Exorcising self acceleration

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Based on work with Ruth Gregory and Christos Charmousis

Outline of the talk:

- The dark energy problem
- Modified gravity and “self acceleration”
- The CGP model
- Linearised vacuum perturbations and asymptotic stability
- Cosmological behaviour

The Dark Energy Problem

Gravity is well described by four-dimensional General Relativity

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi G T_{ab}$$

on the following scale

$$0.1\ mm \lesssim r \lesssim 10^{26}\ cm$$

We have also observed that the universe is currently accelerating!

Can we account for this acceleration in 4D GR?

Yes, but we need to include some exotic contribution to T_{ab}
eg: a cosmological constant Λ

To agree with observation, we need $\Lambda \sim 10^{-12}(eV)^4$

But, what is the natural scale for Λ?

For a field theory cut-off at the Planck scale, m_{pl},
we would expect that $\Lambda \sim m_{pl}^4$.

This is 10^{120} times the observed value!!!!!!

Perhaps cosmic speed up is actually a sign of new gravitational physics kicking in at large distances

So instead of focussing on what to put in the RHS of Einstein's Equation, perhaps we should think about modifying the LHS?

Any theory of modified gravity must pass each of the following tests

1. Naturalness

2. Agreement with experiment

3. Theoretical consistency

Eg:

Brans-Dicke gravity: 1 is OK, 2 fails unless $\omega_{BD} > 40000$, and 3 is OK

DGP model: 1 is OK, 2 is debatable, and 3 definitely fails!

Modified gravity and self acceleration

Suppose we have no vacuum energy ($\rho_{vac} = 0$), then in 4D GR, the Friedmann equation is given by:

$$\rho = 6m_{pl}^2 H^2$$

In a *modified* theory of gravity this is replaced by a *modified* Friedmann equation:

$$\rho = F(H^2)$$

In 4D GR, the vacuum solution has to be flat ($H = 0$).

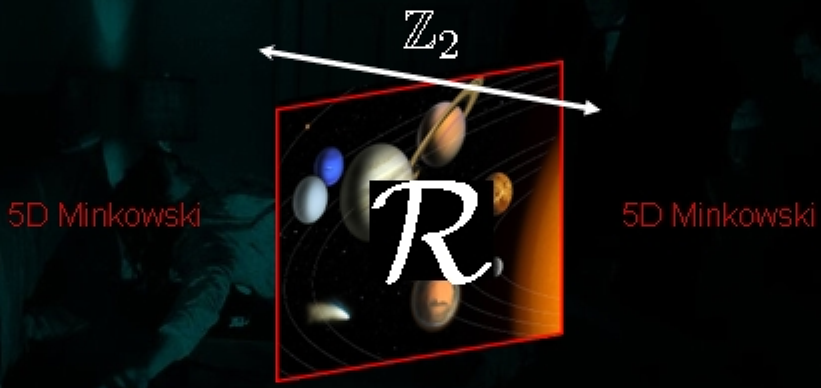
In modified gravity, we can have *self accelerating* vacua, with $H = H_0 > 0$.

This happens if there exists $H_0 > 0$, satisfying $F(H_0) = 0$

COSMIC ACCELERATION IS ENTIRELY DUE TO MODIFICATION OF GRAVITY

The DGP model: an example of self acceleration

We live on a single 3-brane embedded in an infinite empty space



Curvature can be "induced" on the brane by loop corrections to brane stress-energy, or by finite width corrections

The DGP model is described by the following action

$$S = 2M^3 \int_{\mathrm{bulk}} \sqrt{-g}R(g) + 4M^3 \int_{\mathrm{brane}} \sqrt{-q}K + \int_{\mathrm{brane}} \sqrt{-q}\left(m_{pl}^2 \mathcal{R}(q) + \mathcal{L}_{\mathrm{matter}}\right)$$

A theory of gravity, modified in the infra-red, with two cosmological branches:

$$F(H^2) = 6m_{pl}^2 H^2 \pm 12M^3 H$$

The minus branch is ***self accelerating*** since there exists

$$H_0 = 2M^3/m_{pl}^2 \text{ with } F(H_0) = 0$$

Remember the 3 tests:

1. Naturalness? ✓

2. Agreement with experiment? ?

3. Theoretical consistency? ✗

Large ... duced ... ature tra ... ravity on the ... ne ... , mimic ...

Linea ... tensor theory so sola ... fail.

But can't really ... earised theory on solar system ... , so…

Perturbations about the self-accelerating solution contain a ghost

Ghost is in the scalar sector

It has negative kinetic energy and if it couples to ordinary fields, will destabilise the solution by continually transferring energy into those fields

See hep-th/0604086 (Gregory, Charmousis, Kaloper, A.P)
hep-th/0512097 (Gorbunov, Koyama, Sibiryakov)

Will self acceleration generically lead to ghosts?

No proof, but I suspect the answer is "Yes"

This is because perturbations about self accelerating solutions will typically have an ultra-light graviton propagating on a dS background.

It is well known that if the mass of the graviton

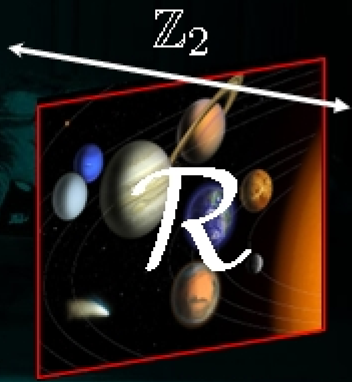
$$0 < m^2 < 2H^2$$

then the scalar (helicity-0) component will be a ghost

A possible alternative to self-acceleration

Vacuum is flat rather than de Sitter but can still get acceleration due to modification of gravity at some stage in cosmic evolution

The CGP model



Our action is given by:

$$S = \sum_{i=1,2} M_i^3 \int_{\mathcal{M}_i} \sqrt{-g}(R(g) - 2\Lambda_i) + 2M_i^3 \int_{\partial\mathcal{M}_i} \sqrt{-q}K^{(i)}$$

$$+ \int_{\text{brane}} \sqrt{-q}\left(m_{pl}^2 \mathcal{R}(q) - \sigma + \mathcal{L}_{\text{matter}}\right)$$

Note that we can have $\Lambda_1 \neq \Lambda_2$ and even $M_1 \neq M_2$

Equations of motion in $\mathcal{M}_i$ are just Einstein equations

$$E_{ab} = G_{ab}(g) + \Lambda_i g_{ab} = 0$$

Brane equations of motion are the continuity equation

$$\Delta q_{\mu\nu} = 0$$

and the Israel equations

$$\Theta_{\mu\nu} = 2(M^3(K_{\mu\nu} - Kq_{\mu\nu})) + m_{pl}^2 \mathcal{G}_{\mu\nu}(q) + \frac{\sigma}{2} q_{\mu\nu} = \frac{1}{2} T_{\mu\nu}$$

Background solution

We take a Poincare slicing of AdS in each region of the bulk

$$ds^2 = a^2(y)(dy^2 + \eta_{\mu\nu} dx^\mu dx^\nu)$$

where

$$a(y) = \frac{1}{1 - \theta k y}, \qquad \theta = \pm 1, \qquad \Lambda = -6k^2$$

Each region corresponds to $0 < y < y_{\max}$, where

$$y_{\max} = \begin{cases} 1/k & \text{for } \theta = 1 \qquad \text{(AdS boundary)} \\ \infty & \text{for } \theta = -1 \qquad \text{(AdS horizon)} \end{cases}$$

The brane is at $y = 0$, and the boundary conditions there give

$$\sigma = -12 \langle M^3 \theta k \rangle$$

This fine tuning guarantees a Minkowski vacuum

Linearised vacuum perturbations and asymptotic stability

As in the DGP model, we need to consider perturbations in order to check the stability of the vacuum solution

We need to eliminate those solutions that contain a perturbative ghost

The bulk metric becomes

$$ds^2 = a^2(y)\,[\,\eta_{ab} + h_{ab}(x,y)]\,dx^a dx^b$$

In $\mathcal{M}_i$, the brane position becomes $y = f_i(x)$

We can decompose perturbation into tensors, vectors and scalar wrt the 4D diffeomorphism group (Poincare).

$$\begin{aligned} h_{\mu\nu} &= h^{\mathrm{TT}}_{\mu\nu} + 2\partial_{(\mu} F_{\nu)} + 2\partial_\mu \partial_\nu E + 2A\,\eta_{\mu\nu} \\ h_{\mu y} &= B_\mu + \partial_\mu B \\ h_{yy} &= 2\phi \end{aligned}$$

F_μ and B_μ are Lorentz-gaugevectors, $h^{\mathrm{TT}}_{\mu\nu}$ is a transverse-tracefree tensor.

Now consider vacuum fluctuations $\delta E_{ab} = 0, \quad \delta \Theta_{\mu\nu} = 0$

Assume tensors, vectors and scalars DO NOT MIX.
Treat EOM independently

Ghosts typically appear in scalar sector, so lets focus on that...

The bulk equations of motion give $\delta E_{ab}^{\mathrm{scalar}} = 0$, where

$$\delta E_{\mu\nu}^{\mathrm{scalar}} = \left[-\partial_\mu \partial_\nu + \eta_{\mu\nu}\partial^2\right](2X + Y) + 3\,\eta_{\mu\nu}\left[\partial_y + 3\frac{a'}{a}\right]\left(X' - \frac{a'}{a}Y\right)$$

$$\delta E_{\mu y}^{\mathrm{scalar}} = -3\partial_\mu\left(X' - \frac{a'}{a}Y\right)$$

$$\delta E_{yy}^{\mathrm{scalar}} = 3\partial^2 X + 12\frac{a'}{a}\left(X' - \frac{a'}{a}Y\right)$$

$$X = A - \frac{a'}{a}(B - E'), \qquad Y = \phi - \frac{[a(B - E')]'}{a}$$

These equations are easily solved on each side of the brane to give

$$X = \frac{U(x)}{a^2}, \qquad Y = -\frac{2U(x)}{a^2}$$

where $\partial^2 U = 0$.

We have 4 scalar degrees of freedom

2x radion: U_1 and U_2, 2x brane bending: f_1 and f_2

and two boundary conditions (continuity of metric +Israel junction conditions)…
..so should be left with two scalars….

Easy to see why <u>two</u> scalars if one introduces regulator branes at

$$y = y_* < y_{\max}$$

Remaining d.o.f are f luctuations in the proper distance between the branes

In Gaussian Normal coordinates, with the brane fixed at $y = 0$, we have

$$h_{\mu y} = h_{yy} = 0, \qquad h_{\mu\nu} = h^{(U)}_{\mu\nu} + h^{(A)}_{\mu\nu}$$

where

$$\begin{aligned} h^{(U)}_{\mu\nu} &= \frac{1}{2k^2}(1 - a^{-4})\partial_\mu\partial_\nu U \\ h^{(A)}_{\mu\nu} &= -\frac{1}{k^2}(1 - a^{-2})\partial_\mu\partial_\nu A + 2A\ \eta_{\mu\nu} \\ A &= U - k^2 f \end{aligned}$$

This mode behaves like a massless tensor. No mixing with tensors if bulk is infinite

The continuity condition, and the Israel equations now require that

$$\Delta A = 0, \qquad \left\langle \frac{M^3\theta}{k}(U - A) + m_{pl}^2 A \right\rangle = 0$$

The f luctuation in proper distance to the regulator brane is given by

$$\delta z_* = \frac{\theta}{k}\left[U a(y_*)^{-2} - A\right]$$

4D Effective Action

We can calculate the 4D effective action by integrating out the region between the branes and the regulators

$$S_{\text{eff}} = -6 \int d^4x \left\langle M^3 \;\; \delta z_* \partial^2 U(x) \right\rangle$$

We can remove the regulator branes by taking $y_* \to y_{\max}$, then

$$\delta z_* \to \begin{cases} -A/k & \text{whenever } \theta k > 0 \\ \infty & \text{whenever } \theta k \leq 0 \end{cases}$$

This implies that when $\theta k \leq 0$, mode U is non-normalisable.

AdS boundary in the bulk → Normalisable radion
AdS horizon in the bulk → No normalisable radion
Minkowksi bulk → No normalisable radion

Having removed the regulators, and imposed the boundary conditions, we find the following:

1. If we have the AdS horizon in *both* sides of the bulk, then all scalars decouple.

   (this has a finite volume bulk, so no IR modification of gravity-less interesting)

2. If we have a Minkowski bulk on *either* side, then all scalars decouple.

3. Otherwise, (only) one scalar d.o.f survives and the effective action is

$$S_{\text{eff}} = 6\left[m_{pl}^2 - \left\langle \frac{M^3\theta}{k} \right\rangle\right] \int d^4x (\partial A)^2$$

To avoid a ghost, we need $\chi = m_{pl}^2 - \left\langle \frac{M^3\theta}{k} \right\rangle \leq 0$

χ measures the strength of the coupling to the trace of the energy momentum tensor, since schematically we have

$$-\chi\partial^2 A \sim T$$

$\chi \to \infty$ limit: the radion completely decouples.

This corresponds to the case where $k_1 k_2 = 0$
so at least one side of the bulk is Minkowski.

$\chi \to 0$ limit: the strong coupling or conformal limit

Cannot support non-conformal matter source $(T \neq 0)$
to linear order in perturbation theory

Linearised theory breaks down due to strong coupling,
geometry responds non-linearly!

Can think of conformal limit as a limit of enhanced symmetry.

Linearised field equations in bulk and on brane become invariant under

$$h_{ab} \to h_{ab} + h^{(f)}_{ab}$$

where

$$h^{(f)}_{\mu\nu} = (1 - a^{-2})\partial_\mu \partial_\nu f - 2k^2 f \ \eta_{\mu\nu}, \qquad h^{(f)}_{\mu y} = h^{(f)}_{yy} = 0$$

pure gauge in the bulk, but not on the brane ... beyond the usual diffeos

Summary

To get IR modification of gravity, need infinite volume bulk,
then we can treat scalars independently

At most one scalar d.o.f survives
This corresponds to fluctuations in the brane's centre of mass motion

Scalar is a ghost if $\chi > 0$, and then vacuum solution is unstable

Scalar *decouples* if either side of the bulk is Minkowski ($\chi \to \infty$)

Scalar is eliminated by an extra symmetry in conformal limit (χ=0)

Cosmological behaviour

As is well known in braneworld gravity, cosmological branes correspond to branes moving through the bulk.

Exact solutions can be easily found by gluing bulk solutions together across the moving brane

Without going into too much detail….

Put some matter on the brane, with equation of state $p = \omega \rho$, satisfying the SEC,

$$\rho \geq 0, \qquad \rho + 3p \geq 0$$

We get conservation of energy, and a *modified* Friedmann equation

$$\dot{\rho} = -3H(\rho + p), \qquad \rho = F(H^2)$$

In our model

$$F(H^2) = 6m_{pl}^2 H^2 - 12 \left\langle M^3 \theta \left(\sqrt{H^2 + k^2} - k \right) \right\rangle$$

Note that $F(0) = 0$, as expected since Minkowski space should be a vacuum solution

Want a consistent cosmology that approaches the *Minkowski* vacuum as $\rho \to 0$

We therefore need to assume

$$F(H^2) > 0, \quad F'(H^2) \geq 0 \qquad \text{for } 0 < H < H_{\max}$$

Cosmology must also pass through a standard $4D$ phase.

Guaranteed if $H_{\max}$ is large enough, as then $\rho \sim 6m_{pl}^2 H^2$ for $H \sim H_{\max}$

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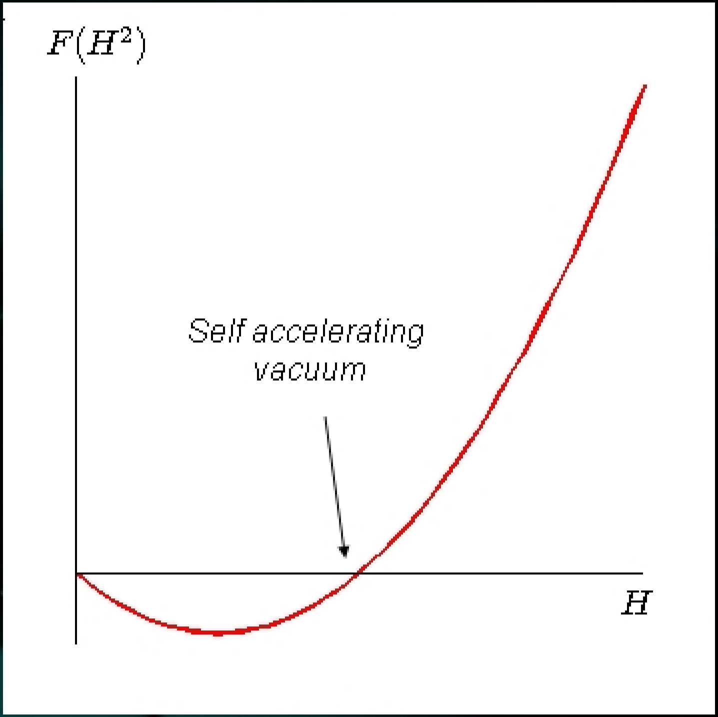

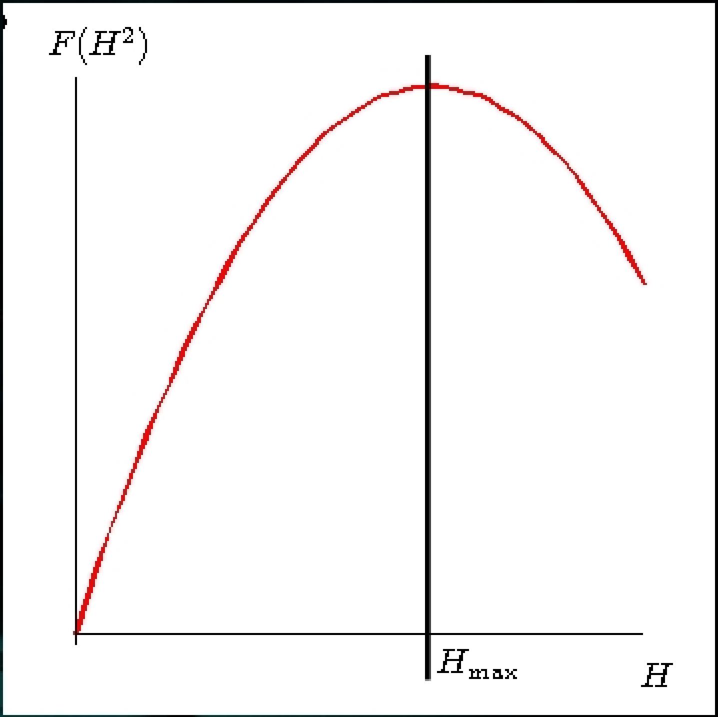
$F(H^2)$
H_{max}
H

Note that we certainly need $F'(0) = 6\chi \geq 0$

But remember if $\chi > 0$, we have a radion ghost!!!!

A physically safe radion corresponds to an unphysical cosmology!

(and vice versa)

This surprising result can be traced to the absence of a tensor zero mode

No tensor zero mode, so large distance/late time behaviour dominated by radion (if it exists)

We can use the Gauss-Codazzi equations to find the projection of the Einstein tensor on the brane. After linearising we obtain…..

$$\delta\left(\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}q_{\mu\nu}\right) = \frac{1}{2\chi}T_{\mu\nu}$$

This EOM would follow from an effective action of form $S_{\text{eff}} = \int \sqrt{-q}\,(\chi\mathcal{R} + L_{\text{matter}})$

so χ is really the effective 4D Planck scale.

For a consistent $4D$ cosmology, need $\chi > 0$.

But in the absence of a tensor zero mode, the conformal mode has nothing to mix with in the far infra-red, giving rise to a dangerous "conformal ghost".

Therefore, the only cosmological solutions we may consider are:

Those for which the radion *decouples*
ie if either side of the bulk is Minkowski

or

Those for which radion is eliminated by an extra symmetry
ie the conformal limit , $\chi=0$

What do we need for acceleration?

Strictly speaking we need the *effective* equation of state parameter

$$\omega_{eff} < -\frac{1}{3}$$

although from observation we would like to push it closer to -1.

The effective equation of state parameter is given by

$$\omega_{eff} = -1 + \mathcal{C}(H^2)(1+\omega), \qquad \mathcal{C}(H^2) = \frac{F(H^2)}{H^2 F'(H^2)}$$

Assuming that the matter satisfies the SEC $(1 + 3\omega \geq 0)$,

then acceleration can be achieved if $\mathcal{C}(H^2)$ falls below one.

Acceleration with a decoupled radion

Assume we have a Minkowksi bulk on one side,
and an AdS bulk on the other

If this includes AdS horizon,
include AdS boundary
can't get acceleration

In this case

$$F(H^2) = 6m_{pl}^2 H^2 - 6M_1^3\left(\sqrt{H^2 + k_1^2} - k_1\right) + 6M_2^3 H$$

If $M_1 < M_2$, then $\mathcal{C}(H^2) \geq 1$ for all $H > 0$

NO ACCELERATION

Low Planck scale means graviton can propagate easily away from AdS boundary, towards brane

If $M_1 > M_2$, then *can* get $\mathcal{C}(H^2) < 1$ over some range of H

COSMIC ACCELERATION!!!

For large H,

$$\mathcal{C} = 1 - \left[\frac{M_1^3 - M_2^3}{m_{\rm pl}{}^2}\right] H^{-1} + \mathcal{O}(H^{-2})$$

$F(H^2)$ increases monotonically if

$$m_{pl}^2 \geq \frac{M_1^3}{2k_1}\left(1 - \frac{M_2^2}{M_1^2}\right)^{\frac{3}{2}}$$

If this is not satisfied, can't get acceleration

If this is satisfied, *can* get acceleration. Get most when close to equality

Bound is a necessary, but not sufficient condition for acceleration

Numerical checks suggest we can get $C(H^2)$ down to about 0.43

For pressureless matter $(\omega = 0)$, this gives $\omega_{eff} \sim -0.57$

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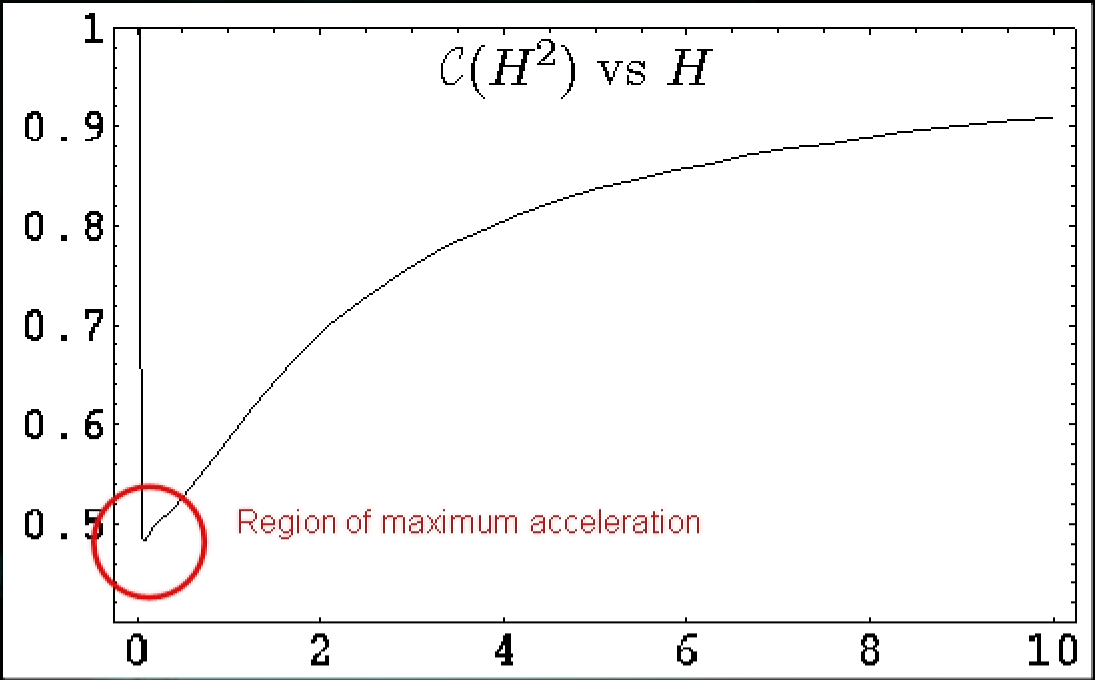
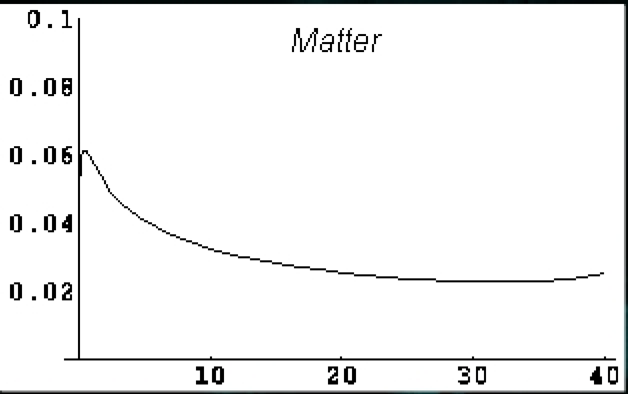
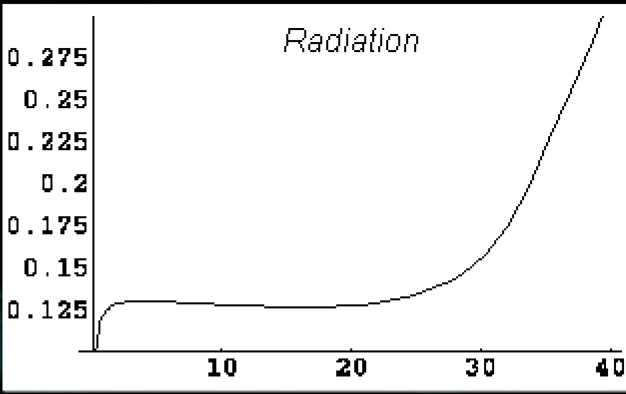


$C(H^2)$ vs H
Region of maximum acceleration
1
0.9
0.8
0.7
0.6
0.5
0
2
4
6
8
10

Evolution of comoving Hubble radius in time





Significant period of acceleration

More like "coasting"

Model naturally explains why acceleration can only occur after matter domination

Acceleration in the conformal limit

$$\chi = m_{pl}^2 - \left\langle \frac{M^3\theta}{k} \right\rangle = 0$$

Greatest acceleration occurs at late times (small H). Taylor expanding $F(H^2)$ near $H = 0$, we get

$$\rho = H^2 \cancel{F'}(0) + \frac{H^4}{2} F''(0) + \frac{H^6}{6} F'''(0) + \mathcal{O}\left(H^8\right)$$

where

$$F'(0) = 6\chi, \qquad F''(0) = 3\left\langle \frac{M^3\theta}{k^3} \right\rangle, \qquad F'''(0) = -\frac{9}{2}\left\langle \frac{M^3\theta}{k^5} \right\rangle$$

For small H, $\rho \propto H^4$ and so $a(t) \sim t^{\frac{4}{3(1+\omega)}}$ and $\mathcal{C}(H^2) \sim \frac{1}{2} + \mathcal{O}(H^2)$

Can fine tune so that $\rho \propto H^6$. Then $a(t) \sim t^{\frac{2}{1+\omega}}$ and $\mathcal{C}(H^2) \sim \frac{1}{3} + \mathcal{O}(H^2)$

Summary

Can get power law acceleration at late times due to modification of gravity

Acceleration only occurs when matter is present. Vacuum brane is Minkowski!

This is achieved *without* introducing ghosts, unlike self accelerating models

Ghost are avoided because radion either decouples,
or is eliminated by an extra symmetry kicking in.

Can increase amount of acceleration by introducing
Gauss-Bonnet correction in the bulk, but lose naturalness