1-Loop Renormalization Group Invariants in the MSSM

M. Carena, PD, N. R. Shah, and C. E. M. Wagner, Phys. Rev. D **82**, 075005 (2010)
arxiv:1006.4363

M. Carena, PD, N. R. Shah, and C. E. M. Wagner, in preparation

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Outline

$$X = X(m_{\tilde{f}}, m_{H_u}, m_{H_d}, M_a, g_a); \;\; \beta_X = 0$$

- Motivation
- Assumptions, Approximations, Previous Studies
- Constructing Renormalization Group Invariants (RGIs)
- Applications to many-parameter SUSY-breaking models
  - Generic Flavor-Blind input
  - General Gauge Mediation
- Applications to few-parameter models
  - Minimal Gauge Mediation

SUPERSYMMETRY

FIND T MSSM OR NMSSM OR SUPERSYMMETRY OR SUPERSYMMETRIC OR SUSY OR SPARTICLE OR LSP OR NLSP OR CHARGINO OR NEUTRALINO OR SQUARK OR SLEPTON OR GAUGINO OR GLUINO OR SUPERGRAVITY OR MSUGRA OR R-PARITY

FIND T MSSM OR NMSSM OR SUPERSYMMETRY OR SUPERSYMMETRIC OR

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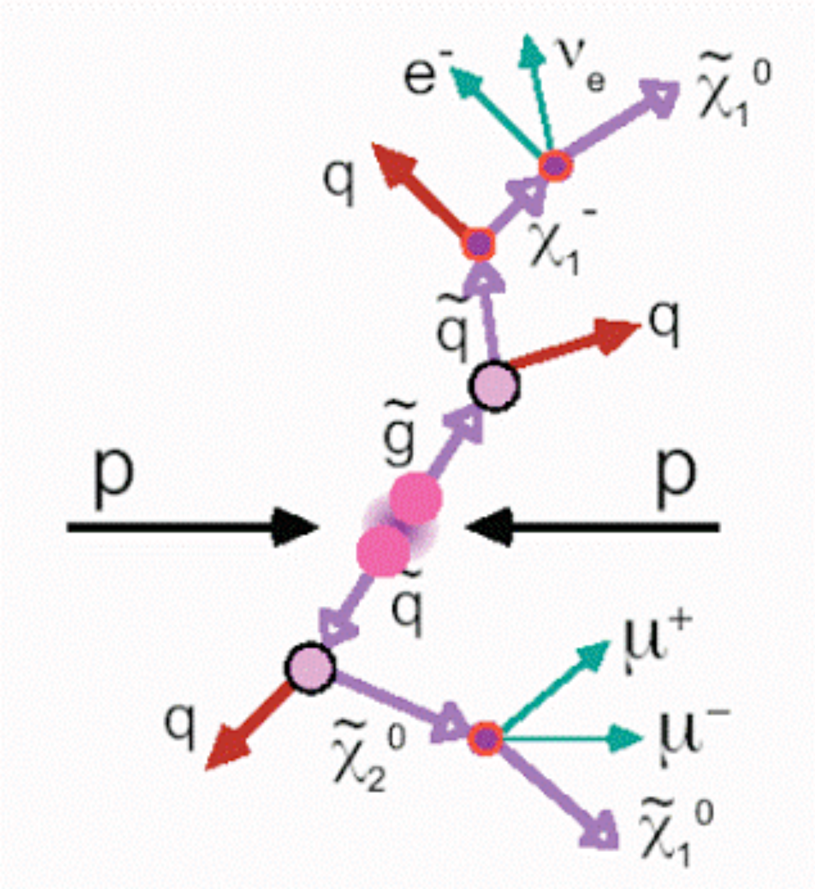
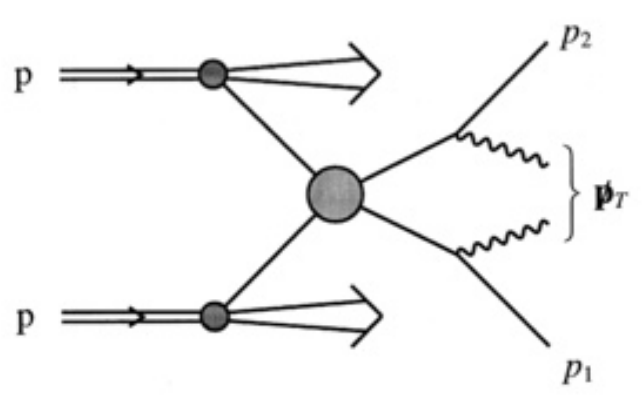
Many nice properties:

gauge coupling unification, radiative EWSB, DM candidate, light Higgs, hierarchy stabilization...

In An Ideal World

Supersymmetry Discovered at the LHC!





Step 1: Sparticle Pole Mass Determination

- Endpoint methods & long cascade decays.
  Invariant mass distributions of visible final-state particles have endpoints set by masses of on-shell sparticles in cascades.

  Hinchliffe, Paige, Shapiro, Soderqvist, Yao 1997

- Mass relation method
  Use mass shell conditions with pair production & decay chains to reconstruct kinematics

  Cheng, Gunion, Han, Marandella, McElrath 2007

- M_{T2} & pair production, short decay chain.
  parent particle mass related to endpoint of kinematic variable M_{T2}

$$m_{\tilde{l}}^2 \geq M_{T2}^2 \equiv \min_{\slashed{p}_1 + \slashed{p}_2 = \slashed{p}_T} \left[\max\{m_T^2(p_{Tl^-}, \slashed{p}_1), m_T^2(p_{Tl^+}, \slashed{p}_2)\}\right]$$

Lester, Summers 1999; Cheng, Han 2008

Can get many masses with these methods.

Weiglein et al. 2006


Now What?

SUSY-Breaking

Sparticles are heavier than SM partners → SUSY must be broken

Broken explicitly in the Minimal Supersymmetric Standard Model (MSSM) by "soft terms" that do not reintroduce the hierarchy problem

$$M_1,\ M_2,\ M_3,\ \mathbf{m}^2_{\tilde{Q}},\ \mathbf{m}^2_{\tilde{u}},\ \mathbf{m}^2_{\tilde{d}},\ \mathbf{m}^2_{\tilde{L}},\ \mathbf{m}^2_{\tilde{e}},\ m^2_{H_u},\ m^2_{H_d},\ A_{ijk},\ B_\mu$$

Mostly these feed into the sparticle mass matrices and can be extracted from measurements of pole masses and mixing angles.

But the MSSM is only an effective theory, and to get the right flavor structure (diagonal m^2), SUSY should be broken spontaneously.

Spontaneous SUSY-Breaking

Spontaneous SUSY-breaking usually implies:

- Additional fields to generate <F> (no MSSM candidates)
- Need higher dimension operators to couple <F> to gauginos
- Want higher dimension operators to couple <F> to scalars (avoid sum rules)
- → Breaking occurs in a “hidden sector” not coupled directly to the MSSM
- → Breaking is communicated to the MSSM through physics that decouples at the “messenger scale” M. (borrowing terminology from gauge mediation)

- What structures might govern the transport of SUSY-breaking into the MSSM?
- How can we test them?
- What is M?

SUSY-Breaking

Two well-studied methods to study the mechanism transmitting SUSY-breaking:

<u>Top-down</u>

Assume a model & scale, fit high-scale parameters to low-scale data e.g. SFITTER: Lafaye, Plehn, Zerwas

<u>Bottom-up</u>

Invert pole masses to get running masses, RG evolve until some structure starts to appear e.g. Kneur & Sahoury 2009

Both methods are effective, but also somewhat complicated: many coupled RGEs to evolve.

Top-down offers little intuition and may be sensitive to the number of high-scale parameters.

Bottom-up is very sensitive to unmeasured low-scale parameters due to the coupled nature of the RGEs.

SUSY-Breaking: RGIs

A third, complimentary approach is to use 1-loop Renormalization Group Invariant (RGI) combinations of the soft parameters.

- Allows systematic construction of a large class of RGI sum rules for a given model
- Sum rules test models, other RGIs reconstruct messenger scale parameters
- Entirely algebraic → simple to use
- Only approximate, and assumes pole→soft conversion has already been accomplished, but easy to incorporate some threshold & 2-loop effects
- In some cases can determine the scale M

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Assumptions & Approximations

General Assumption in building RGIs:

- The MSSM is the correct effective theory between the electroweak and messenger scales (MSSM β-functions only)

MSSM Assumptions:

- Minimal Flavor Violation, 1st and 2nd generation soft masses degenerate at M
- No new sources of CP violation in the soft sector

Approximations:

- β-functions approx flavor-diagonal in the quark basis
- Neglect 1st and 2nd generation Yukawa and soft trilinear couplings

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MFV+diagonal β-functions → soft sfermion masses flavor-diagonal

Yukawa approx → 1st and 2nd generation soft masses remain degenerate

Low-Scale Thresholds

For RGIs, want soft masses all at the scale M_c of the heaviest sparticle so that full MSSM β-functions are valid

Can use pole masses to obtain running soft masses at the scale of the pole masses.

In many models the squarks and gluino will typically be at or close to M_c already.

Soft masses of weakly-coupled sparticles run slowly at low scales and can be approximated at M_c via

$$m_i^2(M_c) \approx m_i^2(m_{i,pole}) - \frac{1}{2\pi^2} C_2(i) g_2^2(M_2) |M_2|^2 \log \frac{M_c}{M_2}$$

generally a percent-level shift.

To a good approximation, can just run gauge couplings to M_c with Standard Model β-functions.

Previous Work

RGI sum rules mentioned in many formal studies of softly-broken N=1 gauge theories, as well as in phenomenological studies of a variety of specific models (CMSSM, SUSY-GUTs, AMSB, GMSB, general flavor-blind models, etc...)

Martin & Ramond 1993
Kawamura, Kobayashi, Kubo 1997
Kazakov 1997
Hisano & Shifman 1997
Jack, Jones, Pickering 1997
Arkani-Hamed, Giudice, Luty, Rattazzi 1997
Carena, Huiti, Kobayashi 2000
Kobayashi & Yoshioka 2000
Ananthanarayan & Pandita 2005
Demir 2005
Kane, Kumar, Morrissey, Toharia 2007
Meade, Seiberg, Shih 2009
Balazs, Li, Nanopoulos, Wang 2010
etc...

Our work:

- a more complete list of the MSSM RGIs
- application of both sum rules and high-scale parameter reconstruction
- analysis of 2-loop effects and experimental uncertainties

Illustrate in the contexts of General and Minimal Gauge Mediation.

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MSSM RG Equations

- $$16\pi^2 \frac{dm_i^2}{dt} = \sum_{jk} y_{ijk}^* y^{ijk}(m_i^2 + m_j^2 + m_k^2 + A_{ijk}^* A^{ijk}) - 8\sum_a C_a(i) g_a^2 |M_a|^2 + \frac{6}{5} Y_i g_1^2 D_Y,$$

$$D_Y \equiv \mathrm{Tr}(Y m^2) = \sum_{\text{gen}} (m_{\tilde{Q}}^2 - 2m_{\tilde{u}}^2 + m_{\tilde{d}}^2 - m_{\tilde{L}}^2 + m_{\tilde{e}}^2) + m_{H_u}^2 - m_{H_d}^2$$

$$t \equiv \log(\mu/M_Z).$$

MSSM RG Equations

$$D_Y \equiv \mathrm{Tr}(Ym^2)$$
$$= \sum_{\text{gen}} (m^2_{\tilde{Q}} - 2m^2_{\tilde{u}} + m^2_{\tilde{d}} - m^2_{\tilde{L}} + m^2_{\tilde{e}}) + m^2_{H_u} - m^2_{H_d}$$

- $16\pi^2 \partial_t D_Y = g_1^2 D_Y (2\,\mathrm{Tr}_n I_1(n) - 6C_1(G)) = \frac{66}{5} g_1^2 D_Y$
- $16\pi^2 \partial_t g_r^2 = g_r^2\, g_r^2 (2\,\mathrm{Tr}_n I_r(n) - 6C_r(G))$
- $16\pi^2 \partial_t M_r = g_r^2 M_r (2\,\mathrm{Tr}_n I_r(n) - 6C_r(G))$

Vanishes in some models, making $D_Y = 0$ an invariant sum rule

Constructing Invariants

Two Classes:

- Gauge-coupling independent RGIs
- Gauge-coupling dependent RGIs

Gauge Coupling-Indep. RGIs

Try to construct linear combinations of the scalar soft masses with vanishing β–functions

$$D \equiv \mathrm{Tr}\, Q_i m_i^2$$

0 if Q_i are the charges of a classical U(1) symmetry of Yukawas → 3 conditions (y_t, y_b, y_τ)

$$16\pi^2 \frac{dD}{dt} = \sum_{ijk} Q_i y^*_{ijk} y^{ijk} (m_i^2 + m_j^2 + m_k^2 + A^*_{ijk} A^{ijk})$$
$$-8 \sum_{a,i} Q_i C_a(i) g_a^2 |M_a|^2 + \frac{6}{5} \sum_i Q_i Y_i g_1^2 D_Y$$

0 if Q_i has vanishing mixed anomalies with SM gauge groups → 3 conditions

1 extra condition

GAUGE COUPLING-INDEP. RGIS

$$16\pi^2\frac{dD}{dt} = \sum_{ijk} Q_i y^*_{ijk} y^{ijk}(m_i^2 + m_j^2 + m_k^2 + A^*_{ijk}A^{ijk}) - 8\sum_{a,i} Q_i C_a(i) g_a^2 |M_a|^2 + \frac{6}{5}\sum_i Q_i Y_i g_1^2 D_Y$$

12 independent scalar soft masses - 7 conditions = 5 RGIs

B and L anomalous, but can be made non-anomalous by family-nonuniversal charge assignments. Also cancels D_Y piece!

- $$D_{B_{13}} \equiv D_{B_1} - D_{B_3} = 2m^2_{\tilde{Q}_1} - m^2_{\tilde{u}_1} - m^2_{\tilde{d}_1} - 2m^2_{\tilde{Q}_3} + m^2_{\tilde{u}_3} + m^2_{\tilde{d}_3}$$
- $$D_{L_{13}} \equiv D_{L_1} - D_{L_3} = 2m^2_{\tilde{L}_1} - m^2_{\tilde{e}_1} - 2m^2_{\tilde{L}_3} + m^2_{\tilde{e}_3}$$

Gauge Coupling-Indep. RGIs

$$16\pi^2 \frac{dD}{dt} = \sum_{ijk} Q_i y^*_{ijk} y^{ijk} (m_i^2 + m_j^2 + m_k^2 + A^*_{ijk} A^{ijk}) - 8 \sum_{a,i} Q_i C_a(i) g_a^2 |M_a|^2 + \frac{6}{5} \sum_i Q_i Y_i g_1^2 D_Y$$

Hypercharge is non-anomalous independently for each generation and Higgs, but $\mathrm{Tr}\ Y^2 = 11$

$$\left.\begin{aligned} \mathrm{Tr}\ Y_1^2 &= 10/3 \\ \mathrm{Tr}\ Y_{3H}^2 &= 13/3 \end{aligned}\right\}$$ Take weighted difference between generations

- $$D_{Y_{13H}} \equiv D_{Y_1} - \frac{10}{13} D_{Y_{3H}}$$
  $$= m^2_{\tilde{Q}_1} - 2m^2_{\tilde{u}_1} + m^2_{\tilde{d}_1} - m^2_{\tilde{L}_1} + m^2_{\tilde{e}_1}$$
  $$- \frac{10}{13}(m^2_{\tilde{Q}_3} - 2m^2_{\tilde{u}_3} + m^2_{\tilde{d}_3} - m^2_{\tilde{L}_3} + m^2_{\tilde{e}_3} + m^2_{H_u} - m^2_{H_d})$$

Gauge Coupling-Indep. RGIs

$$16\pi^2 \frac{dD}{dt} = \sum_{ijk} Q_i y^*_{ijk} y^{ijk} (m_i^2 + m_j^2 + m_k^2 + A^*_{ijk} A^{ijk}) - 8 \sum_{a,i} Q_i C_a(i) g_a^2 |M_a|^2 + \frac{6}{5} \sum_i Q_i Y_i g_1^2 D_Y$$

To find the last two symmetries, think of E_6 breaking:

$$E_6 \to SU(5) \times U(1)_\chi \times U(1)_\psi$$

- $$D_{\chi_1} \equiv 4D_{Y_1} - 5D_{(B-L)_1} = -6m^2_{\tilde{Q}_1} - 3m^2_{\tilde{u}_1} + 9m^2_{\tilde{d}_1} + 6m^2_{\tilde{L}_1} - m^2_{\tilde{e}_1}$$

restriction to 1st generation → indep of others

$U(1)_\psi$ is anomalous when restricted to the MSSM and involves the Higgs, preventing a family-nonuniversal combination.

- $$D_Z \equiv 3m^2_{\tilde{d}_3} + 2m^2_{\tilde{L}_3} - 2m^2_{H_d} - 3m^2_{\tilde{d}_1}$$

only m_{Hd} → indep of others

Z is a linear combination of $U(1)_\psi$ and $U(1)_\chi$ with $Q_{\tilde{L}_1} = 0$

GAUGE COUPLING-INDEP. RGIs

Including Gaugino Masses: 3 more Q_i $\quad I \equiv \mathrm{Tr}\, Q_i m_i^2 + \sum_a Q_a M_a^2$

$$16\pi^2 \frac{dI}{dt} \sim -4 \sum_a g_a^2 M_a^2 \left(\sum_i 2Q_i C_a(i) - Q_a B_a \right) + \dots$$

→ Anomaly cancelation condition modified

3 new DOF, no extra conditions → 3 RGIs

- $I_{M_1} \equiv M_1^2 - \frac{33}{8}(m_{\tilde{d}_1}^2 - m_{\tilde{u}_1}^2 - m_{\tilde{e}_1}^2),$
- $I_{M_2} \equiv M_2^2 + \frac{1}{24}(9(m_{\tilde{d}_1}^2 - m_{\tilde{u}_1}^2) + 16m_{\tilde{L}_1}^2 - m_{\tilde{e}_1}^2),$
- $I_{M_3} \equiv M_3^2 - \frac{3}{16}(5m_{\tilde{d}_1}^2 + m_{\tilde{u}_1}^2 - m_{\tilde{e}_1}^2).$

} First gen. & gauginos only!

$$I_{M_{12}} = I_{M_1} + 11 I_{M_2} = M_1^2 + \frac{11}{3}(3M_2^2 + 2m_{\tilde{L}_1}^2 + m_{\tilde{e}_1}^2)$$

Gauge Coupling-Dependent RGIs

$$16\pi^2 \partial_t D_Y = g_1^2 D_Y (2\,\mathrm{Tr}_n I_1(n) - 6C_1(G)) = \frac{66}{5} g_1^2 D_Y$$

$$16\pi^2 \partial_t g_r^2 = g_r^2\, g_r^2 (2\,\mathrm{Tr}_n I_r(n) - 6C_r(G))$$

$$16\pi^2 \partial_t M_r = g_r^2 M_r (2\,\mathrm{Tr}_n I_r(n) - 6C_r(G))$$

$$\partial_t X = aX,\ \ \partial_t Y = aY \rightarrow \partial_t (X/Y) = 0$$

→ 6 more RGIs from ratios with gauge couplings

Gauge Coupling-Dependent RGIs

- $I_{g_2} \equiv \frac{1}{g_1^2} - \frac{33}{5g_2^2}$ ≈ -10.9
- $I_{g_3} \equiv \frac{1}{g_1^2} + \frac{33}{15g_3^2}$ ≈ 6.2

convert gauge couplings at any scale into each other

- $I_{B_r} \equiv M_r/g_r^2$ equal in models with GUT-scale gaugino mass unification

- $I_{Y\alpha} \equiv \frac{D_Y}{g_1^2}$

14 RGIs

Invariant	Symmetry	Dependence on Soft Masses
$D_{B_{13}}$	$B_1 - B_3$	$2(m^2_{\tilde{Q}_1} - m^2_{\tilde{Q}_3}) - m^2_{\tilde{u}_1} + m^2_{\tilde{u}_3} - m^2_{\tilde{d}_1} + m^2_{\tilde{d}_3}$
$D_{L_{13}}$	$L_1 - L_3$	$2(m^2_{\tilde{L}_1} - m^2_{\tilde{L}_3}) - m^2_{\tilde{e}_1} + m^2_{\tilde{e}_3}$
D_{χ_1}	χ_1	$3(3m^2_{\tilde{d}_1} - 2(m^2_{\tilde{Q}_1} - m^2_{\tilde{L}_1}) - m^2_{\tilde{u}_1}) - m^2_{\tilde{e}_1}$
$D_{Y_{13H}}$	$Y_1 - \frac{10}{13} Y_{3H}$	$m^2_{\tilde{Q}_1} - 2m^2_{\tilde{u}_1} + m^2_{\tilde{d}_1} - m^2_{\tilde{L}_1} + m^2_{\tilde{e}_1} - \frac{10}{13}$ (1<->3+H)
D_Z	Z	$3(m^2_{\tilde{d}_3} - m^2_{\tilde{d}_1}) + 2(m^2_{\tilde{L}_3} - m^2_{H_d})$
$I_{Y\alpha}$	Y	$(m^2_{H_u} - m^2_{H_d} + \sum_{gen}(m^2_{\tilde{Q}} - 2m^2_{\tilde{u}} + m^2_{\tilde{d}} - m^2_{\tilde{L}} + m^2_{\tilde{e}}))/g_1^2$
I_{B_r}		M_r/g_r^2
I_{M_1}		$M_1^2 - \frac{33}{8}(m^2_{\tilde{d}_1} - m^2_{\tilde{u}_1} - m^2_{\tilde{e}_1})$
I_{M_2}		$M_2^2 + \frac{1}{24}(9(m^2_{\tilde{d}_1} - m^2_{\tilde{u}_1}) + 16m^2_{\tilde{L}_1} - m^2_{\tilde{e}_1})$
I_{M_3}		$M_3^2 - \frac{3}{16}(5m^2_{\tilde{d}_1} + m^2_{\tilde{u}_1} - m^2_{\tilde{e}_1})$
I_{g_2}		$1/g_1^2 - 33/(5g_2^2)$
I_{g_3}		$1/g_1^2 + 33/(15g_3^2)$

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GENERIC FLAVOR-BLIND MODELS

$D_{B_{13}} = 0$ and $D_{L_{13}} = 0$ → direct tests of the flavor-blind hypothesis

10 mass parameters + 3 gauge couplings =13 degrees of freedom @ M
12 nonzero RGIs
→ can reconstruct everything as an algebraic function of one unknown, M

$$M_1 = g_1^2 I_{B_1}, \qquad M_2 = g_2^2 I_{B_2}, \qquad M_3 = g_3^2 I_{B_3}$$

$$m_{\tilde{L}}^2 = -\frac{1}{440}(26D_{Y_{13H}} + 11D_{\chi_1} + 20((g_1^4 I_{B_1}^2 + 33 g_2^4 I_{B_2}^2) - (I_{M_1} + 33 I_{M_2}) + g_1^2 I_{Y\alpha})),$$

$$m_{H_d}^2 = m_{\tilde{L}}^2 - \frac{1}{2}D_Z, \quad m_{H_u}^2 = m_{\tilde{L}}^2 - \frac{1}{2}D_Z - \frac{13}{11}D_{Y_{13H}} + \frac{g_1^2}{11}I_{Y\alpha}, \quad m_{\tilde{e}}^2 = \frac{1}{220}(26D_{Y_{13H}} + 11D_{\chi_1} - 20(2(g_1^4 I_{B_1}^2 - I_{M_1}) - g_1^2 I_{Y\alpha})),$$

$$m_{\tilde{u}}^2 = -\frac{1}{990}(78D_{Y_{13H}} + 33D_{\chi_1} + 20(4((g_1^4 I_{B_1}^2 - 11 g_3^4 I_{B_3}^2) - (I_{M_1} - 11 I_{M_3})) + 3 g_1^2 I_{Y\alpha})),$$

$$m_{\tilde{d}}^2 = \frac{1}{1980}(78D_{Y_{13H}} + 33D_{\chi_1} - 20(2((g_1^4 I_{B_1}^2 - 44 g_3^4 I_{B_3}^2) - (I_{M_1} - 44 I_{M_3})) - 3 g_1^2 I_{Y\alpha})),$$

$$m_{\tilde{Q}_1}^2 = \frac{1}{3960}(78D_{Y_{13H}} - 627D_{\chi_1} - 20((g_1^4 I_{B_1}^2 + 297 g_2^4 I_{B_2}^2 - 176 g_3^4 I_{B_3}^2) - (I_{M_1} + 297 I_{M_2} - 176 I_{M_3}) - 3 g_1^2 I_{Y\alpha})).$$

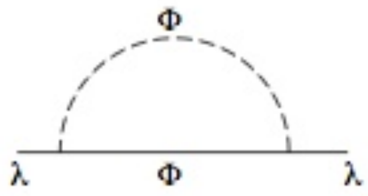
$g_a(t_M) = [g_a(t_0)^{-2} - B_a(t_M - t_0)/8\pi^2]^{-\frac{1}{2}}$ → only t_M remains unknown

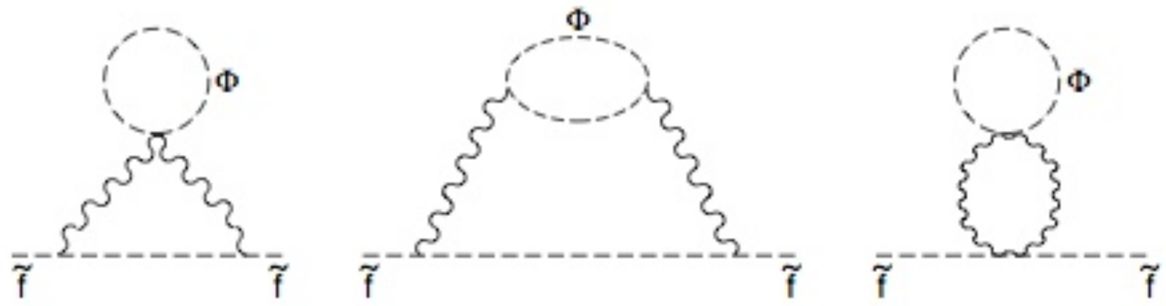
Can then bound all parameters by requiring 5<Log(M/GeV)<16

Gauge Mediation

★ Gauge mediation is a method of transmitting SUSY-breaking to the MSSM with only gauge interactions → automatically flavor-blind Dine, Nelson, Nir, Shirman Giudice, Rattazzi and many others

A traditional implementation, "Minimal Gauge Mediation," uses messenger particles Φ of mass M. The Φ couple to the SUSY-breaking vev and to the gauge bosons and gauginos of the MSSM.





, etc...

Integrating out the Φ produces the MSSM soft terms at M.

But messengers are not necessary. "General Gauge Mediation" Meade, Seiberg, Shih encompasses a broader class of models built on the same principle ★

General Gauge Mediation

The MSSM couples to current-current correlation functions in the hidden sector. Soft masses are parametrized by six constants A_r and B_r at a characteristic mass scale M:

$$m_{\tilde{f}}^2 = \sum_{r=1}^{3} g_r^4 C_r(f) A_r$$

$$M_r = g_r^2 M B_r$$

Generating μ and Bμ of the right size probably requires additional interactions between the Higgs doublets and the hidden sector, which may also shift the soft Higgs masses:

$$m_{H_u}^2 = m_{\tilde{L}_3}^2 + \boxed{\delta_u}, \qquad m_{H_d}^2 = m_{\tilde{L}_3}^2 + \boxed{\delta_d}$$

8 mass parameters + 3 gauge couplings = 11 unknowns @ M

RGIs in GGM

Invariant	GGM Value
$D_{B_{13}} \equiv 2m^2_{\tilde{Q}_1} - m^2_{\tilde{u}_1} - m^2_{\tilde{d}_1} - 2m^2_{\tilde{Q}_3} + m^2_{\tilde{u}_3} + m^2_{\tilde{d}_3}$	0
$D_{L_{13}} \equiv 2m^2_{\tilde{L}_1} - m^2_{\tilde{e}_1} - 2m^2_{\tilde{L}_3} + m^2_{\tilde{e}_3}$	0
$D_{\chi_1} \equiv -6m^2_{\tilde{Q}_1} - 3m^2_{\tilde{u}_1} + 9m^2_{\tilde{d}_1} + 6m^2_{\tilde{L}_1} - m^2_{\tilde{e}_1}$	0

$$D_{\chi_1} = \mathrm{Tr}\ \chi_1 m^2 \sim \mathrm{Tr}\ \chi_1 C_G = \mathrm{Tr}\ \chi_1 t_a^G t_a^G = 0$$

*CMSSM: $D_{\chi_1} = 5m_0^2$

if χ commutes with t_a — mixed anomaly cancellation

→ 3 sum rules in GGM

14 RGIs – 3 sum rules = 11 nonzero RGIs for parameter reconstruction

can in principle reconstruct everything, including M from the $g_r(M)$

RGIs in GGM

Invariant	GGM Value
$D_{Y_{13H}} \equiv m^2_{\tilde{Q}_1} - 2m^2_{\tilde{u}_1} + m^2_{\tilde{d}_1} - m^2_{\tilde{L}_1} + m^2_{\tilde{e}_1} - \frac{10}{13}(m^2_{\tilde{Q}_3} - 2m^2_{\tilde{u}_3} + m^2_{\tilde{d}_3} - m^2_{\tilde{L}_3} + m^2_{\tilde{e}_3} + m^2_{H_u} - m^2_{H_d})$	$-\frac{10}{13}(\delta_u - \delta_d)$
$D_Z \equiv 3m^2_{\tilde{d}_3} + 2m^2_{\tilde{L}_3} - 2m^2_{H_d} - 3m^2_{\tilde{d}_1}$	$-2\delta_d$
$I_{Y_\alpha} \equiv m^2_{H_u} - m^2_{H_d} + \sum_{gen}(m^2_{\tilde{Q}_i} - 2m^2_{\tilde{u}_i} + m^2_{\tilde{d}_i} - m^2_{\tilde{L}_i} + m^2_{\tilde{e}_i})$	$(\delta_u - \delta_d)/g_1^2$

$$\delta_u = -\frac{1}{2}\left(D_Z + \frac{13}{5}D_{Y_{13H}}\right), \qquad \delta_d = -\frac{1}{2}D_Z$$

if $\delta_u \neq \delta_d$: $g_1^2(M) = -\frac{13}{10}\frac{D_{Y_{13H}}}{I_{Y\alpha}}$ → can extract M!

$$\log\frac{M}{M_0} = \frac{8\pi^2}{b_1}\left(\frac{1}{g_1^2(M_0))} - \frac{1}{g_1^2(M))}\right)$$

RGIs in GGM

Invariant	GGM Value
$I_{B_r} \equiv M_r/g_r^2$	MB_r
$I_{M_1} \equiv M_1^2 - \frac{33}{8}(m_{\tilde{d}_1}^2 - m_{\tilde{u}_1}^2 - m_{\tilde{e}_1}^2)$,	$g_1^4((MB_1)^2 + \frac{33}{10}A_1)$
$I_{M_2} \equiv M_2^2 + \frac{1}{24}(9(m_{\tilde{d}_1}^2 - m_{\tilde{u}_1}^2) + 16m_{\tilde{L}_1}^2 - m_{\tilde{e}_1}^2)$,	$g_2^4((MB_2)^2 + \frac{1}{2}A_2)$
$I_{M_3} \equiv M_3^2 - \frac{3}{16}(5m_{\tilde{d}_1}^2 + m_{\tilde{u}_1}^2 - m_{\tilde{e}_1}^2)$.	$g_3^4((MB_3)^2 - \frac{3}{2}A_3)$

Gauge couplings are at M

$$MB_r = I_{B_r}$$

$$A_1 = \frac{10}{33}\left(\frac{I_{M_1}}{g_1^4} - I_{B_1}^2\right) \qquad A_2 = 2\left(\frac{I_{M_2}}{g_2^4} - I_{B_2}^2\right) \qquad A_3 = -\frac{2}{3}\left(\frac{I_{M_3}}{g_3^4} - I_{B_3}^2\right)$$

If $g_1(M)$ known from $\frac{D_{Y_{13H}}}{I_{Y\alpha}}$, can get $g_2(M)$ and $g_3(M)$ from I_{g_2} and I_{g_3}

→ All mass parameters of GGM known in term of RGIs

If not, can again bound all parameters by requiring 5<Log(M/GeV)<16

2-Loop Corrections

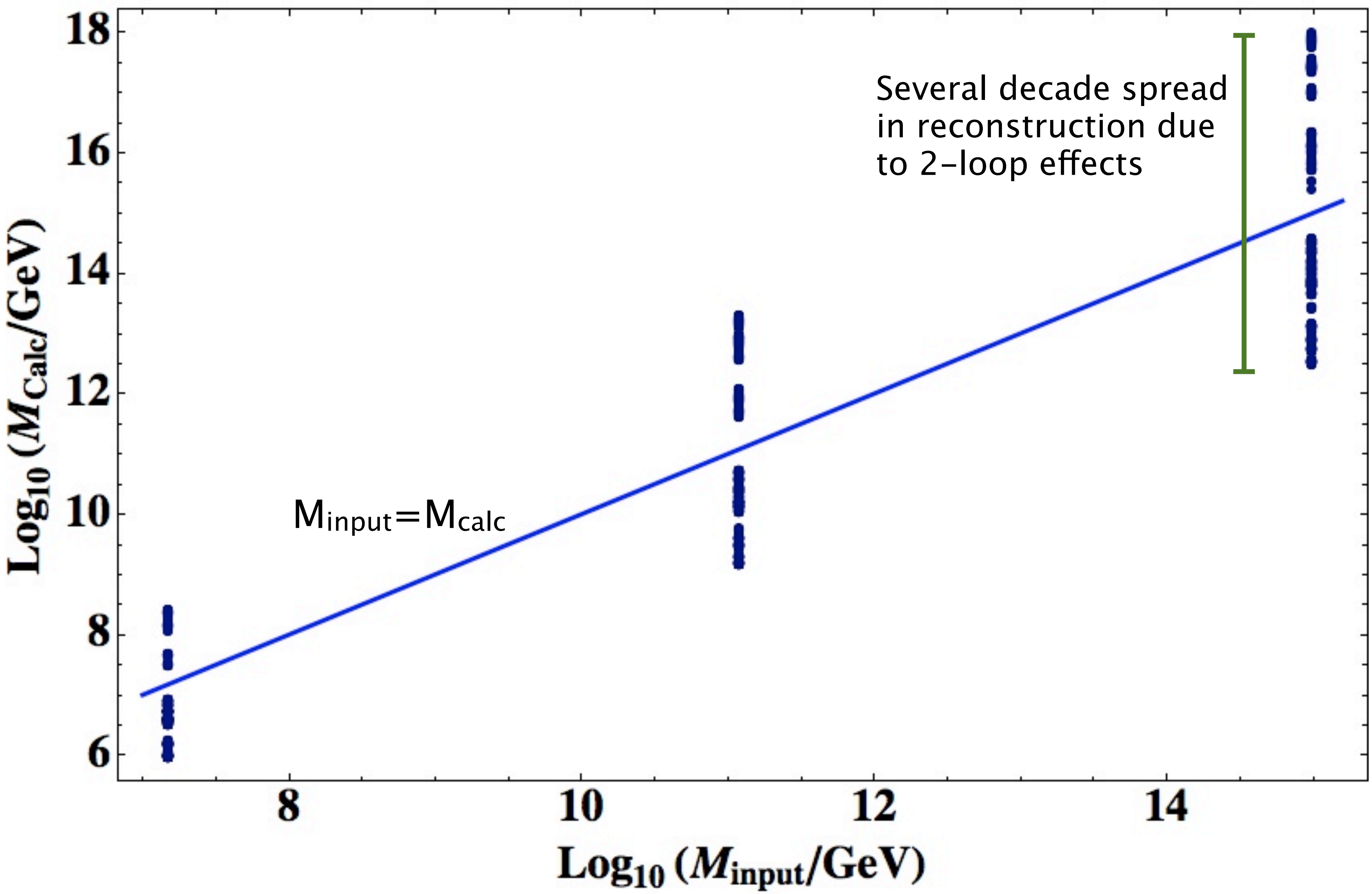
Neglecting 2-loop effects introduces systematic uncertainties. To estimate the size of these effects: scan over GGM input at M and RG-evolve to low scale with full 2-loop β-functions.

$$0.1 \leq A_r \leq 1.0\ \text{TeV}^2; \qquad 0 \leq \delta_{u,d} \leq 1.0\ \text{TeV}^2;$$

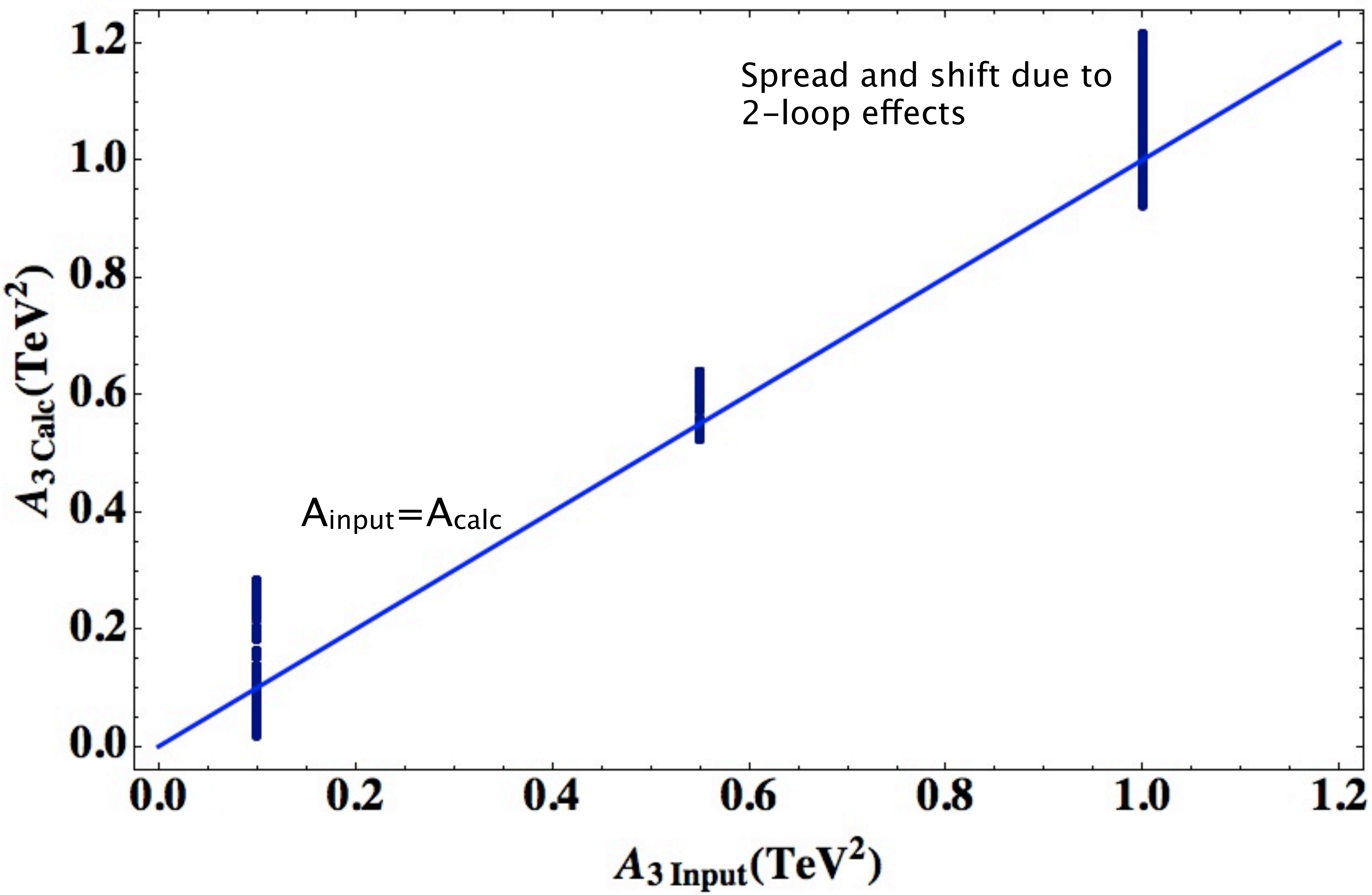
$$0.1 \leq MB_r \leq 1.0\ \text{TeV}; \qquad 2 \leq \tan\beta \leq 50;$$

$$10^6 \lesssim M \lesssim 10^{15}\ \text{GeV}.$$

2-Loop Corrections: Reconstruction of M



2-Loop Corrections: Reconstruction of A_3



2-Loop Corrections

Can improve accuracy with a simple algebraic approximation to the full 2-loop corrections Δ :

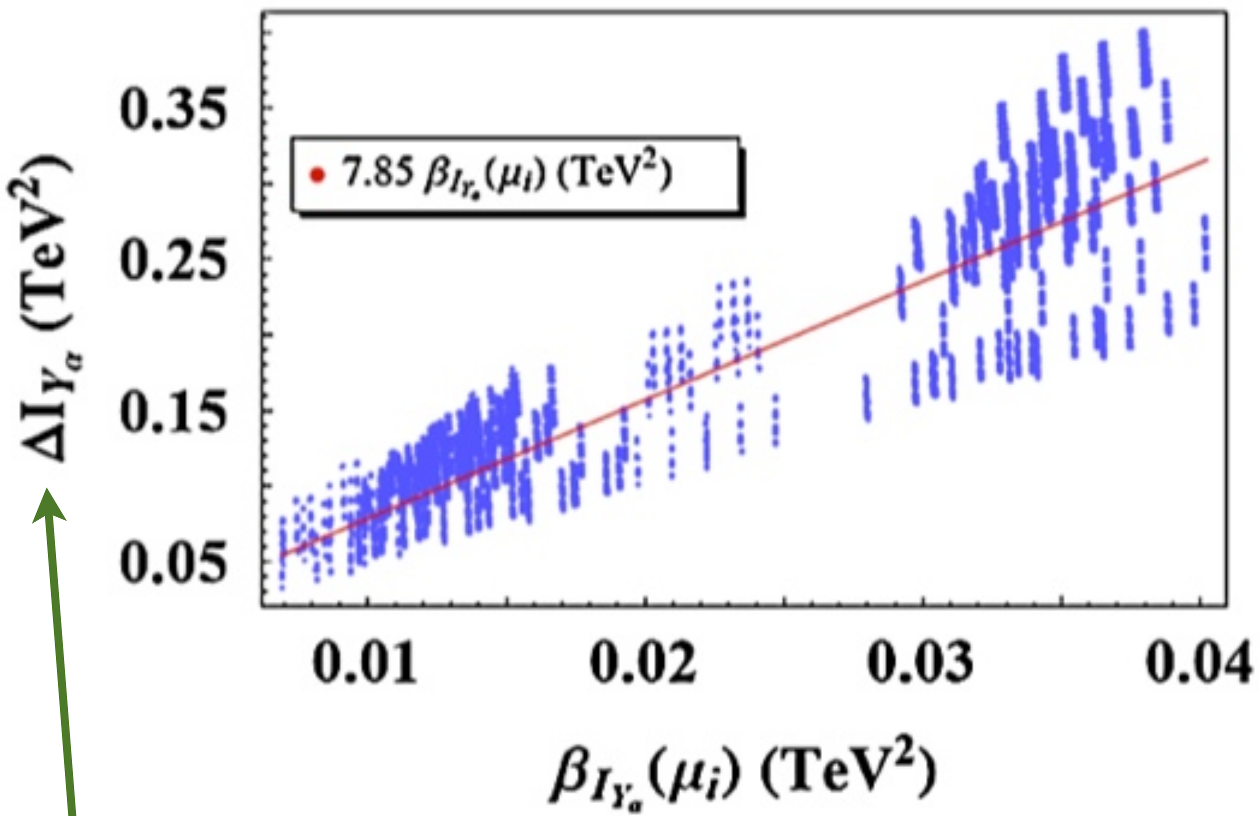
$$\mathrm{RGI} \longrightarrow \mathrm{RGI} + \langle\Delta\rangle$$

$$\langle\Delta\rangle \equiv \beta_2 \times \log(M_{\mathrm{intermediate}}/m_{\tilde{d}_1})$$

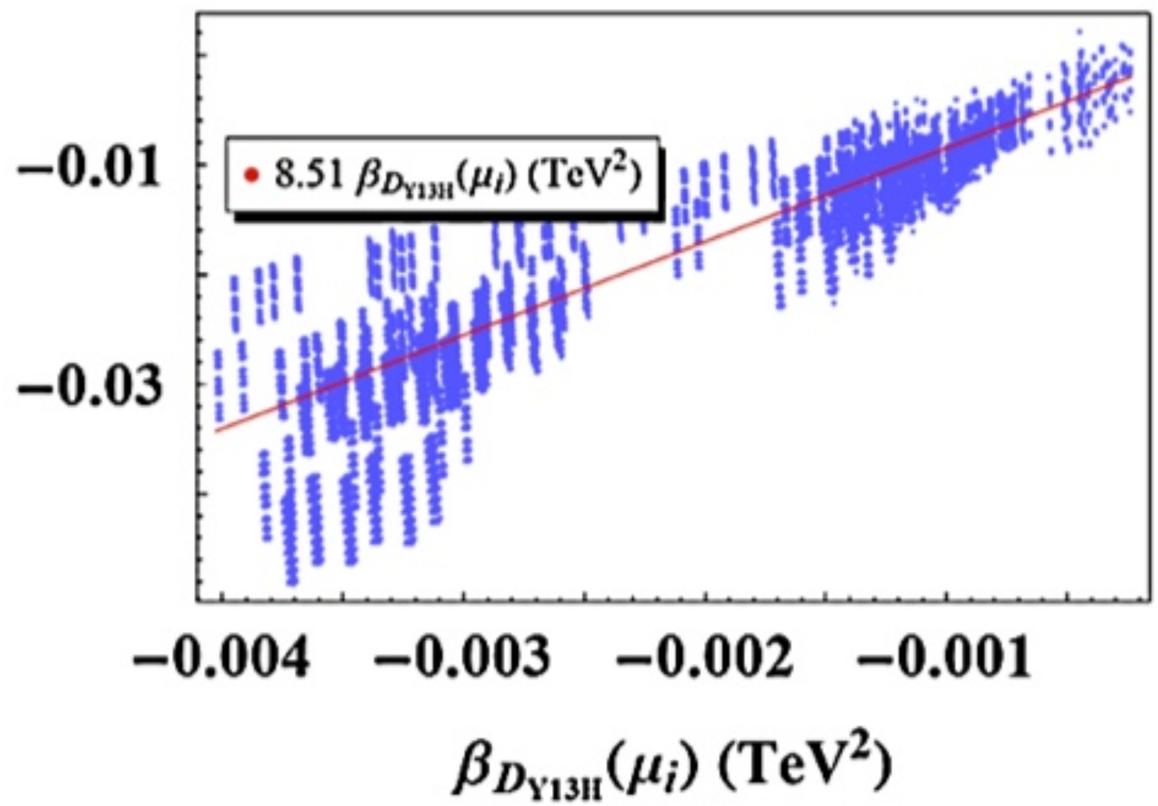
$$\beta_2 \longrightarrow \beta_2|_{m_{weak}\to 0,\ m_{strong}\to m_{\tilde{d}_1},\ g_1\to 0,\ A_{ijk}\to 0}$$

2-Loop Corrections

$$I_{Y\alpha} \equiv \frac{D_Y}{g_1^2}$$



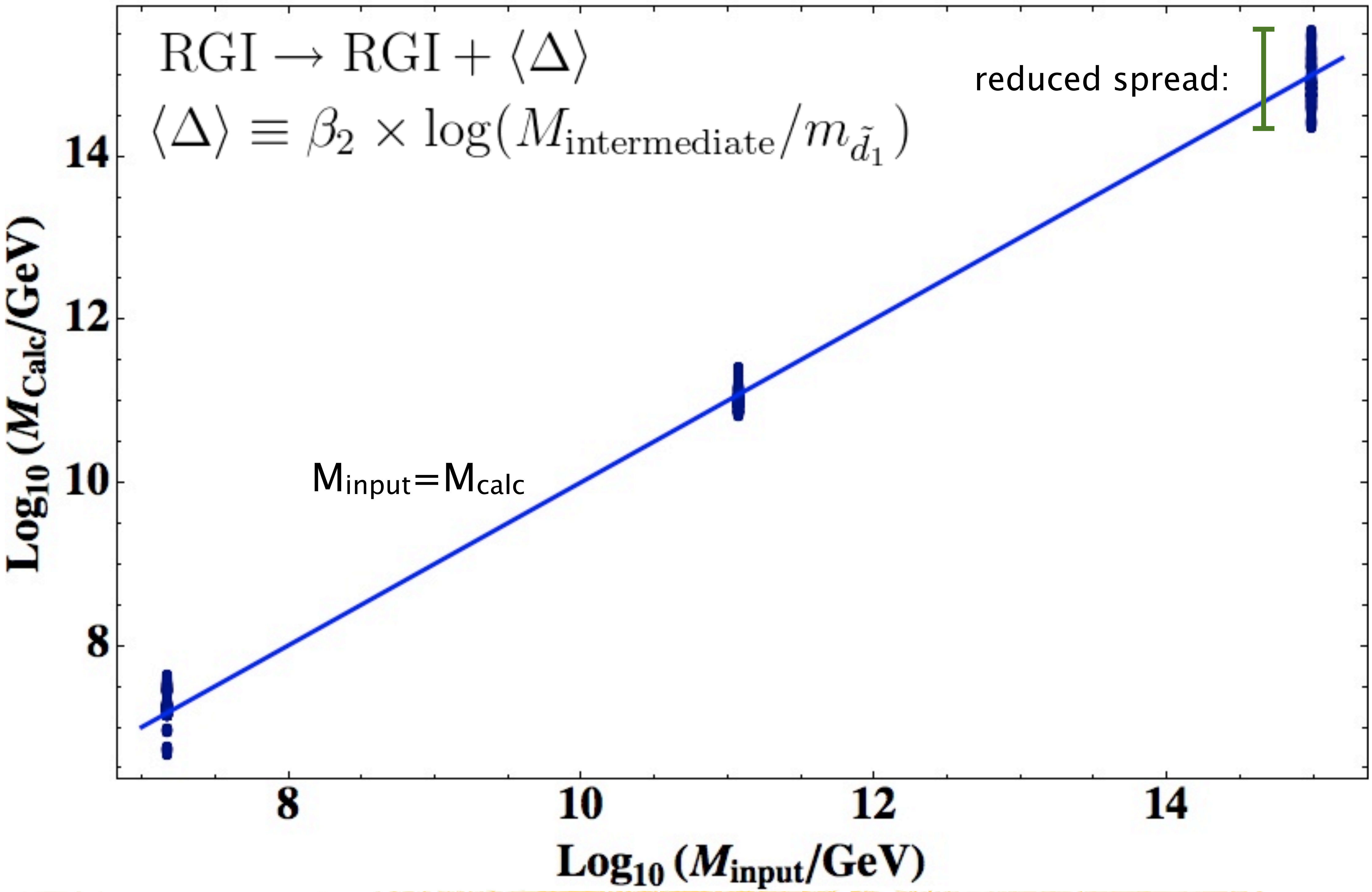
$$D_{Y_{13H}} \equiv D_{Y_1} - \frac{10}{13} D_{Y_{3H}}$$



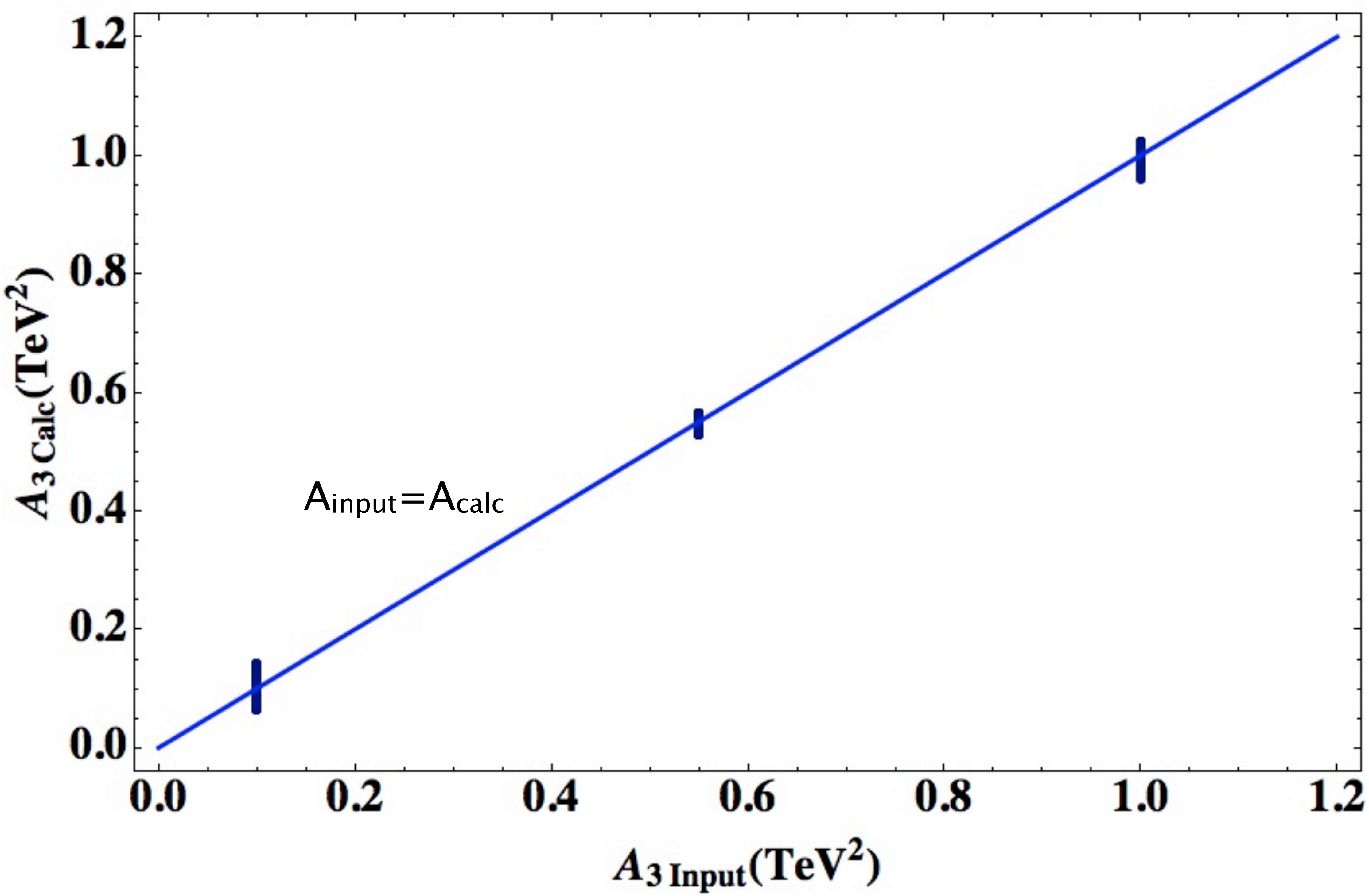
full 2-loop correction
(high minus low)

approx β-function

2-LOOP CORRECTIONS: RECONSTRUCTION OF M



2-Loop Corrections: Reconstruction of A_3



Uncertainties in Sum Rules

No matter what method is used, finding M in GGM requires precision measurements. (Essentially all soft scalar masses involved in any approach, logarithmic sensitivity.)

Sum rules can be tested with less precise measurements.

Assign universal uncertainties to the soft masses at low scale → errors in RGIs will scale linearly (10% taken as a baseline)

Qualitative estimates: ignore correlations (combine in quadrature)

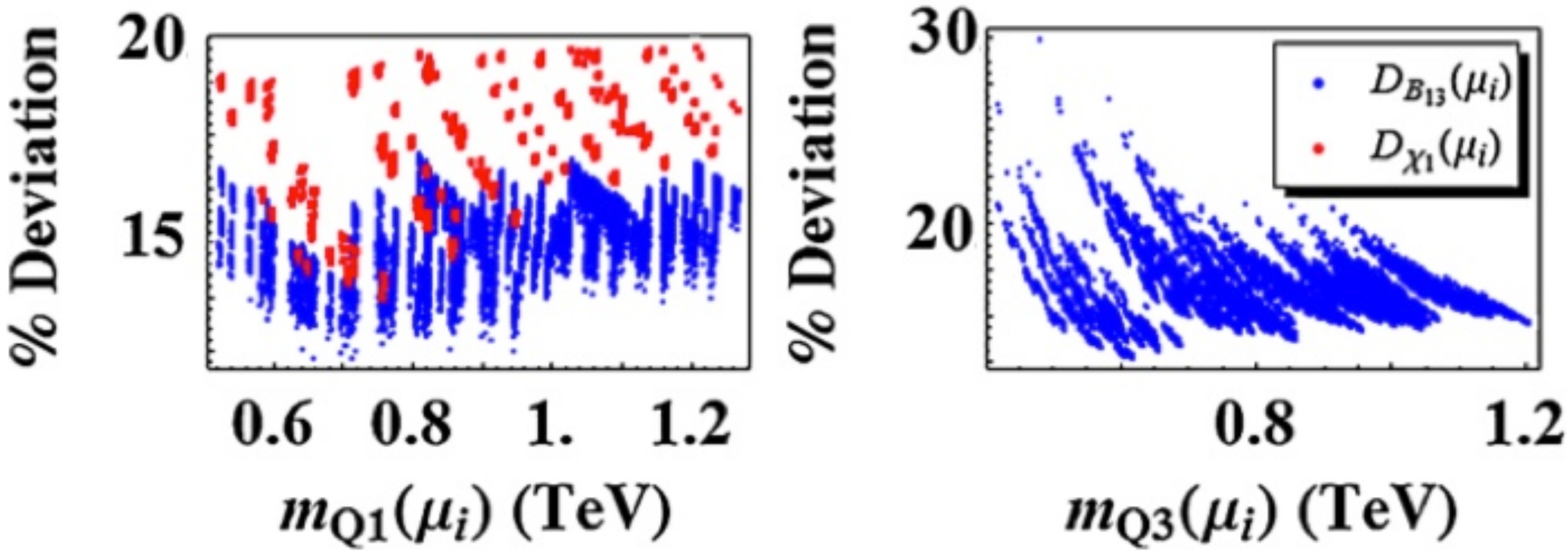
$$0.1 \le A_r \le 1.0\ \text{TeV}^2; \qquad 0 \le \delta_{u,d} \le 1.0\ \text{TeV}^2;$$

$$0.1 \le MB_r \le 1.0\ \text{TeV}; \qquad 2 \le \tan\beta \le 50;$$

$$10^6 \lesssim M \lesssim 10^{15}\ \text{GeV}.$$

Effects of Uncertainties in Sum Rules

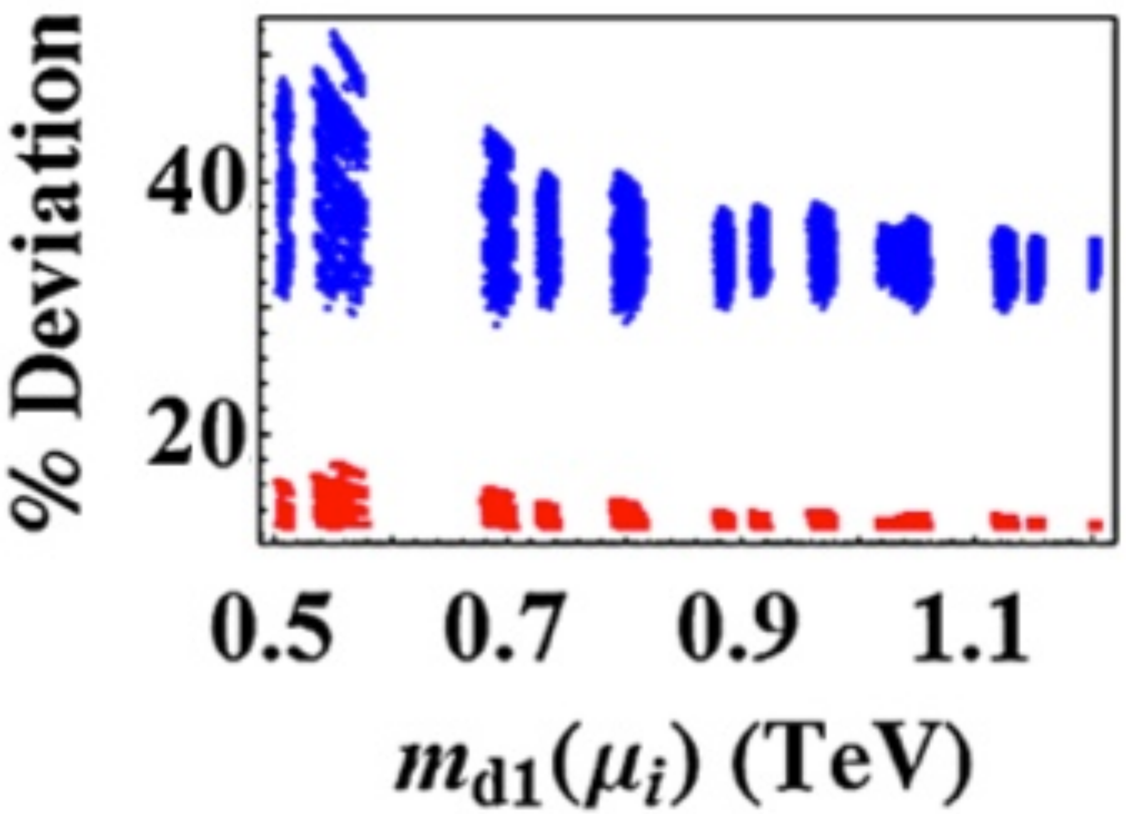
Mass deviations from GGM spectrum required to violate a sum rule by > 1σ

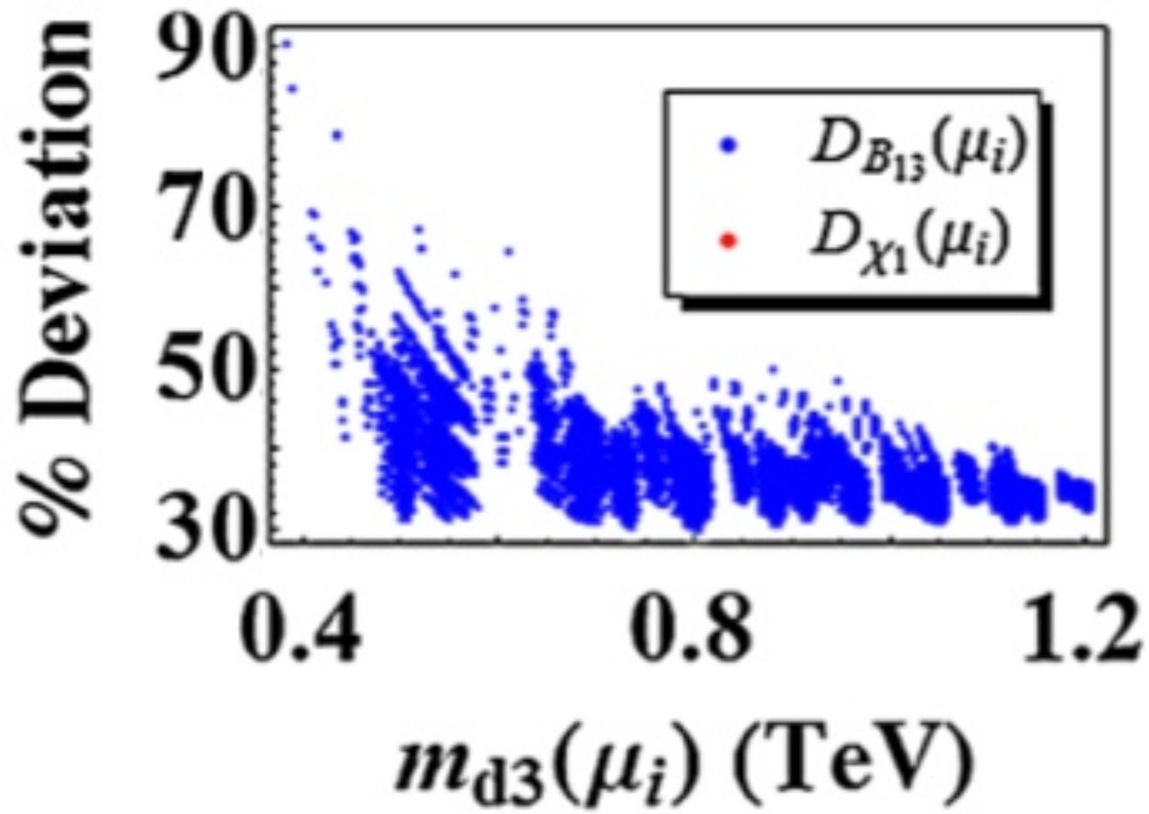


$$D_{B_{13}} \equiv 2m^2_{\tilde{Q}_1} - m^2_{\tilde{u}_1} - m^2_{\tilde{d}_1} - 2m^2_{\tilde{Q}_3} + m^2_{\tilde{u}_3} + m^2_{\tilde{d}_3}$$

$$D_{\chi_1} \equiv -6m^2_{\tilde{Q}_1} - 3m^2_{\tilde{u}_1} + 9m^2_{\tilde{d}_1} + 6m^2_{\tilde{L}_1} - m^2_{\tilde{e}_1}$$

Effects of Uncertainties in Sum Rules





Strong sensitivity of D_χ might give a way to distinguish m_{d1} and m_{u1} (assuming it vanishes for one permutation)

$$D_{B_{13}} \equiv 2m^2_{\tilde{Q}_1} - m^2_{\tilde{u}_1} - m^2_{\tilde{d}_1} - 2m^2_{\tilde{Q}_3} + m^2_{\tilde{u}_3} + m^2_{\tilde{d}_3}$$

$$D_{\chi_1} \equiv -6m^2_{\tilde{Q}_1} - 3m^2_{\tilde{u}_1} + 9m^2_{\tilde{d}_1} + 6m^2_{\tilde{L}_1} - m^2_{\tilde{e}_1}$$

Outline

$$X = X(m_{\tilde{f}}, m_{H_u}, m_{H_d}, M_a, g_a); \;\; \beta_X = 0$$

- Motivation
- Assumptions, Approximations, Previous Studies
- Constructing RGIs
- Applications to many-parameter models
  - Generic Flavor-Blind input
  - General Gauge Mediation
- Applications to few-parameter models
  - Minimal Gauge Mediation

Models with Fewer DOF

GGM: lots of high-scale free parameters → need many precise low-scale measurements to reconstruct them all

But many RGIs depend only on a subset of parameters.
- What can be done with less experimental input?

In this case it will be more effective to test models with fewer degrees of freedom.

Example: Minimal Gauge Mediation

MGM

Restrict GGM to a subspace described by the constraints:

$$A_1 = A_2 = A_3 \equiv A$$
$$MB_1 = MB_2 = MB_3 \equiv B$$
$$A = 2B^2$$

Messenger scale parameters become M, B, δ_u, δ_d (fixing $N_{mess}=1$)

$$A = \frac{2}{(4\pi)^4}\Lambda^2, \ \Lambda \equiv F/M$$

5 new constraints imply:

- simpler high-scale parameter reconstruction
- new sum rules that can be formulated in terms of the RGIs
  - given some low-scale masses, predict others
  - given enough low-scale masses, test sum rules and use to constrain more general high scale models

Higgs sector parameters may be difficult to measure, and 3rd generation involves mixing angles: Focus on RGIs that depend only on the 1st gen. + gauginos

RGIs in MGM

RGI	MGM Value
$I_{B_r} \equiv M_r/g_r^2$	B
$I_{M_1} \equiv M_1^2 - \frac{33}{8}(m_{\tilde{d}_1}^2 - m_{\tilde{u}_1}^2 - m_{\tilde{e}_1}^2),$	$\frac{33}{5} g_1^4 B^2$ ← at M
$I_{M_2} \equiv M_2^2 + \frac{1}{24}(9(m_{\tilde{d}_1}^2 - m_{\tilde{u}_1}^2) + 16 m_{\tilde{L}_1}^2 - m_{\tilde{e}_1}^2),$	$2 g_2^4 B^2$
$I_{M_3} \equiv M_3^2 - \frac{3}{16}(5 m_{\tilde{d}_1}^2 + m_{\tilde{u}_1}^2 - m_{\tilde{e}_1}^2).$	$-2 g_3^4 B^2$
$D_{\chi_1} = -6 m_{\tilde{Q}_1}^2 - 3 m_{\tilde{u}_1}^2 + 9 m_{\tilde{d}_1}^2 + 6 m_{\tilde{L}_1}^2 - m_{\tilde{e}_1}^2$	0

With these 1st gen + gaugino RGIs, we can encode one sum rule inherited from GGM and 4/5 new sum rules implied by equality of A_r and B_r parameters

MGM Parameter Reconstruction & Sum Rules

gaugino relations give B and familiar sum rules of GUT-scale gaugino mass unification:

$$B = I_{B_3}\,,\quad I_{B_1} = I_{B_2} = I_{B_3}$$

sfermion relations give M and two less familiar sum rules:

$$\sqrt{\frac{5I_{M_1}}{38I_B^2}} = g_1^2(M) \qquad \sqrt{\frac{I_{M_2}}{2I_B^2}} = g_2^2(M) \qquad \sqrt{\frac{-I_{M_3}}{2I_B^2}} = g_3^2(M)$$

$$C_1 \equiv \sqrt{\frac{38I_B^2}{5I_{M_1}}} - \frac{33}{5}\sqrt{\frac{2I_B^2}{I_{M_2}}} - I_{g_2} \equiv 0,$$

$$C_2 \equiv \sqrt{\frac{38I_B^2}{5I_{M_1}}} + \frac{11}{5}\sqrt{\frac{-2I_B^2}{I_{M_3}}} - I_{g_3} \equiv 0.$$

+ (more) accurate reconstruction of M!

PREDICTING AN MGM SPECTRUM

What masses will be measured first at the LHC?

G. Weiglein *et al.*, Phys. Rep. **426**, 47 (2006)

exhaustive SPS1A mSUGRA study --> translate basic ideas to MGM

Guess:

M_3 -- produce many gluinos, tag b-jets from off-shell decays through bottom squarks

$m_{\tilde{e}_1}$ -- often light enough to appear on-shell in cascade decays from the gluino through χ_2^0, leptons in the final state

$m_{\tilde{Q}_1}$ -- squarks are too heavy in MGM to be on-shell in gluino cascades, but may get from cascades starting from $\tilde{Q}_1$. Also decays through χ_2^0 giving another handle on $m_{\tilde{e}_1}$

Possibly also M_1 and M_2 from on-shell χ_1^0 and χ_2^0 in the cascades, although neutralino mass matrix also involves μ and $\tan\beta$

3rd generation masses require mixing elements to extract running parameters

PREDICTING AN MGM SPECTRUM

what can we do with [$M_3 \quad m_{\tilde{e}_1} \quad m_{\tilde{Q}_1}$] ?

$$\frac{M_3}{g_3^2} - \frac{M_1}{g_1^2},\ \frac{M_3}{g_3^2} - \frac{M_2}{g_2^2},\ C_1,\ C_2,\ D_{\chi_1} = 0$$

depend on

$$M_1,\ M_2,\ M_3,\ m_{\tilde{Q}_1},\ m_{\tilde{u}_1},\ m_{\tilde{d}_1},\ m_{\tilde{L}_1},\ m_{\tilde{e}_1}$$

can solve for the remaining unknown low-scale 1st generation + gaugino masses, and then use them to get messenger scale with $\sqrt{\frac{-I_{M_3}}{2I_B^2}} = g_3^2(M)$

$$g_3^4(M) = \frac{\mathcal{C}}{162} g_3^4(M_c) \left[1 - \frac{32}{3\mathcal{C}} \frac{g_3^4(M)}{g_3^4(M_c)}\right] \left[1 - \left(1 + 3\frac{g_3^2(M_c)}{g_2^2(M_c)}\right) \frac{g_3^2(M)}{g_3^2(M_c)}\right]^2$$

$$\mathcal{C} = \frac{1}{M_3^2}\left(6m_{Q_1}^2 - m_{e_1}^2 - \frac{5}{33}M_1^2 + 9M_2^2 - \frac{16}{3}M_3^2\right)$$

only one solution is physically realistic

Predicting an MGM Spectrum

Example: $A = 2B^2 = 0.3\ \mathrm{TeV}^2$, M=10^7 GeV (Input)

given [$M_3 \quad m_{\tilde{e}_1} \quad m_{\tilde{Q}_1}$],
reconstruct the rest from constraint equations.
universal 5% errors on
$M_3 \quad m_{\tilde{e}_1} \quad m_{\tilde{Q}_1}$

	Calculated	Data
M_3 (GeV)		446.8
$m_{\tilde{Q}_1}$ (GeV)		641.6
$m_{\tilde{e}_1}$ (GeV)		114.0
$g_1(M)$	0.5153 ± 0.0465	0.5159
$g_2(M)$	0.6647 ± 0.0400	0.6679
$g_3(M)$	0.9093 ± 0.1090	0.9144
M_1 (GeV)	84.4 ± 12.5	84.2
M_2 (GeV)	158.5 ± 24.0	159.4
$m_{\tilde{L}_1}$ (GeV)	221.3 ± 52.0	227.2
$m_{\tilde{u}_1}$ (GeV)	611.6 ± 36.5	608.4
$m_{\tilde{d}_1}$ (GeV)	607.5 ± 41.0	604.7

$\mathrm{Log}_{10}(M/\mathrm{GeV}) \approx 7 \pm 3$ (Calc)

$$D_{Y_{13H}} \equiv D_{Y_1} - \frac{10}{13} D_{Y_{3H}} = -\frac{10}{13}(\delta_u - \delta_d)$$

$$I_{Y_\alpha} \equiv \frac{2D_{Y_1} + D_{Y_{3H}}}{g_1^2(M)} = \frac{1}{g_1^2(M)}(\delta_u - \delta_d)$$

$$D_{Y_{3H}}(M_c) = \frac{13}{10}\left(1 + \frac{20}{13}\frac{g_1^2(M)}{g_1^2(M_c)}\right)\left(1 - \frac{g_1^2(M)}{g_1^2(M_c)}\right)^{-1}(m_{\tilde{d}_1}^2 + m_{\tilde{e}_1}^2 - m_{\tilde{L}_1}^2 + m_{\tilde{Q}_1}^2 - 2m_{\tilde{u}_1}^2)$$

$$\delta_u - \delta_d = \frac{33}{10}\left(1 - \frac{g_1^2(M)}{g_1^2(M_c)}\right)^{-1}(m_{\tilde{d}_1}^2 + m_{\tilde{e}_1}^2 - m_{\tilde{L}_1}^2 + m_{\tilde{Q}_1}^2 - 2m_{\tilde{u}_1}^2).$$

TESTING MGM

If all the 1st generation and gaugino masses are measured, then they can be inserted directly into the constraint equations and MGM can be tested.

Say $D_{\chi_1} = 0$. → Have some confidence that gauge mediation is at work.

Assume $I_{B_1} = I_{B_2} = I_{B_3}$ holds within errors (sensitive test, but holds in any model with gaugino mass unification at M_{GUT})

What about the sfermion sector- how effective are C_1 and C_2 at detecting deviations from MGM into the more general parameter space of GGM?

Introduce 3 parameters $x_i \equiv A_i/2B_i^2$. MGM satisfies $x_i=1$.

Useful to reformulate constraints to try to minimize exp errors.

Testing MGM

$$C_3 \equiv \frac{5I_{M_1}}{38I_B^2} + \frac{50I_{M_3}/I_B^2}{\left(22 - 5I_{g_3}\sqrt{-2I_{M_3}/I_B^2}\right)^2}$$

$$C_4 \equiv \frac{I_{M_2}}{2I_B^2} + \frac{2178I_{M_3}/I_B^2}{\left(22 - 5(I_{g_3} - I_{g_2})\sqrt{-2I_{M_3}/I_B^2}\right)^2}$$

Equivalent to C1 and C2 when both vanish (no new information), and not much better in terms of uncertainties

But: $C_5 \equiv \frac{38}{5}C_3 + 22C_4$

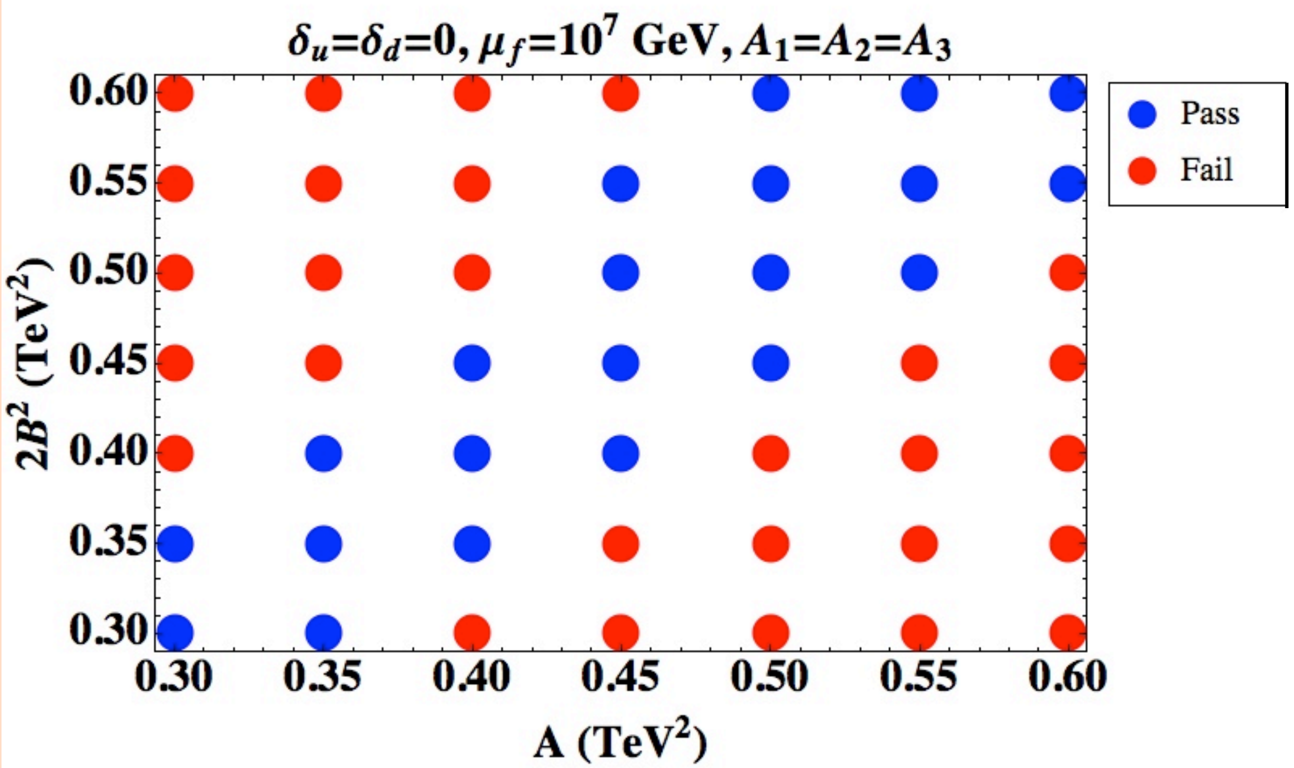
depends on $I_{M_{12}}$ and I_{M_3}, where $I_{M_{12}} = I_{M_1} + 11I_{M_2}$

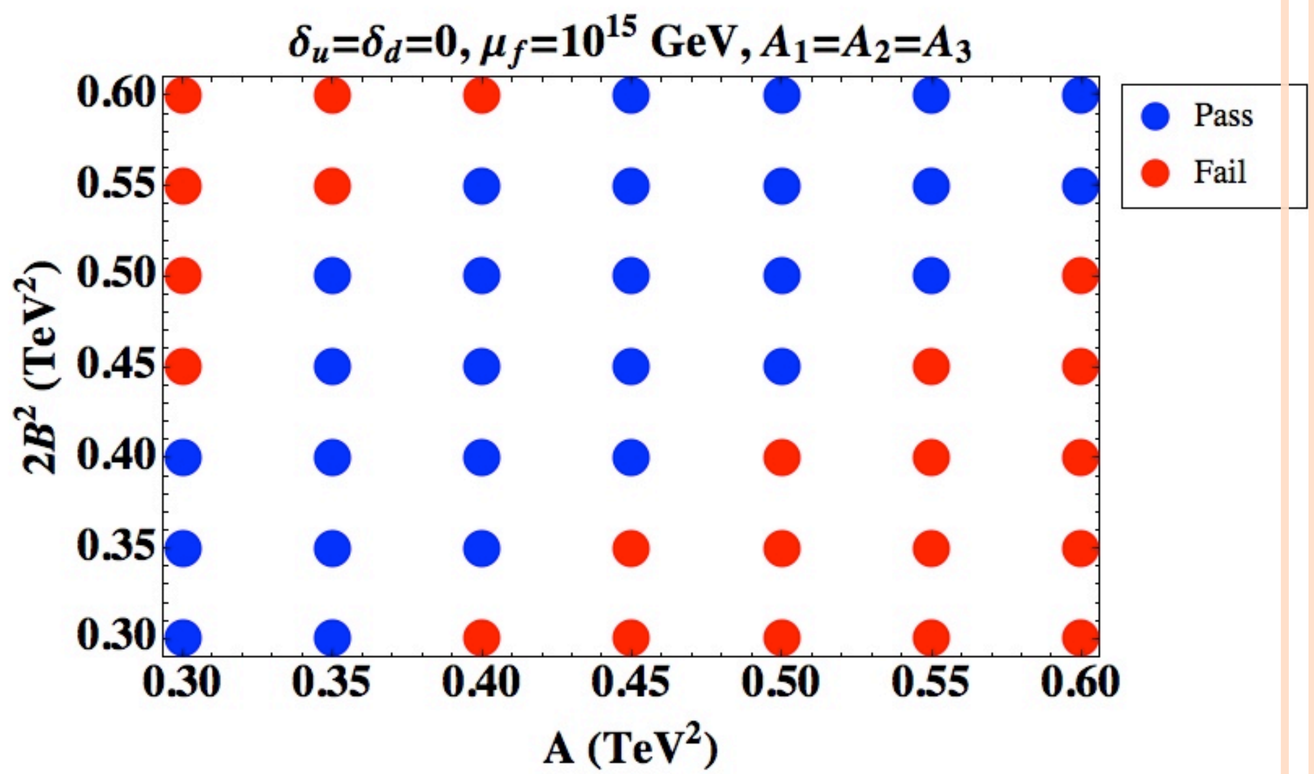
$$= M_1^2 + \frac{11}{3}\left(3M_2^2 + 2m_{\tilde{L}_1}^2 + m_{\tilde{e}_1}^2\right)$$

$I_{M_{12}}$ typically has small errors because it depends only on weakly-interacting particles → C_5 will be a powerful discriminator

Testing MGM

Fix $x_1=x_2=x_3=x$, scan over x and B assuming 5% uncertainties. Apply $C_1=C_2=C_5=0$





Sensitive to ~10% deviations of A from $2B^2$, tested mainly by C_5

Tested by C_5 for $x_3 > 1$, $C_{1,2}$ for $x_3 < 1$

Relevant for a modification of MGM with N sets of SU(5) messenger multiplets: $AN=2B^2$.

GGM Models Satisfying $C_1=C_2=0$

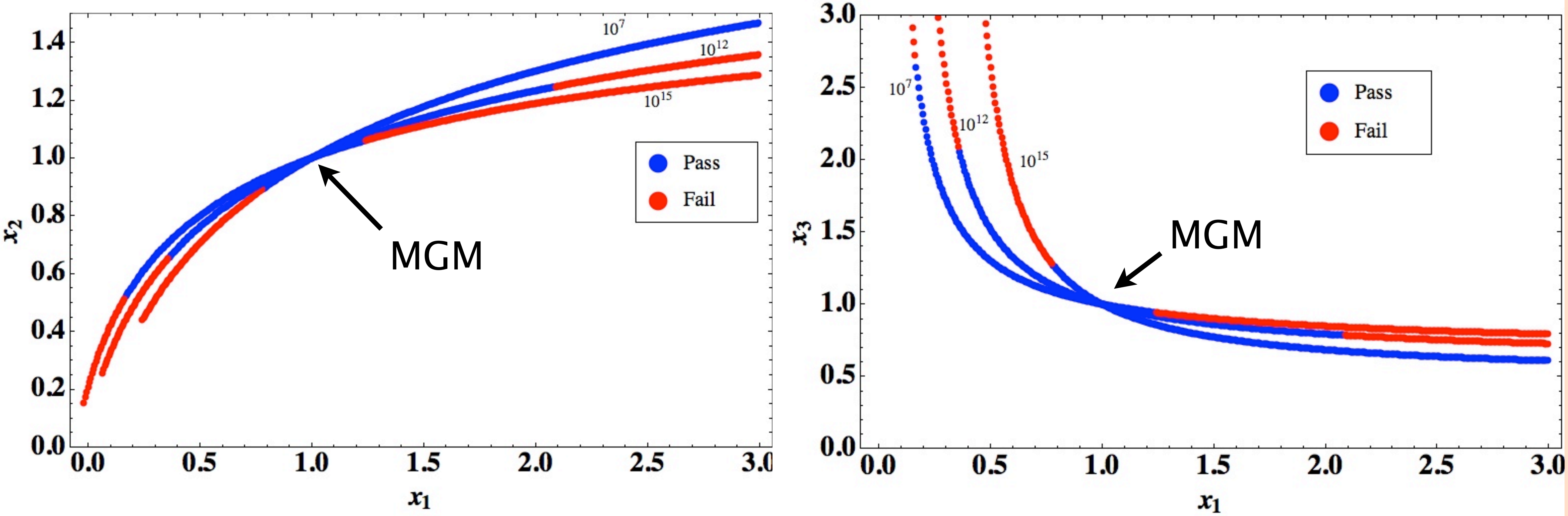
Regardless of how the constraints are formulated, we can only make 2 with these RGIs. → never fully implement $x_i=1$. (one of the three I_{Mr} had to be used to extract g(M), and then could not be used to make the third constraint.)

→ should be some GGM models that satisfy the constraints, but do not satisfy $x_i=1$. Not simple deviations like in previous plots. For a given M, these models fall along a curve that satisfies

$$C_1 = \frac{1}{g_1^2(M)}\left(1-\frac{33}{38}(1-x_1)\right)^{-1/2} - \frac{33}{5}\frac{1}{g_2^2(M)}\left(1-\frac{1}{2}(1-x_2)\right)^{-1/2} - I_{g_2} = 0$$

$$C_2 = \frac{1}{g_1^2(M)}\left(1-\frac{33}{38}(1-x_1)\right)^{-1/2} + \frac{11}{5}\frac{1}{g_3^2(M)}\left(1-\frac{3}{2}(1-x_3)\right)^{-1/2} - I_{g_3} = 0$$

GGM Models Satisfying $C_1=C_2=0$



Can still constrain the curves with the requirement that the messenger scale reconstructed from $\sqrt{\frac{-I_{M_3}}{2I_B^2}} = g_3^2(M)$ lies between about 10^7 and 10^{16} GeV. (Below 10^7 have additional constraints on M from NLSP decays to gravitino) $0.25 \lesssim g_3^4(M) \lesssim 1$

CONCLUSIONS

- 1-loop Renormalization group invariant quantities in the MSSM provide a simple, clear method for obtaining information about the mechanism that transmits SUSY-breaking

- RGIs facilitate the construction of a large class of sum rules in a given model, and give an algebraic method to estimate high-scale parameters

- Easy to approximate 2-loop corrections and low-scale thresholds

- Have studied here in the context of gauge mediation, but can be applied to other models and can in principle be extended to non-minimal models of TeV-scale SUSY: NMSSM, etc

Backup

The Right-Handed Sneutrino

When we fixed "MSSM β-functions only," we excluded the possibility of a RH (s)neutrino. What if it exists? In principle, h_ν and "Dirac" soft mass enter β-functions

Seesaw: $m_\nu \sim \mathcal{O}\left(\frac{h_\nu^2 \sin^2\beta v^2}{M_R}\right)$ → if $m_\nu \lesssim 1\ \mathrm{eV}$,

$$h_\nu \lesssim 0.1 \text{ for } M_R \lesssim 10^{11}\ \mathrm{GeV}$$

Or, maybe $M_R > M$. Then RH (s)neutrinos can be integrated out.

→ can neglect in β-functions unless $M_R \ll M$ **and** h_ν too large to ignore.

Note that soft mass is not even generated at M in GGM.

EWSB

What are natural values for delta_u and delta_d (ie mhu2 and mhd2)?

We are agnostic about the values of B and mu at M.

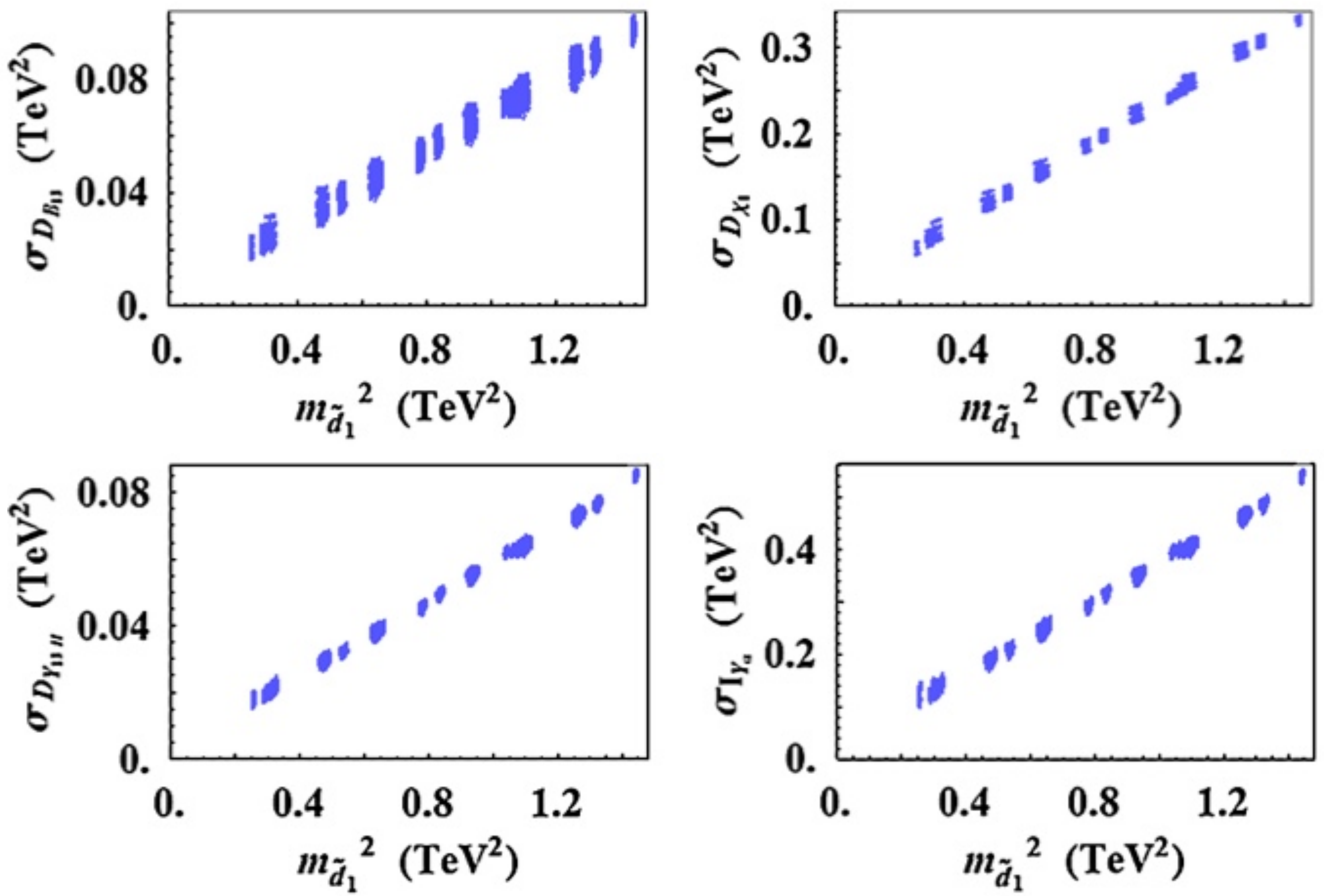
Then fine-tuning is minimized for larger delta_u, which drives down mu at the electroweak scale.

But EWSB is only achieved if mhu2-mhd2 is not to large, so deltau-deltad is also bounded.

In general to get the messenger scale in GGM we needed large |deltau-deltad|

As in our scans, this can be achieved with positive deltau and larger deltad, at the same or better level of fine-tuning as in the MSSM with universal BCs for the Higgs masses.

Experimental Errors in RGIs

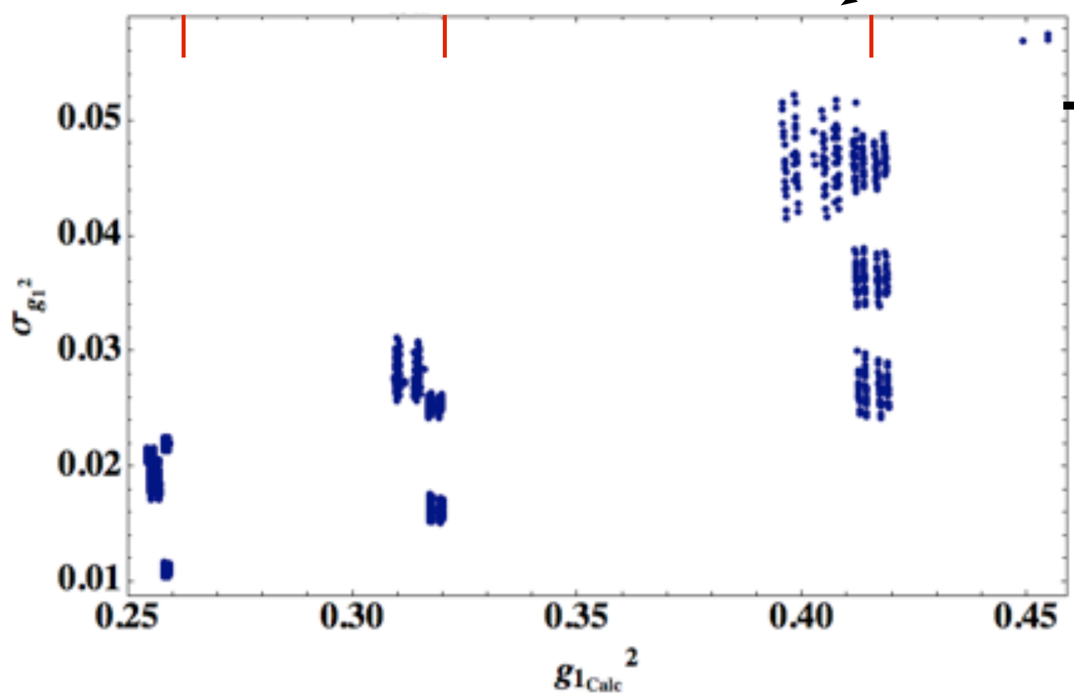


with universal soft mass errors, uncertainties in RGIs usually controlled by masses of colored particles due to typical hierarchy $m_{weak} < m_{strong}$

Experimental Errors in Reconstructed M

Three input scan values

Spread in reconstruction due to 2-loop effects

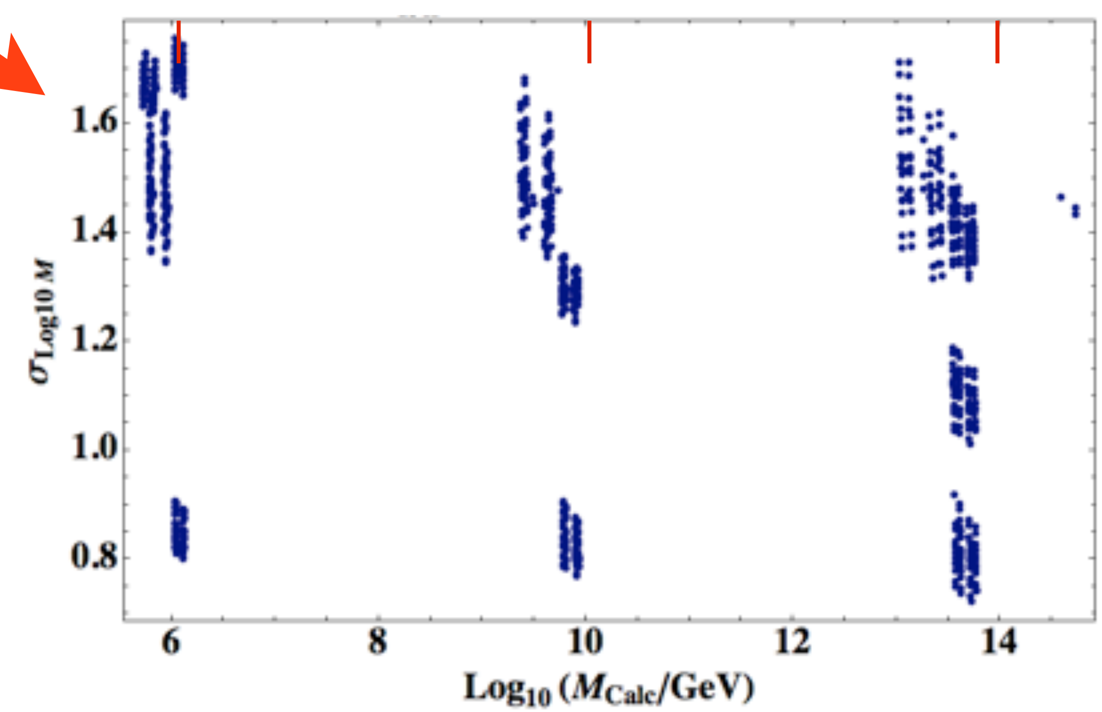


largest exp errors & 2-loop shifts for high M + large A_3, B_3

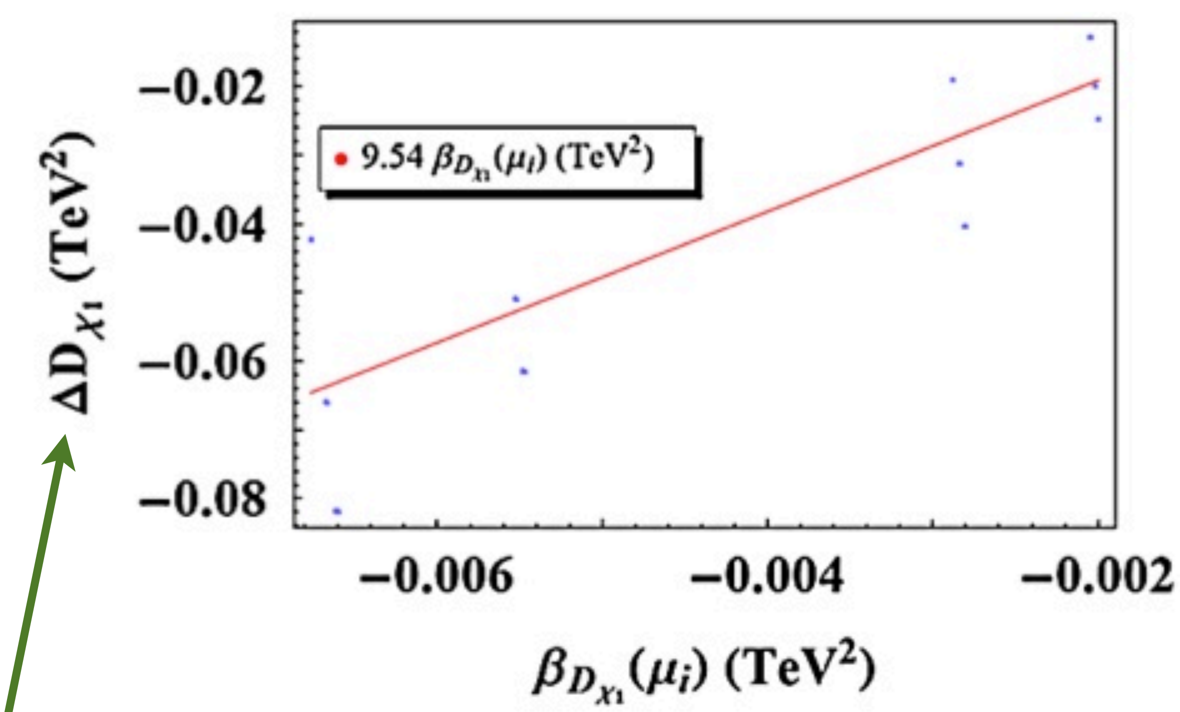
Only attempt A_r, g(M) reconstruction when $\frac{|D_{Y_{13H}}|}{m^2_{squark}} \gtrsim 1$

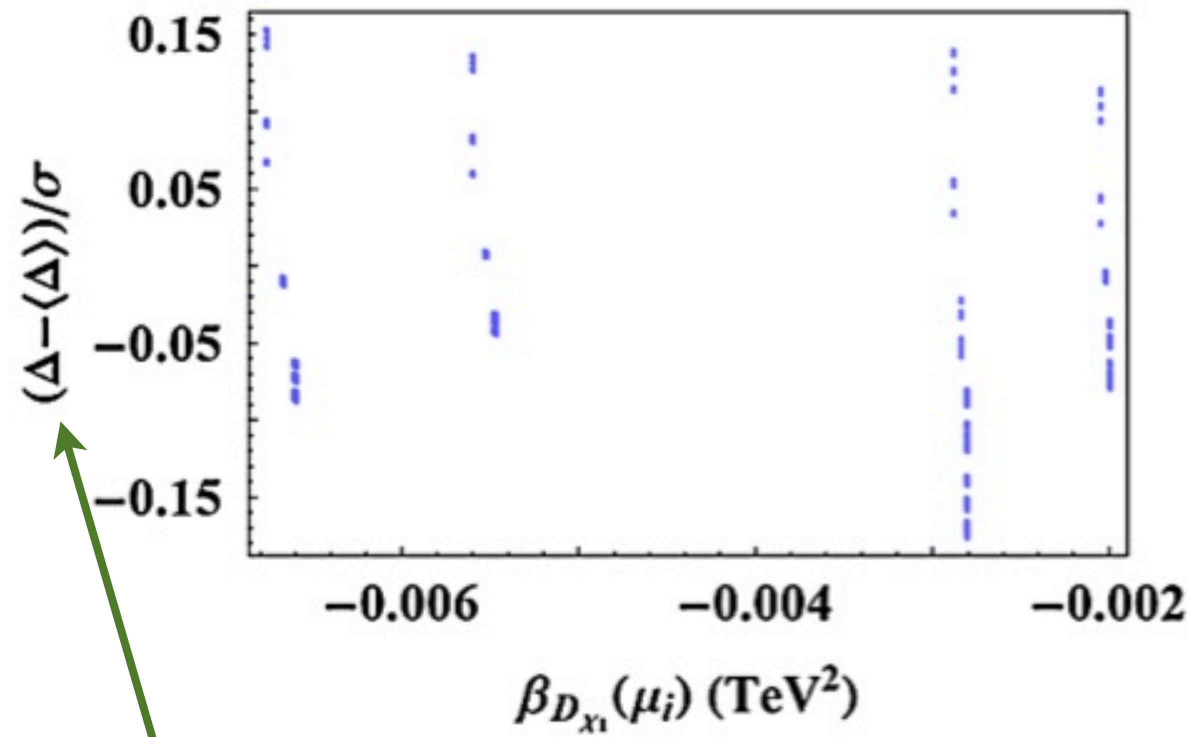
$$g_1^2(M) = -\frac{13}{10}\frac{D_{Y_{13H}}}{I_{Y\alpha}}$$

$$\log\frac{M}{M_0} = \frac{8\pi^2}{b_1}\left(\frac{1}{g_1^2(M_0))} - \frac{1}{g_1^2(M))}\right)$$



2-LOOP EFFECTS IN RGIs



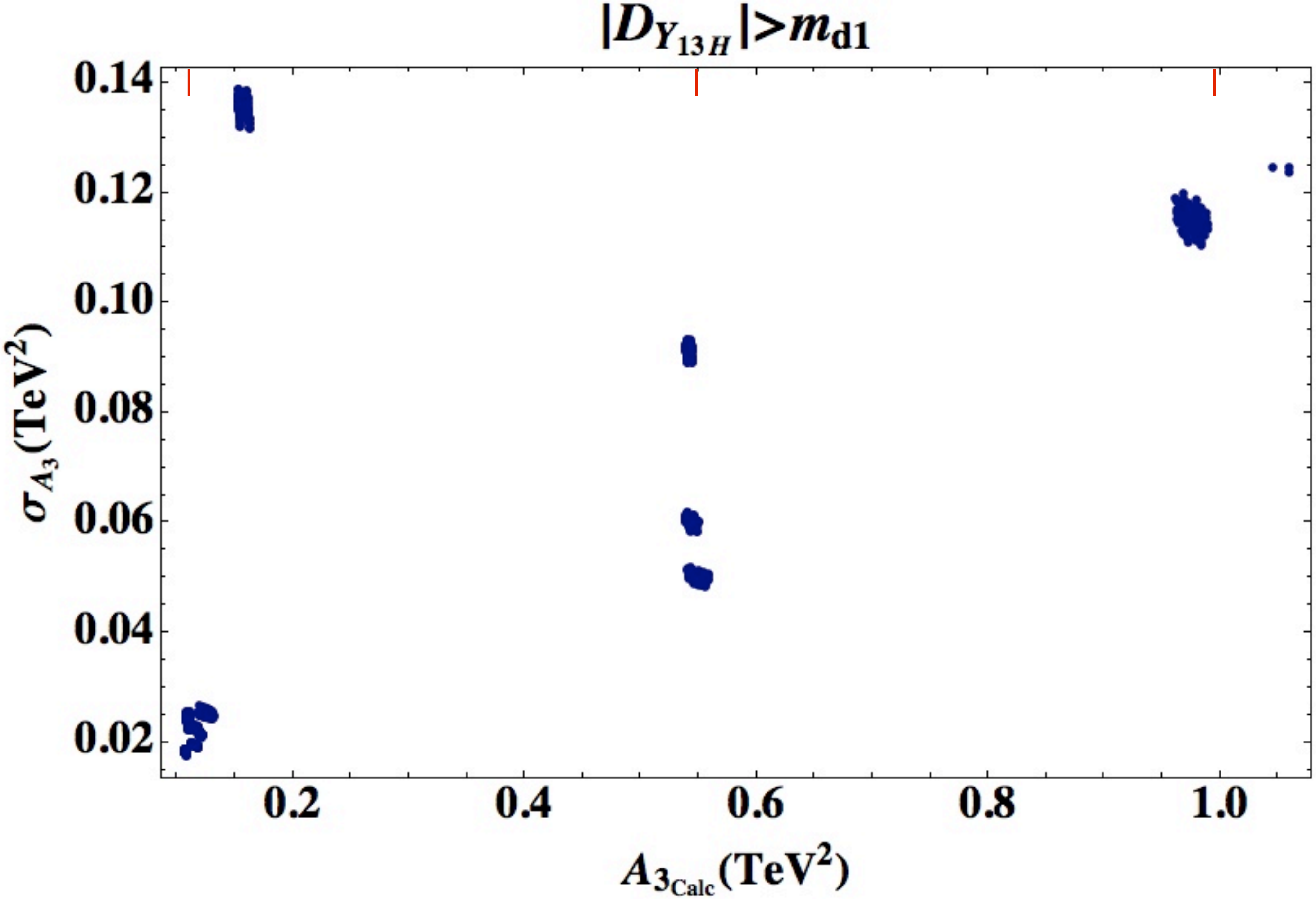


full 2-loop correction

approx β-function

residual 2-loop corrections usually smaller than statistical uncertainties (universal 1%)

EXPERIMENTAL ERRORS IN A3



$$A_3 = -\frac{2}{3}\left(\frac{I_{M_3}}{g_3^4} - I_{B_3}^2\right) \qquad I_{M_3} \equiv M_3^2 - \frac{3}{16}(5m_{\tilde{d}_1}^2 + m_{\tilde{u}_1}^2 - m_{\tilde{e}_1}^2)$$