Linking Leptogenesis and Neutrino Oscillation

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Baryon Number Asymmetry in SM

- Within SM:

  CP violation in quark sector not sufficient to explain the observed matter-antimatter asymmetry of the Universe

- CP phase in CKM matrix: Shaposhnikov, 1986; Farrar, Shaposhnikov, 1993

$$B \simeq \frac{\alpha_w^4 T^3}{s} \delta_{CP} \simeq 10^{-8} \delta_{CP} \qquad \delta_{CP} \simeq \frac{A_{CP}}{T_C^{12}} \simeq 10^{-20}$$

  - effects of CP violation suppressed by small quark mixing

$$A_{CP} = (m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_u^2 - m_t^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2)(m_d^2 - m_b^2) \cdot J$$

$$\longrightarrow \quad B \sim 10^{-28}$$

too small to account for the observed $B \sim 10^{-10}$

Various Baryogenesis Mechanisms

- GUT Baryogenesis:
  - B-number violation unavoidable
  - sufficient CP violation
  - time scale of heavy particle decays ⇔ intrinsically out-of-eq
  - require high reheating T ⇔ gravitino overproduction
- electroweak Baryogenesis:
  - additional sources of CPV
  - strong 1st order phase transition ⇒ $m_H < 120$ GeV
- neutrino oscillation opens up a new possibility:

Leptogenesis

Fukugita, Yanagida, 1986

Compelling Neutrino Oscillation Evidences

Atmospheric Neutrinos:

SuperKamiokande (up-down asymmetry, L/E, θz dependence of μ-like events)

dominant channel: $\nu_\mu \to \nu_\tau$

next: K2K, MINOS, CNGS (OPERA)

Solar Neutrinos:

Homestake, Kamiomande, SAGE, GALLEX/GNO, SK, SNO, BOREXINO, KamLAND

dominant channel: $\nu_e \to \nu_{\mu,\tau}$

next: BOREXINO, KamLAND, ...

LSND:

dominant channel: $\overline{\nu}_\mu \to \overline{\nu}_e$

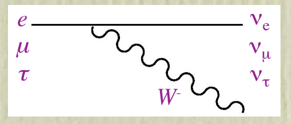
MiniBOONE -- negative result (2007)

Parameters for 3 Light Neutrinos

- three neutrino mixing $$\nu_{\ell L} = \sum_{j=1}^{3} U_{\ell j} \nu_{jL} \quad \ell = e,\ \mu,\ \tau$$

- mismatch between weak and mass eigenstates

$$\mathcal{L}_{cc} = (\ \overline{\nu}_1, \quad \overline{\nu}_2, \quad \overline{\nu}_3\ )\gamma^\mu U^\dagger \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} W_\mu^+$$



$$\nu_{1,\,2,\,3} \longrightarrow m_{1,\,2,\,3}$$

- PMNS matrix

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix} \qquad V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atm reactor solar

- Dirac CP-violating phase: $\delta = [0, 2\pi]$
- Majorana CP-violating phases: $\alpha_{21},\ \alpha_{31}$

Current Status of Oscillation Parameters

- oscillation probability: $P(\nu_a \to \nu_b) = \left|\langle \nu_b | \nu,\ t\rangle\right|^2 \simeq \sin^2 2\theta\ \sin^2\left(\frac{\Delta m^2}{4E} L\right)$
- 3 neutrinos global analysis: Maltoni, Schwetz, Tortola, Valle (updated Sep 2007)

  solar+KamLAND+CHOOZ+atmospheric+K2K+Minos

$$\sin^2\theta_{23} = 0.5\ (0.38 - 0.64), \quad \sin^2\theta_{13} = 0\ (< 0.028) \qquad \sin^2\theta_{12} = 0.30\ (0.25 - 0.34)$$

$$\Delta m^2_{23} = (2.38^{+0.2}_{-0.16}) \times 10^{-3}\ \mathrm{eV}^2, \quad \Delta m^2_{12} = (8.1 \pm 0.6) \times 10^{-5}\ \mathrm{eV}^2$$

- indication for non-zero θ13: $\sin^2\theta_{13} = 0.016 \pm 0.010\ (1\sigma)$ Fogli, Lisi, Marrone, Palazzo, Rotunno, June 2008
- Tri-bimaximal Neutrino Mixing:

Harrison, Perkins, Scott, 1999

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\sin^2\theta_{\mathrm{atm,\,TBM}} = 1/2 \qquad \sin\theta_{13,\mathrm{TBM}} = 0.$$

$$\sin^2\theta_{\odot,\mathrm{TBM}} = 1/3 \qquad \tan^2\theta_{\odot,\mathrm{TBM}} = 1/2$$

$$\tan^2\theta_{\odot,\mathrm{exp}} = 0.429$$

new KamLAND result: $\tan\theta^2_{\odot,exp} = 0.47^{+0.06}_{-0.05}$

Discovery phase into precision phase for some oscillation parameters

Neutrino Mass Spectrum

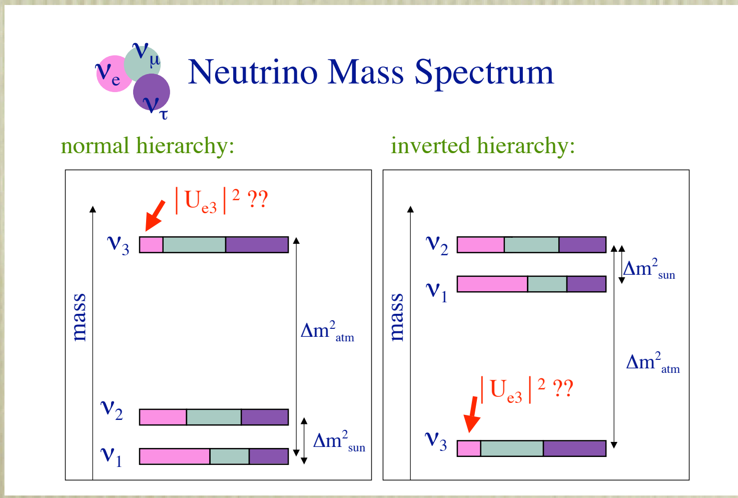
- search for absolute mass scale:
  - end point kinematic of tritium beta decays:

$$\text{Tritium} \rightarrow He^3 + e^- + \overline{\nu}_e$$

Mainz: $m_\nu < 2.2$ eV
KATRIN: increase sensitivity ~ 0.2 eV

  - WMAP + 2dFRGS + Lyα: $\sum(m_{\nu_i}) < (0.7\text{-}1.2)$ eV
  - neutrinoless double beta decay

current bound: $| < m > | < (0.19 - 0.68)$ eV (CUORICINO, Feb 2008)



The known unknowns:

- How small is θ_{13}? (ν_e component of ν_3)
- $\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, $\theta_{23} = \pi/4$? (ν_3 composition)
- Neutrino mass hierarchy (Δm_{13}^2) ?
- CP violation in neutrino oscillations?

Seesaw Mechanism

- a natural way to generate small neutrino masses

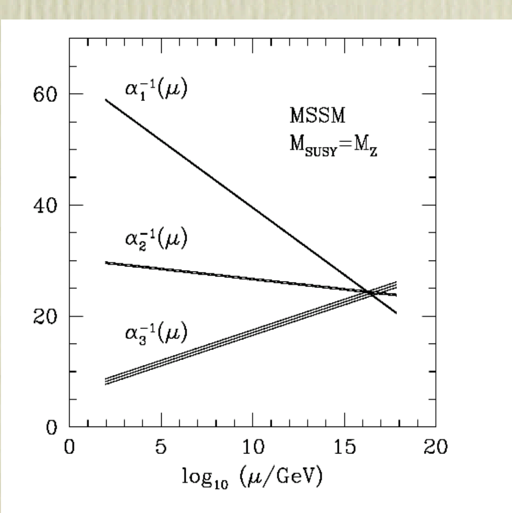
  Minkowski, 1977; Gell-mann, Ramond, Slansky,1981;Yanagida, 1979; Mohapatra, Senjanovic, 1981

- possibility to link origin of BAU to neutrino oscillation through leptogenesis
- Introduce right-handed neutrinos, which are SM gauge singlets

  [predicted in many GUTs, e.g. SO(10)]
- The Lagrangian: $\mathcal{L}_Y = f_{ij}\overline{e}_{R_i}\ell_{L_j}H^\dagger + h_{ij}\overline{\nu}_{R_i}\ell_{L_j}H - \frac{1}{2}(M_R)_{ij}\overline{\nu}^c_{R_i}\nu_{R_j} + h.c.\ .$
- integrating out N_R: effective mass matrix

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

light neutrino mass: $m_\nu \sim \frac{m_D}{M_R} m_D$



$$m_\nu \sim \sqrt{\Delta m^2_{atm}} \sim 0.05\ \mathrm{eV},\ m_D \sim m_t \sim 172\ \mathrm{GeV}$$

$$\Rightarrow M_R \sim 10^{15}\mathrm{GeV}$$ ~ M_{GUT}

seesaw ⇒ Neutrinos are Majorana fermions

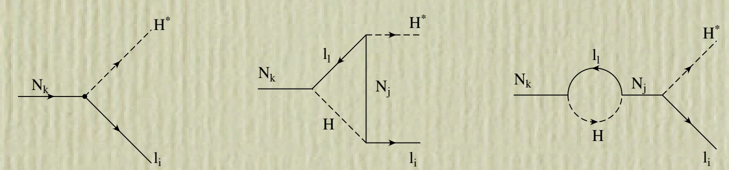
⇒ Lepton Number violation

Leptogenesis

Fukugita, Yanagida, 1986

- implemented in the context of seesaw mechanism
- out-of-equilibrium decays of RH neutrinos produce primordial lepton number asymmetry

Luty, 1992; Covi, Roulet, Vissani, 1996; Flanz et al, 1996; Plumacher, 1997; Pilaftsis, 1997;



$$\epsilon_1 = \frac{\sum_\alpha \left[\Gamma(N_1 \to \ell_\alpha H) - \Gamma(N_1 \to \overline{\ell}_\alpha \overline{H})\right]}{\sum_\alpha \left[\Gamma(N_1 \to \ell_\alpha H) + \Gamma(N_1 \to \overline{\ell}_\alpha \overline{H})\right]}$$

- sphaleron processes: $\Delta L \to \Delta B$

- the asymmetry

Buchmuller, Plumacher, 1998;
Buchmuller, Di Bari, Plumacher, 2004

$$Y_B = \frac{n_B - n_{\overline{B}}}{s} \sim 8.6 \times 10^{-11} \qquad Y_B \simeq 10^{-2} \epsilon \kappa$$

$$\kappa: \text{ efficiency factor} \sim (10^{-1} - 10^{-3})$$

$\Phi^+ + \ell^- \to N_1$, $\quad \ell^- + \Phi^+ \to \Phi^- + \ell^+$, etc.

Realizations of Leptogenesis

- Standard Leptogenesis with Type-I seesaw + hierarchical RH neutrino mass spectrum
  Fukugita, Yanagida, 1986

- Leptogenesis with Type-II seesaw
  Joshipura, Paschos, Rodejohann, 2001; Hambye, Senjanovic, 2004; Antusch, King, 2004, ...

- Resonant leptogenesis: near degenerate RH neutrino mass spectrum
  Pilaftsis, 1997; ...

- soft leptogenesis
  Grossman, Kashti, Nir, Roulet, 2003; D'Ambrosio, Giudice, Raidal, 2003; Boubekeur, 2002; Boubekeur, Hambye, Senjanovic, 2004, ...

$$\epsilon = \left(\frac{4\Gamma_1 B}{\Gamma_1^2 + 4B^2}\right)\frac{\mathrm{Im}(A)}{M_1}\delta_{B-F}$$

A, B: SUSY CP-violating phases
lose connection to neutrino oscillation

- Dirac leptogenesis
  Dick, Lindner, Ratz, Wright, 2000; Murayama, Pierce, 2002; ...

Testing Leptogenesis?

Sakharov conditions:

- out-of-equilibrium (expanding Universe)
- Lepton number violation (neutrinoless double beta decay)
- CP violation

Connection to Low Energy Observables

- Lagrangian at high energy (in the presence of RH neutrinos)

$$\mathcal{L} = \overline{\ell}_{L_i} i\gamma^\mu \partial_\mu \ell_{L_i} + \overline{e}_{R_i} i\gamma^\mu \partial_\mu e_{R_i} + \overline{N}_{R_i} i\gamma^\mu \partial_\mu N_{R_i}$$
$$+ f_{ij}\overline{e}_{R_i}\ell_{L_j} H^\dagger + h_{ij}\overline{N}_{R_i}\ell_{L_j} H - \frac{1}{2} M_{ij} N_{R_i} N_{R_j} + h.c.$$

in f_{ij} and M_{ij} diagonal basis →

h_{ij} general complex matrix: { 9-3 = 6 mixing angles; 9-3 = 6 physical phases }

- Low energy effective Lagrangian (after integrating out RH neutrinos)

$$\mathcal{L}_{eff} = \overline{\ell}_{L_i} i\gamma^\mu \partial_\mu \ell_{L_i} + \overline{e}_{R_i} i\gamma^\mu \partial_\mu e_{R_i} + f_{ii}\overline{e}_{R_i}\ell_{L_i} H^\dagger + \frac{1}{2}\sum_k h^T_{ik} h_{kj} \ell_{L_i}\ell_{L_j} \frac{H^2}{M_k} + h.c.$$

in f_{ij} diagonal basis →

h_{ij} symmetric complex matrix: { 6-3 = 3 mixing angles; 6-3 = 3 physical phases }

- high energy → low energy:
  numbers of mixing angles and CP phases reduced by half

Connection to Low Energy Observables

- diagonal basis for charged lepton and RH neutrino mass matrices
- neutrino Dirac Yukawa interactions: $h = V_R^{\nu\,\dagger}\mathrm{diag}(h_1, h_2, h_3)V_L^\nu$
- CP asymmetry parametrized by (orthogonal parametrization)

(Casas & Ibarra, 2001)

$m = \mathrm{diag}(m_1, m_2, m_3)$ (light neutrino masses)

$M = \mathrm{diag}(M_1, M_2, M_3)$ (RH neutrino masses)

$R = vM^{-1/2}hUm^{-1/2}$ R: phases in RH sector

$$hh^\dagger v^2 = V_R^{\nu\,\dagger}\mathrm{diag}(h_1^2, h_2^2, h_3^2)V_R^\nu v^2 = M^{1/2}RmR^\dagger M^{1/2}$$

$$h = \frac{1}{v}M^{1/2}Rm^{1/2}U^\dagger$$

combination relevant for leptogenesis in 1-flavor approximation

R: high energy parameters
U: low energy MNS

Connection to Low Energy Observables

hierarchical RH neutrinos: $M_1 \ll M_2 \ll M_3$

One Flavor Approximation: T > 10^12 GeV

- individual lepton number asymmetry

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j |R_{1j}|^2}, \qquad v = 174 \text{ GeV}$$

where the effective masses $\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2 v^2}{M_1} = \left|\sum_k R_{1k} m_k^{1/2} U_{lk}^*\right|^2, \quad l = e, \mu, \tau$

- out-of-equilibrium temperature

  $Y_{e,\mu,\tau}$ all small $\; Y_l \; H^c(x)\overline{l_R}(x)\psi_{lL}$ out of equilibrium at $T \sim M_1 > 10^{12}$ GeV

  ➡ $L_{e,\mu,\tau}$ not distinguishable

- Boltzmann equations for $n(N_1)$ and $\Delta L = \Delta(L_e + L_\mu + L_\tau)$
- resulting lepton number asymmetry

$$\varepsilon_1 = \sum_l \varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im}\left(\sum_{j,k} m_j^2 R_{1j}^2\right)}{\sum_k m_k |R_{1k}|^2} \qquad \widetilde{m_1} = \sum_l \widetilde{m_l} = \sum_k m_k |R_{1k}|^2$$

Connection to Low Energy Observables

- one-flavor approximation

real R, complex U:
non-vanishing low energy CPV (h)
vanishing leptogenesis

presence of low energy leptonic CPV (neutrino oscillation, neutrinoless double beta decay)	$\not\Leftrightarrow$	leptogenesis ≠ 0

- no model independent connection can exist

Abada, Davidson, Josse-Michaux, Losada, Riotto, 2006;
Nardi, Nir, Roulet, Racker, 2006

Flavor matters?

leptogenesis at $T \sim M_1 < 10^{12}$ GeV:

three flavors distinguishable (different $T_{eq} = Y^2 M_{pl}$)

non-universal wash-out factors

Connection to Low Energy Observables

- At $M_1 \sim T \sim 10^{12}$ GeV: Y_τ - in equilibrium, $Y_{e,\mu}$ - not
- At $M_1 \sim T \sim 10^{9}$ GeV: Y_τ, Y_μ - in equilibrium, Y_e - not
- two flavor regime: $M_1 \sim 10^9 - 10^{12}$ GeV

  $\varepsilon_{1\tau}$ and $(\varepsilon_{1e} + \varepsilon_{1\mu}) \equiv \varepsilon_2$ evolve independently

- three flavor regime $M_1 < 10^9$ GeV

  $\varepsilon_{1\tau}$, ε_{1e} and $\varepsilon_{1\mu}$ evolve independently

  - asymmetry associated with each flavor   Pascoli, Petcov, Riotto, 2006

$$\epsilon_\alpha = -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{\beta\rho} m_\beta^{1/2} m_\rho^{3/2} U^*_{\alpha\beta} U_{\alpha\rho} R_{1\beta} R_{1\rho}\right)}{\sum_\beta m_\beta |R_{1\beta}|^2}$$

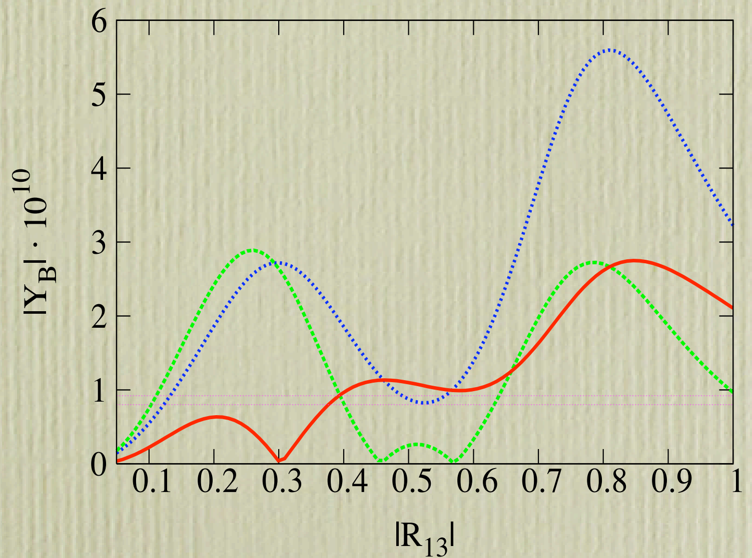
leptogenesis ≠ 0 → low energy CPV ≠ 0

Connection to Low Energy CPV

- including flavor effects: for inverted mass spectrum, low energy CP phases can be important under certain conditions

Molinaro, Petcov, 2008

$m_1 < m_2 < m_3$ (NO(NH))

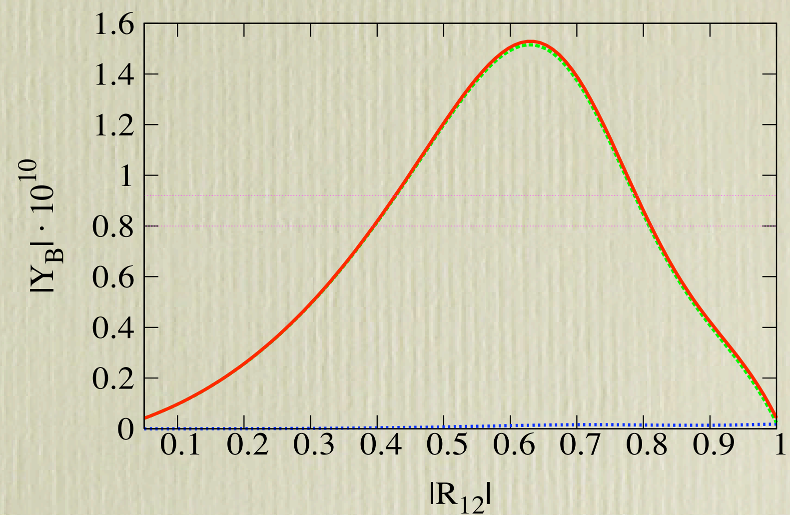


R CPV, blue

U CPV, green

total $|Y_B|$ (red line)

$m_3 \ll m_1 < m_2$ (IH))



$\alpha_{21} = \pi/2$, $|R_{11}| \cong 1.0$

$M_1 = 10^{11}$ GeV $\quad s_{13} = 0.2$ and $\delta = \pi$

CPV: α_{21} in U and R phases

Connection in Specific Models

- models for neutrino masses:
  additional symmetries or textures
  ➡ reduce the number of parameters
  ➡ connection can be established

- texture assumption
  2x3 seesaw model

- all CP violation can come from a single source
  minimal left-right model with spontaneous CP violation

- implications of tri-bimaximal neutrino mixing
  A4 model

Seesaw with 2 RH Neutrinos

- cancellation of Witten anomaly
  - ➡ leptonic SU(2) horizontal symmetry
  - ➡ two RH neutrinos
  - ➡ 2x3 seesaw mechanism

Kuchimanchi & Mohapatra, 2002

- Lagrangian

Frampton, Glashow, Yanagida, 2002

$$\mathcal{L} = \frac{1}{2}(N_1 N_2)\begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}\begin{pmatrix} N_1 \\ N_2 \end{pmatrix} + (N_1 N_2)\begin{pmatrix} a & a' & 0 \\ 0 & b & b' \end{pmatrix}\begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix} H + h.c. \,,$$

- effective neutrino mass matrix

$$\begin{pmatrix} \frac{a^2}{M_1} & \frac{aa'}{M_1} & 0 \\ \frac{aa'}{M_1} & \frac{a'^2}{M_1} + \frac{b^2}{M_2} & \frac{bb'}{M_2} \\ 0 & \frac{bb'}{M_2} & \frac{b'^2}{M_2} \end{pmatrix}$$

a, b, b' are real and $a' = |a'|e^{i\delta}$

Seesaw with 2 RH Neutrinos

- bi-large mixing angle $a' = \sqrt{2}a$ $b = b'$ $a^2/M_1 \ll b^2/M_2$

$$m_{\nu_1} = 0, \quad m_{\nu_2} = \frac{2a^2}{M_1}, \quad m_{\nu_3} = \frac{2b^2}{M_2}$$

$$U = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \qquad \theta \simeq m_{\nu_2}/\sqrt{2}m_{\nu_3}$$

- relation between sign of baryonic asymmetry and sign of CP violation in neutrino oscillation

$$\xi_{osc} = -\frac{a^4 b^4}{M_1^3 M_2^3}(2 + Y^2)\xi_B \propto -B$$

$$(B \propto \xi_B = Y^2 a^2 b^2 \sin 2\delta)$$

Sources of CP Violation

- Manifestations of CP violation
  - weak scale CPV (kaon, B-meson, neutrino oscillation, ...)
  - cosmological BAU
  - strong CP problem

  ⇒ can they come from a common origin??

- Explicit CP violation
  - complex Yukawa couplings
- Spontaneous CP violation
  - complex VEV

Models with Spontaneous CP Violation

M-C.C & Mahanthappa, 2005

- minimal left-right model:
- gauge symmetry

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \\ \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \ ,$$

- particle content

$$Q_{i,L} = \begin{pmatrix} u \\ d \end{pmatrix}_{i,L} \sim (1/2, 0, 1/3), \qquad Q_{i,R} = \begin{pmatrix} u \\ d \end{pmatrix}_{i,R} \sim (0, 1/2, 1/3)$$

$$L_{i,L} = \begin{pmatrix} e \\ \nu \end{pmatrix}_{i,L} \sim (1/2, 0, -1), \qquad L_{i,R} = \begin{pmatrix} e \\ \nu \end{pmatrix}_{i,R} \sim (0, 1/2, -1)$$

- minimal higgs sector

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \sim (1/2,\ 1/2,\ 0) \qquad \Delta_L = \begin{pmatrix} \Delta_L^+/\sqrt{2} & \Delta_L^{++} \\ \Delta_L^0 & -\Delta_L^+/\sqrt{2} \end{pmatrix} \sim (1,\ 0,\ 2) \qquad \Delta_R = \begin{pmatrix} \Delta_R^+/\sqrt{2} & \Delta_R^{++} \\ \Delta_R^0 & -\Delta_R^+/\sqrt{2} \end{pmatrix} \sim (0,\ 1,\ 2)$$

- in general, 4 complex VEV's

$$\langle \Phi \rangle = \begin{pmatrix} \kappa e^{i\alpha_\kappa} & 0 \\ 0 & \kappa' e^{i\alpha_{\kappa'}} \end{pmatrix}, \quad \langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\alpha_L} & 0 \end{pmatrix}, \ \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R e^{i\alpha_R} & 0 \end{pmatrix}$$

Models with Spontaneous CP Violation

- Lagrangian invariant under unitary transformations

$$U_L = \begin{pmatrix} e^{i\gamma_L} & 0 \\ 0 & e^{-i\gamma_L} \end{pmatrix}, \qquad U_R = \begin{pmatrix} e^{i\gamma_R} & 0 \\ 0 & e^{-i\gamma_R} \end{pmatrix}$$

$$\psi_L \to U_L \psi_L, \qquad \psi_R \to U_R \psi_R$$

$$\Phi \to U_R \Phi U_L^\dagger, \qquad \Delta_L \to U_L^* \Delta_L U_L^\dagger, \qquad \Delta_R \to U_R^* \Delta_R U_R^\dagger$$

- VEVs transform accordingly

$$\kappa \to \kappa e^{-i(\gamma_L - \gamma_R)}, \quad \kappa' \to \kappa' e^{i(\gamma_L - \gamma_R)}, \quad v_L \to v_L e^{-2i\gamma_L}, \quad v_R \to v_R e^{-2i\gamma_R}$$

- only two physical phases:

$$< \Phi > = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha_{\kappa'}} \end{pmatrix}, \qquad < \Delta_L > = \begin{pmatrix} 0 & 0 \\ v_L e^{i\alpha_L} & 0 \end{pmatrix}, \quad < \Delta_R > = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$

$\alpha_{\kappa'} \Rightarrow$ all CPV in quark sector (contributions to lepton sector negligible for high seesaw scale)

$\alpha_L \Rightarrow$ all CPV in lepton sector

Models with Spontaneous CP Violation

M-C.C & Mahanthappa, 2005

- all leptonic CP violation from a single phase
  - the three CP-violating phases in MNS matrix are functions of the intrinsic phase α_L
- the phase α_L enters
  - neutrino oscillation

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} Re(U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i}) \sin^2\left(\Delta m^2_{ij}\frac{L}{4E}\right) + 2\sum_{i>j} J^{\rm lep}_{\rm CP} \sin^2\left(\Delta m^2_{ij}\frac{L}{4E}\right)$$

$$J^{\rm lep}_{\rm CP} = -\frac{Im(H_{12}H_{23}H_{31})}{\Delta m^2_{21}\Delta m^2_{32}\Delta m^2_{31}}, \quad H \equiv (M^{eff}_\nu)(M^{eff}_\nu)^\dagger$$

$$\boxed{J^{lep}_{CP} \sim \sin\alpha_L}$$

  - neutrino-less double beta decay

$$|\langle m_{ee}\rangle|^2 = m_1^2\,|U_{e1}|^4 + m_2^2\,|U_{e2}|^4 + m_3^2\,|U_{e3}|^4 + 2m_1m_2\,|U_{e1}|^2\,|U_{e2}|^2\cos\alpha_{21}$$
$$+2m_1m_3\,|U_{e1}|^2\,|U_{e3}|^2\cos\alpha_{31} + 2m_2m_3\,|U_{e2}|^2\,|U_{e3}|^2\cos(\alpha_{31}-\alpha_{21})$$

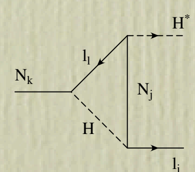
  - leptogenesis

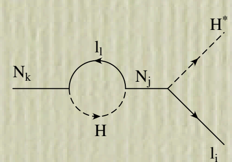
Models with Spontaneous CP Violation

- triplet leptogenesis:
  - $N_1 \to \ell + H^\dagger$ $\qquad \epsilon = \dfrac{\Gamma(N_1 \to \ell + H^\dagger) - \Gamma(N_1 \to \overline{\ell} + H)}{\Gamma(N_1 \to \ell + H^\dagger) + \Gamma(N_1 \to \overline{\ell} + H)}$
  - $\Delta^* \to \ell + \ell$ $\qquad \epsilon = \dfrac{\Gamma(\Delta_L^* \to \ell + \ell) - \Gamma(\Delta_L \to \overline{\ell} + \overline{\ell})}{\Gamma(\Delta_L^* \to \ell + \ell) + \Gamma(\Delta_L \to \overline{\ell} + \overline{\ell})}$
- natural scenario: Δ^* heavier than N_1 ➔ N_1 decay dominant
- two contributions

M-C.C & Mahanthappa, 2005

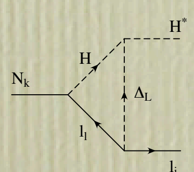
  - usual diagrams (type I contribution) $\qquad \mathcal{M}_D = O_R M_D$





$$\epsilon^{N_1} = \frac{3}{16\pi}\left(\frac{M_{R_1}}{v^2}\right) \cdot \frac{\mathrm{Im}\Big(\mathcal{M}_D \,(M_\nu^I)^* \,\mathcal{M}_D^T\Big)_{11}}{(\mathcal{M}_D \mathcal{M}_D^\dagger)_{11}} = 0$$

  - new diagram (type II contribution)

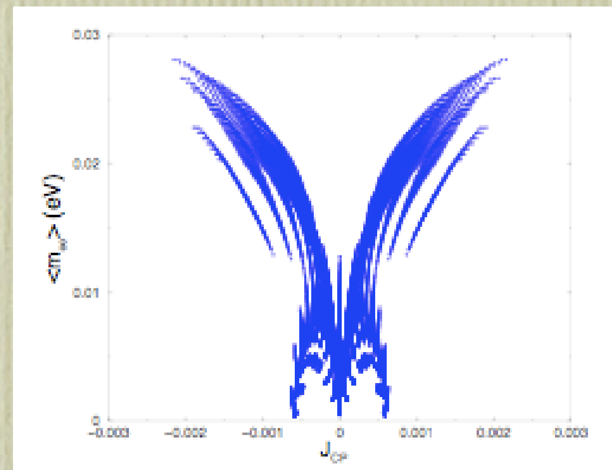


$$\epsilon^{\Delta_L} = \frac{3}{16\pi}\left(\frac{M_{R_1}}{v^2}\right) \cdot \frac{\mathrm{Im}\Big(\mathcal{M}_D \,(M_\nu^{II})^* \,\mathcal{M}_D^T\Big)_{11}}{(\mathcal{M}_D \mathcal{M}_D^\dagger)_{11}} \sim \sin \alpha_L$$

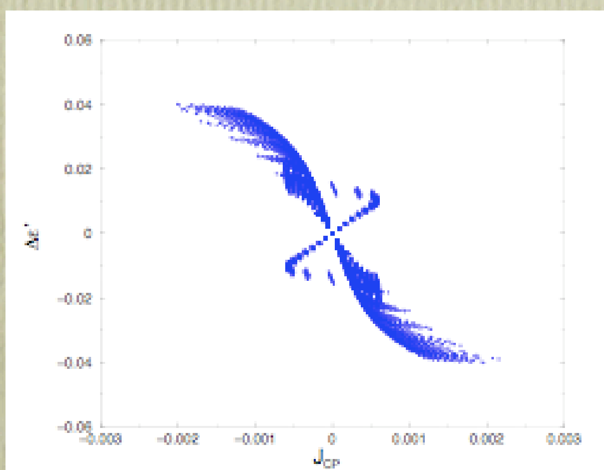
independent of choice of $U_{L,R}$

Models with Spontaneous CP Violation

M.-C.C & Mahanthappa, 2005



- predict small θ_{13}
- in large J_{cp} regime:
  strong correlation between J_{cp} and $\langle m_{ee} \rangle$
- J_{cp}: $(0 - 10^{-3})$
- $\langle m_{ee} \rangle$: $(10^{-4} \sim 10^{-2})$ eV; current limit ~ 0.1 eV



- symmetry between 2nd & 4th quadrants
- in large J_{cp} regime: strong correlation between J_{cp} and $\Delta\epsilon'$ (even without flavor effects)
- total amount of lepton number asymmetry

$$\epsilon = 10^{-2} \times \Delta\epsilon' < (10^{-5} - 10^{-4})$$

- no wash-out $\quad \dfrac{\Gamma_{N_1}}{H|_{T=M_1}} = \dfrac{1}{0.01 \text{ eV}} \dfrac{(M_D M_D^\dagger)_{11}}{M_1} < 1$

$$\frac{(M_D M_D^\dagger)_{11}}{M_1} \propto \left(\frac{m_c}{m_t}\right)^2 v_L < \mathcal{O}(10^{-7}) \text{ eV}$$

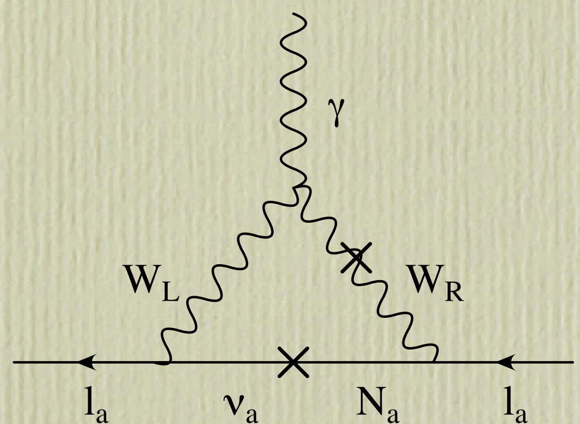
observed BAU ⇒ $J_{cp} \sim 10^{-5}$

Models with Spontaneous CP Violation

M.-C.C & Mahanthappa, 2007

- with an additional U(1) symmetry

  ⇒ can lower seesaw scale to 10^6 GeV (and below)



$$d_e \simeq -\frac{e\alpha}{4\pi M_W^2}\frac{\kappa\kappa'}{v_R^2 - v_L^2}\text{Im}(e^{-i\alpha_{\kappa'}} M_D)_{ee}$$

$$\simeq 10^{-19} \times r\left(\frac{GeV}{v_R}\right)^2\left(\frac{|(M_{\nu_D})_{ee}|}{MeV}\right)\sin(2\alpha_{\kappa'}) \quad \text{e-cm}$$

- relation between CPV in quark & lepton sectors
- electron EDM ~ 10^{-32} e-cm

Models with Spontaneous CP Violation

- SM + D° (vectorial quark) + S (singlet scalar) Branco, Parada, Rebelo, 2003

$$\langle\phi^0\rangle = \frac{v}{\sqrt{2}}, \qquad \langle S\rangle = \frac{V\exp(i\alpha)}{\sqrt{2}}$$

$$(f_q S + f_q{}' S^*)\overline{D^0_L} d^0_R + \tilde{M}\overline{D^0_L} D^0_R$$ →quark CPV

$$\frac{1}{2}\nu_R^{0T} C(f_\nu S + f_\nu{}' S^*)\nu_R^0$$ →leptonic CPV

- SCPV in SO(10) Achiman, 2004, 2008

<126> complex: break (B-L) $\overline{\Delta} =< \overline{\Sigma}(1,1,0) > \; = \frac{\sigma}{\sqrt{2}} e^{i\alpha}$

$$Y_\ell^{ij}\nu_R^i\overline{\Delta}\nu_R^j$$

  - no symmetry reason why <S> is the only complex VEV

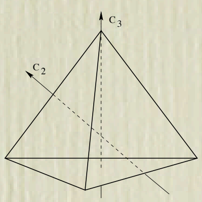
Models with Tri-bimaximal Neutrino Mixing

- global neutrino oscillation data strongly suggests TBM mixing pattern

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$\boxed{\sin\theta_{13,\rm TBM} = 0}$ ⟷ Leptogenesis?

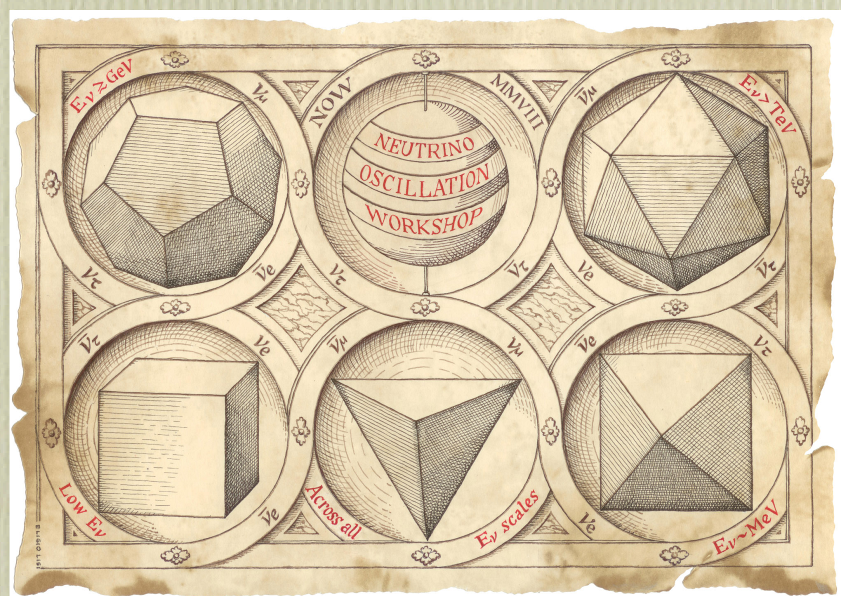
- TBM mixing can arise from underlying symmetry
  - S3 : less constrained Mohapatra, Nasri, Yu, 2006
  - Z7 x Z3 Luhn, Nasri, Ramond, 2007
  - A4: Ma, 2004; Altarelli, Feruglio, 2006
    - tri-bimaxmal mixing results from group theory!
    - no CKM mixing
  - $^{(d)}$T: double covering of A4 Carr, Frampton, 2007; Feruglio, Hedgedorn, Lin, Merlo, 2007
    - retain predictivity of A4 in neutrino sector
    - realistic CKM in SU(5) x $^{(d)}$T M.-C.C & Mahanthappa, 2007



Perfect Geometric Solids & Family Symmetries

solid	faces	vert.	Plato	Hindu	sym.
tetrahedron	4	4	fire	Agni	A_4
octahedron	8	6	air	Vayu	S_4
cube	6	8	earth	Prithvi	S_4
icosahedron	20	12	water	Jal	A_5
dodecahedron	12	20	quintessence	Akasha	A_5

Table from E. Ma, talk at WHEPP-9, Bangalore

A5  A5

S4 S4

A4

Tri-bimaximal Neutrino Mixing

- Neutrino mass matrices:

$$M = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix} \longrightarrow \sin^2 2\theta_{23} = 1 \qquad \theta_{13} = 0$$

solar mixing angle NOT fixed

  - S_3 Mohapatra, Nasri, Yu, 2006; ...
  - D_4 Grimus, Lavoura, 2003; ...
  - μ-τ symmetry Fukuyama, Nishiura, '97; Mohapatra, Nussinov, '99; Ma, Raidal, '01; ...

- if $A+B = C + D \longrightarrow \tan^2\theta_{12} = 1/2$ TBM pattern

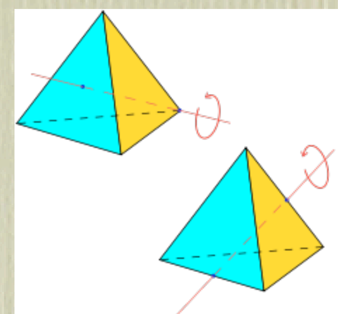
  - A_4 Ma, '04; Altarelli, Feruglio, '06; .....
  - $Z_3 \times Z_7$ Luhn, Nasri, Ramond, 2007

[Other discrete groups: Hagedorn, Lindner, Plentinger; Chen, Frigerio, Ma; and many others...]

recent claim: S_4 unique group for TBM [C.S. Lam, 2008]

Non-abelian Finite Family Symmetry

- TBM mixing matrix: can be realized in finite group family symmetry based on A4 Ma & Rajasekaran, '01
- even permutations of 4 objects
  - (1234) →(4321)
  - (1234) → (2314)
- invariance group of Tetrahedron



- orbifold compactification: Altarelli, Feruglio, '06

  6D → 4D on T2/Z2
- four in-equivalent representations: 1, 1′, 1″, 3
- Tri-bimaximal mixing arise: Ma, '04; Altarelli, Feruglio, '06; .....
  - three families of lepton doublets ~ 3
  - RH charged leptons ~ 1, 1′, 1″

Non-abelian Finite Family Symmetry

- fermion charge assignments:

$$\begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix}_L \sim 3, \quad e_R \sim 1, \quad \mu_R \sim 1'', \quad \tau_R \sim 1' \qquad \xi \sim 3, \quad \eta \sim 1 \qquad \langle \xi \rangle = \xi_0 \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

- SM Higgs ~ singlet under $^{(d)}T$
- operator for neutrino masses: $\frac{HHLL}{M}\left(\frac{\langle\xi\rangle}{\Lambda} + \frac{\langle\eta\rangle}{\Lambda}\right)$
- TBM neutrino mixing from A4 CG coefficients

$$M_\nu = \frac{\lambda v^2}{M_x} \begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix}$$

Form diagonalizable!

$$V_\nu^T M_\nu V_\nu = \text{diag}(u + 3\xi_0,\, u,\, -u + 3\xi_0)\frac{v_u^2}{M_x}$$

$$V_\nu = U_{\text{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

-- no adjustable parameters
-- neutrino mixing from CG coefficients!

- charged lepton mass matrix: diagonal $\langle \phi \rangle = \phi_0 \Lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
- no quark CKM mixing!!

The Double Tetrahedral $^{(d)}T$ Symmetry

- consider double covering of A_4
- Classified as a candidate family symmetry that can arise from Type-II B String theories

Frampton, Kaphart, 1995, 2001

- can account for quark sector:

Carr, Frampton, 2007;
Feruglio, Hedgedorn, Lin, Merlo, 2007

exist in A_4: $1,\ 1',\ 1'',\ 3$ ⟶ TBM for neutrinos

not in A_4: $2,\ 2',\ 2''$ ⟶ 2 +1 assignments for quarks

- Combined with GUT: $^{(d)}T \times SU(5)$ GUT

M.-C.C & K.T. Mahanthappa
Phys. Lett. B652, 34 (2007)

  - only 9 operators allowed: highly predictive model

SU(5) x $^{(d)}$T Model

M.-C.C & K.T. Mahanthappa
Phys. Lett. B652, 34 (2007)

- CKM mixing matrix

$$M_u = \begin{pmatrix} i\phi_0'^3 & \frac{1-i}{2}\phi_0'^3 & 0 \\ \frac{1-i}{2}\phi_0'^3 & \phi_0'^3 + (1-\frac{i}{2})\phi_0^2 & y'\psi_0\zeta_0 \\ 0 & y'\psi_0\zeta_0 & 1 \end{pmatrix} y_t v_u$$

$y'\psi_0\zeta_0 \to V_{cb}$

$$M_d = \begin{pmatrix} 0 & (1+i)\phi_0\psi_0' & 0 \\ -(1-i)\phi_0\psi_0' & \psi_0 N_0 & 0 \\ \phi_0\psi_0' & \phi_0\psi_0' & \zeta_0 \end{pmatrix} y_b v_d \phi_0,$$

$\phi_0\psi_0' \to V_{ub}$

$$\theta_c \simeq \left|\sqrt{m_d/m_s} - e^{i\alpha}\sqrt{m_u/m_c}\right| \sim \sqrt{m_d/m_s},$$

Georgi-Jarlskog relations ⇒ $V_d \neq I$

SU(5) ⇒ $M_d = (M_e)^T$

⇒ corrections to TBM related to θ_c

- MNS matrix:

$$M_e = \begin{pmatrix} 0 & -(1-i)\phi_0\psi_0' & \phi_0\psi_0' \\ (1+i)\phi_0\psi_0' & -3\psi_0 N_0 & \phi_0\psi_0' \\ 0 & 0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0$$

⇒

$$\theta_{12}^e \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3}\sqrt{\frac{m_d}{m_s}} \sim \frac{1}{3}\theta_c$$

$$U_{\rm MNS} = V_{e,L}^\dagger U_{\rm TBM} = \begin{pmatrix} 1 & -\theta_c/3 & * \\ \theta_c/3 & 1 & * \\ * & * & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,\rm TBM} - \frac{1}{2}\theta_c\cos\beta$$

$\beta \to$ leptonic CPV

$$\theta_{13} \simeq \theta_c/3\sqrt{2}$$

GJ relations in SU(5) ⇒ new QLC relation!

Quark-Lepton Complementarity

lepton mixing

parameter	Best-fit value	3σ range
θ_{12}	33.2^o	$28.7^o - 38.1^o$
θ_{23}	45^o	$35.7^o - 55.6^o$
θ_{13}	2.6^o	$0 - 12.5^o$

quark mixing

parameter	Best-fit value	3σ range
θ_c	12.88^o	$12.75^o - 13.01^o$
θ^q_{23}	2.36^o	$2.25^o - 2.48^o$
θ^q_{13}	0.21^o	$0.17^o - 0.25^o$

$$\theta_{12} + \theta_c = 45^o$$

Raidal, '04; Smirnov & Minakata, '04

quark-lepton complementarity relation

↕

quark-lepton unification?

more generally:

$$\theta_{12} + \theta_C \left(\underbrace{\frac{1}{\sqrt{2}} + \frac{\theta_C}{4}}_{k} \right) \simeq \frac{\pi}{4}$$

See: Talk by Walter Winter

Plentinger, Seidl, Winter, 08; Frampton, Matsuzaki, 08; King 05; King Antusch, 05

RG effects: $\Delta\theta_c \sim \theta_c^4$

MSSM: normal hierarchy $\Delta\theta_{12} < 0.1°$ Schmidt & Smirnov, '06

Motivate measurements of neutrino mixing angles to at least the accuracy of the measured quark mixing angles

Neutrino Mass Sum Rule

- sum rule among three neutrino masses: $m_1 - m_3 = 2m_2$
- including CP violation:

$$\begin{aligned} m_1 &= u_0 + 3\xi_0 e^{i\theta} \\ m_2 &= u_0 \\ m_3 &= -u_0 + 3\xi_0 e^{i\theta} \end{aligned}$$

$$\begin{aligned} \Delta m^2_{atm} &\equiv |m_3|^2 - |m_1|^2 = -12 u_0 \xi_0 \cos\theta \\ \Delta m^2_{\odot} &\equiv |m_2|^2 - |m_1|^2 = -9\xi_0^2 - 6 u_0 \xi_0 \cos\theta \end{aligned}$$

- leads to sum rule

$$\Delta m^2_{\odot} = -9\xi_0^2 + \frac{1}{2}\Delta m^2_{atm} \longrightarrow \Delta m^2_{atm} > 0$$

normal hierarchy predicted!!

- constraint on Majorana phases:

$$0 > \cos\theta > -\frac{3}{2}\frac{\xi_0}{u_0}$$

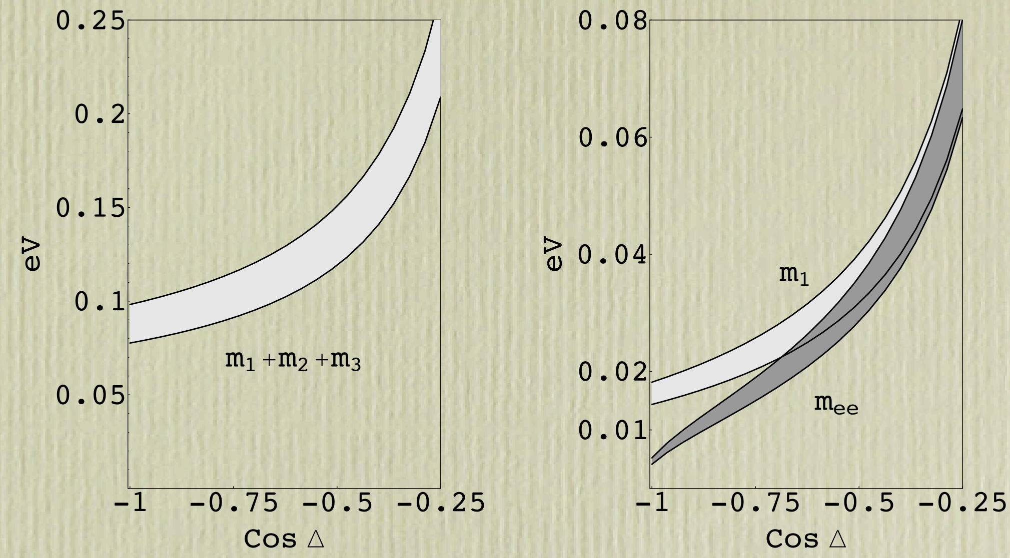
- neutrino-less double beta decay:

$$\begin{aligned} \xi_0 &= \frac{1}{3}\sqrt{(\frac{1}{2} - r)\Delta m^2_{atm}} \\ u_0 &= -\frac{1}{4\cos\theta}\sqrt{\frac{\Delta m^2_{atm}}{(\frac{1}{2} - r)}} \end{aligned}$$

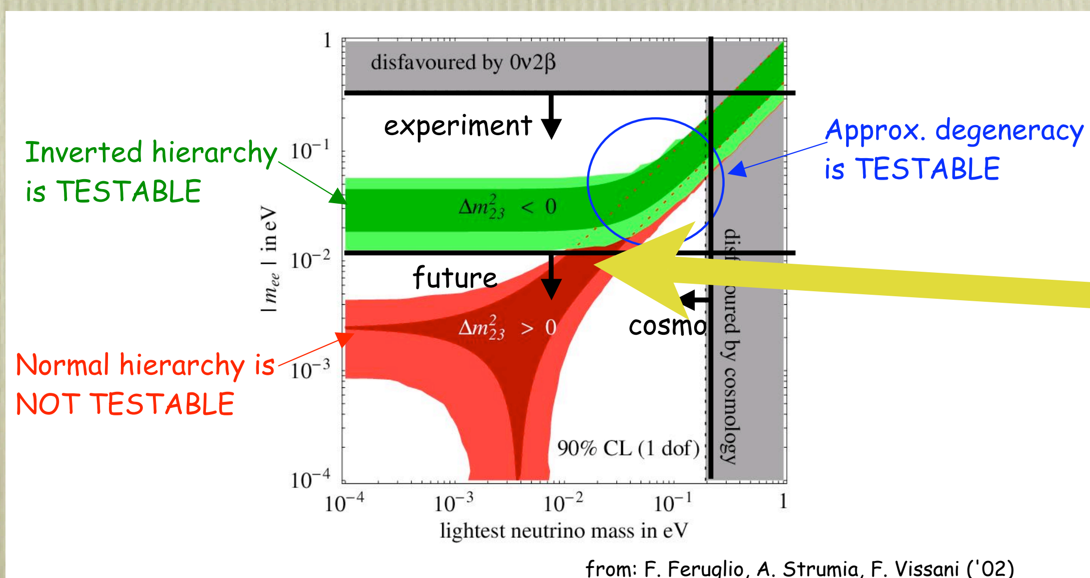
$$r \equiv \Delta m^2_{\odot} / \Delta m^2_{atm}$$

$$|\langle m_{ee}\rangle|^2 = \Big[-\frac{1+4r}{9} + \frac{1}{8(1-2r)\cos^2\theta}\Big]\Delta m^2_{atm}$$

Models with Tri-bimaximal Neutrino Mixing



For A4: Altarelli et al, 2006



prediction in
A_4 and $^{(d)}T$ models

Models with Tri-bimaximal Neutrino Mixing

- TBM mixing arises from underlying broken discrete symmetries (A_4, Z_7 x Z_3, $^{(d)}T$) through type-I seesaw Jenkins, Manohar, 2008
  - ➡ exact TBM mixing

    $\sin\theta_{13} = 0 \;\Rightarrow\; J_{CP}^{lep} \propto \sin\theta_{13} = 0$

    CP violation through Majorana phases: $\alpha_{21},\ \alpha_{31}$
  - ➡ no leptogenesis as $Im(y_D y_D^\dagger) = 0$
  - ➡ true even when flavor effects included
  - corrections to TBM pattern due to high dim operators

    small symmetry breaking parameter $\eta \ll 1$:

    $$\sin\theta_{13} \sim \eta \sim 10^{-2},\ \epsilon \sim 10^{-6} \text{ can be generated}$$
- type-II seesaw contribution in S_3 Mohapatra, Yu, 2006
  - exact TBM limit: $\varepsilon_2^{II} \simeq -\frac{3}{8\pi}\frac{m_1 M_2 \sin\varphi_1}{v^2 \sin^2\beta}$ φ_1 : one of the Majorana phases

Quantum Boltzmann Equations

Buchmuller, Fredenhagen, 2000;
Simone, Riotto 2007;
Lindner, Muller 2007

- Classical vs Quantum Boltzmann equations:
  - collision terms: involving quantum interference
  - time evolution: quantum mechanical treatment

- Classical Boltzmann equations:

  scattering independent from previous one

$$\frac{\partial n_{N_1}}{\partial t} = -\langle \Gamma_{N_1} \rangle \left(n_{N_1} - n_{N_1}^{\rm eq} \right),$$

$$\langle \Gamma_{N_1} \rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{f_{N_1}^{\rm eq}}{n_{N_1}^{\rm eq}} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{|\mathcal{M}(N_1 \to \ell H)|^2}{2\omega_\ell 2\omega_H \omega_{N_1}} (2\pi)\delta\left(\omega_{N_1} - \omega_\ell - \omega_H\right)$$

Quantum Boltzmann Equations

- Quantum Boltzmann equations:

Schwinger, 1961; Mahanthappa, 1962; Bakshi, Mahanthappa, 1963; Keldysh, 1965

  - Closed-Time-Path (CTP) formulation for non-equilibrium QFT
  - involve time integration for scattering terms

➡ "memory effects": time-dependent CP asymmetry

$$\frac{\partial n_{N_1}}{\partial t} = -\langle \Gamma_{N_1}(t) \rangle n_{N_1} + \langle \widetilde{\Gamma}_{N_1}(t) \rangle n_{N_1}^{\mathrm{eq}},$$

$$\langle \Gamma_{N_1}(t) \rangle = \int_0^t dt_z \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{f_{N_1}^{\mathrm{eq}}}{n_{N_1}^{\mathrm{eq}}} \Gamma_{N_1}(t),$$

$$\Gamma_{N_1}(t) = 2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{|\mathcal{M}(N_1 \to \ell H)|^2}{2\omega_\ell 2\omega_H \omega_{N_1}} \cos\left[\left(\omega_{N_1} - \omega_\ell - \omega_H \right) (t - t_z) \right]$$

Quantum Boltzmann Equations

- time scale of Kernel << relaxation time scale ~ $1/\Gamma_{N1}$

  Classical Boltzmann eqs ≈ Quantum Boltzmann equation

- In resonant leptogenesis: $\Delta M = (M_2 - M_1) \sim \Gamma_{N2}$

  Kernel time scale ~ $1/\Delta M > 1/\Gamma_{N1}$ possible

  ⇒ quantum Boltzmann equations important!!

Conclusions

- Leptogenesis: promising mechanism for BAU
- connection between leptogenesis & low energy CPV processes generally does not exist in a model independent way
  - statement weakened when flavor effects included
- models for neutrino mass: reduced number of parameters, allowing connection
  - 2x3 seesaw
  - models with SCPV: single source for all CPV
  - TBM mixing pattern compatible with leptogenesis, if
    - higher order corrections included; or
    - type-II seesaw
- Quantum Boltzmann equations?