A string dual perspective on quark-gluon plasmas of QCD-like theories

José D. Edelstein

U. Santiago de Compostela
& CECS, Valdivia

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Based on joint work with:

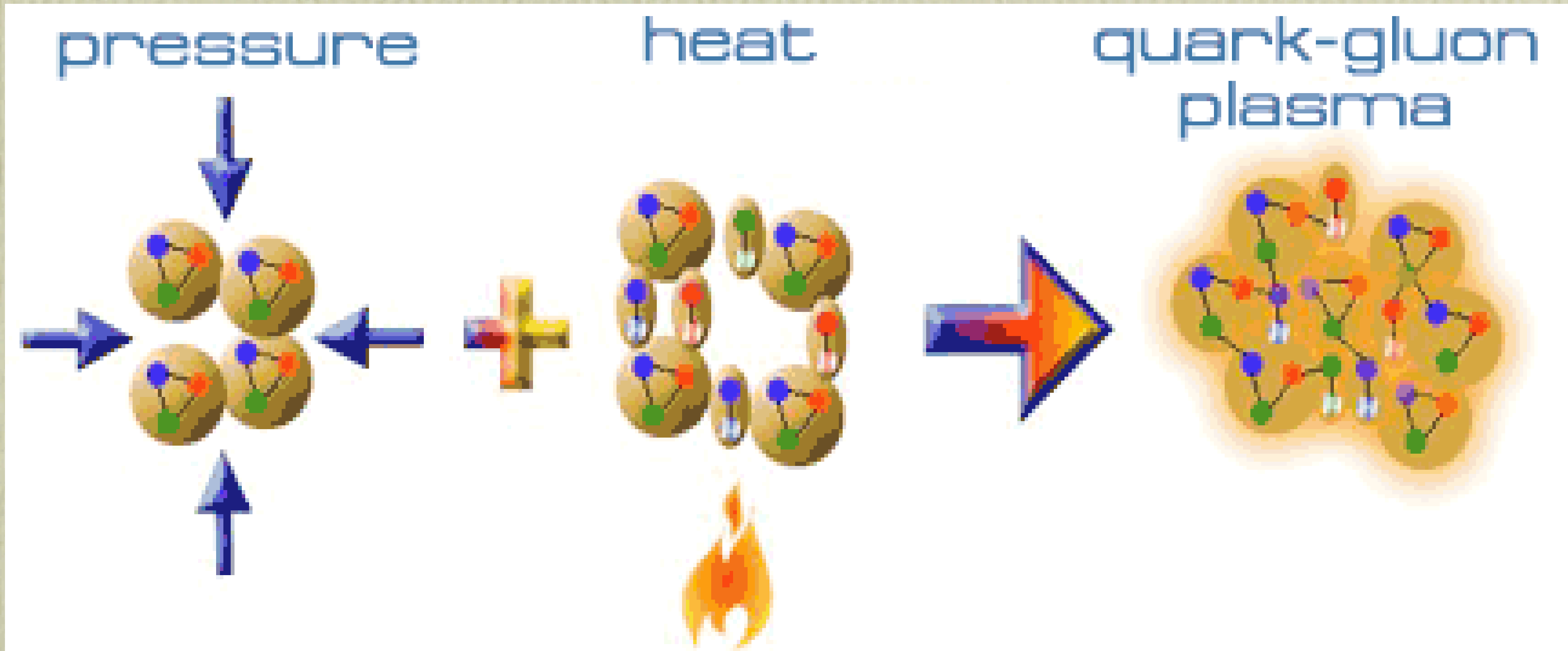
- **Néstor Armesto and Javier Mas (Santiago de Compostela), JHEP 09 (2006) 039, *hep-ph/0606245***
- **Gaetano Bertoldi (Swansea), Francesco Bigazzi (Brussels) and Aldo Cotrone (Barcelona), *hep-th/0702225***

Plan of the talk

- Introduction: some features of the (strongly coupled) Quark-Gluon Plasma
- Selected experimental signals: elliptic flow and jet quenching
- AdS/CFT at finite temperature and sQGP
- The jet quenching parameter
- Unquenched flavor: non-critical QGP
- Unquenched flavor: wrapped fivebranes QGP
- Discussion and concluding remarks

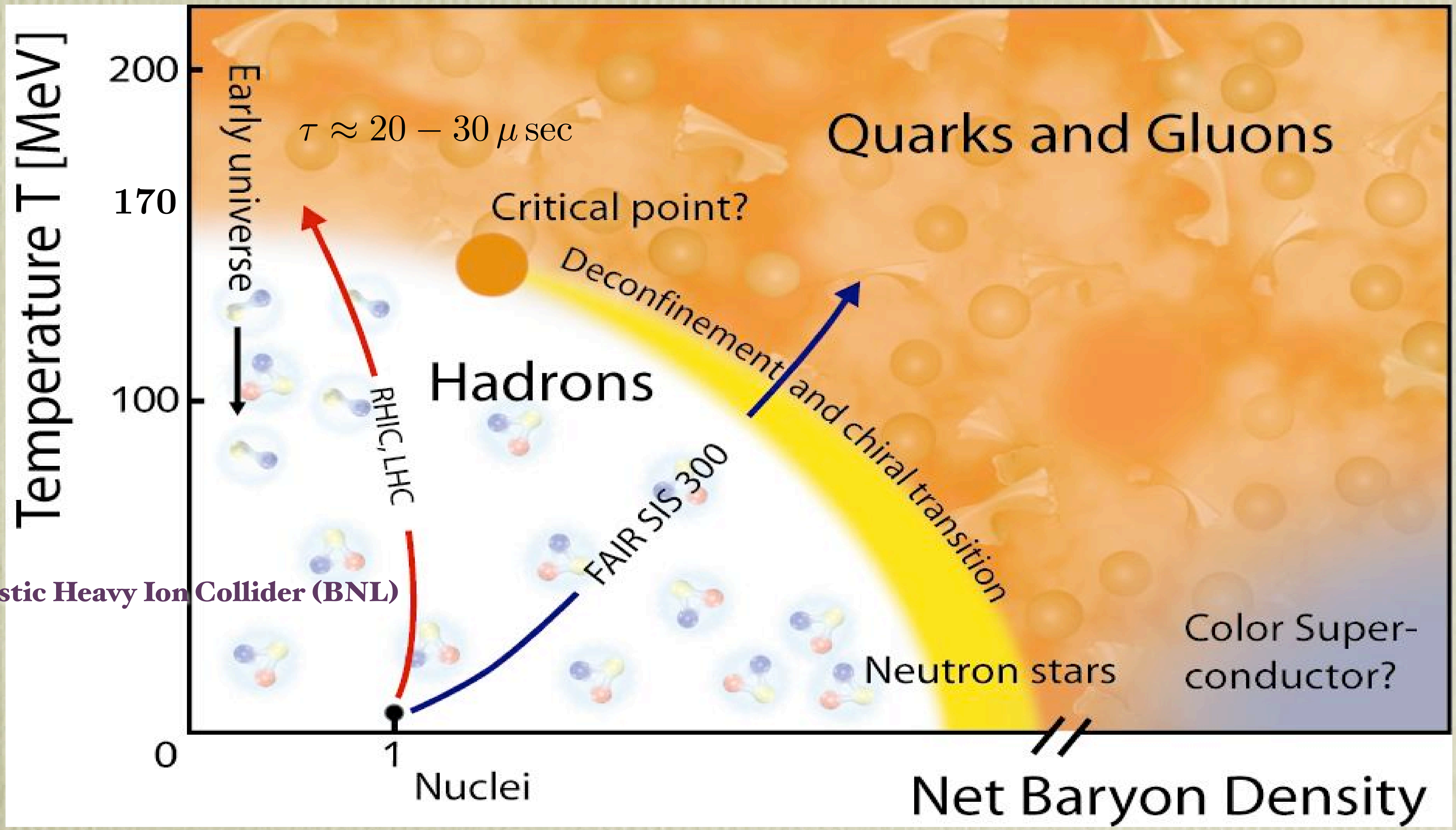
What is the Quark-Gluon Plasma?

It is a phase of QCD conjectured to exist at high temperature and density



$$\mathbf{T}_c \approx 10^{12}\ {}^o\mathbf{K}$$

The QCD Phase Diagram

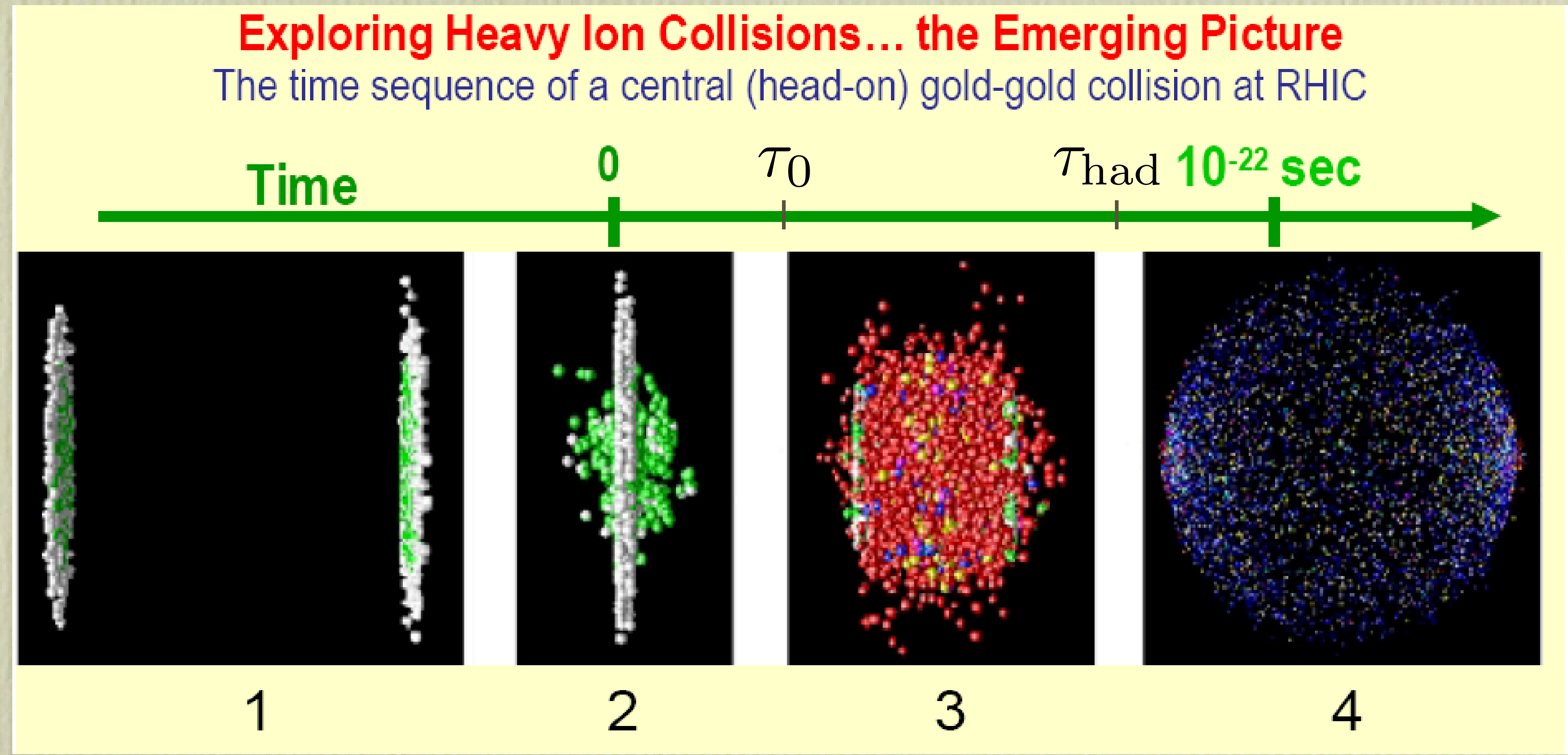


Relativistic Heavy Ion Collider (BNL)

Figure obtained from the Facility for Antiproton and Ion Research

How is the Quark-Gluon Plasma produced?

Head-on Au+Au collision at (center of mass) energies of $\sqrt{s} \simeq 200\,\mathrm{GeV/nucleon}$ **at RHIC**



Fast thermalization at $\tau_0 \leq 1$ fm

pQCD predicts (parton-parton collisions) $\tau_0 \sim 3$ fm

Heinz, 2002

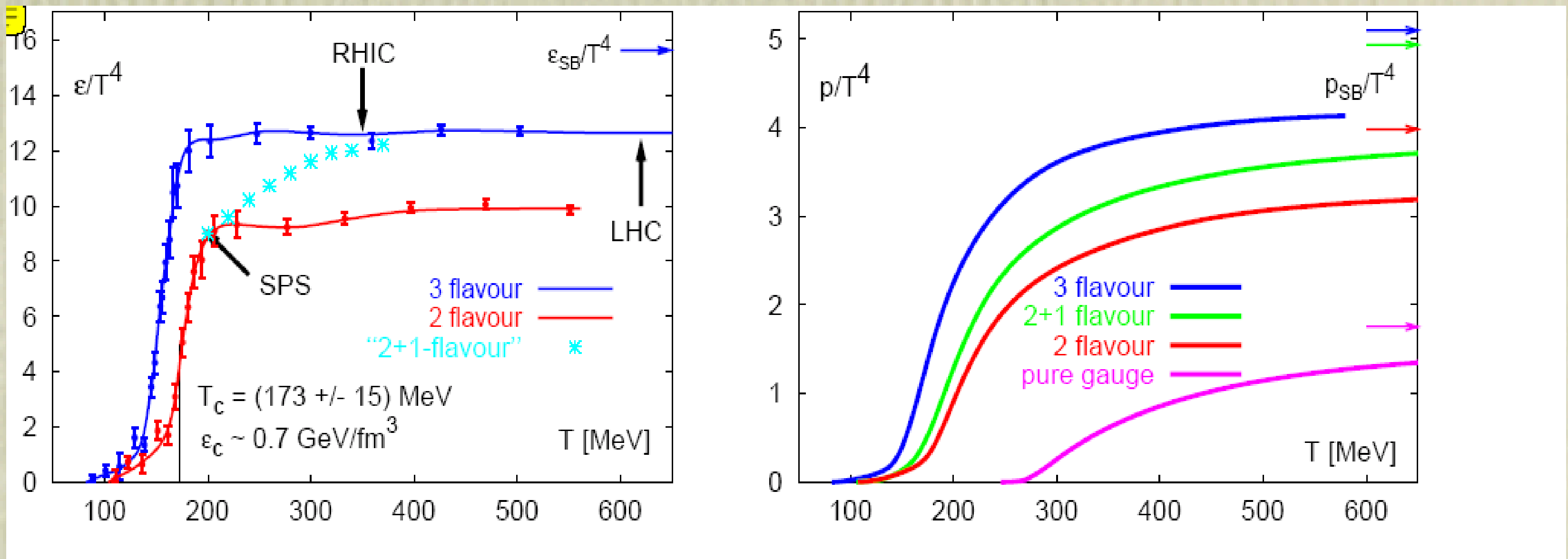
Conformal behavior and hadronization

Lattice data supports an equation of state

$$\epsilon = 3p$$

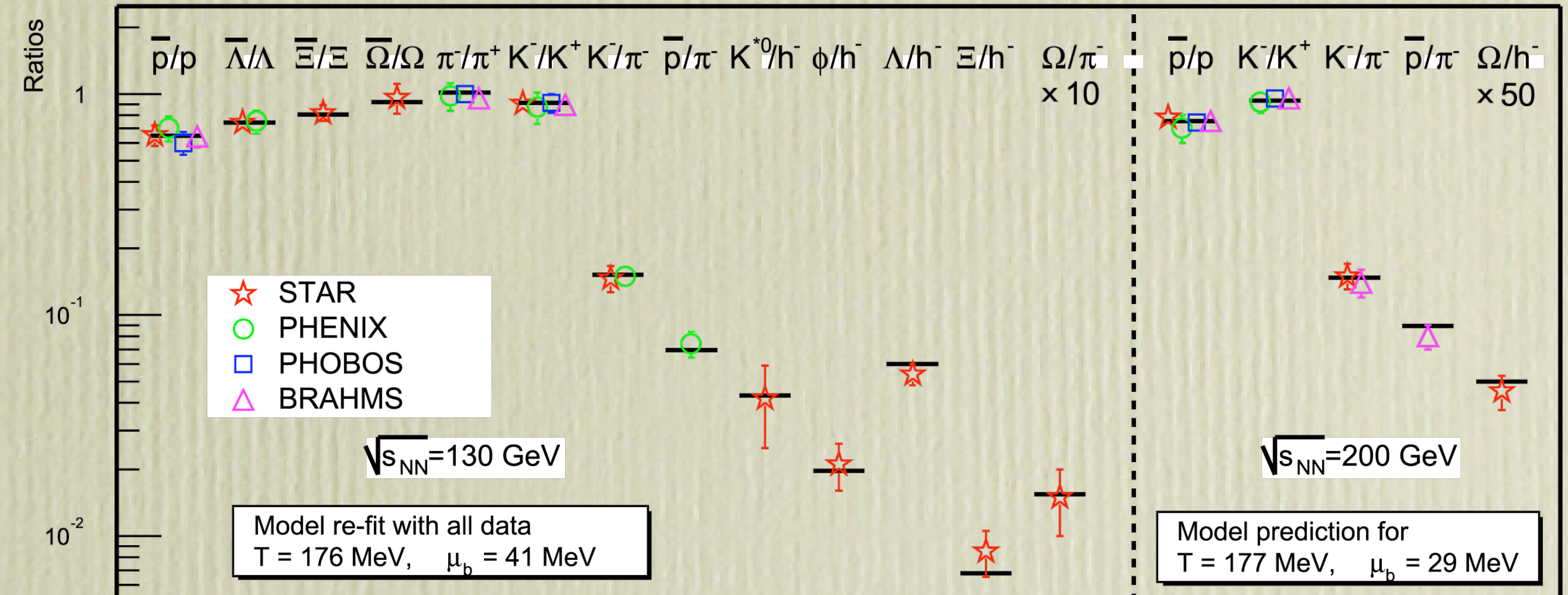
for $\mathrm{T} \geq 2\ \mathrm{T}_c$

Karsch-Laermann, 2003



Relative abundances of detected particles provides

$$\mathrm{T}_{\mathrm{freezeout}} \approx 176\ \mathrm{MeV}$$



Braun-Munzinger et al., PLB 518 (2001) 41 D. Magestro (updated July 22, 2002)

Some features of QGP: I. Elliptic Flow

In off-center collisions, the heated overlap region is elongated. Collective interactions produce pressure gradients that result in an anisotropy of produced hadrons w.r.t. the reaction plane:

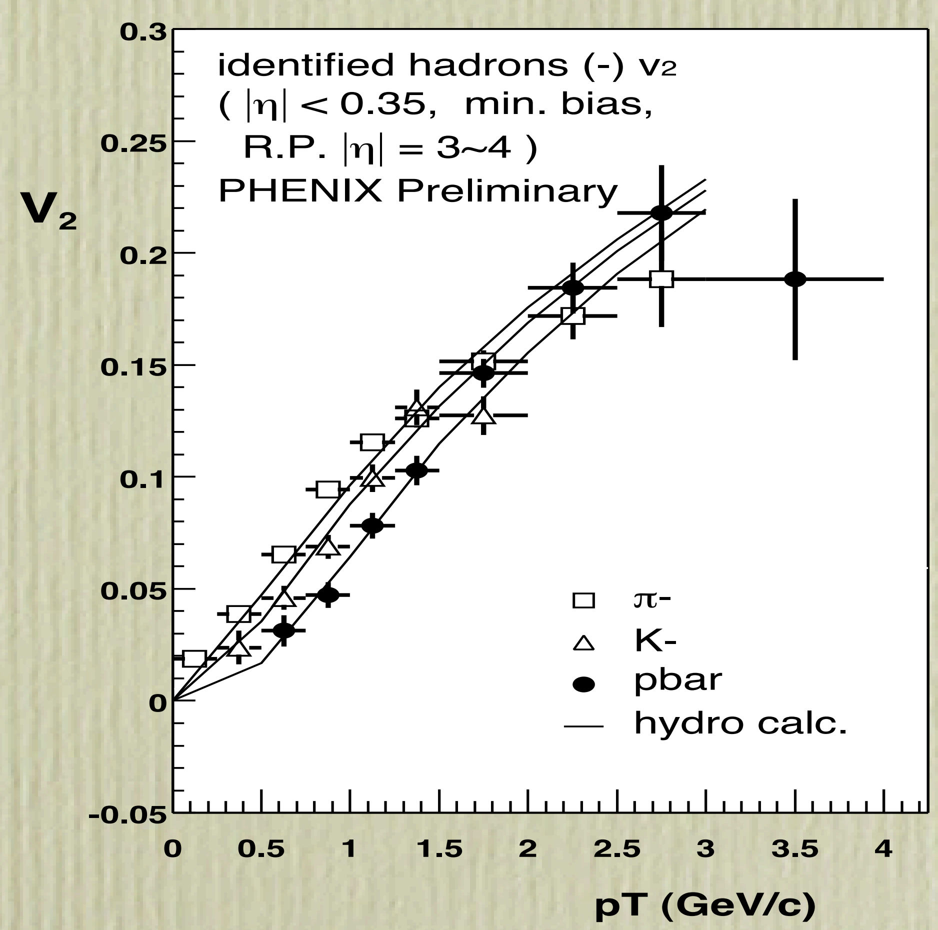
Animation by J. Mitchell (BNL)

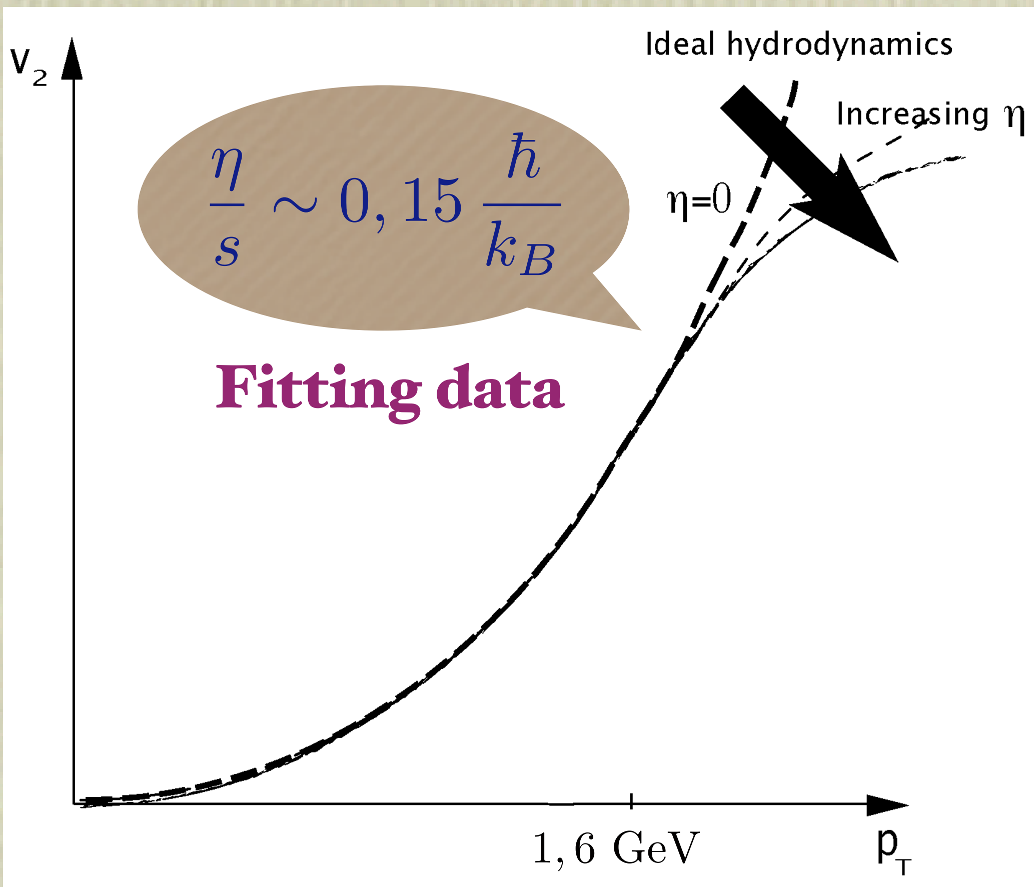
The fireball expands (in thermal and hydrodynamical equilibrium) under its own pressure and cool while expanding. It is much larger than a single gold nucleus with a lifetime of order 10 fm.

Elliptic Flow and Shear Viscosity

The elliptic Flow is characterized by the anisotropy parameter $v_2 = v_2(p_T, b, A)$

$$\frac{dn}{d\phi} \propto 1 + \underline{v_2} \cos 2\phi$$





More than a gas of quarks and gluons, it seems an almost perfect liquid!

Some features of QGP: II. Jet Quenching

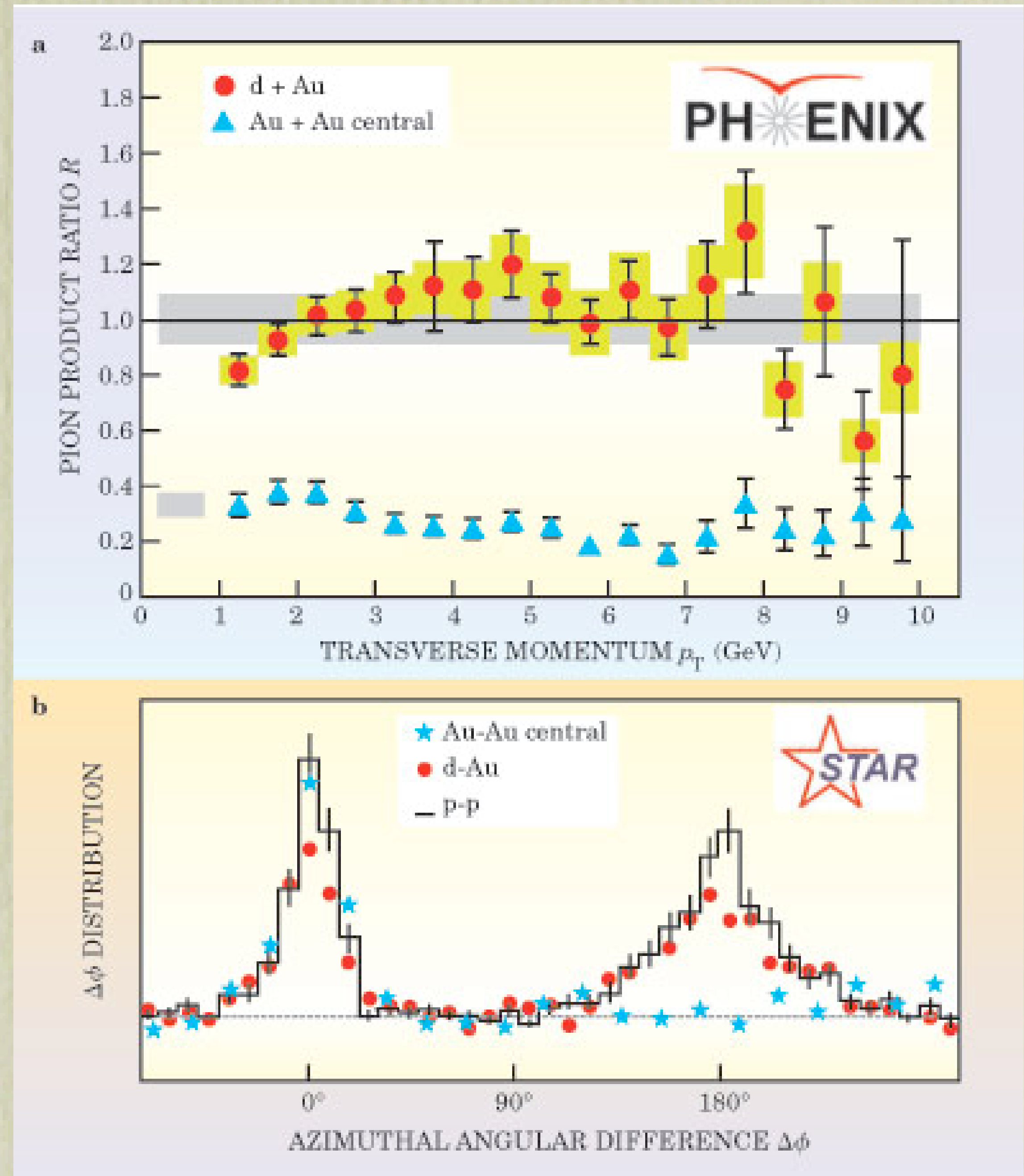
Hard scattering is seen for the first time in nuclear collisions. There is a supression in the observed back-to-back high p_t jets in Au+Au vs. p+p collisions

- **R is the ratio of number of jets to those seen in p+p collisions (scaled to account for the number of participating nucleons)**

- **Departure from R=1 indicates that partons kicked up by hard scatterings are slowed by the hot medium**

- **Correlating azimuthal angles among high p_t particles produced in the same event. The peak at $\Delta\phi=0$ indicates partners in the same jet as the trigger**

- **The recoil peak at 180º indicating back-to-back jets in p+p and d+Au collisions, is absent/displaced in Au+Au collision**



Some features of QGP: II. Jet Quenching

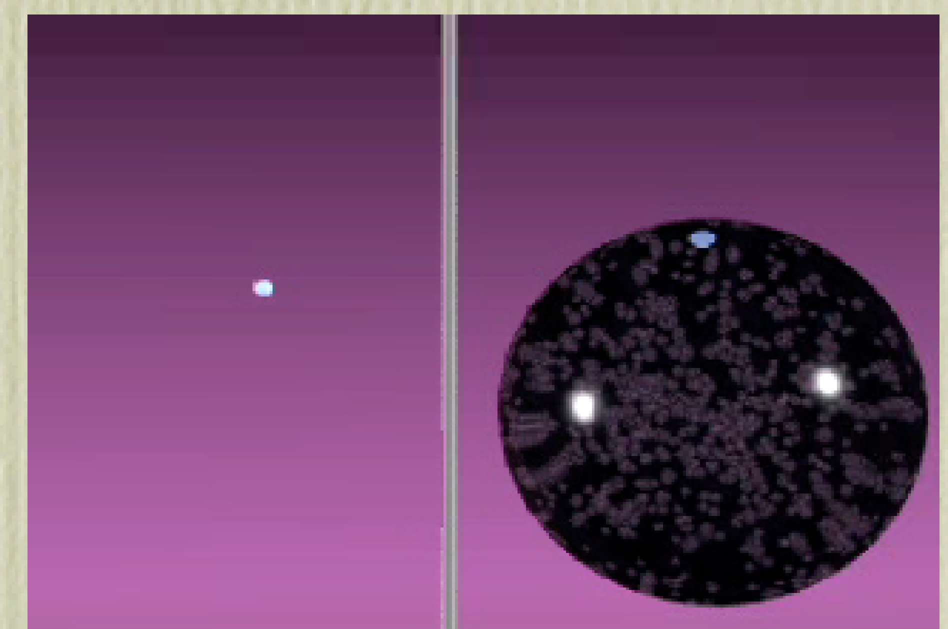
The observed deficit of high-energy jets seems to be the result of a slowing down, damping or quenching of the most energetic partons as they propagate through the QGP

Bjorken, 1983

The rate of energy loss should be spectacular: several GeV per fm instead of a few MeV per centimeter (cold nuclear matter). This can be seen as follows:

Animation by J. Mitchell (BNL)

Back-to-back collision in vacuum



Back-to-back collision in a hot medium

$$\hat{q} = (10 \pm 5)\,\frac{\mathrm{GeV}^2}{\mathrm{fm}}$$

Fitting data

The energy loss of a hard parton in QCD is parameterized as follows:

The average squared transverse momentum transferred to the hard parton, per mean free path, is a transport coefficient called $\hat{q}$

A strongly interacting QGP?

There are further interesting features in the physics of QGP:

- **Diffusion constants**
- J/ψ **and other heavy meson's melting**
- **Thermal spectral functions**
- **Further transport properties**

If we assume a weakly interacting QGP (λ<<1), and use perturbative QCD, we get:

$$\frac{\eta}{s} \approx \frac{1}{\lambda^2 \, \log\frac{1}{\lambda}} \frac{\hbar}{k_B} \gg 1 \qquad\qquad \hat{q} \approx 1 \, \frac{\mathrm{GeV}^2}{\mathrm{fm}}$$

but we have seen earlier that

$$\frac{\eta}{s} \approx 0.15 \, \frac{\hbar}{k_B} \qquad\qquad \hat{q} \approx (10 \pm 5) \, \frac{\mathrm{GeV}^2}{\mathrm{fm}}$$

This is a significant mismatch! It suggests a strongly interacting Quark-Gluon Plasma: sQGP

How can we deal with a sQGP?

Lattice? Well, this would cover the window given by small N_c and large λ

However, it is not suitable to study real-time dynamics of a strongly interacting QCD plasma

Besides, hydrodynamics with $\eta \neq 0$ is also very hard

What about AdS/CFT?

QCD		N=4 SYM
Confinement, Stable particles, Scattering	T=0 ⟷ No relation	Conformal, No particles, No S-matrix
Non-Abelian plasma, (gluons + fundamental matter), No confinement, Debye screening, Finite spatial correlation length	T≠0 ⟷ Very similar!	Non-Abelian plasma (gluons + adjoint matter), No confinement , Debye screening, Finite spatial correlation length

The only available tool: AdS/CFT?!

It would be the perfect tool but:

- We only know how to compute when $N_c \to \infty$ and $\lambda \to \infty$ (not terribly bad)
- It is hard to deal with dynamical quarks beyond the quenched approximation, $N_f \ll N_c$
- The more tractable case is the unphysical N=4 super Yang-Mills theory
- In general, it is hard to get rid of supersymmetry, conformal invariance and, roughly speaking, to pick the right supergravity dual of QCD

Nevertheless:

- Finite temperature already breaks supersymmetry

AdS/CFT at finite temperature → black holes (black branes)!

$T_H \,\&\, S_{BH}$ ⟷ $T \,\&\, S$

- There might be universal features that we can learn about

The gravity dual of finite T gauge theories

Soon after Maldacena, it was proposed that finite T implies a string background

$$ds^2 = H^{-1/2}(r)\left[-f(r)dt^2 + d\vec{x}^2\right] + H^{1/2}(r)\left[f^{-1}(r)dr^2 + r^2 d\Omega_5^2\right]$$

$$H(r) = 1 + \frac{L^4}{r^4} \qquad f(r) = 1 - \frac{r_H^4}{r^4} \qquad r_H < r \ll L$$ **black 3-brane**

Compactification on S⁵ leads to an AdS black hole in 5d

It is not hard to compute:

$$T_H = \frac{r_H}{\pi L^2} \ll \frac{1}{L} \qquad S_{BH} = \frac{3}{4} S_{\rm pSYM}$$

Is there something wrong with the latter? NO, it tells us that

$$S(\lambda) = f(\lambda)\, S_{\rm pSYM}$$ **such that** $$f(0) = 1 \;\text{ and }\; f(\infty) = \frac{3}{4}$$

Indeed, finite coupling corrections suggest a smooth interpolation

Shear viscosity revisited

Kubo formulas allow us to calculate transport coefficientes from microscopic models:

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x\, e^{i\omega t} \langle [T_{xy}(t,x), T_{xy}(0,0)] \rangle$$

Now, this correlator can be computed by means of AdS/CFT:

$$\frac{\eta}{s} = \frac{1}{4\pi} \frac{\hbar}{k_B} \approx 0.08 \frac{\hbar}{k_B}$$

Policastro-Son-Starinets, 2000

compatible with the values measured at RHIC!!!

It is amusing to check that the leading finite `t Hooft coupling correction reads:

$$\frac{\eta}{s} = \left[\frac{1}{4\pi} + \frac{135\,\zeta(3)}{32\pi\, 2^{3/2}} \lambda^{-3/2} \right] \frac{\hbar}{k_B}$$

Buchel-Liu-Starinets, 2004

that seems to smoothly interpolate between the weak/strong coupling results

This strongly supports the use of AdS/CFT to describe RHIC physics

The viscosity bound: a conjecture

This result holds for any gravity dual (no matter the amount of supersymmetry and field content!), at least for the cases worked out so far! Even with chemical potential.

Buchel, 2004

Mas, 2006

Son-Starinets, 2006

It also holds when massless quarks are introduced in the quenched approximation

Mateos-Myers-Thomson, 2006

Conjecture:

For all relativistic quantum field theories at finite temperature,

$$\frac{\eta}{s} \geq \frac{1}{4\pi}\,\frac{\hbar}{k_B}$$

and saturated for gauge/gravity duals!

Kovtun-Son-Starinets, 2004

Multiple soft scattering of a parton in sQGP

In order to study the jet quenching phenomenon, we must first provide an appropriate phenomenological description of the relevant physics

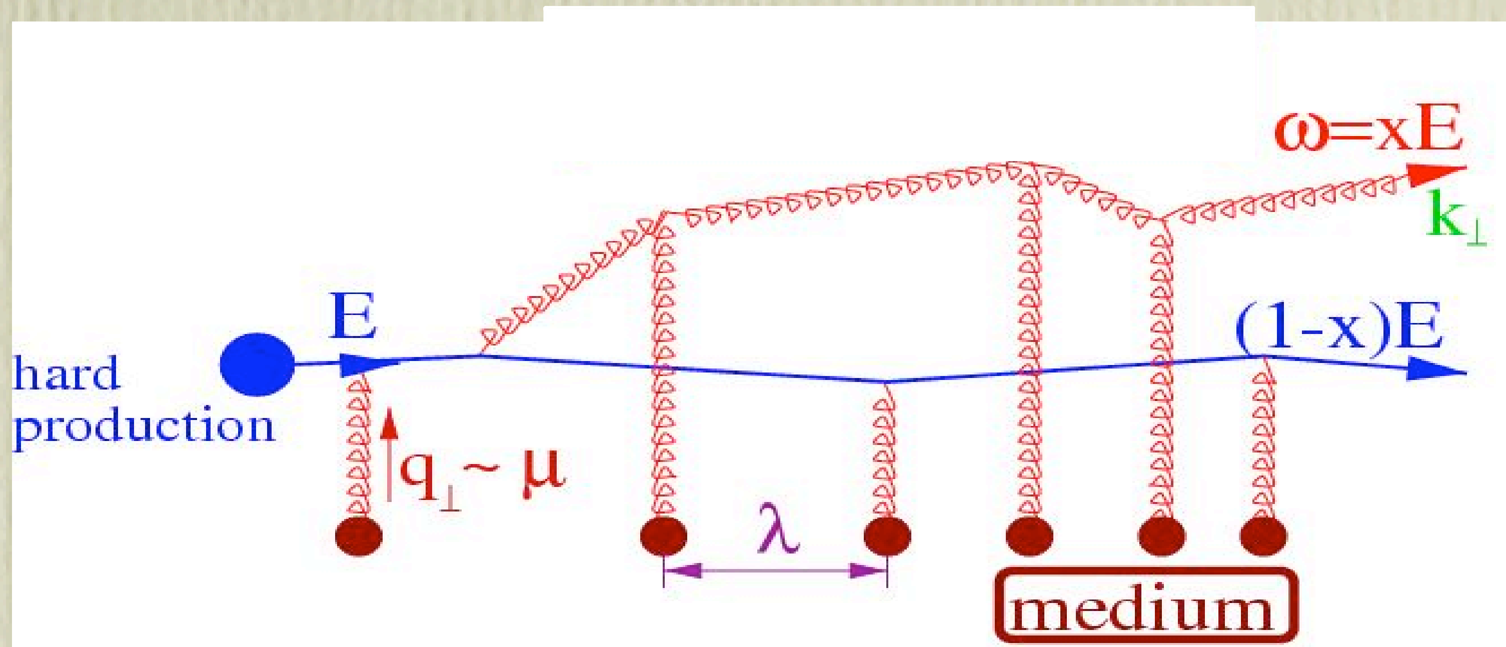
There are several models of radiative energy loss for a parton moving on a medium

Landau-Pomeranchuk, 1953
Migdal, 1956
Zakharov, 1997
Baier-Dokshitzer-Mueller-Peigné-Schill, 1997

We will assume:

- **Almost straight trajectory $\theta \simeq 0$**
- **Transverse Brownian motion**
- **$x \ll 1$**
- **Eikonal approximation**
- **Wave length << λ << Size medium**
- **Dipole approximation**

Nikolaev-Zakharov, 1994



Eikonal approximation: relativistic probes

Wiedemann, 2000

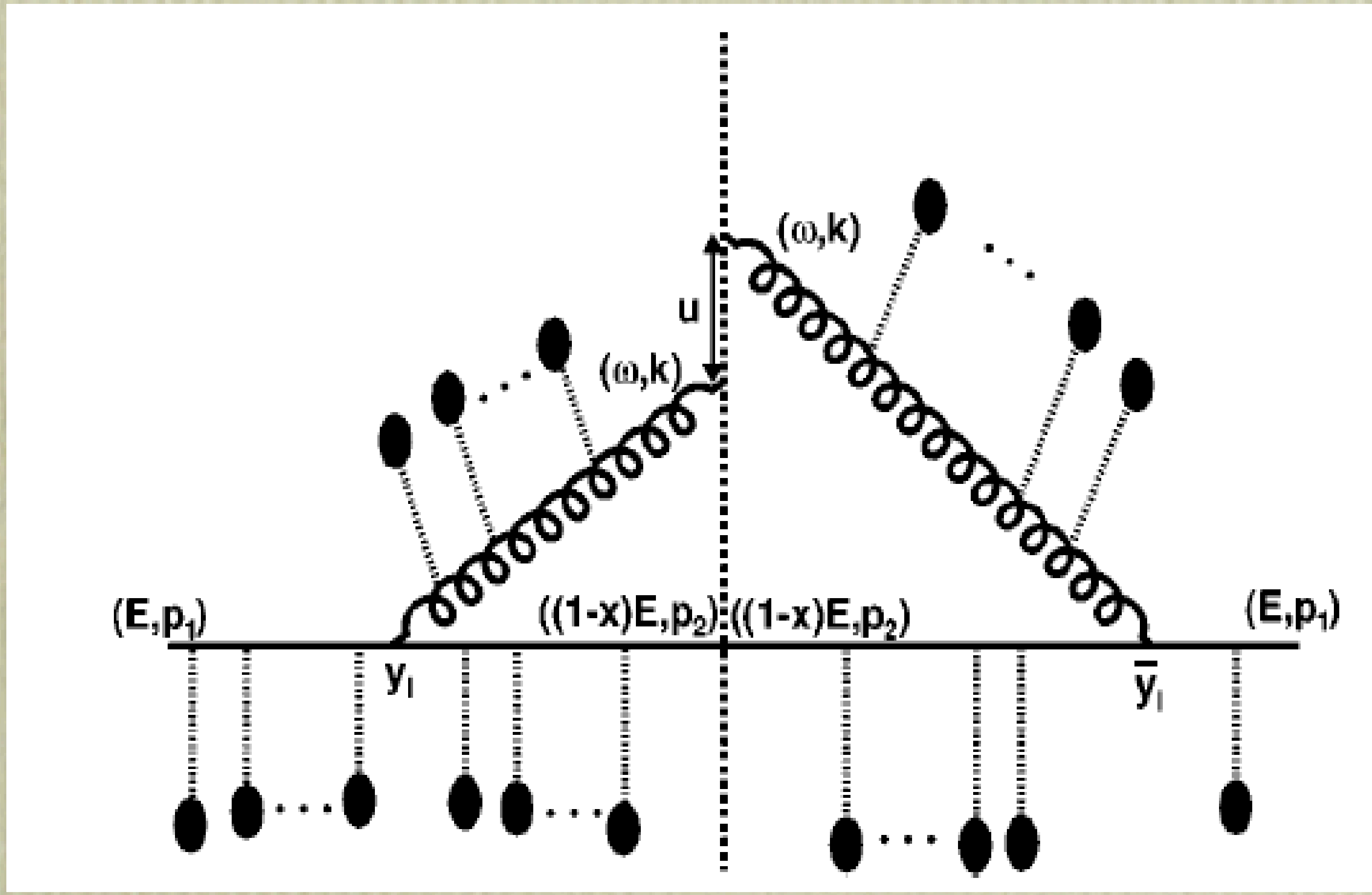
Casimir factor: quarks/gluons

Density of scattering centers

Dipole cross section

$$\omega \frac{dI}{d\omega} = \frac{\alpha_s C_R}{(2\pi)^2 \omega^2} 2\mathrm{Re} \int_{\xi_0}^{\infty} dy_l \int_{y_l}^{\infty} d\bar{y}_l \int d^2\mathbf{u} \int_0^{\chi\omega} d^2\mathbf{k}\, e^{-i\mathbf{k}\cdot\mathbf{u}}\, e^{-\frac{1}{2}\int_{\bar{y}_l}^{\infty} d\xi\, n(\xi)\,\sigma(\mathbf{u})}$$

$$\times \frac{\partial}{\partial \mathbf{y}} \cdot \frac{\partial}{\partial \mathbf{u}} \int_{\mathbf{y}=0}^{\mathbf{u}=\mathbf{r}(\bar{y}_l)} \mathcal{D}\mathbf{r} \exp\left[i \int_{y_l}^{\bar{y}_l} d\xi \frac{\omega}{2} \left(\dot{\mathbf{r}}^2 - \frac{n(\xi)\sigma(\mathbf{r})}{i\omega} \right) \right]$$



A compact formula after some approximations:

- **multiple soft scattering, Brownian motion, harmonic oscillator**

  Baier-Dokshitzer-Mueller-Peigné-Schill, 1997

$$n(\xi)\sigma(\mathbf{r}) \approx \frac{1}{2} \hat{q}(\xi)\, \mathbf{r}^2$$

- **opacity expansion, hard scattering**

  Gyulassy-Levai-Vitev, 2000

$$[n(\xi)\sigma(\mathbf{r})]^N$$

Eikonal approximation: a formula for q̂

For a static medium, the jet quenching parameter is time independent:

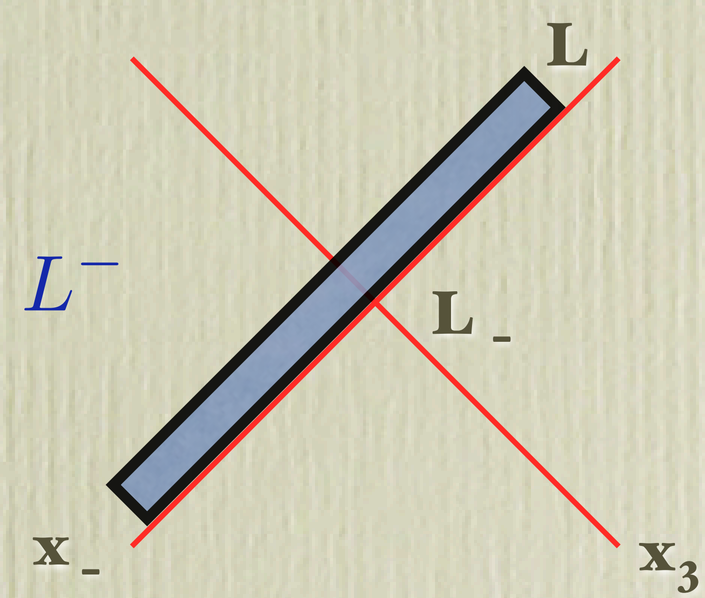
After some further approximations and a lengthy computation:

Kovner-Wiedemann, 2001

$$\langle W^A(\mathcal{C})\rangle \equiv \exp\left[-\frac{1}{4}\hat{q}L^-L^2\right]$$

for a light-like Wilson loop of the form

$$L \ll L^-$$



AdS/CFT and Wilson loops

At large Nc,

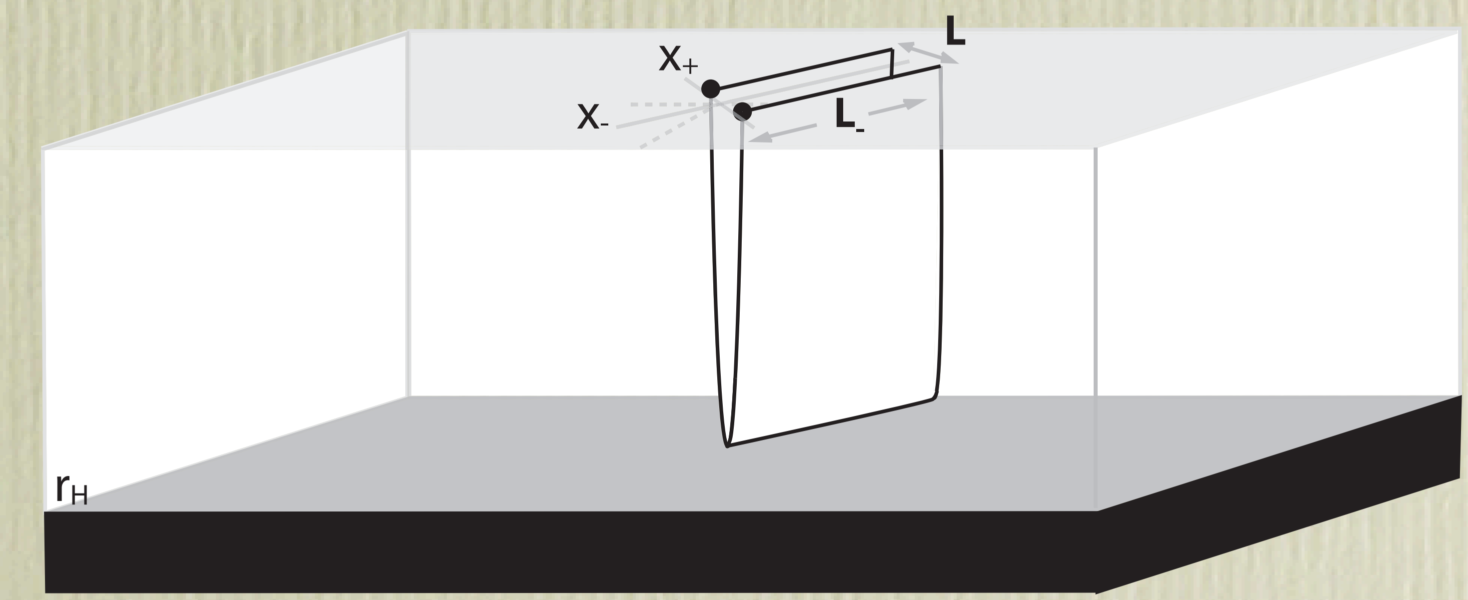
$$\left\langle W^A(\mathcal{C})\right\rangle = \left\langle W^F(\mathcal{C})\right\rangle^2 + \mathcal{O}\left(\frac{1}{N_c}\right)$$

Now, AdS/CFT tells us that it can be computed by evaluating the classical Nambu-Goto action for a string ending on the boundary along the previous light-like contour,

Maldacena, 1998

Rey-Yee, 1998

$$\langle W^F(\mathcal{C})\rangle = \exp\left[-S(\mathcal{C})\right]$$



Non-perturbative computation of q̂

This computation was carried out by Liu-Rajagopal-Wiedemann with the result

Liu-Rajagopal-Wiedemann, 2006

$$\hat{q} = \frac{\pi^{3/2}\,\Gamma(3/4)}{\Gamma(5/4)}\,\underline{\sqrt{\lambda}\,T^3}$$

It seems to measure the temperature but not the number of degrees of freedom

For N_c=3, α_s=.5 and T = 300 MeV: $\hat{q} = 4.5\,\frac{\mathrm{GeV}^2}{\mathrm{fm}}$ **not bad!!!**

However, this is strictly valid for infinite λ, while we have adopted λ=6π

It is necessary to compute finite `t Hooft coupling corrections

Non-perturbative computation of q̂

Let us start from a family of ten dimensional metrics

$$ds^2 = G_{MN}\, dX^M dX^N = -c_T^2\, dt^2 + c_X^2\, dx^i dx_i + c_R^2\, dr^2 + G_{Mn} dX^M dX^n$$

and consider the following light-like Wilson line

$$x^- = \tau\ , \qquad x^2 = \sigma\ , \qquad r = r(\sigma) \qquad\qquad \tau \in (0, L^-) \quad \sigma \in (-\frac{L}{2}, \frac{L}{2}) \quad L^- \gg L$$

From these expressions, the Nambu-Goto action takes the form

$$S = \frac{L^-}{\sqrt{2}\pi\alpha'} \int_0^{L/2} d\sigma\ \left(c_X^2 - c_T^2\right)^{1/2} \left(c_X^2 + c_R^2\, r'(\sigma)^2\right)^{1/2}$$

The energy is a first integral of motion, from which we get the profile

$$r'(\sigma)^2 = \frac{c_X^2}{c_R^2} \left(k\, c_X^2\, (c_X^2 - c_T^2) - 1\right)$$

where "k" is an integration constant $r_0 = r(0) \quad r'(0) = 0$

Non-perturbative computation of q̂

It is not hard to solve the profile equation with the result:

$$\sigma(r) = \int_{r_H}^{r} \frac{c_R}{c_X} \frac{dr}{\left(k\, c_X^2\, (c_X^2 - c_T^2) - 1\right)^{1/2}}$$

The integration constant is linked with L by the relation σ(∞)=L/2

The prescription in LRW calls for the leading behavior with L when LT << 1. This is clearly related to the limit k→∞

$$L = \frac{2\, r_H}{\sqrt{k}} \int_1^\infty \frac{c_R\, d\rho}{c_X^2\, (c_X^2 - c_T^2)^{1/2}} + \mathcal{O}(k^{-3/2})$$

we are now using dimensionless radial coordinate ρ=r/r$_H$. The action reads:

$$S = \frac{r_H\, L^-}{\sqrt{2}\pi\alpha'} \int_1^\infty \frac{\sqrt{k}\, (c_X^2 - c_T^2)\, c_X\, c_R\, d\rho}{\left(k\, c_X^2\, (c_X^2 - c_T^2) - 1\right)^{1/2}}$$

We must still substract the contribution corresponding to the self-energy of the quarks

Non-perturbative computation of q̂

This is given by the NG action for a pair of Wilson lines stretched straight from the boundary to the horizon. The regularized action, to leading order in 1/k, reads

$$S = \frac{L^-}{\sqrt{2}\pi\alpha'}\,\frac{L^2}{8r_H}\left(\int_1^\infty \frac{c_R\,d\rho}{c_X^2\,(c_X^2 - c_T^2)^{1/2}}\right)^{-1}$$

It is now convenient to define

$$c_T^2(\rho) = \frac{1}{\Delta_R}\,\hat{c}_T^2(\rho) \qquad c_X^2(\rho) = \frac{1}{\Delta_R}\,\hat{c}_X^2(\rho) \qquad c_R^2(\rho) = \Delta_R\,\hat{c}_T^2(\rho) \qquad \Delta_R = \left(\frac{(\alpha')^{5-p}\,\lambda}{r_H^{7-p}}\right)^{1/2}$$

From all these formulas we obtain

$$\hat{q} = \frac{1}{\sqrt{2}\pi\lambda}\left(\frac{r_H}{\alpha'}\right)^{6-p}\left(\int_1^\infty \frac{\hat{c}_R\,d\rho}{\hat{c}_X^2\,(\hat{c}_X^2 - \hat{c}_T^2)^{1/2}}\right)^{-1}$$

In the case of non-rotating backgrounds, it can be made more explicit:

$$\hat{q} = \frac{1}{\sqrt{2}\pi}\left[16\pi^2\left(\frac{\sqrt{\hat{c}_T^2(1)\,\hat{c}_R^2(1)}}{\hat{c}_T^2{}'(1)}\right)^2\right]^{\frac{6-p}{5-p}} T^2\,\left(T^2\,\lambda\right)^{\frac{1}{5-p}}\left(\int_1^\infty \frac{\hat{c}_R\,d\rho}{\hat{c}_X^2\,(\hat{c}_X^2 - \hat{c}_T^2)^{1/2}}\right)^{-1}$$

Witten's D4-background at finite T

The fifth dimension is compactified to a circle of radius ℓ. Hence, the four dimensional effective coupling is

$$\tilde{\lambda} = \lambda/\ell \equiv 4\pi\alpha_{SYM} N_c$$

Therefore, we may write for the effective quenching parameter

$$\hat{q} \simeq 20.16\, c\, T^3\, \alpha_{SYM}\, N_c$$

where c = ℓT is the ratio of the thermal and Kaluza-Klein circles. c = 1 signals the confinement/deconfinement transition temperature.

For N_c=3, α_s=.5 and T = 300 MeV: $\hat{q} = 4,14\, \frac{\text{GeV}^2}{\text{fm}}$ **still good, but not universal!!!**

These values are slightly smaller than those in LRW. Still, the 5d origin is reflected in the linear dependence in the `t Hooft coupling

Finite `t Hooft coupling correction

In the gravity side this amounts to stringy corrections. The α' corrected near-extremal D3-brane solution reads

Gubser-Klebanov-Tseytlin, 1998

Pawelczyk-Theisen, 1998

$$\hat{c}_T^2(\rho) = \rho^2(1-\rho^{-4})(1+\gamma\, T(\rho)+\ldots) \qquad \hat{c}_X^2(\rho) = \rho^2(1+\gamma\, X(\rho)+\ldots)$$

$$\hat{c}_R^2(\rho) = \rho^{-2}(1-\rho^{-4})^{-1}\,(1+\gamma\, R(\rho)+\ldots)$$

to first order in $\gamma = \frac{\zeta(3)}{8}\,(\alpha'/R^2)^3 \sim 0.15\,\lambda^{-3/2}$**, with**

$$T(\rho) = \left(-75\rho^{-4} - \frac{1225}{16}\rho^{-8} + \frac{695}{16}\rho^{-12}\right) \qquad X(\rho) = \left(-\frac{25}{16}\rho^{-8}(1+\rho^{-4})\right) \qquad R(\rho) = \left(75\rho^{-4} + \frac{1175}{16}\rho^{-8} - \frac{4585}{16}\rho^{-12}\right)$$

The final result in this case is:

Decreases! A good interpolation?

$$\hat{q}(\lambda) = \hat{q}(0)\left(1 - \underline{1.7652\,\lambda^{-3/2}} + \ldots\right)$$

Caveat: Dominant finite `t Hooft coupling corrections are those coming from quantum fluctuations of the world sheet. They contribute as $\lambda^{-1/2}$ **and are quite hard to compute!**

Finite chemical potential: STU black hole

The near horizon metric of rotating black D3-branes with maximal number of angular momenta:

$$ds^2 = \sqrt{\Delta}\left(-\mathcal{H}^{-1} f\, dt^2 + f^{-1} dr^2 + \frac{r^2}{R^2}\, d\vec{x}\cdot d\vec{x}\right) + \frac{1}{\sqrt{\Delta}} \sum_{i=1}^{3} R^2 H_i \left[d\nu_i^2 + \nu_i^2 (d\phi_i + A^i dt)^2\right]$$

where $\nu_1 = \cos\theta_1,\ \nu_2 = \sin\theta_1 \cos\theta_2,\ \nu_3 = \sin\theta_1 \sin\theta_2$, and $\mathcal{H} = H_1 H_2 H_3$,

$$\Delta = \mathcal{H}\sum_{i=1}^{3} \frac{\nu_i^2}{H_i} \qquad H_i = 1 + \frac{q_i}{r^2} \qquad f = \frac{r^2}{R^2}\mathcal{H} - \frac{\mu}{r^2} \qquad A^i = \frac{1}{R}\sqrt{\frac{\mu}{q_i}}(1 - H_i^{-1})$$

Upon KK reduction, this becomes a charged AdS black hole solution of $\mathcal{N} = 2\ U(1)_R^3$ supergravity

$\mathcal{N} = 4\ SU(N)$ SYM at finite temperature and with a chemical potential for the $U(1)_R^3$ symmetry

This is not the baryonic chemical potential!

Finite chemical potential: STU black hole

We can trade the non-extremality parameter μ for the horizon radius

$$\mu = \frac{r_H^4}{R^2}\,\mathcal{H}(r_H)$$

and define the adimensional quantities

$$\kappa_i = \frac{q_i}{r_H^2} \qquad \Delta_R = \frac{R^2}{r_H^2}$$

as before go the dimensionless variable ρ,

$$H_i(\rho) = 1 + \kappa_i \rho^{-2} \qquad f(\rho) = \frac{1}{\Delta_R}\left(\rho^2 \mathcal{H}(\rho) - \rho^{-2}\mathcal{H}(1)\right) \equiv \frac{1}{\Delta_R}\hat{f}(\rho)$$

so that the relevant functions entering the previously derived formula are:

$$\hat{c}_T^2(\rho) = \frac{\sqrt{\Delta}\,\hat{f}}{\mathcal{H}} - \frac{1}{\sqrt{\Delta}}\sum_{i=1}^{3}\frac{\nu_i^2\,\mathcal{H}(1)}{\kappa_i H_i}(H_i - 1)^2 \qquad \hat{c}_X^2(\rho) = \sqrt{\Delta}\,\rho^2 \qquad \hat{c}_R^2(\rho) = \frac{\sqrt{\Delta}}{\hat{f}}$$

The factors in the metric depend on the internal angles

Finite chemical potential: STU black hole

However, the terms above conspire to give

$$\int_1^\infty \frac{\hat{c}_R d\rho}{\hat{c}_X^2\sqrt{\hat{c}_X^2 - \hat{c}_T^2}} = \frac{1}{\mathcal{H}(\infty)}\int_1^\infty d\rho \left(\rho^4 \frac{\mathcal{H}(\rho)}{\mathcal{H}(\infty)} - 1\right)^{-1/2}$$

where all information about the internal angles has dissapeared. Now, given that the Hawking temperature of this solution is given by

$$T = \frac{2 + \sum_{i=1}^3 \kappa_i - \prod_{i=1}^3 \kappa_i}{2\sqrt{\mathcal{H}(1)}}\,\frac{r_H}{\pi R^2}$$

we get the final answer

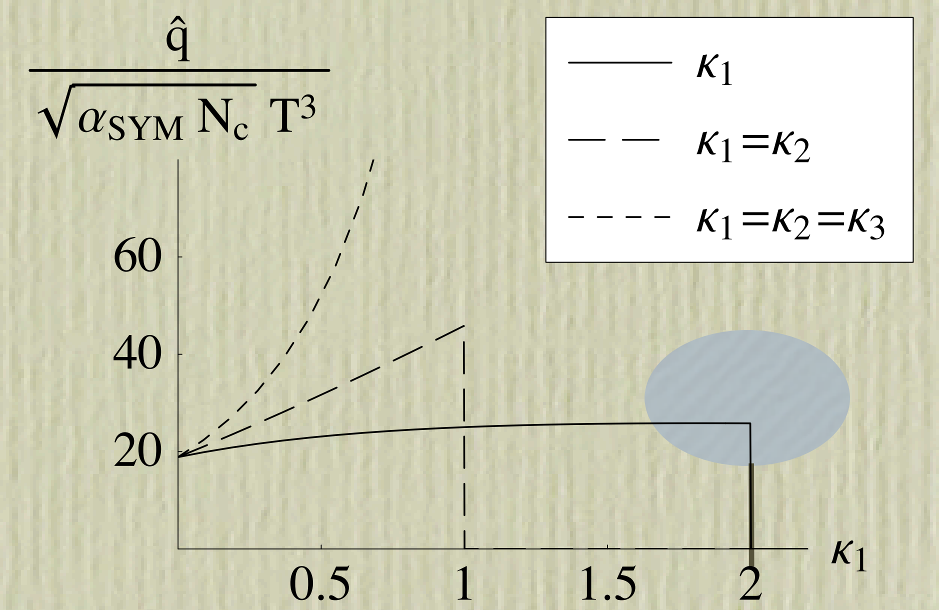
$$\hat{q}(\kappa_i) = \frac{\pi^2 T^3 \sqrt{\lambda}}{\sqrt{2}}\,\mathcal{H}(1)\left(\frac{2\sqrt{\mathcal{H}(1)}}{2 + \sum_{i=1}^3 \kappa_i - \prod_{i=1}^3 \kappa_i}\right)^3 \left(\int_1^\infty d\rho \left(\rho^4 \frac{\mathcal{H}(\rho)}{\mathcal{H}(1)} - 1\right)^{-1/2}\right)^{-1}$$

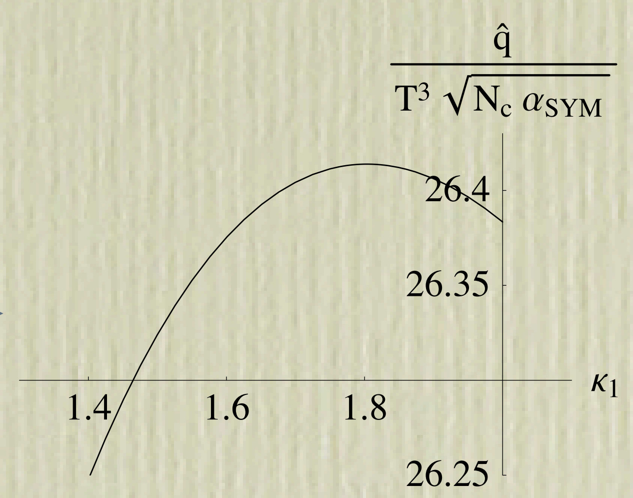
In order to analyze this result, it must be recalled that the domain of thermodynamical stability is bounded by the inequality

$$\kappa_1 + \kappa_2 + \kappa_3 - \kappa_1\,\kappa_2\,\kappa_3 < 2$$

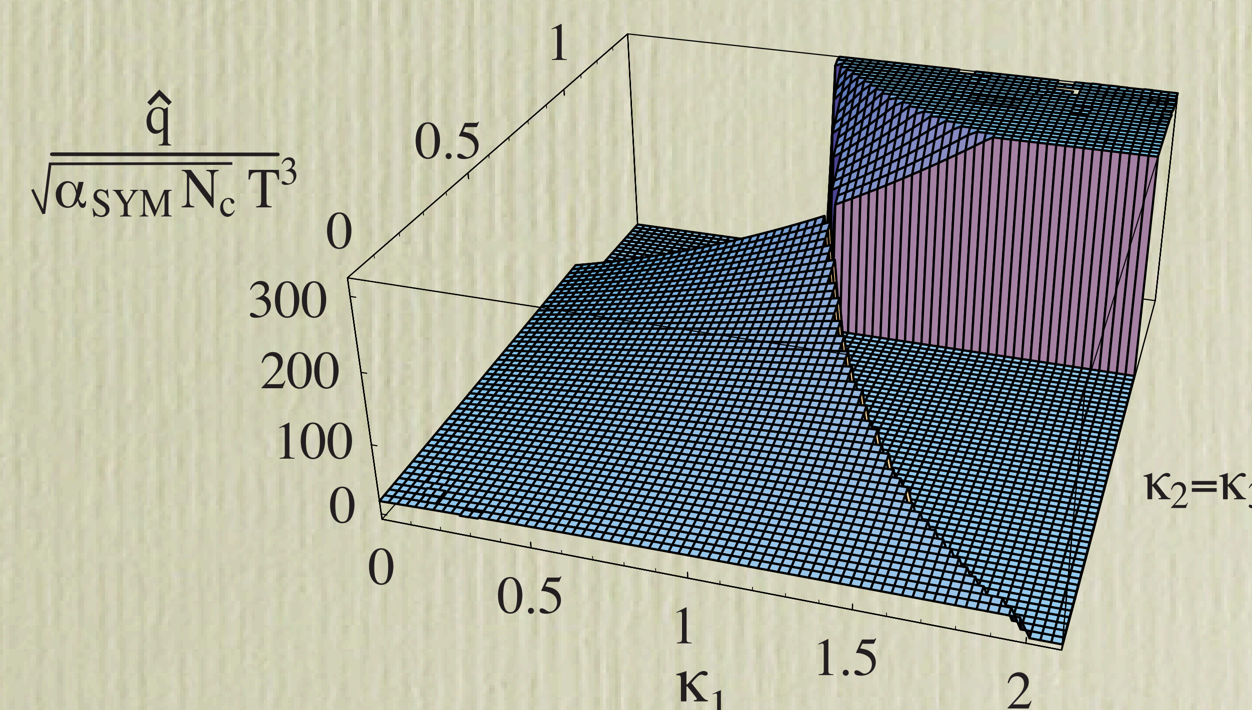
Jet quenching with chemical potential

Let me discuss the results through some plots:





In general:



Expansions in order to better compare

In order to compare with other approaches, it is useful to perform an expansion in terms of quantum field theoretical magnitudes

In particular, the density of physical charge and chemical potential are:

$$\rho_i = \frac{\pi N^2 T_0^3}{8}\sqrt{2\kappa_i}\prod_{i=1}^{3}(1+\kappa_i)^{1/2} \qquad \mu_i \equiv A^i(r)\big|_{r=r_H} = \frac{\pi T_0\sqrt{2\kappa_i}}{1+\kappa_i}\prod_{i=1}^{3}(1+\kappa_i)^{1/2}$$

We should invert in terms of (ρ,T) [canonical ensemble] or (μ,T) [grand canonical ensemble]

This is difficult in the general case. Consider $\kappa_1 = \kappa$ and $\kappa_2 = \kappa_3 = 0$

$$\kappa_C = \xi - \xi^2 + \frac{11}{4}\xi^3 + \ldots \quad \textbf{or} \quad \kappa_{GC} = \zeta + \zeta^2 + \frac{5}{4}\zeta^3 + \ldots \quad \textbf{with} \quad \xi = \left(\frac{4\sqrt{2}\rho}{\pi N^2 T^3}\right)^2 \quad \zeta = \left(\frac{\mu}{\sqrt{2}\pi T}\right)^2$$

This allows to make contact with the results in the literature:

$$\hat{q}_C(\rho) = \hat{q}(0)\left(1 + 0.63\,\xi - 1.08\,\xi^2 + 2.83\xi^3 + \ldots\right) \qquad \hat{q}_{GC}(\mu) = \hat{q}(0)\left(1 + 0.63\,\zeta + 0.18\,\zeta^2 + 0.06\zeta^3 + \ldots\right)$$

A call for massless dynamical quarks

QCD has quarks. These are d.o.f. in the fundamental representation of the gauge group. Notice that, up to this point, we have been using the words QGP for theories without quarks

It is evident that, in order to deal with QCD-like QGPs, we need to be able to accomodate quarks beyond the quenched approximation, i.e. for $N_f \approx N_c$

Some very recent attemps to extrapolate results from N=4 SYM towards QCD, have been shown to apply in a variety of theories without fundamental d.o.f.

Gubser, 2006
Liu-Rajagopal-Wiedemann, 2006

For example, based on the following result, that holds for SCFTs

$$\frac{\hat{q}_{\mathcal{N}=1}}{\hat{q}_{\mathcal{N}=4}} = \sqrt{\frac{s_{\mathcal{N}=1}}{s_{\mathcal{N}=4}}}$$

it has been conjectured that, since QCD's QGP is approximately conformal

Liu-Rajagopal-Wiedemann, 2006

$$\frac{\hat{q}_{\rm QCD}}{\hat{q}_{\mathcal{N}=4}} = \sqrt{\frac{s_{\rm QCD}}{s_{\mathcal{N}=4}}} \simeq 0.63$$

It is an open problem whether this nice result actually persists or not after quarks are introduced

QGP and non-critical holography

Non-critical string duals of 4d gauge theories with large N_c, N_f both at zero and at high temperature

Polyakov, 1999
Klebanov-Maldacena, 2004
Bigazzi-Casero-Cotrone-Kiritsis-Paredes, 2005

The gravity solutions are generically strongly coupled and α' corrections are not subleading

Our optimistic prejudice is that these setups are robust enorugh to capture qualitative features

We have dealt with two cases:

Casero-Paredes-Sonnenschein, 2005

- An AdS_5 black hole dual to finite temperature QCD in the conformal window
- An $AdS_5 \times S^1$ black hole dual to finite temperature SQCD in the Seiberg conformal window

In both models, the color d.o.f. are introduced via N_c D3-brane sources and the backreacted flavor via N_f spacetime filling brane-antibrane pairs

This reproduces the classical $U(N_f) \times U(N_f)$ flavor symmetry expected in the gauge duals with massless fundamental matter

QCD in the conformal window

The 5d model is given by the following solution (in α' = 1 units)

Bigazzi-Casero-Cotrone-Kiritsis-Paredes, 2005

$$ds^2 = \left(\frac{u}{R}\right)^2 \left[\left(1 - \frac{u_H^4}{u^4}\right) dt^2 + dx_i dx_i\right] + (Ru)^2 \frac{du^2}{u^4 - u_H^4}$$

$$R^2 = \frac{200}{50 + 7\rho^2 - \rho\sqrt{200 + 49\rho^2}} \qquad e^{\phi_0} = \frac{\sqrt{200 + 49\rho^2} - 7\rho}{10 Q_c} \qquad F_{(5)} = Q_c \, \mathrm{Vol}(AdS)$$

where

$$\rho \equiv \frac{Q_f}{Q_c} \sim \frac{N_f}{N_c}$$

Notice that $g^2_{\rm QCD}$ **depends on the flavor/color ratio. It is a decreasing function of** ρ **(consistent with the expected behavior in the upper part of the conformal window at zero temperature)**

Furthermore, it is given by

$$g^2_{\rm QCD} = \frac{\mathcal{F}(\rho)}{N_c} \sim \frac{1}{\rho} \qquad \rho \to \infty$$

as expected in the Veneziano limit $N_c \to \infty, \; N_f \to \infty, \; \rho$ fixed

Veneziano, 1976

QCD in the conformal window

The black hole temperature and entropy density read

$$T = \frac{u_H}{\pi R^2} \qquad s = \frac{A_3}{4G_{(5)}} = \frac{\pi^2 R^3 T^3}{e^{2\phi_0}}$$

The free energy can be obtained by suitably renormalizing the Euclidean action

$$I = \frac{1}{16\pi G_{(5)}} \int d^5x \sqrt{g} \left[e^{-2\phi} \left(R + 4(\partial_\mu \phi)^2 + 5 \right) - \frac{1}{5!} F^2_{(5)} - 2Q_f e^{-\phi} \right]$$

Since the dilaton is constant, the DBI term is a cosmological constant and the calculation follows closely its 10d critical counterpart

Witten, 1998

The result is

$$F = TI = -\frac{\pi^2 R^3 T^4}{4e^{2\phi_0}}$$

The energy density, heat capacity and speed of sound can be readily computed:

$$\epsilon = \frac{3\pi^2 R^3 T^4}{4e^{2\phi_0}} \qquad c_V = \frac{3\pi^2 R^3 T^3}{e^{2\phi_0}} \qquad v_s^2 = \frac{s}{c_V} = \frac{1}{3} \,,$$

QCD in the conformal window

The holographic evaluation of the shear viscosity per entropy density gives the universal value

$$\frac{\eta}{s} = \frac{1}{4\pi}\,\frac{\hbar}{k_B}$$

We expect that α' corrections shall modify (increase?) this ratio

The entropy density

$$s \sim 4\pi^2 Q_c^2 T^3 \begin{cases} 1 + \sqrt{2}\rho + \dots & \text{for } \rho \to 0 \\ \frac{343}{250}\sqrt{\frac{7}{5}}\left(\rho^2 + \mathcal{O}(\rho^0)\right) & \text{for } \rho \to \infty \end{cases}$$

The first correction to the pure glue result coincides with earlier but very recent findings

Mateos-Myers-Thomson, 2006

The jet quenching is a monotonically increasing function of ρ

$$\hat{q} \sim \frac{4\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})} T^3 \begin{cases} 1 + \frac{\sqrt{2}}{5}\rho + \dots & \text{for } \rho \to 0 \;, \\ \frac{7}{5} + \mathcal{O}\left(\frac{1}{\rho^2}\right) & \text{for } \rho \to \infty \;. \end{cases}$$

QGP and wrapped fivebranes

A family of black hole solutions corresponding to $N_f = 2\,N_c$, with quartic superpotential, coupled to Kaluza-Klein adjoint matter reads

Casero-Nuñez-Paredes, 2006

$$ds^2 = e^{\Phi_0} z^2 \left[-\mathcal{F} dt^2 + d\vec{x}_3^2 + N_c \alpha' \left(\frac{4}{z^2 \mathcal{F}} dz^2 + \frac{1}{\xi} (d\theta^2 + \sin^2\theta \, d\varphi^2) + \frac{1}{4-\xi} (d\tilde{\theta}^2 + \sin^2\tilde{\theta} \, d\tilde{\varphi}^2) + \frac{1}{4} (d\psi + \cos\theta \, d\varphi + \cos\tilde{\theta} \, d\tilde{\varphi})^2 \right) \right]$$

$$e^{\Phi} = z^2 e^{\Phi_0} \qquad \mathcal{F} = 1 - \frac{z_0^4}{z^4}$$

The temperature and entropy of these black holes are

$$T = \frac{1}{2\pi \sqrt{\alpha' N_c}} \qquad s = \frac{A_8}{4 G_{(10)}} = \frac{8 e^{4\Phi_0} z_0^8 N_c^4}{\xi(4-\xi)} T^3$$

The temperature does not depend on the horizon radius and, thus, on the energy density. The free energy vanishes. The theory is in a Hagedorn phase

Indeed, $T = T_H$ of Little String Theory. The solution suffers from thermodynamical instabilities, as it is the case for flat NS5-branes

Kutasov-Sahakyan, 2000

Buchel, 2001

QGP and wrapped fivebranes

A naive proposal to cure these problems: deal with the QGP of a theory on S^3 since the radius of the sphere provides a scale that naturally should shift T away from T_H

This is not the case: this gives an IR cutoff that cannot remedy the LST behavior

If we insist and compute thermodynamical and transport properties of the would be QGP:

- $$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$
- **Quarks and antiquarks are always screened** $V_{q\bar{q}} = 0$
- **The drag force from trailing strings reads** $\mu\, M_{\rm kin} = 2\pi\,\lambda\,\mathrm{T}^2$ — **instead of $\sqrt{\lambda}$ in N=4 SYM**
- $\hat{q} = 0$

This is puzzling. We have checked that an analog behavior takes place in any QGP resulting from a wrapped fivebrane setup. We call these LST plasmas

Conclusions and Outlook

- **We computed the jet quenching parameter in a variety of cases:**
  - **For finite `t Hooft coupling we got corrections suggesting a smooth interpolation with the perturbative results, such as with the entropy and shear viscosity**
  - **For the thermal deformation of Witten's D4-background, we have obtained slightly smaller values and a different `t Hooft coupling dependence**
  - **We have thoroughly studied the addition of a chemical potential for the gauged R-symmetry. It generically increases the value of the jet quenching parameter.**
  - **We showed how this setup can be extended to quarks of finite mass**
- **We studied the introduction of unquenched fundamental degrees of freedom, i.e., quarks**
  - **In non-critical setups corresponding to QCD and SQCD models in the conformal window**
  - **In wrapped fivebrane setups corresponding to SQCD-like theories**

N=4 SYM theory to N=2 flavor multiplets at finite temperature remains to be an open problem