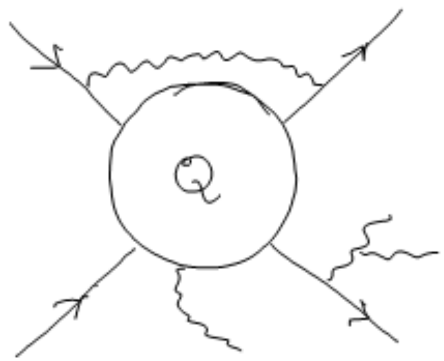
Soft gluons & gaps

Jeff Forshaw

- An introduction to soft gluons
- Jet vetoing and "gaps between jets"
- Superleading logarithms?

JF, A. Kyrieleis, M. Seymour: JHEP 0608:059, 2006.
JF, M. Sjödahl: JHEP 0709:119, 2007.
JF, A. Kyrieleis, M. Seymour: JHEP 0809:128, 2008.
JF, J. Keates, S. Marzani, arXiv:0905.1350.

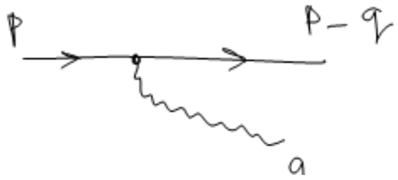
Given a particular hard scattering process we can ask how it will be dressed with additional radiation (perturbatively calculable):



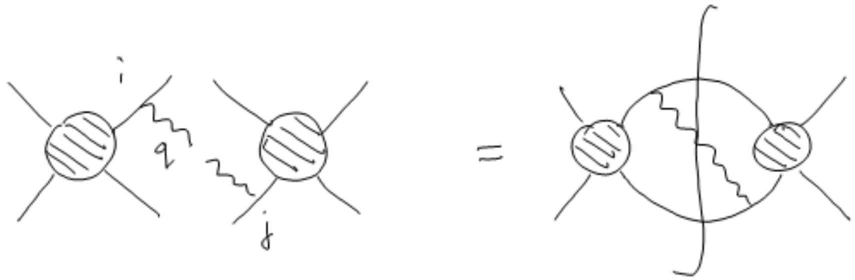
This question may not be interesting a priori because hadronization could wreck any underlying partonic correlations. However experiment reveals that the hadronization process is 'gentle'.

The most important emissions are those involving either <u>collinear</u> quarks/gluons or <u>soft</u> gluons. By important we mean that the usual suppression in the strong coupling is compensated by a large logarithm.

SOFT GLUONS



$$= 2gp_\mu \delta_{\lambda\lambda'} T^a_{ij}$$

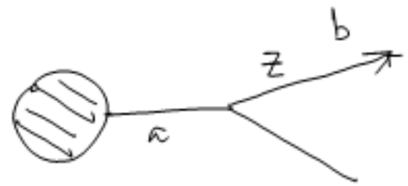


$$d\sigma_{n+1} = d\sigma_n \frac{\alpha_s}{2\pi} \frac{dE}{E} \frac{d\Omega}{2\pi} \sum_{ij} C_{ij} E^2 \frac{p_i \cdot p_j}{p_i \cdot q \; p_j \cdot q}$$

- Only have to consider soft gluons off the external legs of a hard subprocess since internal hard propagators cannot be put on shell.
- Virtual corrections are included analogously….of which more later….
- Only need to consider gluons.
- Colour factor is the “problem”.

COLLINEAR EMISSIONS

Colour structure is easier. It is as if emission is off the parton to which it is collinear ~ "classical branching".



$$d\sigma_{n+1} = d\sigma_n \, \frac{\alpha_s}{2\pi} \frac{dq^2}{q^2} dz \; P_{ba}(z)$$

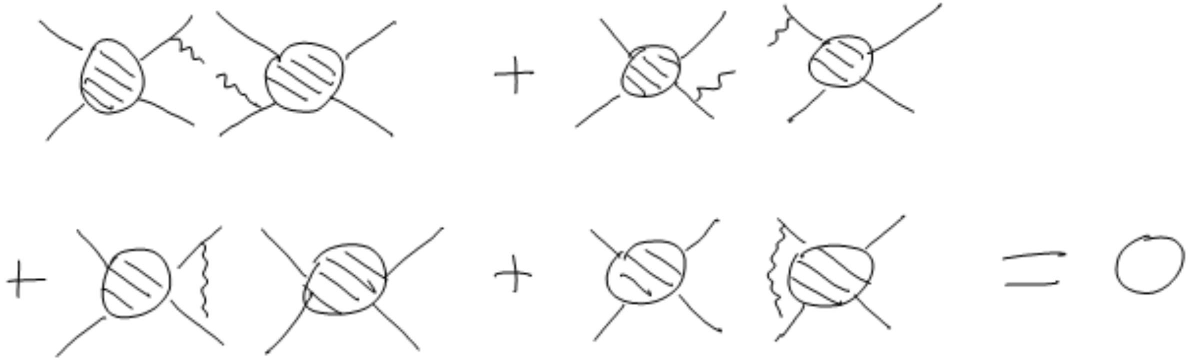
In the Monte Carlos: soft and/or collinear evolution is handled simultaneously using "angular ordered parton evolution".

Conventional wisdom: OK but only in the large N_c approximation where colour simplifies hugely.
Also assumes azimuthal averaging.

Not all observables are affected by soft and/or collinear enhancements

Intuitive: imagine the e^+e^- total cross-section. It cannot care that the outgoing quarks may subsequently radiate additional soft and/or collinear particles (causality and unitarity).

Bloch-Nordsieck: *soft gluon corrections cancel in "sufficiently inclusive" observables.*

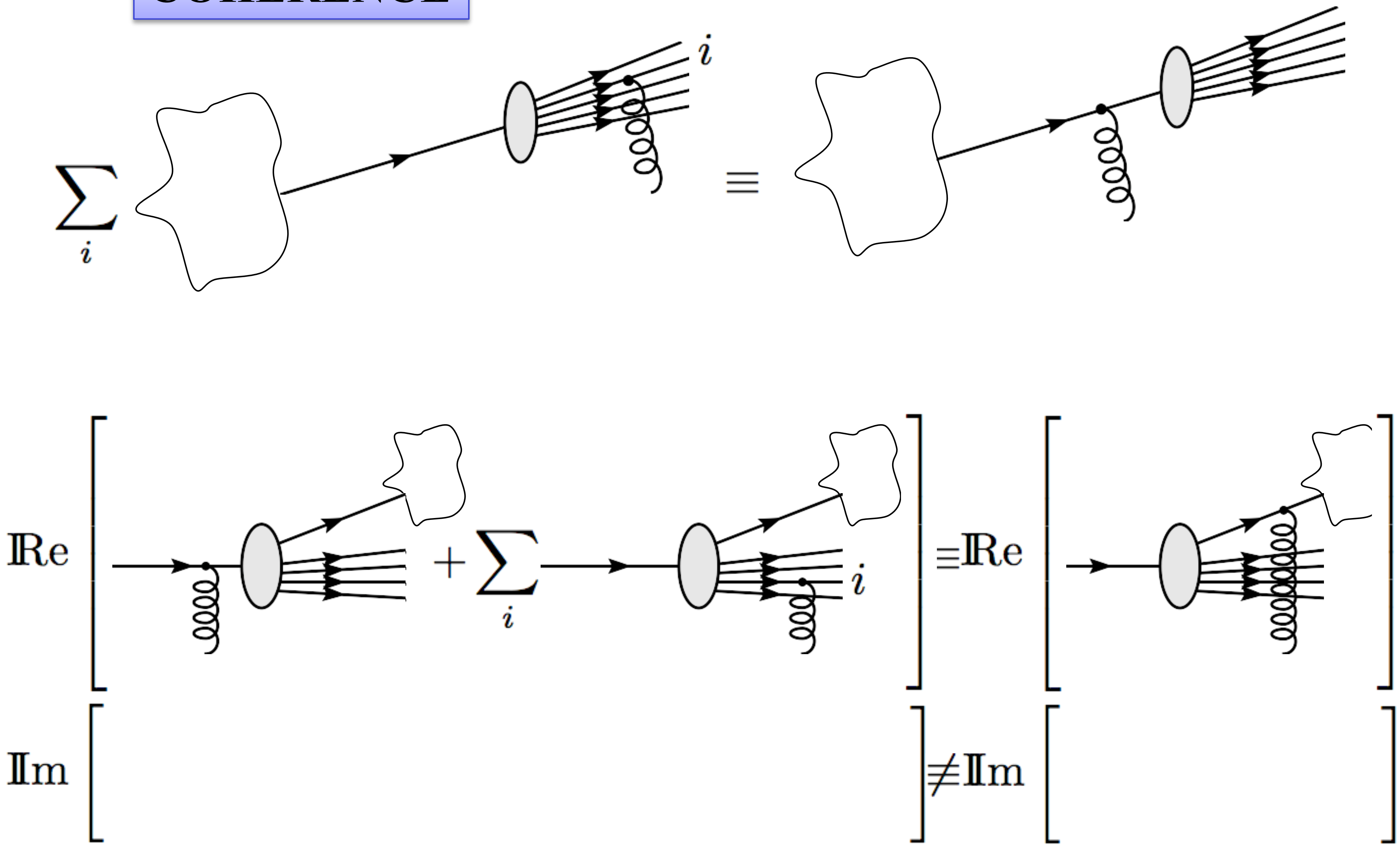


Miscancellation can be induced by restricting the real emissions in some way.

All observables are "sufficiently inclusive" to guarantee that the would-be soft divergence cancels (no detector can detect zero energy particles). But <u>the miscancellation may leave behind a logarithm</u>, e.g. if real emissions are forbidden above μ then virtual corrections give

$$\alpha_s \int_\mu^Q \frac{dE}{E} = \alpha_s \ln \frac{Q}{\mu}$$

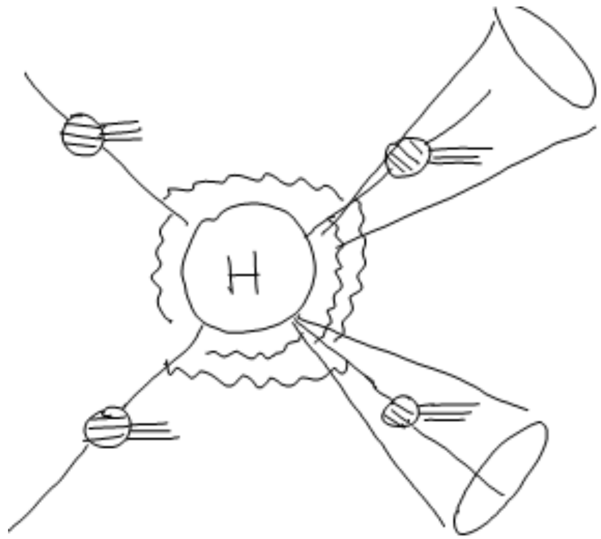
COHERENCE



$\sum_i$
i
$\equiv$
$\mathbb{R}\mathrm{e}$
$+\sum_i$
i
$\equiv \mathbb{R}\mathrm{e}$
$\mathbb{I}\mathrm{m}$
$\neq \mathbb{I}\mathrm{m}$

COHERENCE

It is exploited to factorize collinear emissions from soft, wide angle, gluon emissions.



The failure of the "coherence identity" for the imaginary part will be significant later.

Soft gluon corrections will be important for observables that insist on only *small deviations from lowest order kinematics*.

In such cases real radiation is constrained to a small corner of phase space and BN miscancellation induces large logarithms.

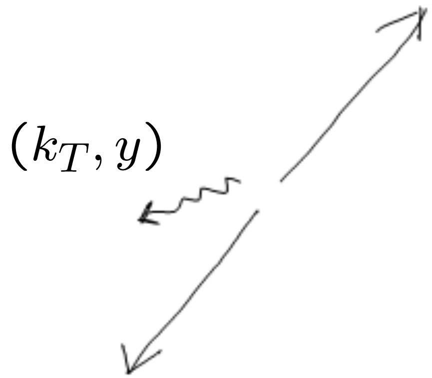
If V measures 'distance' from the lowest order kinematics:

Event shapes such as thrust ($V = 1 - T$)
Production near threshold (top, W/Z) ($V = 1 - M^2/\widehat{s}$)
Drell-Yan at low p_T (W/Z or Higgs) ($V = p_T^2/\widehat{s}$)
Deep-inelastic scattering at large x ($V = 1 - x$)
Gaps between jets....

An example: the **thrust** distribution

$$T = \max \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|}$$

$T = 1$ for $e^+e^- \to q\bar{q}$ in lowest order



Consider

$$f(\tau) = \int_{1-\tau}^{1} dT \frac{1}{\sigma}\frac{d\sigma}{dT}$$

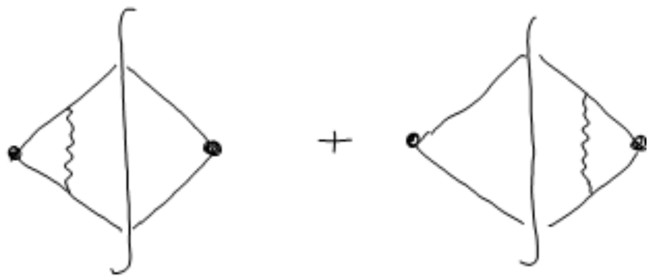
Clearly gluons cannot be emitted at too large an angle if they are to produce a final state which contributes to this integral, i.e. real emissions are forbidden if they satisfy

$$\frac{k_T}{Q} \exp(-|y|) > \tau$$

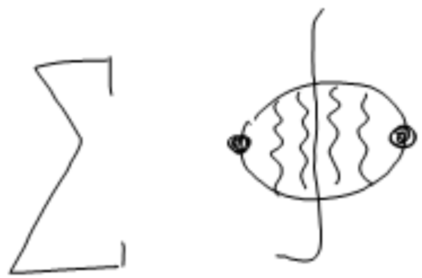
The absence of these real emissions leaves behind uncancelled virtual soft gluon corrections which we must account for….recall that all other emissions cancel between real and virtual graphs due to Bloch-Nordsieck.

The first soft & virtual gluon correction [integrated over the <u>disallowed</u> region for real emission] arises after multiplying the lowest order result by

$$\frac{\alpha_s}{\pi} C_F \int_{\tau Q}^{Q} \frac{dk_T}{k_T} \int_0^{\ln(k_T/(\tau Q))} dy = \frac{\alpha_s}{2\pi} C_F \ln^2 \frac{1}{\tau}$$



$$f(\tau) = 1 - \frac{\alpha_s}{\pi} C_F \ln^2 \frac{1}{\tau}$$



cuts
& number
of gluons

$$f(\tau) = \exp\left(-\frac{\alpha_s}{\pi} C_F \ln^2 \frac{1}{\tau}\right)$$

- Double logarithmic suppression….

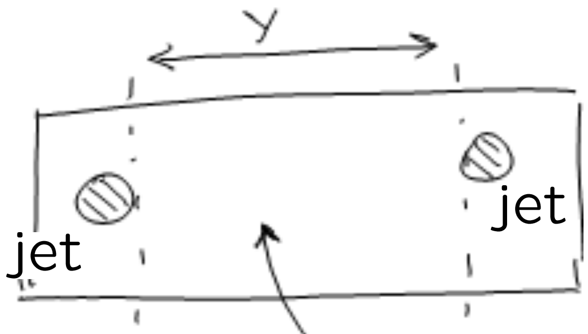
- Double logs because observable restricts gluons which are both soft and collinear (i.e. energy and transverse momentum are small on the scale of the CM energy Q).

- This is a Sudakov suppression. It looks like a poissonian suppression corresponding to the probability not to emit soft-collinear gluons in the forbidden region.

- Next step would be to re-compute to single log accuracy….

Banfi, Salam & Zanderighi: automated resummations
Marchesini & Dokshitzer: classical nature of soft gluon radiation

JET VETOING: "Gaps between jets"



Jets produced with $p_T = Q \gg Q_0$

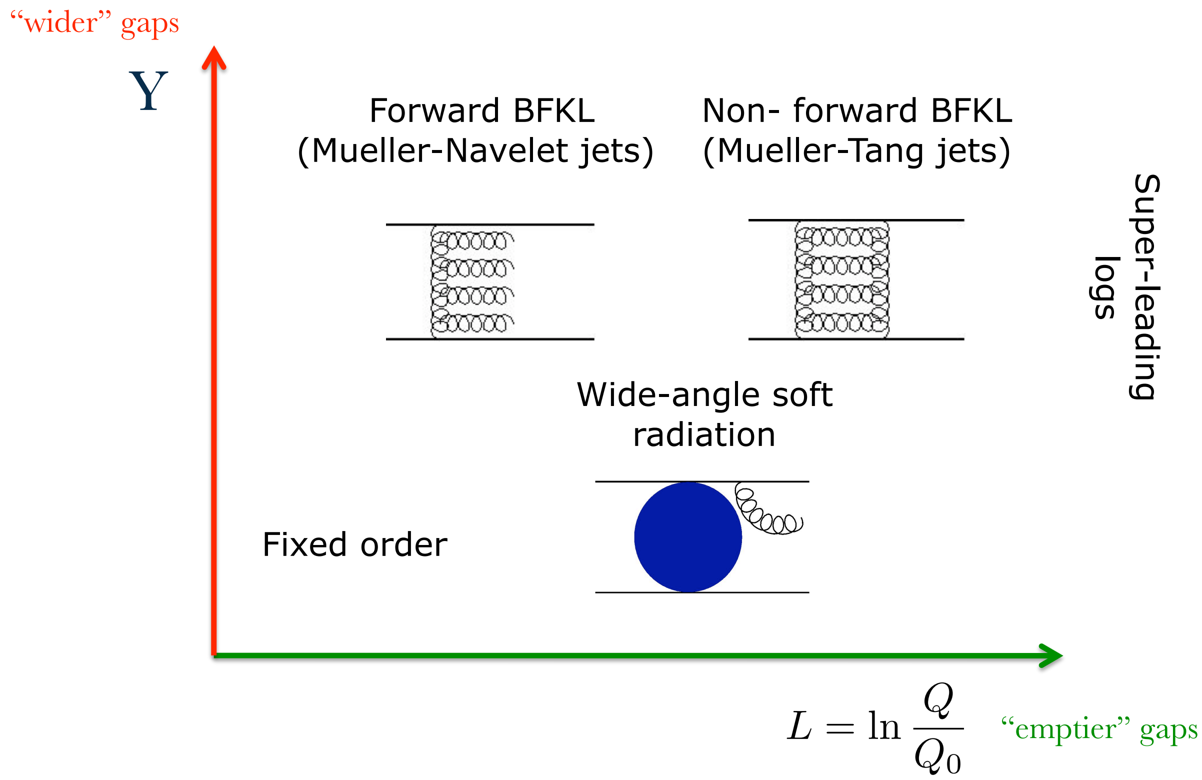
No radiation in between jets with $k_T > Q_0$

Observable restricts emission in the gap region therefore expect

$$\alpha_s^n \ln^n(Q/Q_0)$$

i.e. do not expect collinear enhancement since we sum inclusively over the collinear regions of the incoming and outgoing partons.

The rich physics of "gaps between jets"…..



Real emissions are forbidden in the phase-space region

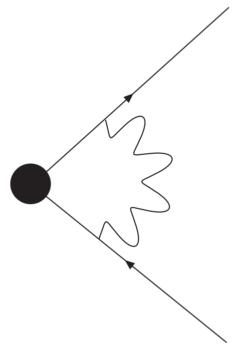
$$-Y/2 < y < Y/2$$

$$k_T > Q_0$$

"*By Bloch-Nordsieck, all other real emissions cancel and we therefore only need to compute the* <u>*virtual*</u> *soft gluon corrections to the primary hard scattering*."

$e^+e^- \to q\bar{q}$ case is very simple:

$$\sigma_{\text{gap}} = \sigma_0 \ \exp\left(-C_F \frac{\alpha_s}{\pi} Y \ln\left(\frac{Q}{Q_0}\right)\right)$$



The virtual gluon is integrated over "in gap" momenta, i.e. the region where real emissions are forbidden.

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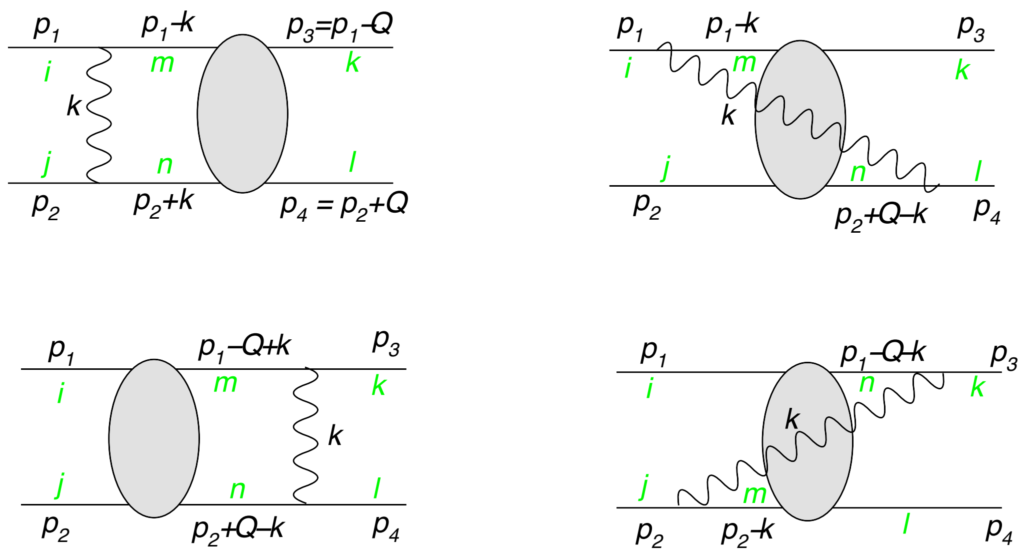


The virtual gluon is integrated over "in gap" momenta, i.e. the region where real emissions are forbidden.

(plus two others)

But this is too naïve….as we shall soon see

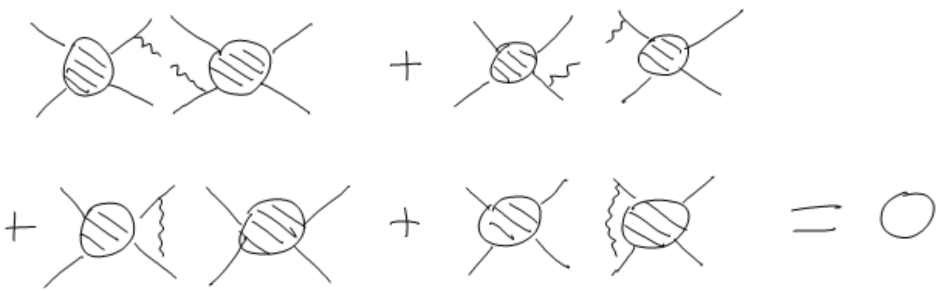
Coulomb gluons

a.k.a. Glauber gluons

- I have skipped over a subtle issue…..the real-virtual cancellation of soft gluons occurs point-by-point in (y, k_T) only between the *real parts* of the virtual correction and the real emission.

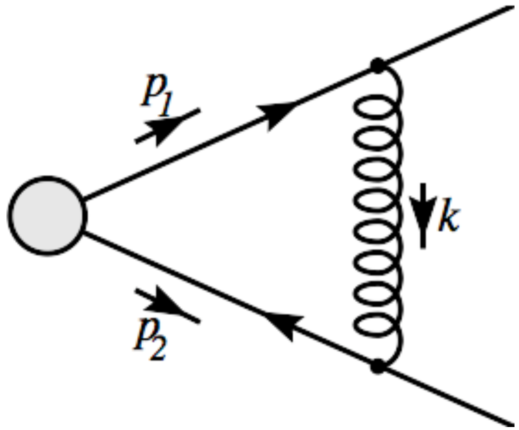
- The imaginary part obviously cancels if the soft gluon is closest to the cut….but what about subsequent evolution? Might this spoil the real-virtual cancellation below Q_0?

- No, it does not. The "non-cancelled" iπ terms exponentiate to produce a pure phase in the amplitude → no physical effect.



e^+e^- revisited

The colour structure is simple enough that the Coulomb gluons lead only to a phase even above Q_0.



$$\mathcal{A}_1 = -\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F\left(Y - i\pi\right)\mathcal{A}_0$$

eikonal $k^2=0$

Coulomb $p_1{}^2=p_2{}^2=0$

$$\mathcal{A} = e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F(Y-i\pi)}\mathcal{A}_0$$

$$\sigma = \mathcal{A}^\star\mathcal{A} = \mathcal{A}_0^\star e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F(Y+i\pi)}e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F(Y-i\pi)}\mathcal{A}_0$$

i π terms cancel

Back to hadron-hadron collisions…

The amplitude can be projected onto a colour basis:

$$(M)_{ij}^{kl} = M^{(1)} C_{ijkl}^{(1)} + M^{(8)} C_{ijkl}^{(8)}$$

$$\begin{aligned} C_{ijkl}^{(8)} &= (T^a)_{ik}(T^a)_{jl} \\ C_{ijkl}^{(1)} &= \delta_{ik}\delta_{jl}. \end{aligned}$$

i.e. $\mathbf{M} = \begin{pmatrix} M^{(1)} \\ M^{(8)} \end{pmatrix}$ and $\sigma = \mathbf{M}^\dagger \mathbf{S}_V \mathbf{M}$

$$\mathbf{S}_V = \begin{pmatrix} N^2 & 0 \\ 0 & \frac{N^2-1}{4} \end{pmatrix}$$

Iterating the insertion of soft virtual gluons builds up the N$^{\text{th}}$ order amplitude:

$$\mathbf{M} = \exp\left(-\frac{2\alpha_s}{\pi} \int_{Q_0}^{Q} \frac{dk_T}{k_T} \, \mathbf{\Gamma} \right) \mathbf{M}_0$$

The factorial needed for exponentiation arises as a result of ordering the transverse momenta of successive soft gluons, i.e.

$$Q_0 \ll k_{T1} \dots \ll k_{TN} \ll Q$$

where the evolution matrix is

$$\mathbf{\Gamma} = \begin{pmatrix} \frac{N^2-1}{4N}\rho(Y,\Delta y) & \frac{N^2-1}{4N^2} i\pi \\ i\pi & -\frac{1}{N} i\pi + \frac{N}{2} Y + \frac{N^2-1}{4N}\rho(Y,\Delta y) \end{pmatrix}$$

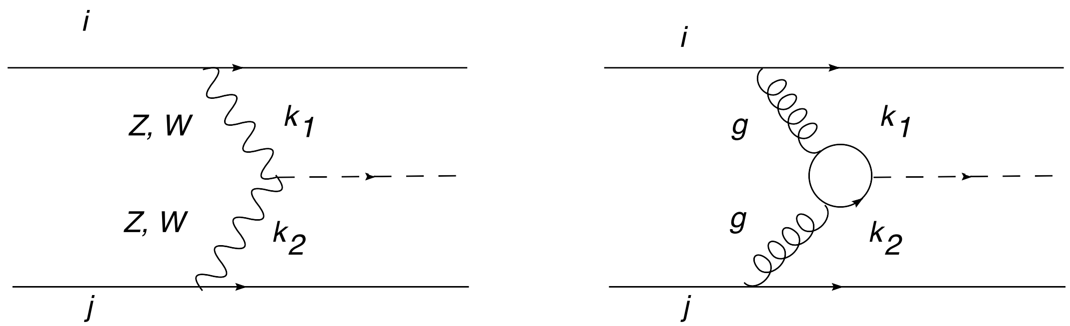
$\Delta y =$ distance between jet centres $\quad Y =$ size of gap

In qq → qq the colour structure is more complicated than e^+e^- and the Coulomb gluons no longer exponentiate into a phase above Q_0 (due to the presence of the real parts of the virtual corrections).

$$\mathbf{\Gamma} = \begin{pmatrix} \frac{N^2-1}{4N}\rho(Y,\Delta y) & \frac{N^2-1}{4N^2} i\pi \\ i\pi & -\frac{1}{N} i\pi + \frac{N}{2} Y + \frac{N^2-1}{4N}\rho(Y,\Delta y) \end{pmatrix}$$

Coulomb gluons are relevant

An example: Higgs plus two jets



- To reduce backgrounds and to focus on the VBF channel, experimenters will make a veto on additional radiation between the tag jets, i.e. no additional jets with

$$k_T \geq Q_0$$

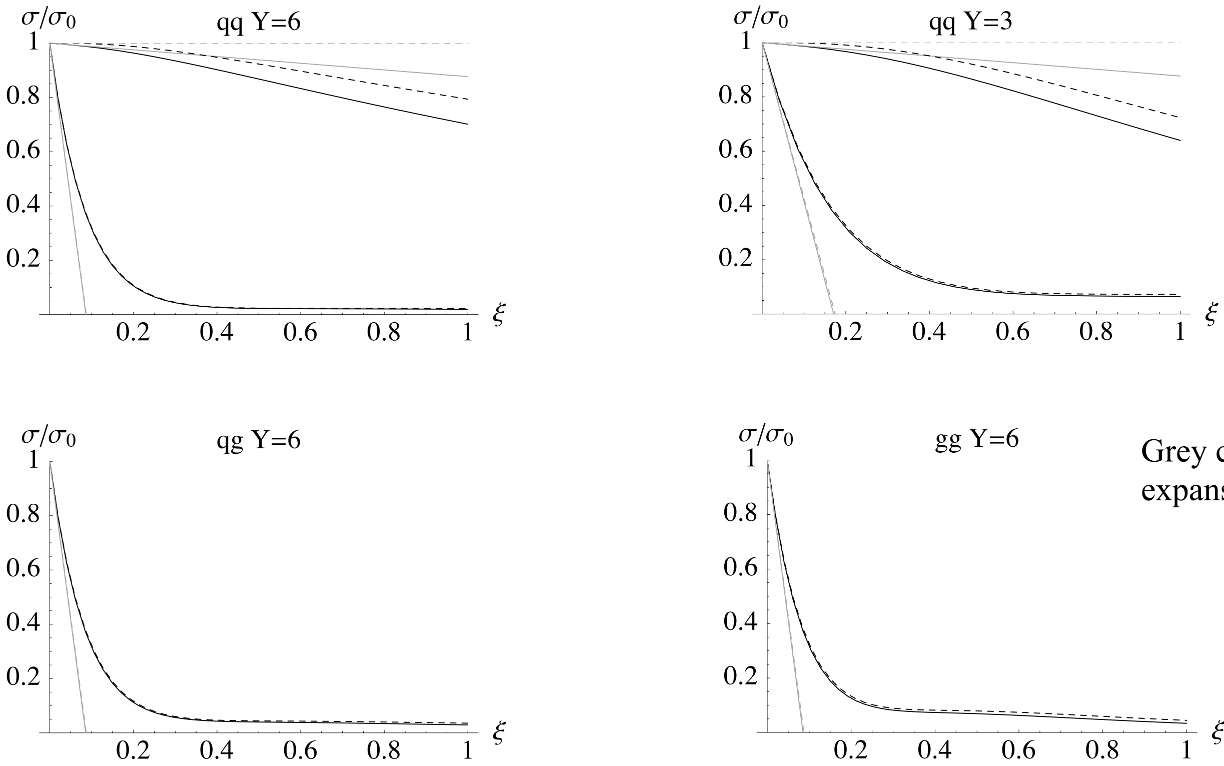
- Soft gluon effects will induce logarithms:

$$\alpha_s^n \ln^n (Q/Q_0)$$

$Q = \text{transverse momentum of tag jets}$

Resummation proceeds almost exactly as for “gaps between jets”

$$\mathbf{M} = \exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\;\mathbf{\Gamma}\right)\mathbf{M}_0 \qquad \rho(Y,\Delta y) \to \frac{1}{2}(\rho(Y,2|y_3|)+\rho(Y,2|y_4|))$$



Grey curves = lowest order expansion of black curves.

Only the colour of the exchange matters.

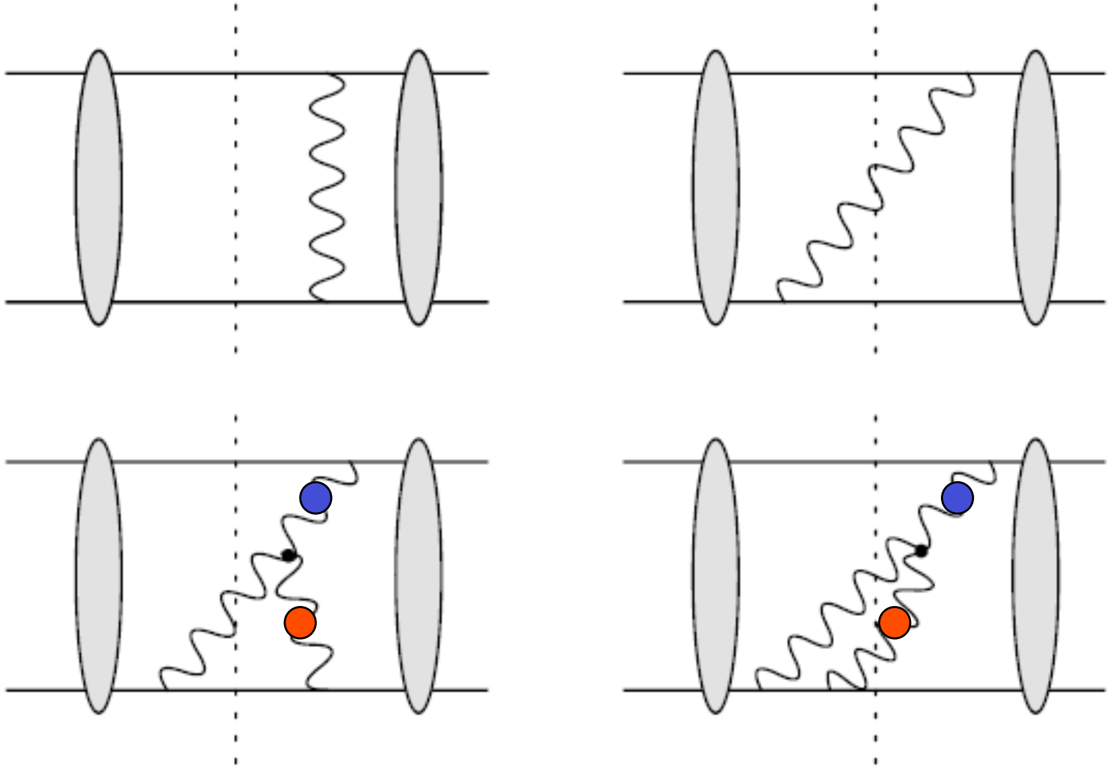
$$\xi = \int_{Q_0}^{Q}\frac{dq}{q}\alpha_s(q) \approx 0.2$$ for 100 GeV jets and a 20 GeV veto, i.e. resummation is important at LHC

JF & Malin Sjödahl (2007)

- Fixed order calculations cannot account adequately for the effect of a veto.

- How much is this physics already present in parton shower Monte Carlos?
  Subleading N_c and Coulomb gluon contributions are missing.

But there is a big fly in the ointment: these observables are *non-global*



Such real & virtual corrections cancel.

But these do not if the gluon marked with a red blob is in the forbidden region: the 2nd cut is not allowed.

So the cancellation does not hold.....

real and virtual

It fails only once we start to evolve emissions (such as those denoted by the blue blob in the above) which lie *outside of the gap* region and which have $k_T > Q_0$

$|y| > Y/2$

If $k_T < Q_0$ then subsequent evolution also has $k_T < Q_0$ and cancellation works.

- The miscancellation is telling us that this observable is sensitive to soft gluon emissions outside of the gap, even though the observable sums inclusively over that region.

- Not a surprise once we realise that emissions outside of the gap can subsequently radiate back into the gap.

- We must therefore include any number of emissions outside of the gap and their subsequent evolution.

- Colour structure makes this impossible using current technology.

- We could aim to compute the all orders non-global corrections in the leading N_c approximation. Dasgupta, Salam, Appleby, Seymour, Delenda, Banfi

- Instead we shall compute the "one hard emission out of the gap" contribution without any approximation on the colour.

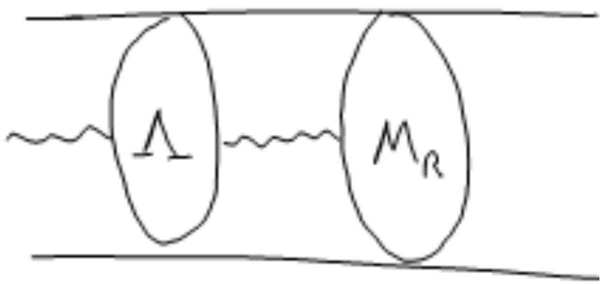
Two new ingredients still sticking to quark-quark scattering

1) How to add a real gluon to the four-parton amplitude

$$\mathbf{M}_R = \boxed{\mathbf{D}} \cdot \mathbf{M}$$

2) How to evolve the resulting five-parton amplitude

$$\mathbf{M}_R(Q_0) = \exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\,\boxed{\mathbf{\Lambda}}\right)\mathbf{M}_R(k_T)$$



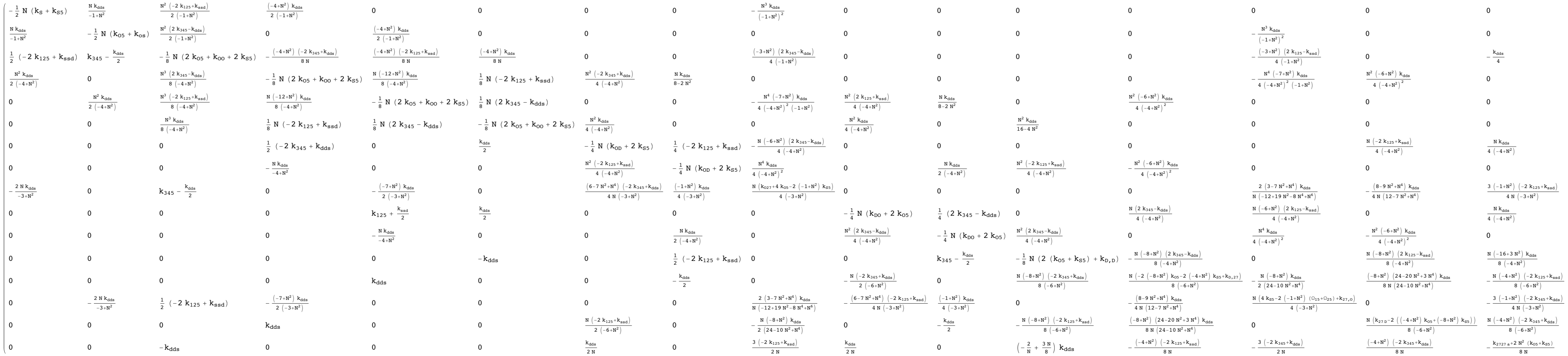
Kyrieleis & Seymour

$$
\mathbf{D}^\mu = \begin{pmatrix} \frac{1}{2}(-h_1^\mu - h_2^\mu + h_3^\mu + h_4^\mu) & \frac{1}{4N}(-h_1^\mu - h_2^\mu + h_3^\mu + h_4^\mu) \\ 0 & \frac{1}{2}(-h_1^\mu - h_2^\mu + h_3^\mu + h_4^\mu) \\ \frac{1}{2}(-h_1^\mu + h_2^\mu + h_3^\mu - h_4^\mu) & \frac{1}{4N}(h_1^\mu - h_2^\mu - h_3^\mu + h_4^\mu) \\ 0 & \frac{1}{2}(-h_1^\mu + h_2^\mu - h_3^\mu + h_4^\mu) \end{pmatrix} \qquad h_i^\mu = \frac{1}{2}k_T\frac{p_i^\mu}{p_i \cdot k}
$$

$$
\begin{aligned}
\mathbf{\Lambda} = & \begin{pmatrix} \frac{N}{4}(Y-i\pi)+\frac{1}{2N}i\pi & \left(\frac{1}{4}-\frac{1}{N^2}\right)i\pi & -\frac{N}{4}s_yY & 0 \\ i\pi & \frac{N}{4}\left(2Y-i\pi\right)-\frac{3}{2N}i\pi & 0 & 0 \\ -\frac{N}{4}s_yY & 0 & \frac{N}{4}(Y-i\pi)-\frac{1}{2N}i\pi & -\frac{1}{4}i\pi \\ 0 & 0 & -i\pi & \frac{N}{4}\left(2Y-i\pi\right)-\frac{1}{2N}i\pi \end{pmatrix} \\
& + \begin{pmatrix} N & 0 & 0 & 0 \\ 0 & N & 0 & 0 \\ 0 & 0 & N & 0 \\ 0 & 0 & 0 & N \end{pmatrix} \frac{1}{4}\rho(Y, 2\left|y\right|) \\
& + \begin{pmatrix} C_F & 0 & 0 & 0 \\ 0 & C_F & 0 & 0 \\ 0 & 0 & C_F & 0 \\ 0 & 0 & 0 & C_F \end{pmatrix} \frac{1}{2}\rho(Y, \Delta y) \\
& + \begin{pmatrix} \frac{N}{4}\left(-\frac{1}{2}\lambda\right) & 0 & \frac{N}{4}\left(-\frac{1}{2}s_y\lambda\right) & \frac{1}{4}\left(\frac{1}{2}s_y\lambda\right) \\ 0 & \frac{N}{4}\left(-\frac{1}{2}\lambda\right) & 0 & \frac{N}{4}\left(\frac{1}{2}s_y\lambda\right) \\ \frac{N}{4}\left(-\frac{1}{2}s_y\lambda\right) & 0 & \frac{N}{4}\left(-\frac{1}{2}\lambda\right) & \frac{1}{4}\left(-\frac{1}{2}\lambda\right) \\ \frac{1}{2}s_y\lambda & \left(\frac{N}{4}-\frac{1}{N}\right)\left(\frac{1}{2}s_y\lambda\right) & -\frac{1}{2}\lambda & \frac{N}{4}\left(-\frac{1}{2}\lambda\right) \end{pmatrix}
\end{aligned}
$$

Recently extended to all five parton amplitudes:

e.g. gg → ggg



Sjödahl (2008)

….for an arbitrary n-parton amplitude:

$$\mathbf{\Gamma} = -\sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \, \Omega_{ij}$$

$$\Omega_{ij} = \frac{1}{2}\left\{ \int_{\text{veto}} \frac{dy\, d\phi}{2\pi} \frac{1}{2} k_T^2 \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)} - i\pi\, \Theta(ij = II \text{ or } FF) \right\}$$

$$\begin{aligned} \mathbf{\Gamma} \;&=\; \frac{1}{2} Y \mathbf{T}_t^2 + \boxed{i\pi \mathbf{T}_1 \cdot \mathbf{T}_2} + \frac{1}{4} \sum_{i \in F} \rho(Y; 2|y_i|) \mathbf{T}_i^2 \\ &+ \frac{1}{2} \sum_{(i<j) \in L} \lambda(Y; |y_i| + |y_j|, |\phi_i - \phi_j|) \mathbf{T}_i \cdot \mathbf{T}_j + \frac{1}{2} \sum_{(i<j) \in R} \lambda(Y; |y_i| + |y_j|, |\phi_i - \phi_j|) \mathbf{T}_i \cdot \mathbf{T}_j \,. \end{aligned}$$

Easy to see it is final state collinear safe but not initial state collinear safe:

i.e. $\mathbf{\Gamma} \sim \mathbf{T}_i + \mathbf{T}_j$ only for *i* and *j* collinear *and* in final state

A surprise: True beyond one-loop (massless case):
2-loop: Mert Aybat, Dixon, Sterman (2006)
3-loop: Dixon (2009)
n-loop? Bern et al (2009), Gardi & Magnea (2009), Becher & Neubert (2009)

Failure at 2-loop in massive case…
Mitov, Sterman & Sung (2009)

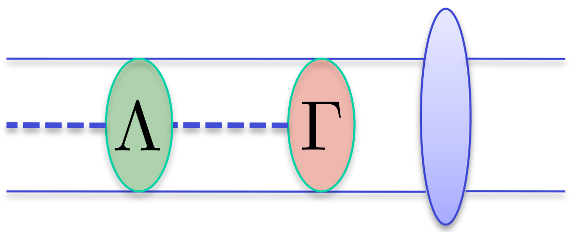
JF, Kyrieleis & Seymour (2008)

The complete cross-section for one real emission outside of the gap is thus

$$\sigma_R = -\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\int_{\text{out}}\frac{dy\, d\phi}{2\pi}$$

$$\mathbf{M}_0^\dagger \exp\left(-\frac{2\alpha_s}{\pi}\int_{k_T}^{Q}\frac{dk'_T}{k'_T}\mathbf{\Gamma}^\dagger\right)\mathbf{D}_\mu^\dagger \exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\mathbf{\Lambda}^\dagger\right)\mathbf{S}_R$$

$$\boxed{\exp\left(-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\mathbf{\Lambda}\right)}\boxed{\mathbf{D}^\mu}\boxed{\exp\left(-\frac{2\alpha_s}{\pi}\int_{k_T}^{Q}\frac{dk'_T}{k'_T}\mathbf{\Gamma}\right)}\boxed{\mathbf{M}_0}$$



And the corresponding contribution when the out-of-gap gluon is virtual is

$$\sigma_V = -\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_T}{k_T}\int\limits_{\text{out}}\frac{dy\;d\phi}{2\pi}$$

$$\left[\mathbf{M}_0^\dagger \exp\left(-\frac{2\alpha_s}{\pi}\int\limits_{Q_0}^{Q}\frac{dk'_T}{k'_T}\mathbf{\Gamma}^\dagger\right)\mathbf{S}_V\right.$$

$$\left.\exp\left(-\frac{2\alpha_s}{\pi}\int\limits_{Q_0}^{k_T}\frac{dk'_T}{k'_T}\mathbf{\Gamma}\right)\boxed{\gamma}\exp\left(-\frac{2\alpha_s}{\pi}\int\limits_{k_T}^{Q}\frac{dk'_T}{k'_T}\mathbf{\Gamma}\right)\mathbf{M}_0 + \text{c.c.}\right]$$

Adds one “out of the gap” virtual gluon

Conventional wisdom: when the out of gap gluon becomes collinear with either incoming quark or either outgoing quark the real and virtual contributions should cancel.

This cancellation operates for **final state collinear emission**:

$$\mathbf{D}^{\mu\dagger}(\mathbf{\Lambda}^{\dagger})^{n-m}\mathbf{S}_R\mathbf{\Lambda}^m\mathbf{D}_\mu + (\mathbf{\Gamma}^{\dagger})^{n-m}\mathbf{S}_V\mathbf{\Gamma}^m\gamma + \gamma^{\dagger}(\mathbf{\Gamma}^{\dagger})^{n-m}\mathbf{S}_V\mathbf{\Gamma}^m = \mathbf{0}$$

But it fails for **initial state collinear emission**:

The problem is entirely due to the emission of Coulomb gluons.

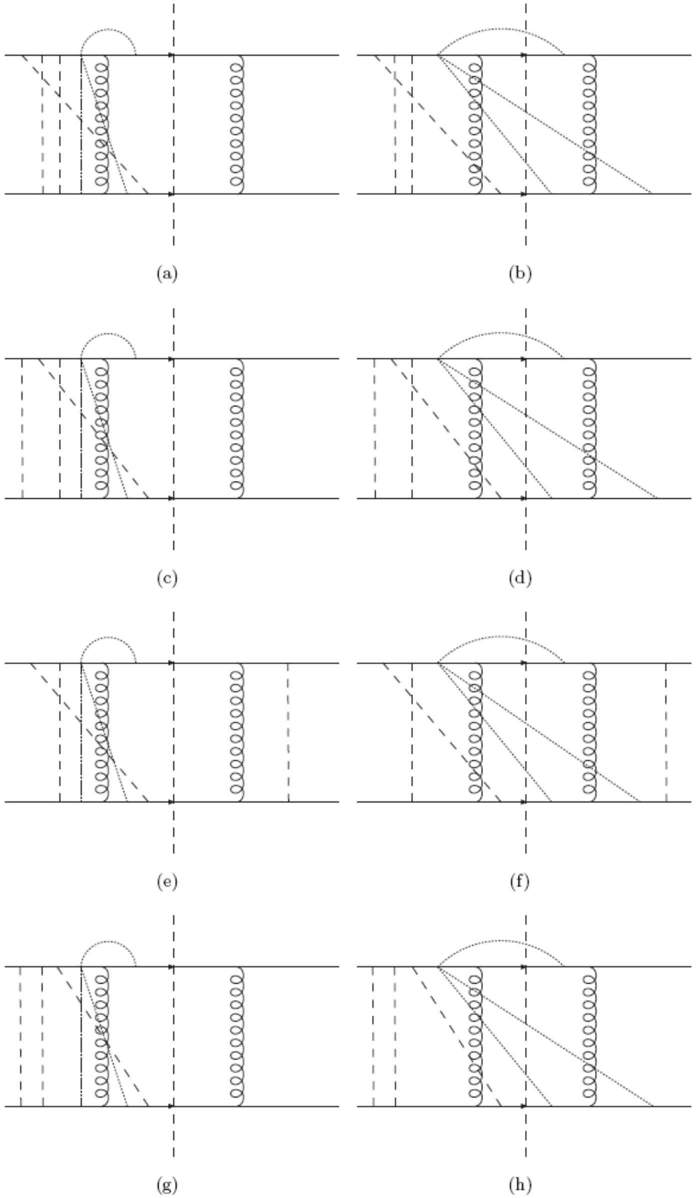
Cancellation *does* occur for $n = 1$, 2 and 3 gluons relative to lowest order but not for larger n. This is the lowest order where the Coulomb gluons do not trivially cancel.

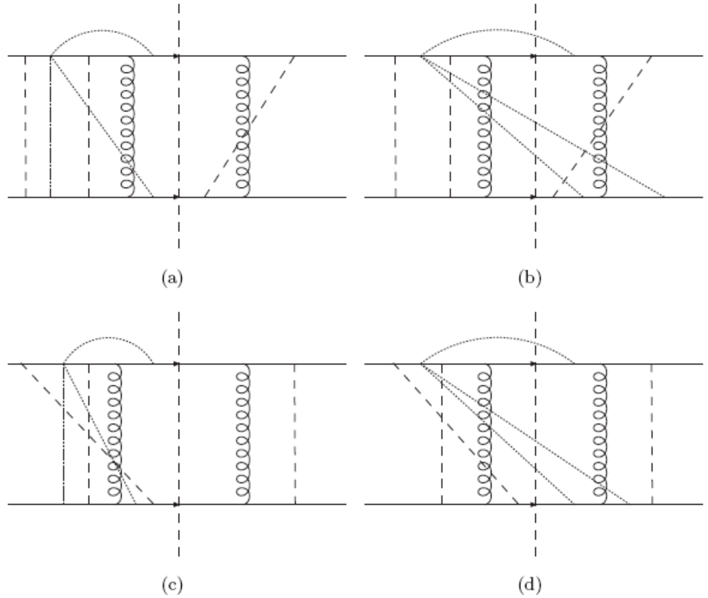
The non-cancelling diagrams (Feynman gauge)…..

Dotted line is the out-of-gap gluon.

Dashed lines are in-gap & Coulomb gluons.

Springs are hard scatter gluons.





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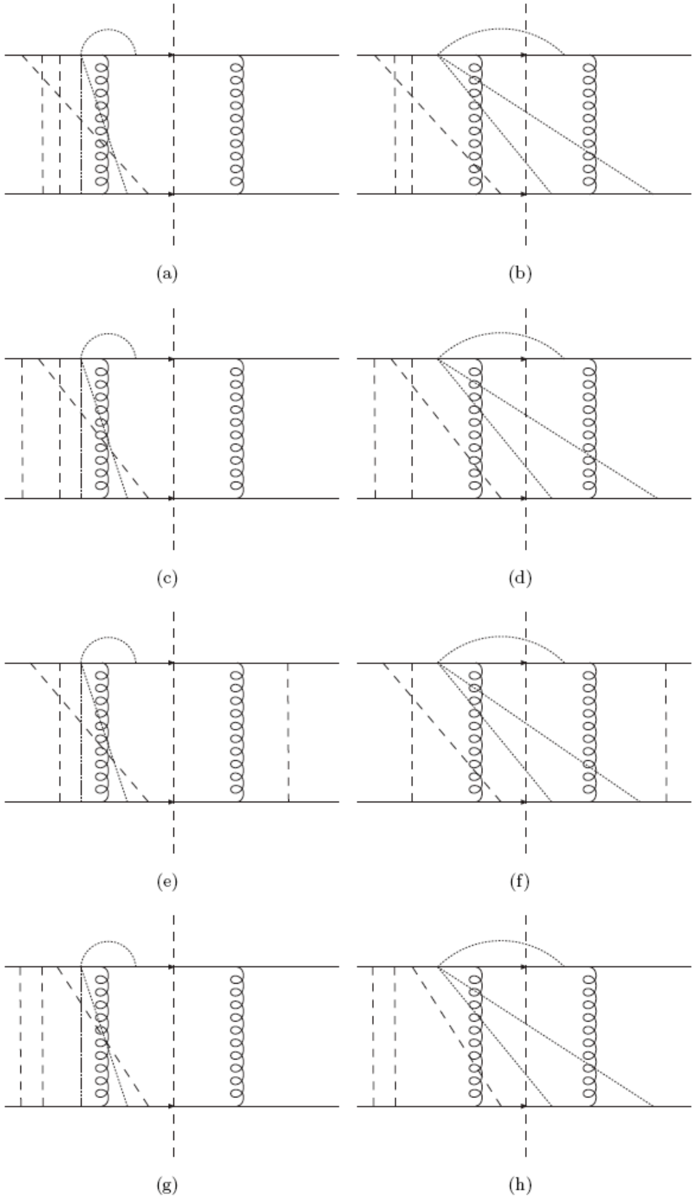
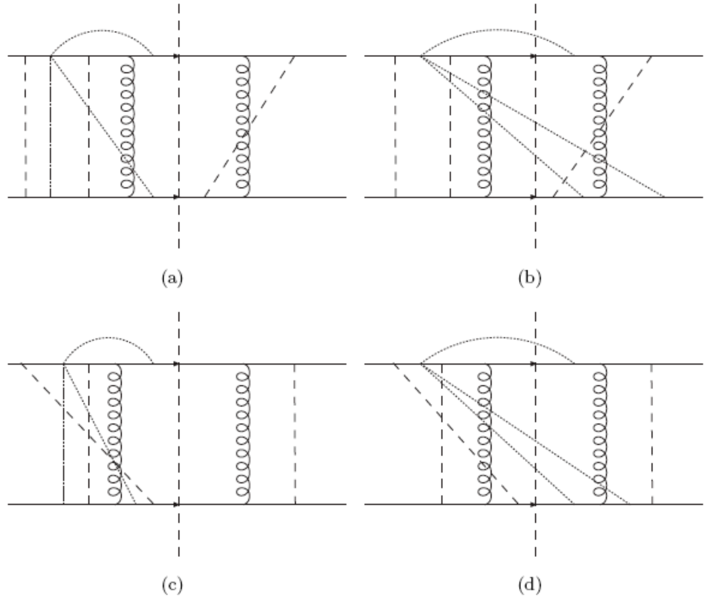
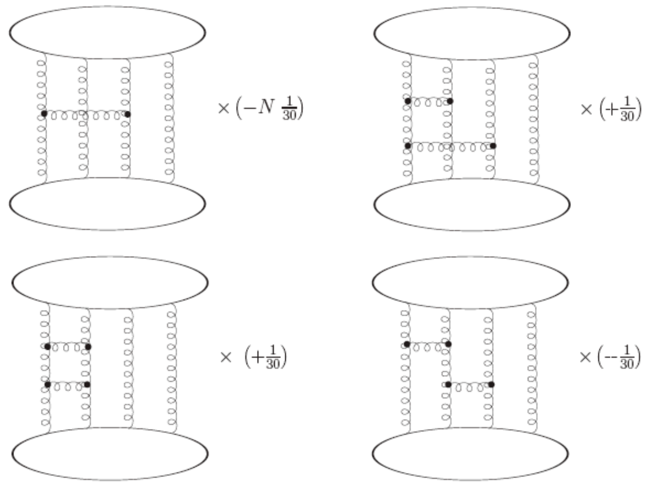






Colour traces ~ small-*x* physics?

What are we to make of a non-cancelling collinear divergence?

$$\sigma \sim \sigma_0 \; \alpha^4 L^4 \pi^2 Y \int_{\text{out}} dy$$

Cannot actually have infinite rapidity with $k_T > Q_0$

Need to go beyond soft gluon approximation in collinear (large rapidity) limit:

$$\int d^2k_T \int_{\text{out}} dy \quad \frac{d\sigma}{dy d^2k_T}\bigg|_{\text{soft}} \to \int d^2k_T \left[\int^{y_{\max}} dy \quad \frac{d\sigma}{dy d^2k_T}\bigg|_{\text{soft}} + \int_{y_{\max}}^{\infty} dy \quad \frac{d\sigma}{dy d^2k_T}\bigg|_{\text{collinear}} \right]$$

$$\int_{y_{\max}}^{\infty} dy \quad \frac{d\sigma}{dy d^2k_T}\bigg|_{\text{collinear}} = \int_{y_{\max}}^{\infty} dy \quad \left(\frac{d\sigma_{\text{R}}}{dy d^2k_T}\bigg|_{\text{collinear}} + \frac{d\sigma_{\text{V}}}{dy d^2k_T}\bigg|_{\text{collinear}} \right)$$

Soft approximation:

$$\int dz \, \frac{1}{2}\left(\frac{1+z^2}{1-z}\right) \to \int dy$$

Real collinear emission:

$$\int_{y_{\max}}^{\infty} dy \quad \frac{d\sigma_{\mathrm{R}}}{dy d^2 k_T}\bigg|_{\text{collinear}} = \int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right) \frac{q(x/z,\mu^2)}{q(x,\mu^2)} A_{\mathrm{R}}$$

$$= \int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right)\left(\frac{q(x/z,\mu^2)}{q(x,\mu^2)} - 1\right) A_{\mathrm{R}} + \int_0^{1-\delta} dz \frac{1}{2}\frac{1+z^2}{1-z} A_{\mathrm{R}}$$

Virtual collinear emission:

$$\int_{y_{\max}}^{\infty} dy \quad \frac{d\sigma_{\mathrm{V}}}{dy d^2 k_T}\bigg|_{\text{collinear}} = \int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right) A_{\mathrm{V}}$$

$$y > y_{\max}$$

implies

$$\delta \approx \frac{k_T}{Q} \exp\left(y_{\max} - \frac{\Delta y}{2}\right)$$

If $A_{\mathrm{R}} + A_{\mathrm{V}} = 0$ then the divergence would cancel leaving behind a regularized splitting, which would correspond to the DGLAP evolution of the incoming quark pdf.

But as we have seen, the Coulomb gluons spoil this cancellation. Instead we have

$$\int_0^{1-\delta} dz \frac{1}{2}\left(\frac{1+z^2}{1-z}\right)(A_{\mathrm{R}}+A_{\mathrm{V}}) = \ln\left(\frac{1}{\delta}\right)(A_{\mathrm{R}}+A_{\mathrm{V}}) + \text{subleading}$$

$$\approx \left(-y_{\max} + \frac{\Delta y}{2} + \ln\left(\frac{Q}{k_T}\right)\right)(A_{\mathrm{R}}+A_{\mathrm{V}})$$

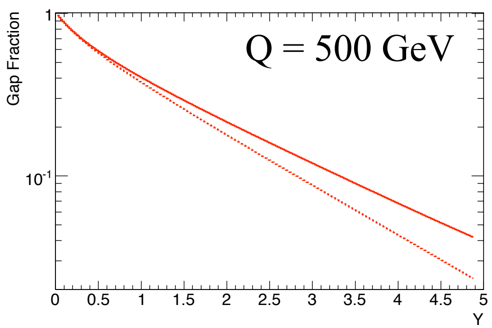
Hence

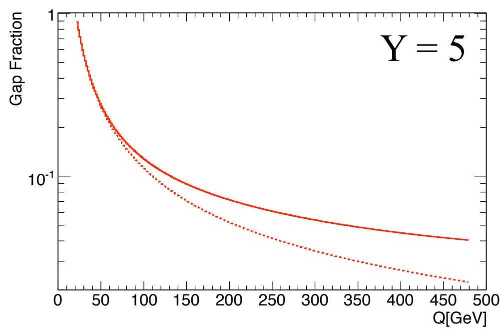
$$\int_{Q_0}^{Q} \frac{dk_T}{k_T} \int_{\text{out}} dy \to \int_{Q_0}^{Q} \frac{dk_T}{k_T} \left(\int^{y_{\max}} dy + (-y_{\max} + \ln \frac{Q}{k_T}\right) = \frac{1}{2} \ln^2 \frac{Q}{Q_0}$$

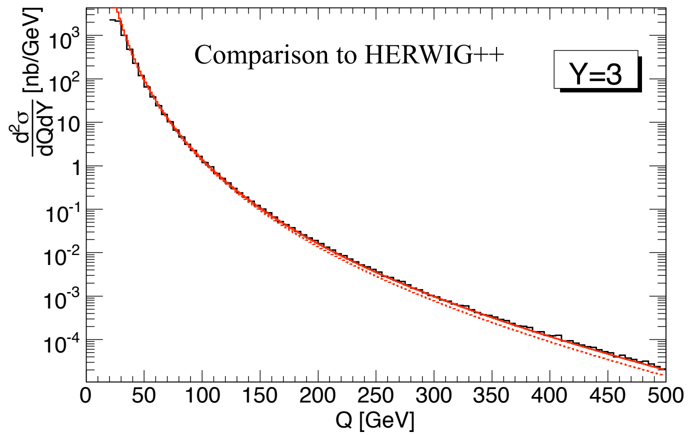
The final result for the "one emission out-of-gap" cross-section is

$$\sigma_{1,\mathrm{SLL}} = -\sigma_0 \left(\frac{2\alpha_s}{\pi}\right)^4 \ln^5\left(\frac{Q}{Q_0}\right) \pi^2 Y \, \frac{(3N^2-4)}{480}$$

Some numbers….first let's take a look at the primary emissions…





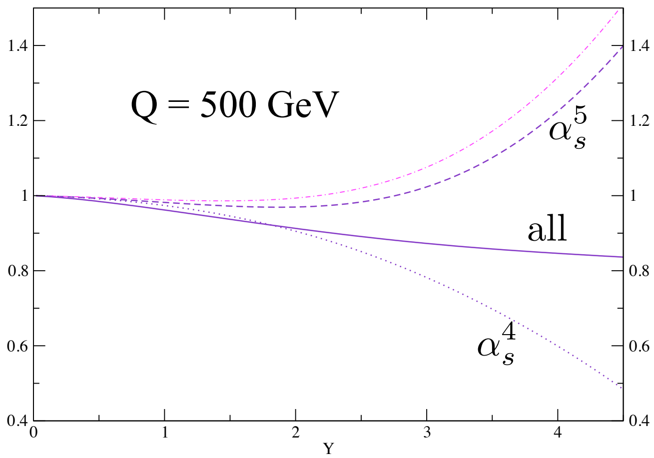


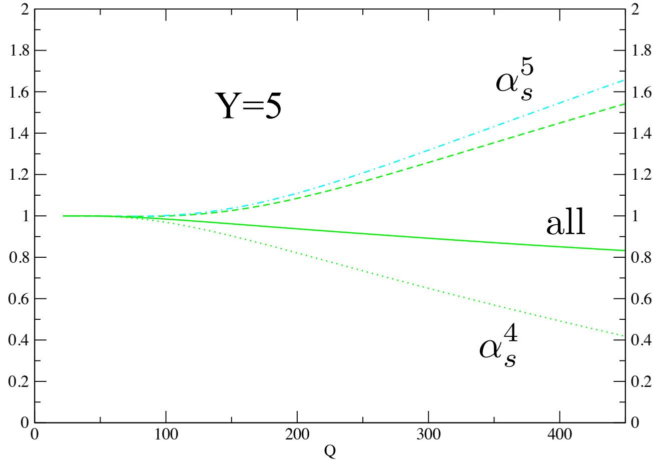
Solid = resummation of primary logs (i.e. "zero out-of-gap gluons")

Dotted = Dropping all Coulomb gluon contributions

Very significant contribution from Coulomb gluons heralds the breakdown of the angular ordered parton shower approach

Now an estimate of the impact due to super-leading logarithms…





Upper curve at $O(\alpha_s^5)$ includes the contribution from two "out-of-gap" gluons

Less than 20% effect for Q < 500 GeV and Y < 5

Concluding comments on super-leading logs:

- Need to add the contribution from $n > 1$ out-of-gap gluons.
- The $\alpha_s^4 L^5$ term we just computed *cannot* be cancelled by an $n > 1$ contribution.
- To get the "leading" logs correct requires a "next-to-leading" calculation of the evolution matrices etc. (Dixon, Mert Aybat, Sterman)
- Shocking: large collinear enhancements in an observable that sums inclusively over the collinear region.
  Conventional wisdom says expect soft enhancement but not soft-collinear, i.e. constitutes a breakdown of collinear factorization ("plus prescription" fails) and of coherence.
- Implications for other observables?
- Are they really there? [remnants? k_T ordering?]

Conclusions

- Existing (partial) resummations, based on parton shower Monte Carlos, are subject to potentially large corrections, especially from Coulomb gluons.
- Moreover, "standard" non-global effects have not yet been included in resummations. Impact on Higgs-plus-two-jet production?
- And the jet algorithm dependence of the primary emissions is non-trivial (Banfi, Dasgupta, Delenda).
- Link to small-x physics is just starting to be explored (Avsar, Hatta, Matsuo; Weigert; Mueller, Marchesini).
- Still a lot of interesting QCD left in the study of soft gluons