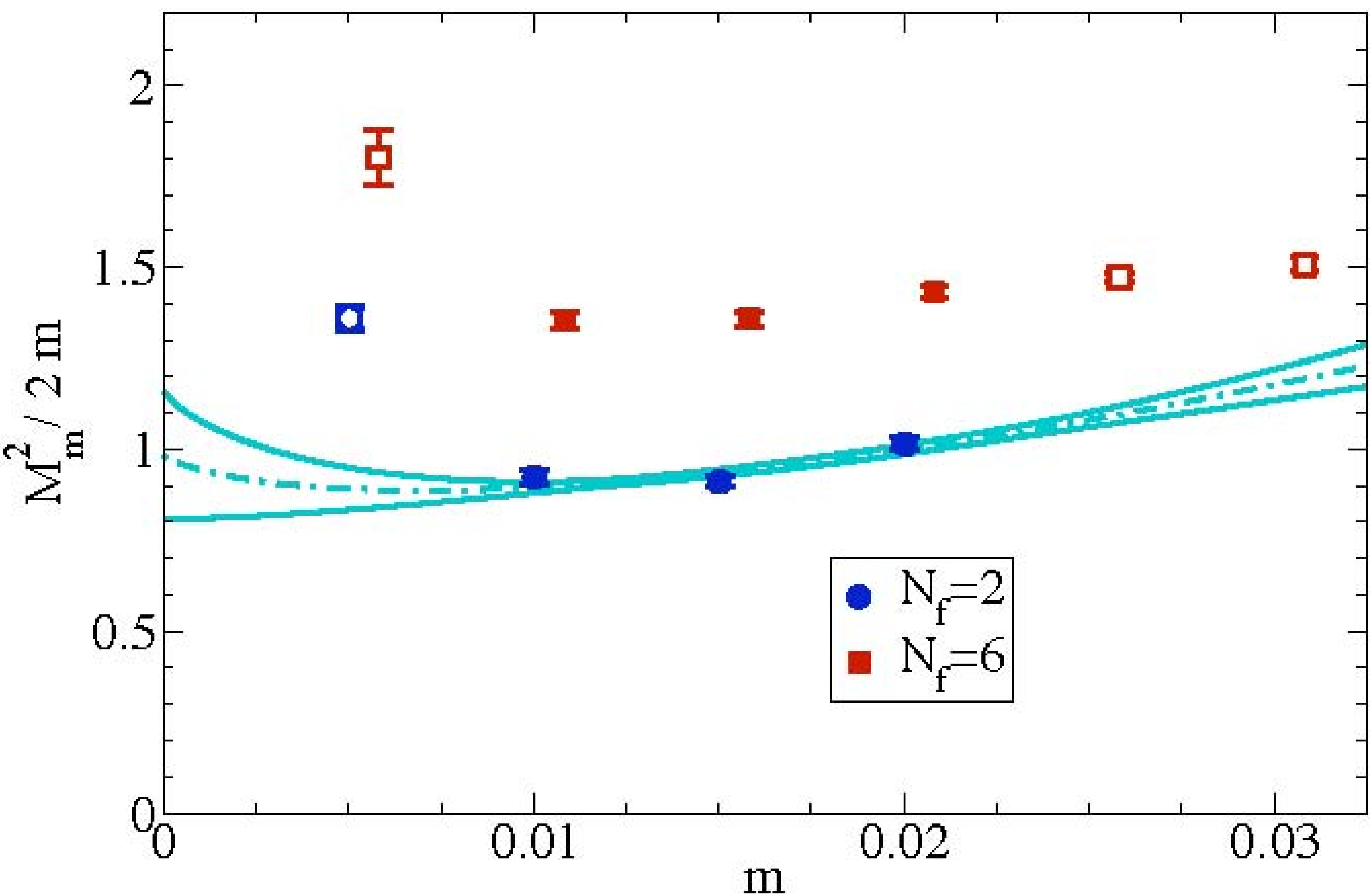
- $\langle \bar{\psi}\, \psi \rangle_m = \langle \bar{\psi}\, \psi \rangle \{1 + zm\, [\alpha_{C1} - ((N_f^2 - 1)/N_f)\log(zm)] + \ldots\}$

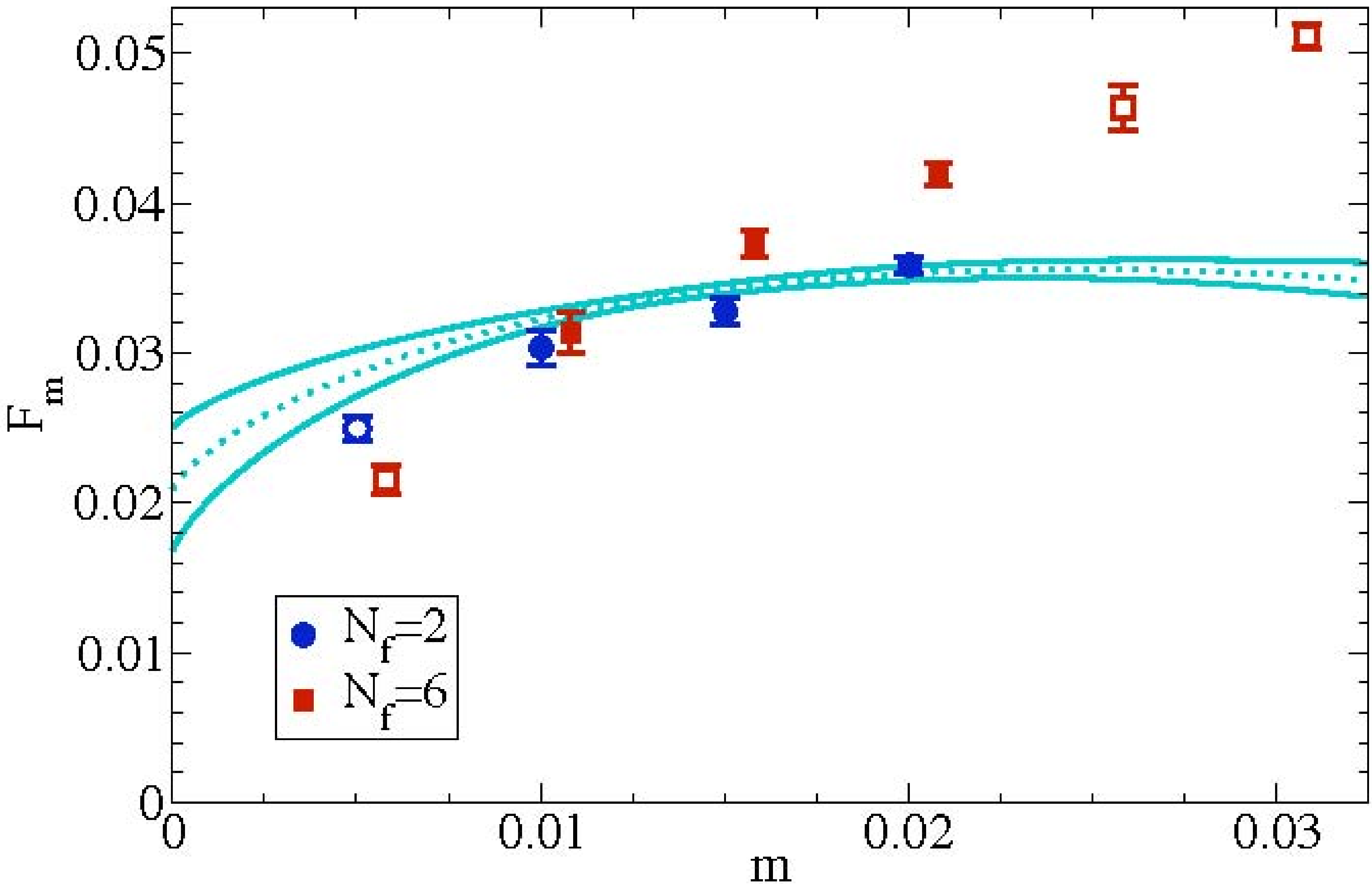
$$M_{Vm} = M_V \{ 1 + \alpha_{R1}\, zm + \alpha_{R3/2}\, (zm)^{3/2} + \ldots\}$$

$$M_{Am} = M_A \{ 1 + \alpha_{A1}\, zm + \alpha_{A3/2}\, (zm)^{3/2} + \ldots\}$$

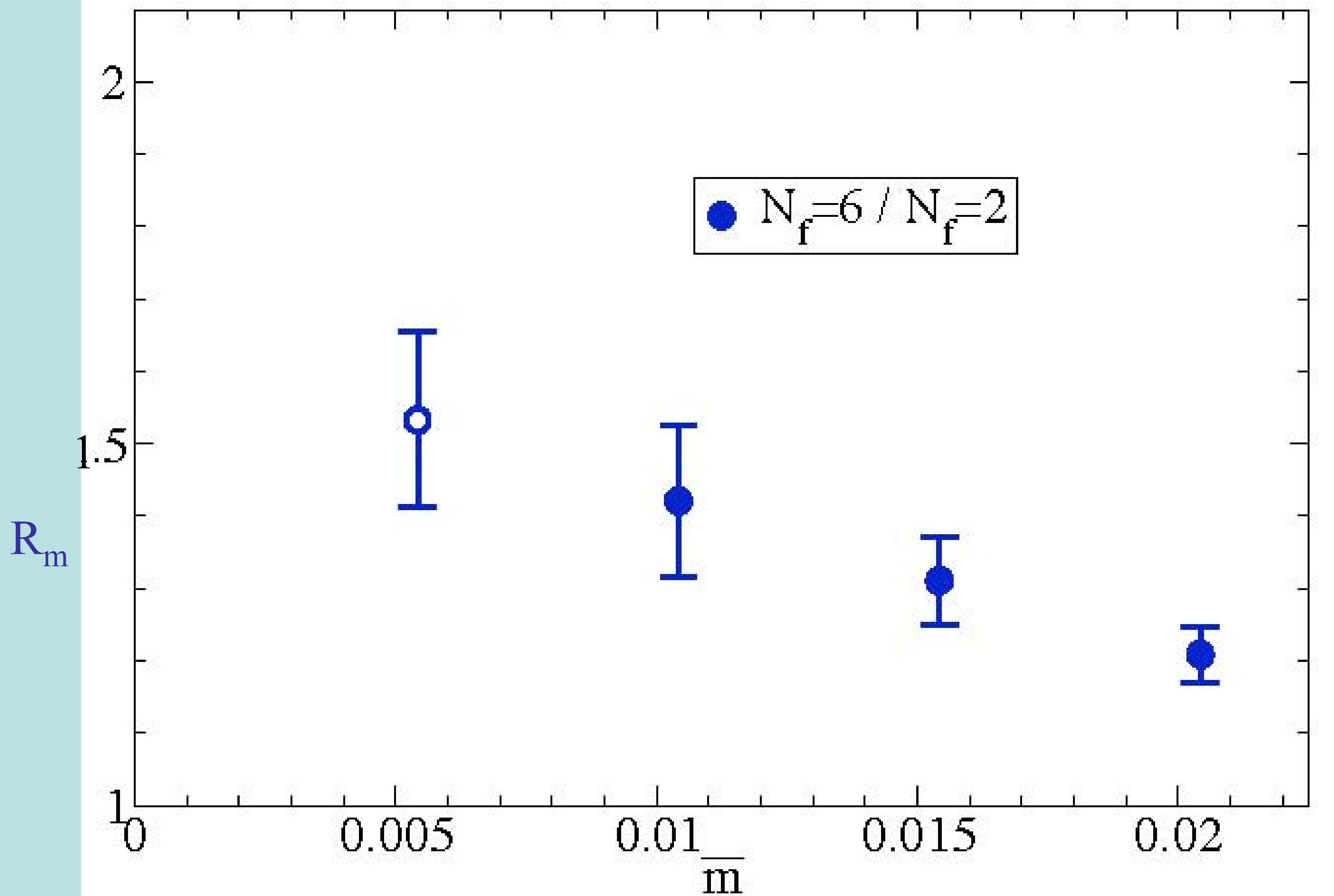


$N_f = 2$: $\beta = 2.7$ $N_f = 6$: $\beta = 2.1$


$M_m^2 / 2m$
m
0
0.5
1
1.5
2
0.01
0.02
0.03
N_f=2
N_f=6



0.05
0.04
0.03
0.02
0.01
0
F_m
0
0.01
0.02
0.03
m
N_f=2
N_f=6



$R_m = [M^2_{Pm}/2mF_m]_{6f} / [M^2_{Pm}/2mF_m]_{2f}$

$N_f = 2$

- Chiral perturbation theory extrapolation:

$$\langle \bar{\psi} \psi \rangle / F^3 = 47.1\ (17.6)$$

QCD Experimental Value: (renormalized to our lattice scheme - Aoki et al hep-lat/0206013)

$$\langle \bar{\psi} \psi \rangle / F^3 = 36.2\ (6.5)$$

$N_f = 6$

Linear Extrapolation →

Very Conservative Lower Bound on $\langle \bar{\psi} \psi \rangle / F^2$

Very Conservative Upper Bound on F

Thus

$$\langle \bar{\psi} \psi \rangle / F^3 \geq 60.0\ (8.0)$$

(4) Resonance Spectrum and the S parameter (LSD)

arXiv: this week

Is S naturally small as $N_f \rightarrow N_{fc}$ due to approximate parity doubling?

$$S = 4\pi\,(N_f/2)[\Pi'_{VV}((0) - \Pi'_{AA}(0)] - \Delta S_{SM}$$

$$= \int_0^\infty \frac{ds}{s}\left\{ (Nf/2)\left[\operatorname{Im}\Pi_{VV}(s) - \operatorname{Im}\Pi_{AA}(s)\right] - \frac{1}{12\pi}\left[1 - \left(1 - \frac{m_{H,ref}}{s}\right)^3 \theta\left(s - m_{H,ref}^2\right)\right]\right\}$$

Peskin and Takeuchi

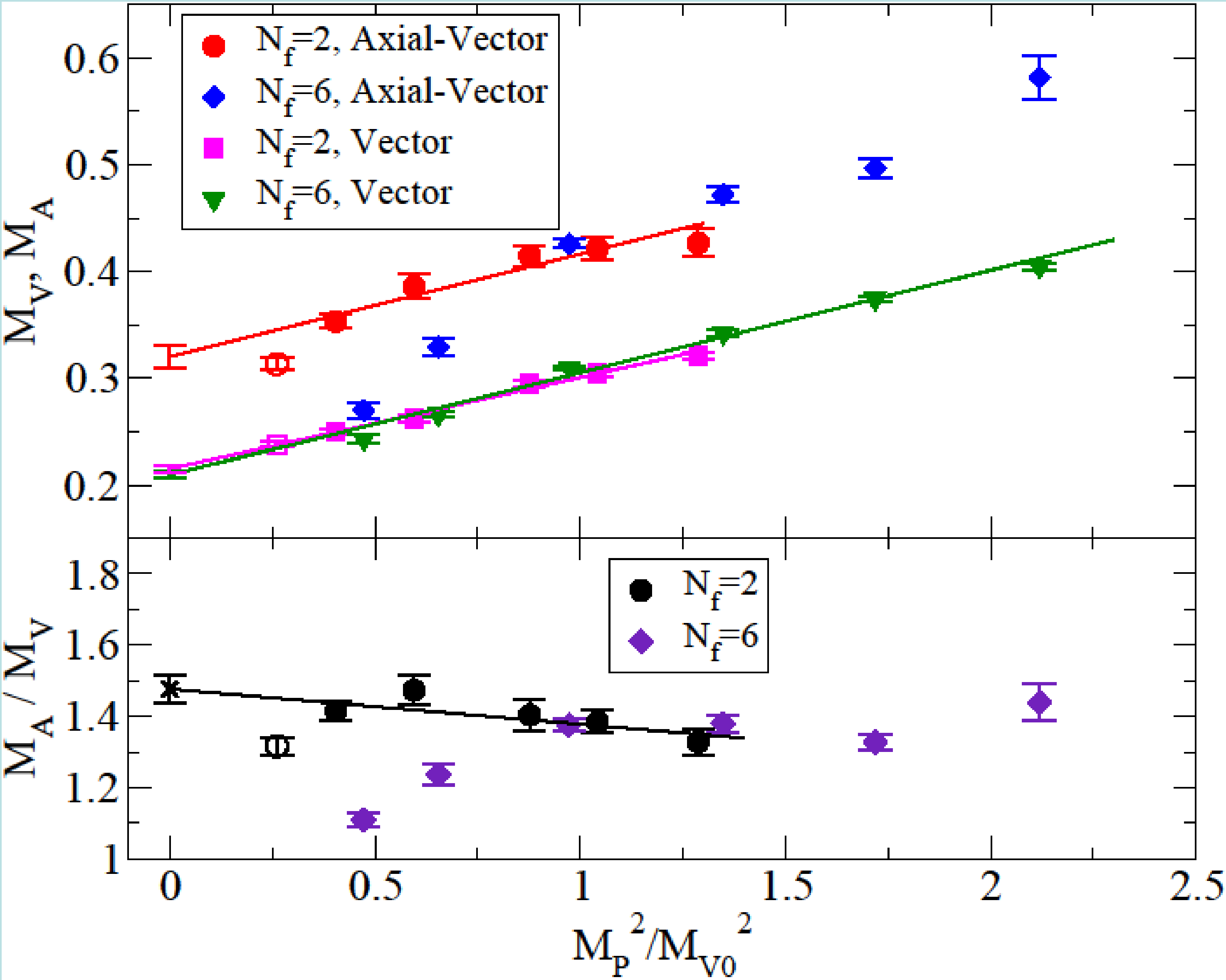
~Same Details

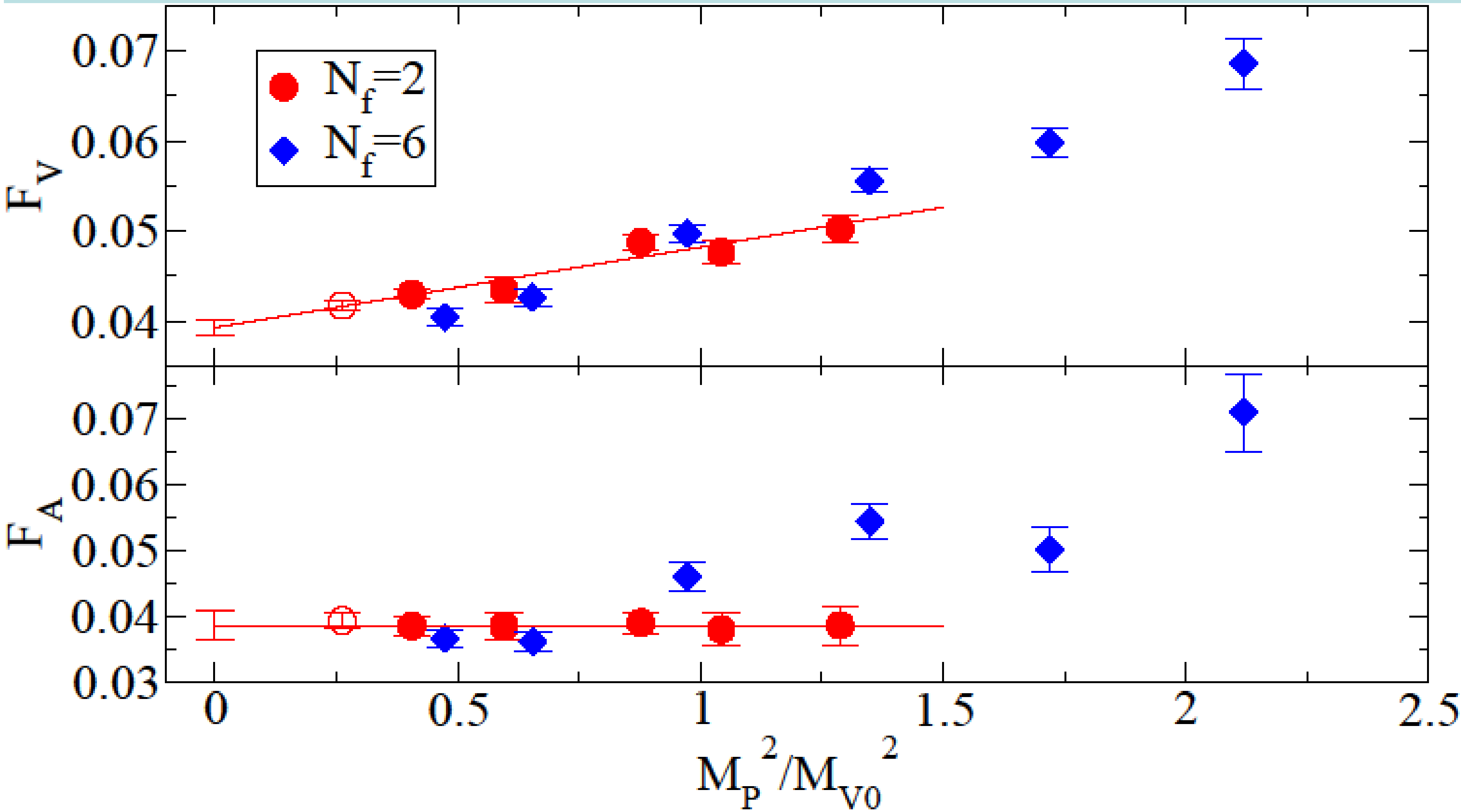
- Domain-wall fermions, Iwasaki improved action
- USQCD: Chroma, CPS

- 32^3 x 64 lattice (L_s = 16)
- m_f = .005, .01, .015, .02, .025 , $m = m_f + m_{res}$

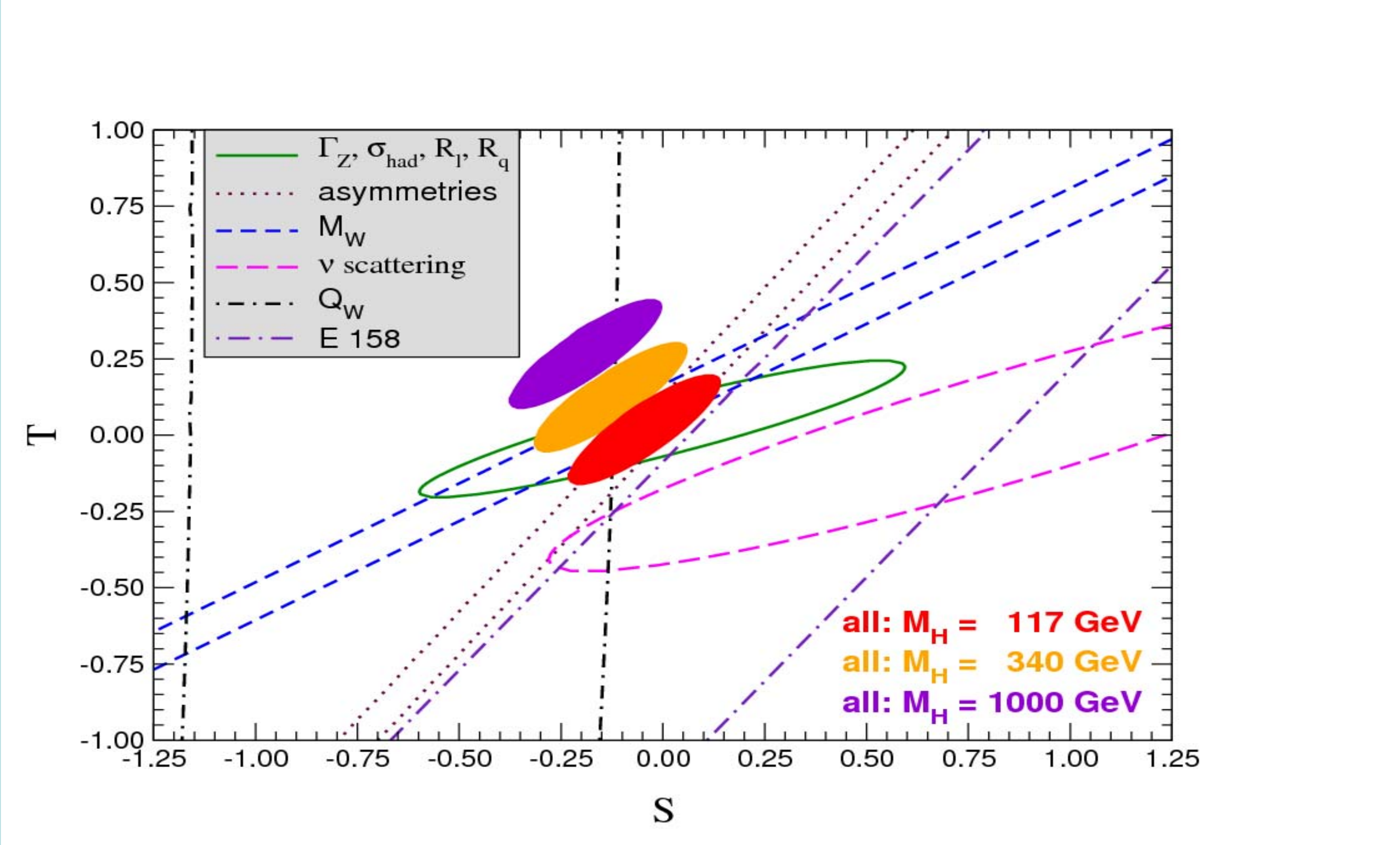
$M_P L > 4$

Vector and Axial-Vector Masses



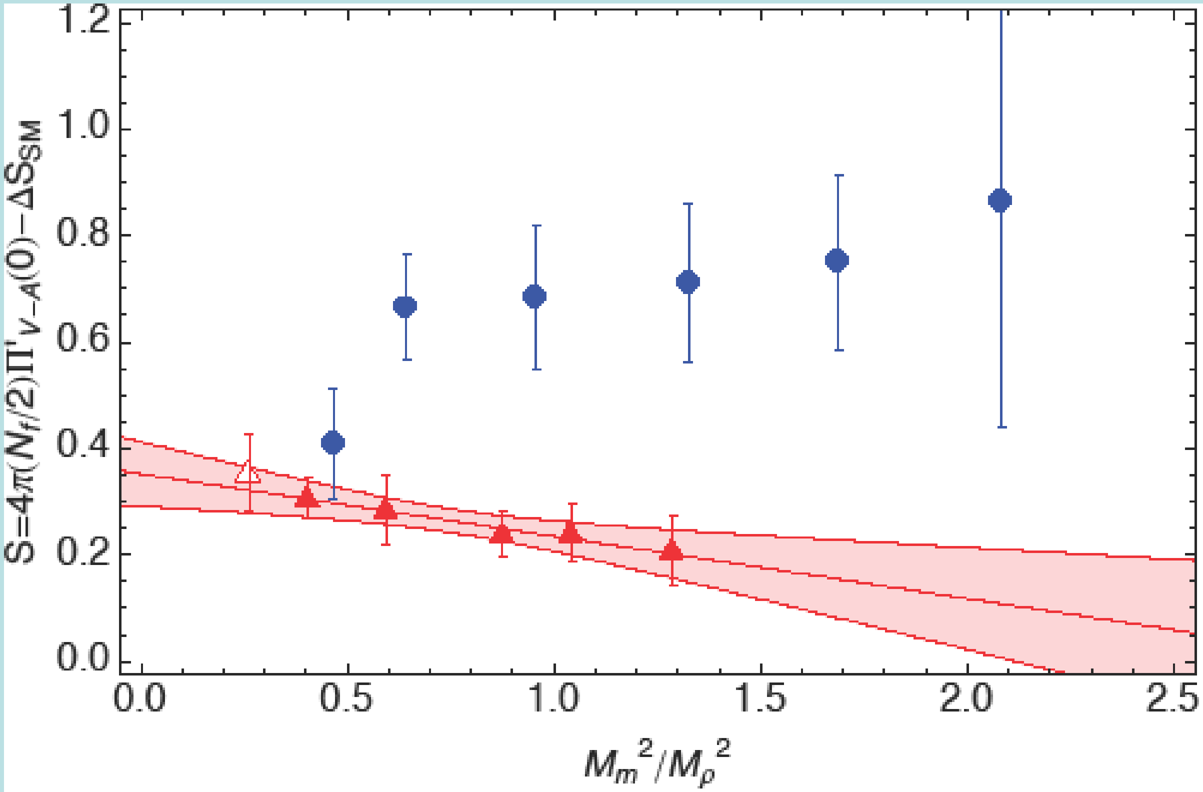

N_f=2
N_f=6
F_V
F_A
0.07
0.06
0.05
0.04
0.03
0
0.5
1
1.5
2
2.5
M_P^2/M_{V0}^2

S and T



$S_{free} = (1/2\pi)(N/3)(N_f/2)$ $S_{QCD} = 0.3\ (N/3)(N_f/2)$

S Parameter



3 EW doublets

Nf = 2: S is smooth

DelDebbio et al arXiv:0909.4931
Shintani et al arXiv:0806.4222

Extrapolation:

Nf = 6: $S \sim 1/12\pi\,[N_f^2/4 - 1]\log(1/m)$
Cut off by PNGB masses

Features

When N_f is increased from 2 to 6:

1. The lightest vector and axial states become more parity doubled.

2. The S parameter per electroweak doublet decreases
(In the chiral limit m → 0, the full answer will depend logarithmically on PNGB masses.)

3. Single pole dominance ($S = 4\pi [F_V^2 / M_V^2 - F_A^2 / M_A^2]$) works to within 20% at $N_f = 2$ and at least as well at $N_f = 6$, showing the relative decrease of S per electroweak doublet.

4. Not true of the WSR's ($F_V^2 - F_A^2 = F_P^2$, ……)

Summary

- Lattice simulations are beginning to teach us about novel features of strongly coupled gauge theories other than QCD.

- So far, lots of focus on the transition toward *infrared* conformal behavior. Relevant to real world?
  Condensate enhancement, tendency to parity doubling, decreased S parameter, ………

Next

1. Push on toward N_{fc}
2. Jump into the conformal window ($N_f > N_{fc}$) (mass-deformed CFT)
3. → SUSY Theories
   Check methods
   Other applications