# Magnetic inflation: realizing natural inflation on a steep potential

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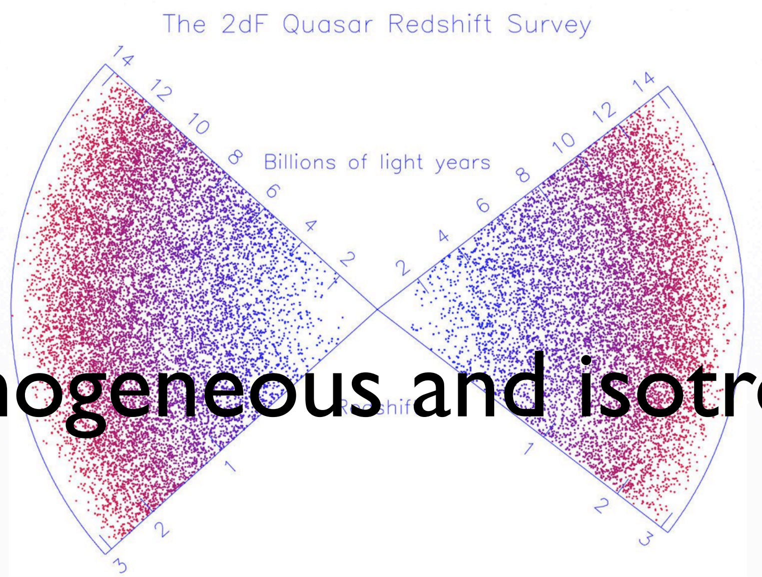
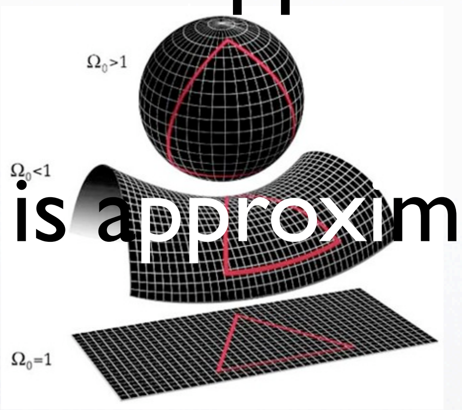
with M. Anber, in preparation

# *5 INTERESTING FACTS ABOUT THE UNIVERSE*

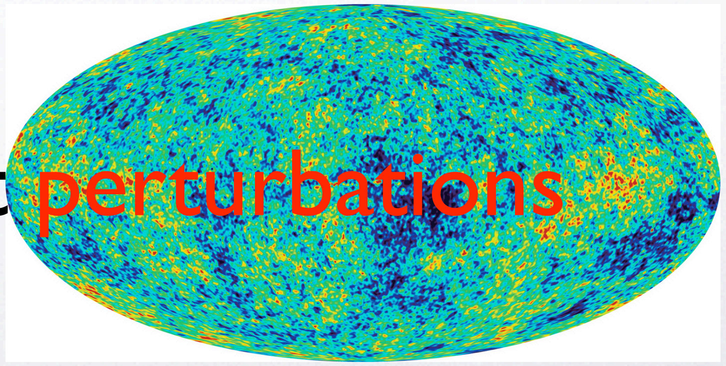



- it is old and very large

- in first approximation it is homogeneous and isotropic

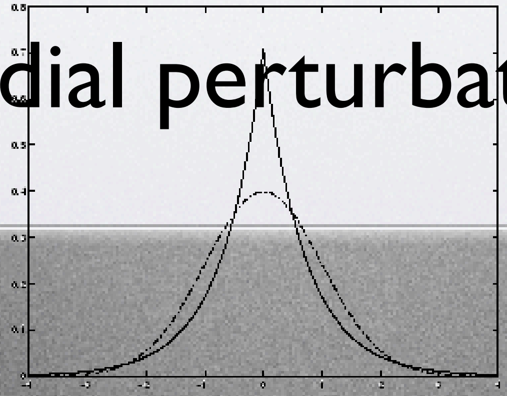



- it is approximately flat



- structure grew out of small, scale invariant perturbations

- spectrum of primordial perturbations was gaussian



All these facts can be explained by

**INFLATION**

:= period of accelerated expansion in the very early Universe

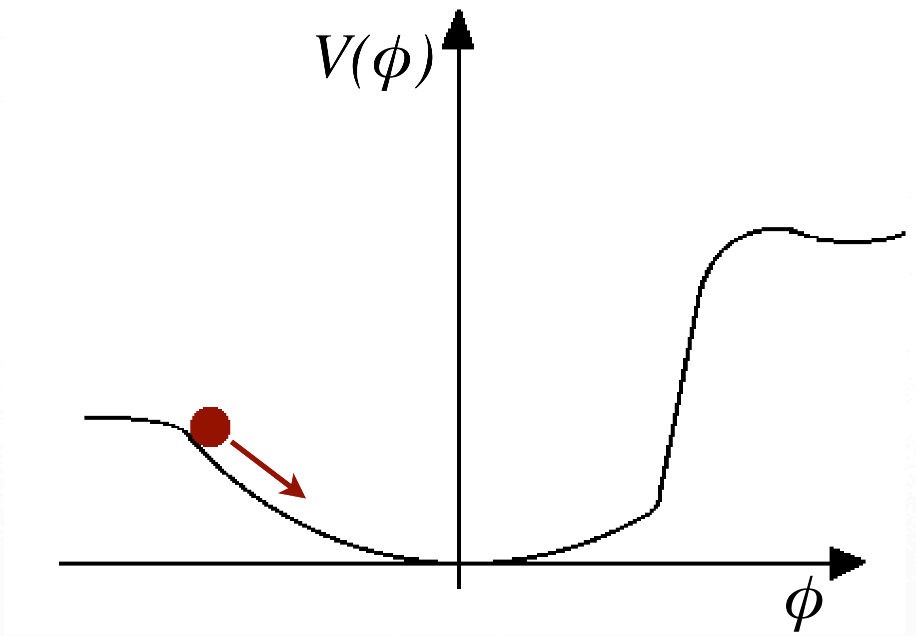
$a$=scale factor of the Universe. Obeys

$$H^2 = \frac{8\pi G}{3}\varrho \equiv \frac{\varrho}{3M_P^2} \qquad H \equiv \frac{\dot{a}}{a}$$

during inflation require $H$~constant

*(not so easy, since $\varrho$ dilutes away for ordinary matter...)*

*How to get some "slowly diluting" matter?*




✓ very early Universe filled by scalar field $\phi$, potential $V(\phi)>0$

✓ to induce acceleration, $V(\phi)$ must be *flat*

$|V'(\phi)| << V(\phi)/M_P$

✓ to have long enough inflation, $V(\phi)$ must *stay* flat for long enough

$|V''(\phi)| << V(\phi)/M_P^2$

Simple way of realizing $|V'(\phi)| << V(\phi)/M_P$, $|V''(\phi)| << V(\phi)/M_P{}^2$: monomial potential, with $\phi$ large enough

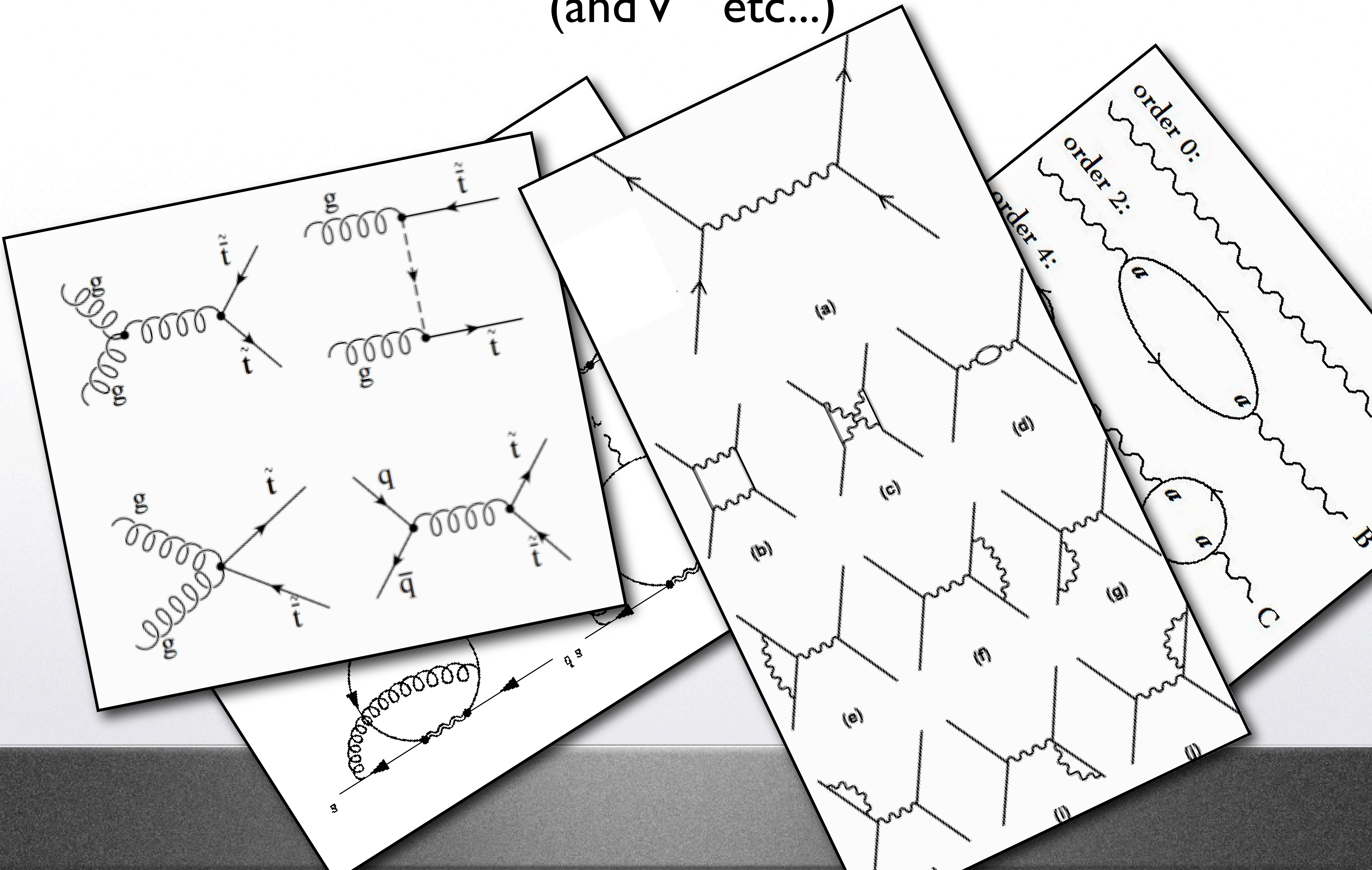
Most famous example: quadratic potential *(chaotic inflation)*

Linde 1983

$$V(\phi)=m^2\,\phi^2\,/2$$

Amplitude of perturbations produced during inflation → $m \sim 10^{13}$ GeV

...but, in general, quantum loops will contribute to V’ and V”
(and V’” etc...)



Radiative corrections can disrupt the inflationary potential in two ways

1- affect the functional form of $V(\phi)$

2- affect value of the parameters that appear in $V(\phi)$

Chaotic inflation example

1- adds terms $\propto\phi^n$, $n=4, 6, \ldots$

2- push $m$ to larger values (e.g. $M_P$ - cf EW hierarchy pbm)

*How to make sure that radiative effects are under control?*

# *The situation is actually not so horrible...*

Smolin 80
Linde 88

**If** we have a theory where $\phi$ interacts only with gravity

**then** quantum corrections are not a problem!

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Indeed: for potential $V(\phi)$, quantum gravity effects are

$\mathcal{O}(1)\, V(\phi)^2/M_P^4$ and $\mathcal{O}(1)\, V''(\phi)\, V(\phi)/M_P^2$

negligible during inflation

---

however, in general there will coupling to other fields

reheating

*How to make sure that radiative effects are under control?*

A very well-known system that contains
"controllably small" quantities is the **Standard Model**:
"small" quantities are protected against radiative effects
by ***symmetries***

If a model has a symmetry, quantum effects cannot violate it
(unless the symmetry is anomalous...)

If the symmetry is broken, quantum effects cannot make the
breaking much larger
(ie the breaking parameter is controllably small)

A field $\phi$ has a *shift symmetry* if the theory that describes it is invariant under the transformation

$$\varphi \longrightarrow \varphi + c$$

($c$=arbitrary constant)

If this symmetry is exact, the only possible potential for $\phi$ is $V(\phi)$=constant

(i.e. a cosmological constant)

*an exact shift symmetry is an overkill...*
*...but we can break the symmetry a bit and generate a potential*

# An (important) example

If $\phi$ is a phase, then shift symmetry ⇔ global U(1)

- Theory with a spontaneously broken global U(1)

$$\mathcal{L} = \partial_\mu H^* \, \partial^\mu H - \lambda \left(|H|^2 - v^2\right)^2$$

- Decompose $H = (v + \delta H) \, e^{i\phi/v}$
  where $\delta H$ is massive and $\varphi$ is a massless Goldstone boson *(pseudoscalar)*

- The global U(1) is broken e.g. by gravitational instantons

$$\delta\mathcal{L} = e^{-S} \, M_P^3 \, (H + H^*) + \ldots$$

( $S$ = instanton action, ∝ $M_P^n$)

- A potential is generated: $\delta V \sim e^{-S} M_P^3 \, v \, \cos(\phi/v)$

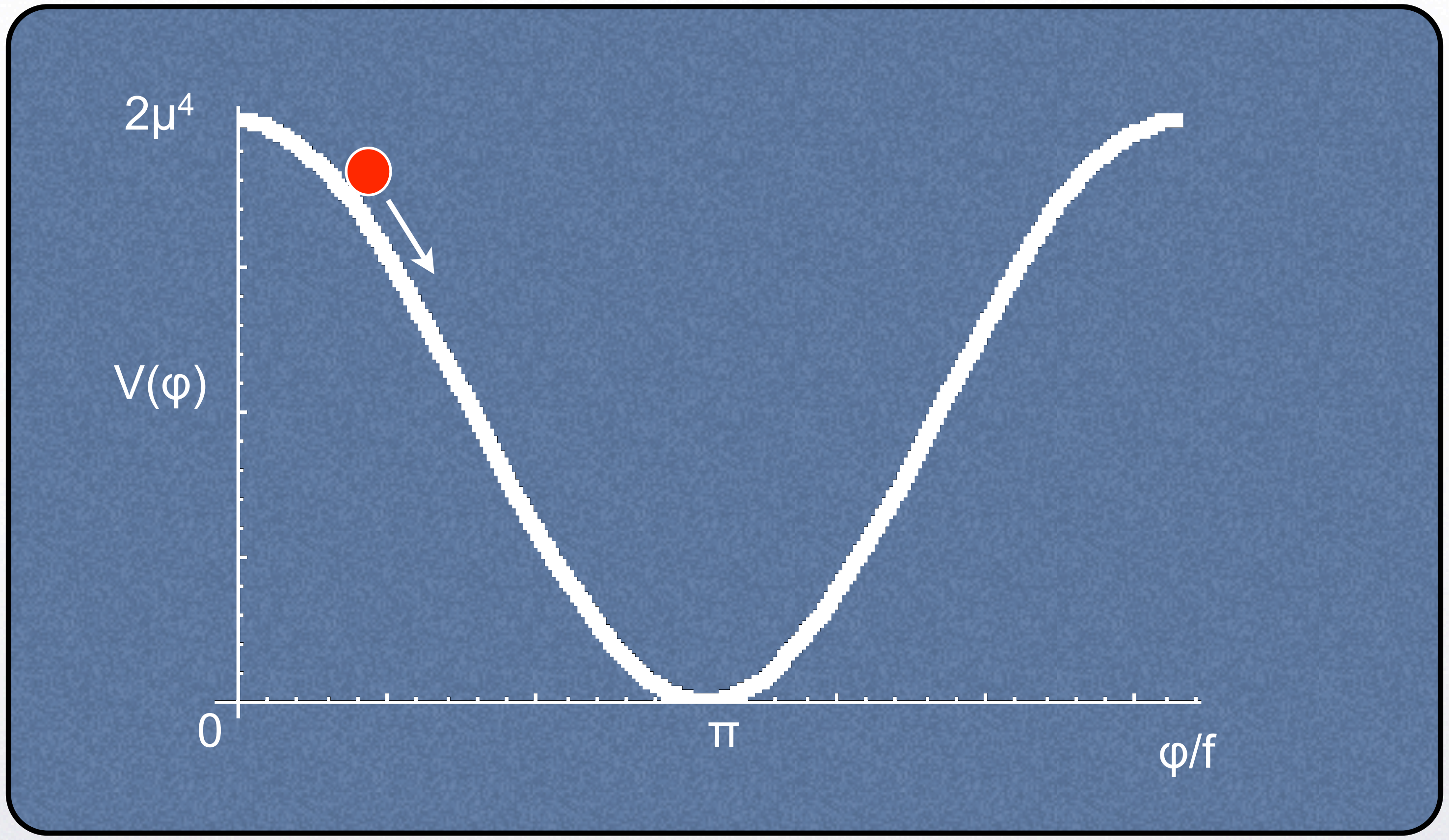
PSEUDO-NAMBU-GOLDSTONE BOSON
PNGB

...using a pNGB as an inflaton...

# Natural inflation

Freese et al 1990

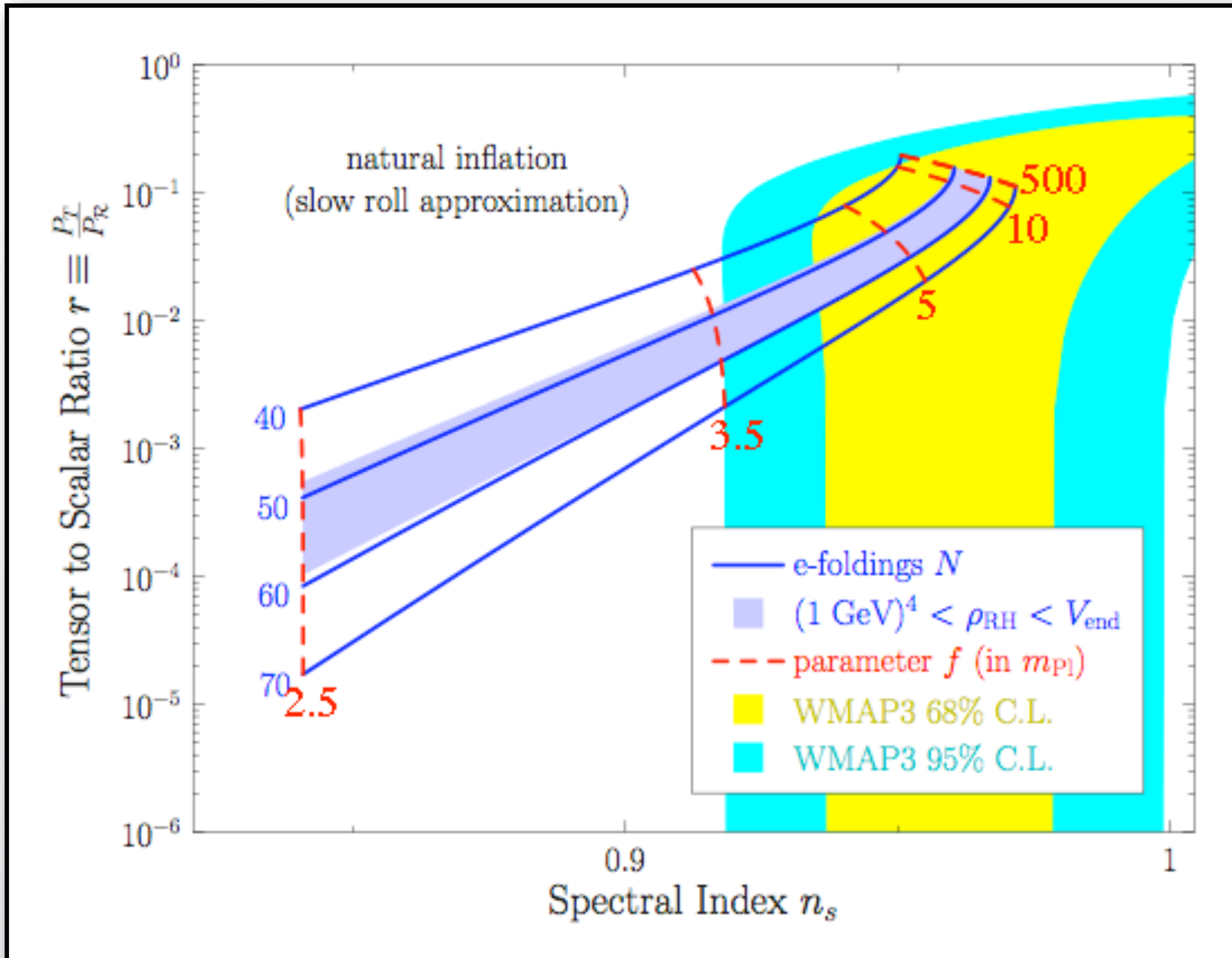
$$V(\varphi)=\mu^4 \left[ \cos(\varphi/f)+1 \right]$$



Because of its radiative stability,

*A pNGB gives an extremely well motivated model of inflation from the point of view of effective field theory*

# What about data?




from Savage et al, 2006

$f > 3.5\ M_P$

# Stringy models of natural inflation?

***YES,*** *in principle*
(string theory contains a plethora of pNGBs)

***However***

Banks, Dine, Fox and Gorbatov 03

String Theory appears to require **$f<M_P$**

$n$-instanton actions contribute $\propto e^{-(n M_P/f)} \cos(n\,\phi/f)$ to pNGB potential

$\Downarrow$

first $f/M_P$ harmonics in $V(\phi)$ matter

# Ways out?

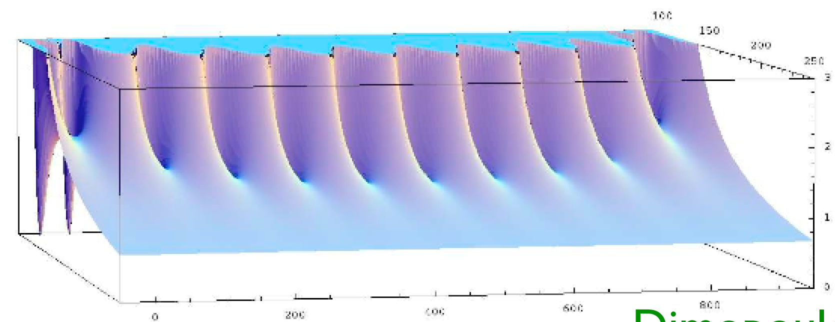
Kim, Nilles and Peloso 2004

- Use *two* pNGBs

$$V = \Lambda_1^4\left[1-\cos\left(\frac{\theta}{f_1}+\frac{\rho}{g_1}\right)\right]+\Lambda_2^4\left[1-\cos\left(\frac{\theta}{f_2}+\frac{\rho}{g_2}\right)\right]$$

Blanco-Pillado et al 2004

- Use pNGBs and moduli



Dimopoulos et al 2005

- Use *many* pNGBs

$$\mathcal{L} = -\sqrt{-g}\sum_{i=1}^{N}\left\{\frac{1}{2}\left(\partial\phi_i\right)^2+\Lambda_i^4\left[1+\cos(\phi_i/f_i)\right]\right\}$$

- ...

...all based on multi field dynamics to generate a flat effective potential

# A different way of approaching the problem...

The inflaton can be slowed down
(even on a steep potential!)
if it ***dissipates*** its kinetic energy

e.g. particle production associated to motion of $\phi$
rate depends on $\dot{\phi}$

# ...back to the origins...

*In the early '70s* (pre-inflation), try to explain isotropy from initial anisotropy by particle production

*Today,* chaotic inflation paradigm allows to ignore primordial anisotropy problem-but still need flat potential

***Particle production***
can help mitigate
the requirement of flat potential

# Trapped inflation (I)

Green et al 2009

**Idea:** field $\chi$ with mass $m_\chi(\phi(t))$

At some time $t_0$, $m_\chi(t_0)=0$, with $\dot{m}_\chi(t_0)\neq 0$.

$\Rightarrow$ Heisenberg inequality $\hbar \gtrsim \Delta E\, \Delta t \sim m_\chi\,(m_\chi/\dot{m}_\chi)$ violated

Concept of number of quanta of $\chi$ not well defined

**Quanta of $\chi$ are produced**

# Trapped inflation (II)

Particles created at expenses of inflaton kinetic energy

(the only useful energy available)

Chung et al 1999

Inflaton rolling is slowed down for ~1 efold

To get 60 efolds, need many production events

Green et al 2009

$$\frac{1}{2}g^2\sum_i(\phi-\phi_i)^2\chi_i^2\ .$$

depending on parameters, 1 to $10^{12}$ events per efold are needed

this structure can be present in some stringy constructions

# A mechanism analogous to trapping is built in natural inflation

Anber and LS
in preparation

Idea: pNGB driving natural inflation is "naturally" coupled to gauge fields

$$\mathcal{L}_{\phi F_{\mu\nu}} = \alpha \frac{\phi}{4 f} \epsilon_{\mu\nu\rho\lambda} F^{\mu\nu} F^{\rho\lambda}$$

$\alpha$=dimensionless constant

Equation for the $U(1)$ field in the presence of $\phi(t)$:

$$\frac{\partial^2 A_\pm}{\partial t^2} + \left( \frac{\vec{k}^2}{a^2} \mp \frac{\alpha}{f} \frac{d\Phi}{dt} \frac{|\vec{k}|}{a} \right) A_\pm = 0$$

$A_\pm$= >ve and <ve helicity comoving modes of the vector potential

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One of the two modes has a ***negative***, ***time dependent*** "mass term"

**Exponential** amplification of one helicity mode

Equation for $A_\pm$ can be solved by assuming $\dot{\phi}$, $H$=constant

Modes with $k/a < \alpha\dot{\phi}/f$

feel tachyonic mass until $k=aH$

⇩

amplification by

$\sim exp[\alpha\dot{\phi}/fH]$

# more precisely...

$$A_+(\tau, \vec{k}) \simeq \frac{1}{\sqrt{2|\vec{k}|}} \left( \frac{|\vec{k}|}{2\,\xi\, a\, H} \right)^{1/4} e^{-2\sqrt{2\xi\,|\vec{k}|/aH} + \pi\,\xi}$$

*Exponential amplification term!*

$$\xi \equiv \frac{\alpha\,\dot{\phi}}{2\,f\,H}$$

# Slowing down the inflaton

backreaction equation

$$\ddot{\phi} + 3\,H\dot{\phi} + V'(\phi) = -\frac{\alpha}{f}\langle\vec{E}\cdot\vec{B}\rangle$$

with

$$\langle\vec{E}\cdot\vec{B}\rangle \propto exp\{\pi\alpha\dot{\phi}/fH\}$$

As φ starts increasing under the effect of the steep potential, the backreaction term gets important, slowing it down.

Slow roll equation⇒ $$V'(\phi) \simeq -\frac{\alpha}{f}\langle\vec{E}\cdot\vec{B}\rangle$$

# the slow roll solution...

$$\dot{\phi} \simeq \frac{f\,H}{\pi\,\alpha}\,\log\left[\frac{M_P^4\,f\,V'(\phi)}{V(\phi)^2}\right]$$

...and a constraint on the model:

$\Delta\phi=\pi f$ from top to bottom of potential

total # efoldings ~ $H\,\Delta\phi/\dot{\phi}$ ~ $\alpha\,\log\left[\frac{M_P^4\,f\,V'(\phi)}{V(\phi)^2}\right]^{-1}$

$\boldsymbol{\alpha \gtrsim 100}$

# Consistency

We have assumed that the contribution to the electromagnetic modes to the Hubble parameter is negligible.

True?

**YES!**

$$\varrho_{EM} \sim \vec{E}^2 + \vec{B}^2 \sim \vec{E} \cdot \vec{B} \sim fV'(\phi)/\alpha$$

$$\varrho_{EM}/V(\phi) \sim fV'(\phi)/(\alpha V(\phi))$$

negligible, for $\alpha >> 1$, unless at the bottom of $V(\phi)$

Reheating?

# ...but... is it really inflation?

Need the slow roll parameters $\varepsilon$ and $\eta << 1$:

$$\epsilon = \frac{\dot{\Phi}^2}{2\,H^2\,M_P^2} \simeq \frac{2\,f^2}{\alpha^2\,M_P^2}$$

$$\eta = 2\,\epsilon - \frac{f}{\pi\alpha}\left(\frac{V''(\Phi)}{V'(\Phi)} - 2\,\frac{V'(\Phi)}{V(\Phi)}\right)$$

# Bottom line - background evolution

For $\alpha > O(100)$, possible to get $\sim 60$ efolds of inflationary expansion

# How to get such a large α?

Choi and Kim 85

One example

Two axions in $E_8 \times E_8$

$$\mathcal{L}_{\text{axions}} = \frac{1}{2}(\partial_\mu a_1)^2 + \frac{1}{2}(\partial_\mu a_2)^2 - \frac{1}{2}\left(\frac{a_1}{M_1} + \frac{a_2}{M_2}\right) F^i_{\mu\nu}\tilde{F}^i_{\mu\nu} - \frac{1}{2}\left(\frac{a_1}{M_1} - \frac{a_2}{M_2}\right) F'^i_{\mu\nu}\tilde{F}'^i_{\mu\nu}$$

$$= \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{2}(\partial_\mu a')^2 - \frac{a}{2M} F\tilde{F} - \frac{a'}{2M'}\left(F\tilde{F} + \frac{M_2^2+M_1^2}{M_1^2-M_2^2} F'\tilde{F}'\right)$$

$$a = (M_1 a_1 + M_2 a_2)/(M_1^2 + M_2^2)^{1/2}$$

$$a' = (M_2 a_1 - M_1 a_2)/(M_1^2 + M_2^2)^{1/2}$$

$$M = \frac{1}{2}(M_1^2 + M_2^2)^{1/2}$$

$$M' = M_1 M_2 (M_1^2 + M_2^2)^{1/2}/(M_1^2 - M_2^2)$$

# Perturbations (I)

Equation for perturbations

$$\delta\ddot{\phi} + 3\,H\,\delta\dot{\phi} + \left(-\nabla^2 + V''(\phi)\right)\delta\phi = -\frac{\alpha}{f}\,\delta\left[\vec{E}\cdot\vec{B}\right]$$

$\gg H^2$

Two contributions to $\delta\left[\vec{E}\cdot\vec{B}\right]$ :

$$\delta\left[\vec{E}\cdot\vec{B}\right] \simeq \left[\vec{E}\cdot\vec{B} - \langle\vec{E}\cdot\vec{B}\rangle\right]_{\delta\phi=0} + \frac{\partial\langle\vec{E}\cdot\vec{B}\rangle}{\partial\dot{\phi}}\,\delta\dot{\phi}$$

intrinsic fluctuations $\delta_{\vec{E}\cdot\vec{B}}$

fluctuations induced by fluctuations in $\phi$

# Perturbations (II)

Fourier-transformed effective equation for perturbations

$$\delta\ddot{\phi}_p + H\left(3 + \frac{\pi\,\alpha\,V'\left(\Phi_0\right)}{3\,f\,H^2}\right)\delta\dot{\phi}_p + \left(\frac{p^2}{a^2} + V''\left(\Phi_0\right)\right)\phi_p = -\frac{\alpha}{f}\,\delta_{\vec{E}\cdot\vec{B}}(p)$$

Solution (using Green function method)

$$\delta\phi_{\vec{p}}\,(t) = -\frac{\alpha}{f}\int dt'\,G\left(t,\,t'\right)\delta_{\vec{E}\cdot\vec{B}}\left(t',\,\vec{p}\right)$$

two point function of inflaton perturbations

Barnaby et al 09

$$\langle 0|\delta\phi_{\vec{p}}\,\delta\phi_{\vec{p}'}\,|0\rangle = \frac{\alpha^2}{f^2}\int dt'\,G\left(t,\,t'\right)\int dt''\,G\left(t,\,t''\right)\,\langle 0|\delta_{\vec{E}\cdot\vec{B}}\left(t',\,\vec{p}\right)\delta_{\vec{E}\cdot\vec{B}}\left(t'',\,\vec{p}'\right)|0\rangle$$

# Perturbations (III)

Inflaton power spectrum related to two point function of $\phi$ by

$$\mathcal{P}_{\mathcal{R}}(p) = \frac{p^3\, H^2\, \langle \phi_{\vec{p}}\, \phi_{\vec{p}'} \rangle}{2\,\pi^2\, \dot{\phi}^2\, \delta^{(3)}(\vec{p} + \vec{p}')}$$

*Spectrum of metric perturbations*

$$\langle \mathcal{R}\mathcal{R} \rangle \cong \frac{4\,/N}{\log\left[M_P^4\, f\, V'/V^2\right]^2} \left(\frac{p}{a\,H}\right)^{2fV''/\pi\alpha V'}$$

for $N$ gauge fields

quasi scale invariant for large $\alpha$

Amplitude $\sim 0.05\ Log(M_P/E_{infl})^{-2}\ N^{-1}$

for $E_{infl} \sim TeV$, need $N \sim 10^5$

but...

did we use the true value of $\delta_{\vec{E}\cdot\vec{B}}$ ?

# More dissipation?

(very much in progress)

Large electromagnetic fields on subhorizon scales

Light particles charged under the $U(1)$s copiously produced

Amplitude of $\delta_{\vec{E}\cdot\vec{B}}$ reduced

Since perturbations in $\phi$ are sensitive only to $\delta_{\vec{E}\cdot\vec{B}}$, amplitude of perturbations reduced

*Nota bene:* this does not affect the slowing down of the zero mode of the inflaton, that is just based on energy conservation

# Cosmological magnetic fields

Anber and LS 06

The model comes with a bonus!

If one of the $U(1)$s is $U(1)_{EM}$,
then magnetic fields of cosmological interest generated

(note: the origin of galactic and cluster magnetic fields is still mysterious)

And with a signature!

The magnetic fields generated this way
should be maximally helical

# Conclusions

- We do not have to be very creative to realize natural inflation on a steep potential (but we need to be fine tuned at *~1%...*)
- Spectrum of perturbations is quasi-scale invariant
- Amplitude tends to be large, but there are ways out (in progress)