Extending Bubbling AdS: Going Beyond the ½ BPS Sector

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Outline

- **Basics of AdS/CFT**
- **Problem with Interactions in AdS/CFT**
  - Interactions on the boundary vs. in the bulk
- **AdS/CFT beyond small perturbations**
  - The ½ BPS sector and the one matrix model **(bubbling)**
  - The role of collective field theory
- **Going beyond the ½ BPS sector**
  - Multi-matrix models: interactions revisited
  - Bubbling with less SUSY? The ¼ and 1/8 BPS sectors
- **Towards Dynamics?** (directions for the future)

WORK IN PROGRESS

The AdS/CFT Conjecture

"Holographic" Duality:

Theory of Gravity ⟷ **Conformal field theory on boundary**

IIB string theory on AdS_5 x S^5 ⟷ **$\mathcal{N}=4$ U(N) SYM in 3+1 d**

(fields) (operators)

Strong / Weak Coupling Duality:

$$\lambda = g_{YM}^2 N \sim g_s N \sim R^4 / l_s^4$$

Trust **perturbative** analysis in **YM** theory when $\lambda = g_{YM}^2 N \sim g_s N \sim \frac{R^4}{l_s^4} << 1$

Classical gravity description reliable when $\frac{R^4}{l_s^4} \sim g_s N \sim g_{YM}^2 N >> 1$

(and ***N*** **large**)

D=4 $\mathcal{N}$=4 U(N) SYM ⟵ **Dictionary** ⟶ **Type IIB SUGRA on $AdS_5 \times S^5$**

SO(2,d) symmetry of d-dim conformal group ⟷ *SO(2,d)* Isometry group of AdS_{d+1}

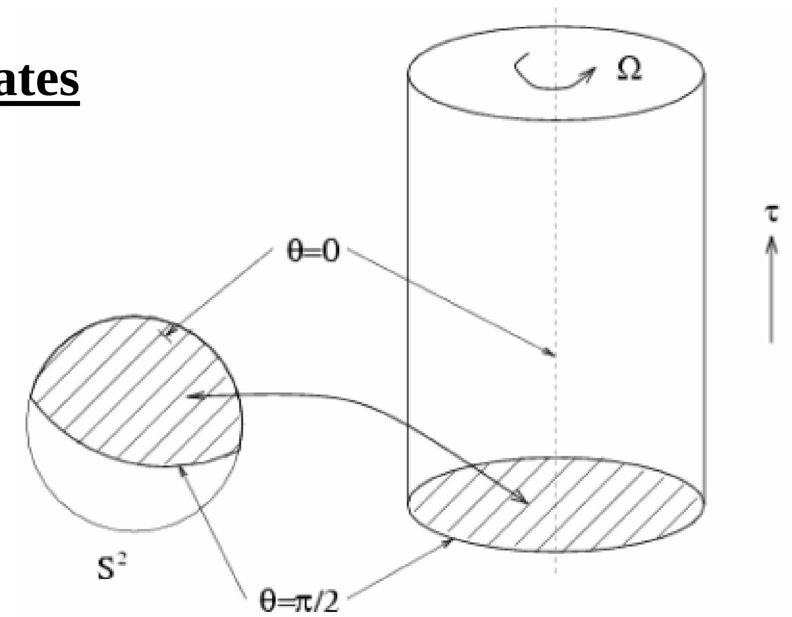
Hyperboloid $X_0^2 + X_{d+1}^2 - \sum_{i=1}^{d} X_i^2 = R^2$

SO(6) R-symmetry ⟷ *SO(6)* isometry of S^5

Simple example: AdS_3 in global coordinates

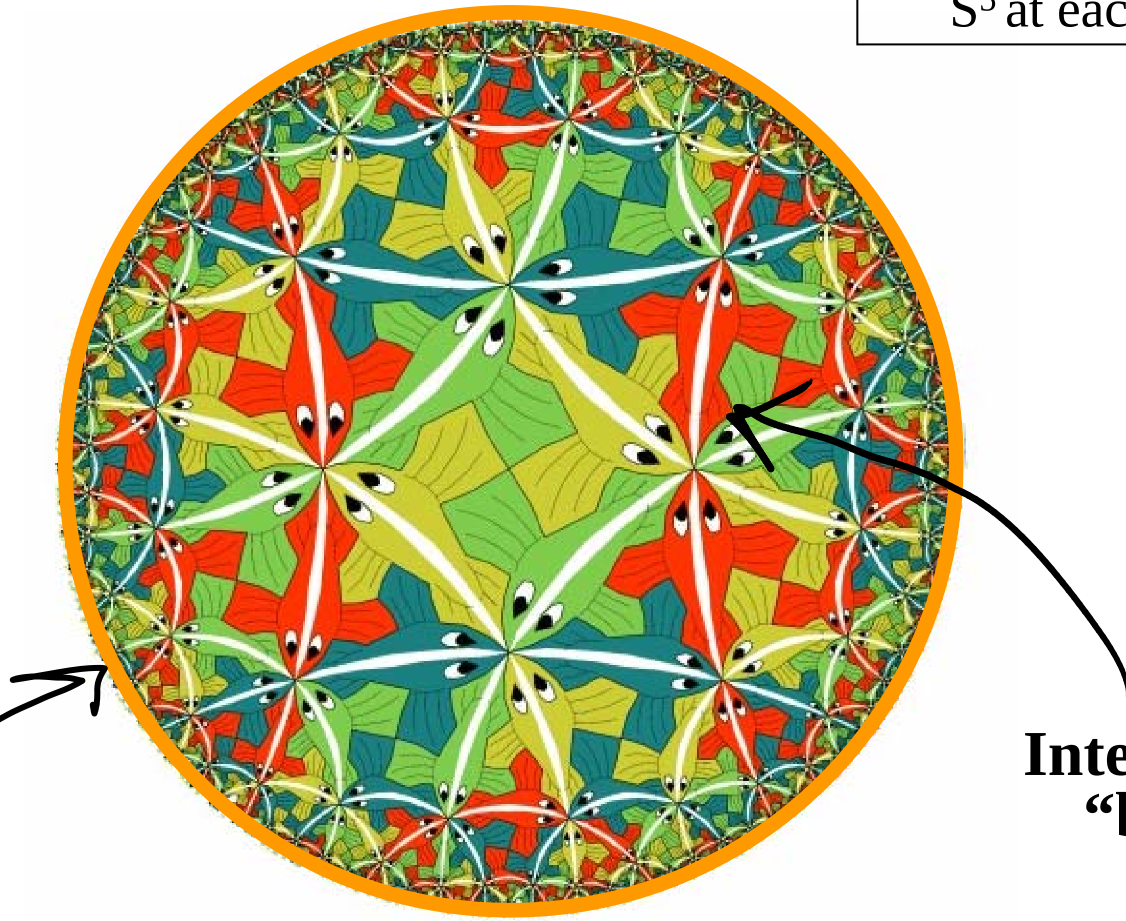
$$ds^2 = \frac{R^2}{\cos^2\theta}(-d\tau^2 + d\theta^2 + \sin^2\theta\ d\Omega^2)$$

Boundary is at θ = π/2 : $R \times S^1$



My favorite AdS picture

S^5 at each point



Interior or “bulk”

boundary

AdS/CFT beyond SUGRA: pp-wave background

So far correspondence only between **SUGRA** and **SYM** (no strings)

<u>Progress:</u> Extending AdS/CFT to **string theory**

BMN
hep-th/0202021

Why plane-wave background ?

String propagation on **pp-wave background** can be solved **exactly**

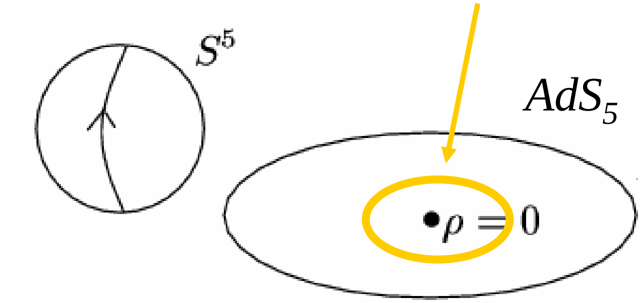
Green-Scharwz light-cone action becomes **quadratic** $\Rightarrow$ can be **quantized**

STRING THEORY on pp-waves	AdS/CFT $\Longleftrightarrow$	**Sector of CFT (large R-charge)**

To get the pp-wave background, start from $AdS_5 \times S^5$:

$$ds^2 = R^2\left[-dt^2\cosh^2\rho + d\rho^2 + \sinh^2\rho\, d\Omega_3^2 + d\psi^2\cos^2\theta + d\theta^2 + \sin^2\theta\, d\Omega'^2_3\right]$$

Focus on "particle" moving **very rapidly** (**large J**) along ψ and sitting near $\rho = \theta = 0$



Systematically:

$$\begin{cases} \tilde{x}^{\pm} = \frac{t \pm \psi}{2} \\ x^+ = \tilde{x}^+ , \quad x^- = R^2 \tilde{x}^- , \quad \rho = \frac{r}{R} , \quad \theta = \frac{y}{R} , \quad R \to \infty \end{cases}$$

$$\boxed{ds^2 = -4dx^+dx^- - (\vec{r}^{\,2} + \vec{y}^{\,2})(dx^+)^2 + d\vec{y}^{\,2} + d\vec{r}^{\,2}}$$

standard plane wave

Main result of BMN : matching of SPECTRUM in large J limit (large R charge)

What about interactions?

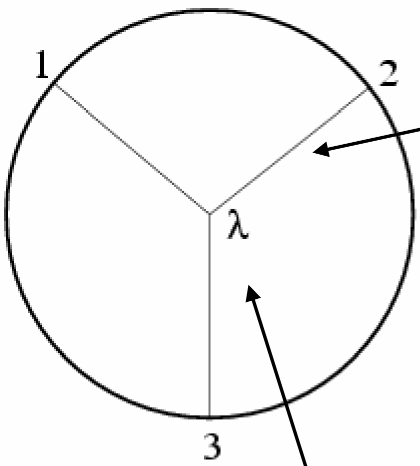
Cubic Interactions

Simple model of interactions:

$$S = \int d^5x \sqrt{g} \left[\sum_i \tfrac{1}{2}(\partial\phi_i)^2 + \tfrac{1}{2} m_i^2 \phi_i^2 + \lambda \phi_1 \phi_2 \phi_3 \right]$$

For fields on **boundary** of AdS, **well-defined prescription** (GKP-W prescription) **:**

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \rangle =$$



bulk-to-boundary propagator

$$\langle \mathcal{O}\mathcal{O} \rangle = \frac{1}{|x_1 - x_2|^{2\Delta}}$$

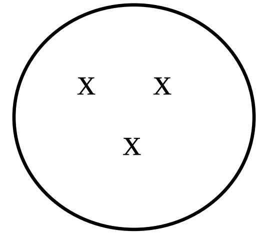
Position of vertex is integrated over AdS

$$\langle \mathcal{O}_1(\vec{x}_1)\mathcal{O}_2(\vec{x}_2)\mathcal{O}_3(\vec{x}_3)\rangle = -\lambda \int d^5x \sqrt{g} K_{\Delta_1}(x;\vec{x}_1) K_{\Delta_2}(x;\vec{x}_2) K_{\Delta_3}(x;\vec{x}_3)$$

$$= \frac{\lambda a_1}{|\vec{x}_1 - \vec{x}_2|^{\Delta_1+\Delta_2-\Delta_3} |\vec{x}_1 - \vec{x}_3|^{\Delta_1+\Delta_3-\Delta_2} |\vec{x}_2 - \vec{x}_3|^{\Delta_2+\Delta_3-\Delta_1}},$$

Interactions inside AdS?

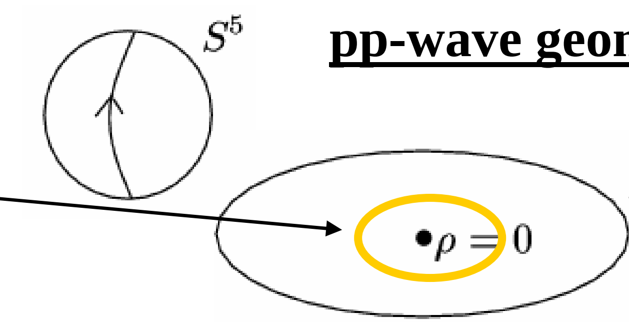
Prescription for calculating interactions **INSIDE** AdS ?



? =

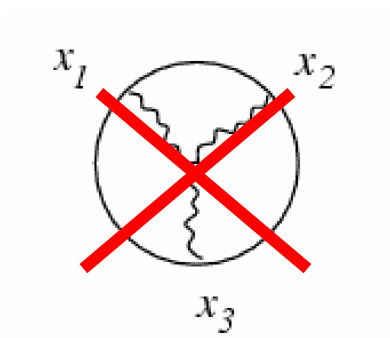
< YM correlator >

DICTIONARY?



pp-wave geometry

In pp-limit **boundary is lost**



There should be a dictionary BUT **bulk-boundary** prescription may **not** be **fundamental or complete**

Approach: construct and study the Hamiltonian

So far…

- To understand AdS/CFT need to set up a **precise dictionary** between states of two theories

- In “original” AdS/CFT **perturbations** on AdS_5 x S^5

Can we go **beyond the perturbative description**?

We may consider SUGRA **solutions** that are **asymptotically AdS_5 x S^5** as **GOOD CANDIDATES for dual states** in the CFT

Hope: carry out this program in the **FULL BPS sector** of the respective theories

First step in this direction:
dictionary for ½ BPS sector of Type IIB string theory (LLM, hep-th/0409174)

What about the problem with AdS interactions?

Natural question: what is the appropriate Hamiltonian?

We will construct the Hamiltonian for :

- ½ **BPS sector** of the theory (well-known)

1 MATRIX MODEL

Motivate H for **more general geometries** (work in progress)

MULTI-MATRIX MODEL

½ BPS geometries in Type IIB

Lin, Lunin, Maldacena
hep-th/0409174

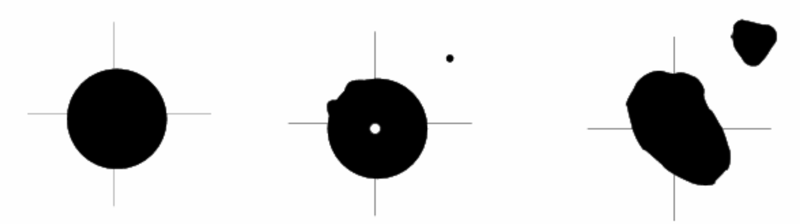
LLM:

- constructed **exact ½ BPS solutions** in type IIB SUGRA
- identified them with the **free fermion picture** of ½ BPS sector of N = 4 SYM

Explicit, **regular** solutions with **SO(4) x SO(4) isometry** ⟺ **Chiral primaries** in $\mathcal{N}$=4 SYM $(\Delta = J)$

⇕

fermionic droplets



geometry ⟺ **Droplet shape**

General idea:

10D Spacetime of form $\quad ds^2_{10} = g_{\mu\nu} dx_\mu dx_\nu + y e^{G} d\Omega_3^2 + y e^{-G} d\tilde{\Omega}_3^2$

Time-like Killing vector $\Longrightarrow$ 3 dimensions really "matter"

Crucial feature: all solutions describable in terms of a **single function** $z(x_1, x_2, y)$:

$$\left(\partial_1^2 + \partial_2^2 + y\partial_y \frac{1}{y}\partial_y\right) z(x_1, x_2, y) = 0$$

Regularity of solutions demands certain **boundary conditions** on **y=0 plane** :

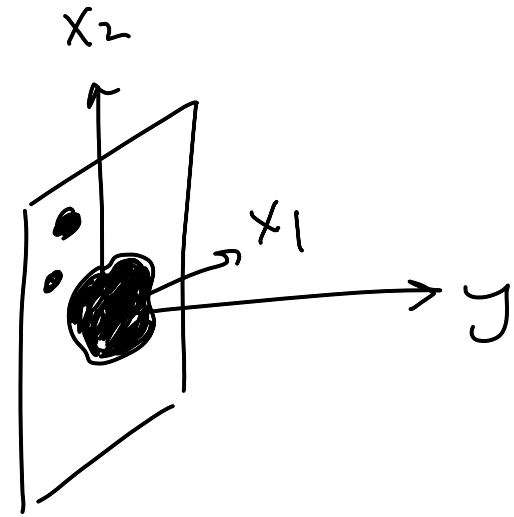
$$z(x_1, x_2, y=0) = \pm\frac{1}{2}$$

meaning of y
(one sphere shrinking smoothly)

Smoothness of solutions : on y=0 plane $\Longleftrightarrow$ black and white color coding of solutions

Boundary conditions on y = 0 plane specify geometry :

$$z(x_1, x_2, y) = \frac{y^2}{\pi} \int_{\mathcal{D}} \frac{z(x_1', x_2', 0)\, dx_1' dx_2'}{[(\mathbf{x} - \mathbf{x}')^2 + y^2]^2}$$



$$\partial_i \partial_i z + y \partial_y \left(\frac{\partial_y z}{y} \right) = 0 \Longrightarrow$$ **Bubbling !**

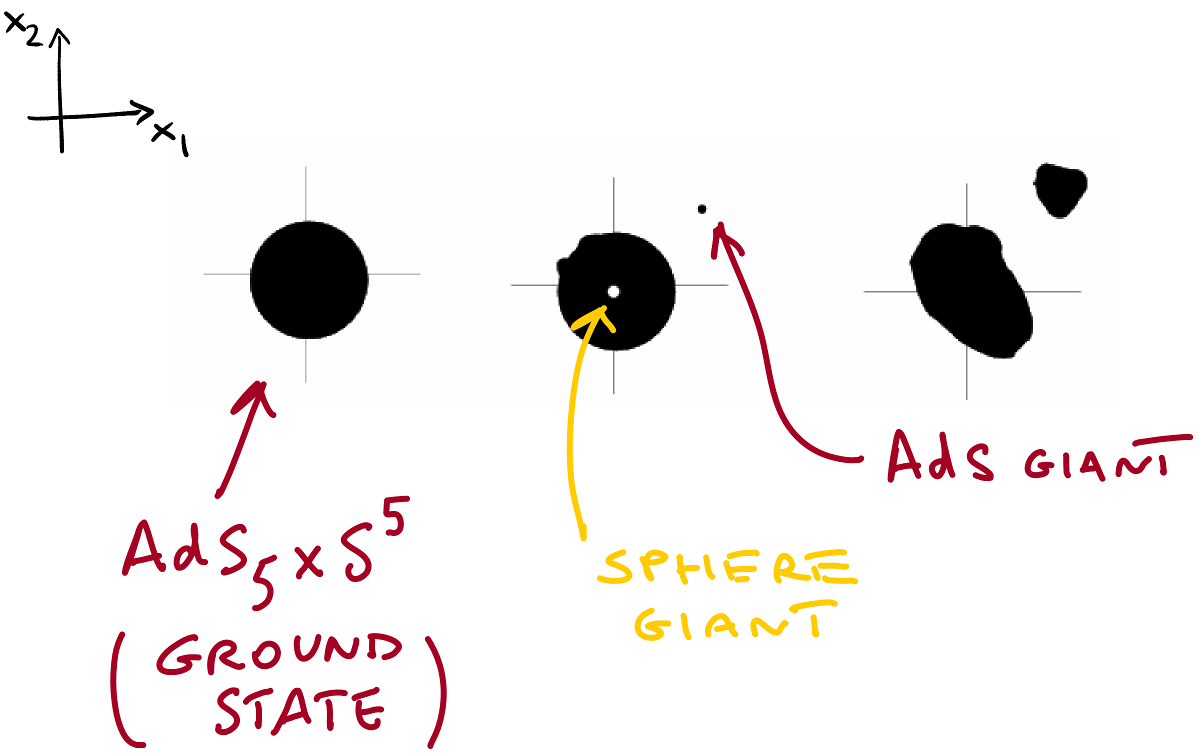
↑ LINEAR !

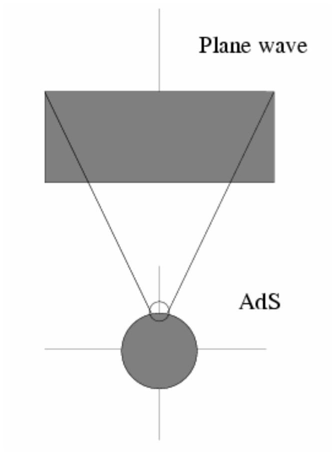
Where is the fermion description?

We will see in detail :

"Special" 2D plane (y=0) identified with **phase space of fermions**

Fermion droplets (= geometries) :





Relation between LLM ansatz and matrix model

Solutions are **BPS**, thus $\Delta = U(1)_R \text{ charge} = J$

Angular momentum and **flux** for all LLM solutions given by :

$$\Delta = J = \frac{1}{16\,\pi^3\,l_P^8}\left[\int_D d^2x\,(x_1^2 + x_2^2) - \frac{1}{2\pi}\left(\int_D d^2x\right)^2\right]$$

$$N = \int d^2x$$

½ **BPS Sector** described by **matrix model** with **harmonic oscillator potential**

Will show this via connection with **collective field theory**

Matrix Model: Reduction to ½ BPS Sector

Why only 1 matrix for ½ BPS sector?

Start from **two-matrix model** :

$$H = \frac{1}{2}\mathrm{Tr}\,(P_1^2 + P_2^2 + X_1^2 + X_2^2) \qquad J = \mathrm{Tr}\,(P_1 X_2 - P_2 X_1).$$

Rewrite in different way: introduce **complex matrices**

$$\begin{aligned} Z &= \frac{1}{\sqrt{2}}(X_1 + iX_2) & \Pi &= \frac{1}{\sqrt{2}}(P_1 + iP_2) = -i\frac{\partial}{\partial Z^\dagger}\,, \\ Z^\dagger &= \frac{1}{\sqrt{2}}(X_1 - iX_2) & \Pi^\dagger &= \frac{1}{\sqrt{2}}(P_1 - iP_2) = -i\frac{\partial}{\partial Z}\,. \end{aligned}$$

With $A = \frac{1}{2}(Z + i\Pi), \quad B = \frac{1}{2}(Z - i\Pi).$

→

$$\boxed{\begin{aligned} H &= \mathrm{Tr}(A^\dagger A + B^\dagger B)\,, \\ J &= \mathrm{Tr}(A^\dagger A - B^\dagger B)\,. \end{aligned}}$$

Constructing states :

$$H = \mathrm{Tr}(A^\dagger A + B^\dagger B)\,,$$
$$J = \mathrm{Tr}(A^\dagger A - B^\dagger B)\,.$$

$$\mathrm{Tr}\,(A^\dagger)^n\,|0>, \qquad E = J = n,$$
$$\mathrm{Tr}\,(B^\dagger)^n\,|0>, \qquad E = -J = n,$$
$$\mathrm{Tr}\,(A^\dagger)^n\,\mathrm{Tr}\,(B^\dagger)^m\,|0>, \qquad E = n+m, \quad J = n-m.$$

E is not J !

½ BPS Sector (E=J) ⟺ **Truncation** to **A** oscillators (no B oscillators) so **SINGLE MATRIX**

These states correspond to :

$Z = \phi_5 + i\phi_6$

chiral primary operators: *e.g.* $\mathrm{Tr}\,Z^{k_1}\mathrm{Tr}\,Z^{k_2}\ldots\mathrm{Tr}\,Z^{k_n}$

BUT a theory of a single matrix can be described by a **collective field theory**

theory of fermions !

Fermion description starts emerging

Collective Field Theory Description

Simple matrix model $$L = \frac{1}{2}\mathrm{Tr}(\dot{M}^2 - M^2)$$

diagonalize *M(t)* → $\lambda_i(t)$ **one-dimensional theory**

Collective field $$\phi(x,t) = \sum_{i=1}^{N} \delta(x - \lambda_i(t))$$

In large N limit appearance of new spatial dimension

Gravity is "EMERGING" (collective phenomenon)

dynamics of *M(t)* ⇒ $$H_{coll} = \int dx \left(\frac{1}{2}\,\partial_x \Pi \;\phi\; \partial_x \Pi + \frac{\pi^2}{6}\phi^3 + \frac{1}{2}\,(x^2 - \mu)\phi \right)$$

(normalization) $$\int dx\;\phi(x) = N$$

Das-Jevicki

We found $$H_{coll} = \int dx \left(\frac{1}{2} \partial_x \Pi \, \phi \, \partial_x \Pi + \frac{\pi^2}{6} \phi^3 + \frac{1}{2} (x^2 - \mu) \phi \right)$$

Introduce new fields : $$\alpha_{\pm}(x,t) = \partial_x \Pi \pm \pi \, \phi(x,t)$$

$$H = \int dx \int_{\alpha_-}^{\alpha_+} d\alpha \, \frac{1}{2} (\alpha^2 + x^2 - \mu)$$

Compare to LLM (geometry) :

$$\begin{cases} \Delta = J = \dfrac{1}{16 \, \pi^3 \, l_P^8} \left[\displaystyle\int_D d^2x \, (x_1^2 + x_2^2) - \frac{1}{2\pi} \left(\int_D d^2x \right)^2 \right] \\ N = \displaystyle\int d^2x \end{cases}$$

With $\mu \sim N$ **we have matching !**

Collective field theory description for ½ BPS SUGRA states

Appropriate Hamiltonian for ½ BPS states !

Back to interactions: the collective field theory cubic vertex

Fluctuations $\phi(x,t) = \phi_0(x) + \frac{1}{\sqrt{\pi}}\partial_x \eta(x,t)$

Static ground state

$$\pi\phi_0 = \sqrt{\mu - x^2}$$

"Time of flight" coordinate $\tau = \int \frac{dx}{\phi_0}, \quad 0 < \tau < \pi$

$$\Rightarrow \quad H = \int d\tau \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\partial_\tau \eta)^2 + \frac{1}{6\pi^2\phi_0^2}\left((\partial_\tau \eta)^3 + 3\Pi\,\partial_\tau \eta\,\Pi\right)\right]$$

This is what we are interested in

Some manipulations:

$$\begin{cases} \alpha = \alpha_+ - \pi\phi_0, & \tau > 0 \\ \quad = -\alpha_- - \pi\phi_0, & \tau < 0 \end{cases} \qquad -\pi < \tau < \pi$$

$$\Rightarrow \quad H^{(3)} = \int_{-\pi}^{\pi} \frac{d\tau}{\phi_0^2}\,\alpha^3(\tau)$$

$$\alpha(\tau) = \sum_n \sqrt{n}\left(e^{in\tau}a_n + e^{-in\tau}a_n^\dagger\right) \quad \Rightarrow \quad H^{(3)} = -4\pi\sqrt{n_1 n_2 n_3}\,(n_1 + n_2 - n_3)\,a_1 a_2 a_3^\dagger + \ldots$$

This matches the corresponding GR calculation ! (see next)

GR side: chiral primary interactions in AdS_5 x S^5

Seiberg et al., hep-th/9806074

$$(\nabla_\mu \nabla^\mu - m_I^2) s^I = \sum_{J,K} \left(D_{IJK} s^J s^K + E_{IJK} \nabla_\mu s^J \nabla^\mu s^K + F_{IJK} \nabla^{(\mu} \nabla^{\nu)} s^J \nabla_{(\mu} \nabla_{\nu)} s^K \right)$$

AdS_5

Chiral primary s^I with mass $m^2 = j(j-4)$.

Derivative couplings can be removed by a **field redefinition :**

$$s^I = s'^I + \sum_{J,K} \left(J_{IJK} s'^J s'^K + L_{IJK} \nabla^\mu s'^J \nabla_\mu s'^K \right)$$

$$(\nabla_\mu \nabla^\mu - m_I^2) s^I = \sum_{J,K} \lambda_{IJK} \, s^J s^K$$

$$\begin{cases} \lambda_{123} = (j_3 - j_1 - j_2)\, 2\kappa \dfrac{\sqrt{j_1 j_2 j_3 (j_3^2 - 1)(j_3 + 2)(j_3 - 2)}}{\sqrt{(j_1^2 - 1)(j_2^2 - 1)(j_1 + 2)(j_2 + 2)}} \times f_{123} \\ f_{123} = \dfrac{1}{\sqrt{2\pi^3}} \dfrac{\sqrt{(j_1 + 1)(j_1 + 2)(j_2 + 1)(j_2 + 2)}}{\sqrt{(j_3 + 1)(j_3 + 2)}}. \end{cases}$$

Highest-weight states $\longrightarrow$ $s = \dfrac{\sqrt{\Delta(\Delta - 1)}}{\pi (\cosh \mu)^\Delta}$.

$$< 3|H_3|12 > \sim (\Delta_3 - \Delta_1 - \Delta_2) \sqrt{\Delta_1 \Delta_2 \Delta_3}\, \delta(j_3 - j_1 - j_2)$$

matches Coll. F.T. cubic vertex

$$H^{(3)} = -4\pi \sqrt{n_1 n_2 n_3}\, (n_1 + n_2 - n_3)\, a_1 a_2 a_3^\dagger$$

Going Beyond the ½ BPS Sector

What have we seen so far?

½ BPS states ⟺ Fermions in harmonic oscillator (collective field theory description)

⤡ ⤢

single-matrix states

Next?

Study **more general states** (outside of ½ BPS Sector)

⇕

multi-matrix states

↓

"Brute force" approach to interactions challenging

Alternative approach?

Interactions for two-matrix states

S.C., A. Jevicki,
R. de Mello Koch
hep-th/0702???

General strategy for reconstructing full AdS interaction :

- **Start from collective field theory vertex** $V_3 \neq 0$
  (assume correct description for multi-matrix states)

- Use SL(2,R) **symmetry** of underlying theory to **generate interactions**
  (find **useful identities** using generators that relate vertices that we know to vertices we don't know)

Feature of SUGRA:

$$V_3 \sim (\Delta_3 - \Delta_1 - \Delta_2)\,\delta(j_1 + j_2 - j_3)$$

vanishes on shell for highest-weight state

Meaning of acting with generators?

R**ewrite the vertex** so that it does not vanish
(**canonical transformation** and **non-linear field redefinition**)

Warm up : a toy model

Consider **particle in two dimensions** as toy model for two matrix states

$$H = -\frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \frac{\omega^2}{2}(x^2 + y^2) \longrightarrow \begin{cases} H = a^\dagger a + b^\dagger b \\ J = a^\dagger a - b^\dagger b \end{cases}$$

States

$$|J, n> = \frac{(a^\dagger)^{J+n}(b^\dagger)^n}{\sqrt{(J+n)!n!}}|0>$$

Recall

$$|J, n> = \frac{1}{\sqrt{(J+n)n}} L_+ |J, n-1>$$

Acting with L_0, L_+, L_- on

$$\int d^2x \frac{1}{\sqrt{\phi_0(\vec{x})}}\, \bar{\Psi}_{J_1,n_1} \Psi_{J_2,n_2} \Psi_{J_3,n_3}$$

$$\int d^2x \frac{1}{\sqrt{\phi_0(\vec{x})}}\, [L_+^{(3)} - L_-^{(1)} - L_-^{(2)} + \frac{1}{2}(L_0^{(3)} - L_0^{(1)} - L_0^{(2)})]\, \bar{\Psi}_{J_1,n_1} \Psi_{J_2,n_2} \Psi_{J_3,n_3} = 0$$

Two matrix interactions in $AdS_5 \times S^5$

Repeat same procedure but for Hamiltonian

$$H = -\frac{1}{2}\left(\frac{\partial^2}{\partial M^2} + \frac{\partial^2}{\partial N^2}\right) + \frac{1}{2}(M^2 + N^2)$$

Eigenfunctions of two-matrix model found by A. Donos, A. Jevicki, J. Rodrigues (hep-th/0507124) :

$$H = -\frac{1}{2}\frac{\partial^2}{\partial M^2} + \frac{1}{2}M^2 + B\frac{\partial}{\partial B}$$

treat as impurity

AdS Result :

Build vertex, act with SL(2,R) generators, and find analog of toy model identity

BOTTOM LINE : SYMMETRIES MAY HELP

Comment on "emergent geometry"

Recall **eigenvalues** of one matrix yielded a **new dimension:**

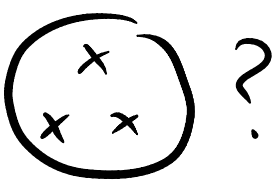
$$M(t) \to \Phi(x,t) = \sum_i \delta(x - \lambda_i)$$

Add **another matrix** ***N(t)*** $\Longrightarrow$ Probe **additional direction** ***y***

LLM probed angle of S^5

LLM used ONE MATRIX to describe AdS_5 x S^5

EXPLAIN



With two matrices, hope to eventually probe radial direction of AdS

BUT still challenge

Detailed bubbling picture of 1/2, 1/4, 1/8 BPS states ?

Work in progress

B. Chen, S.C., A. Donos, F.Lin, H. Lin, J. Liu, D. Vaman, W. Wen, hep-th/0702???

Natural questions:

- Can we extend AdS/CFT dictionary to full BPS spectrum?
- Is there **bubbling** if you have **LESS SUSY** (¼ BPS and 1/8 BPS sectors) ?
- What is the **GAUGE THEORY** picture? **Fermions?**

General SUGRA ansatz for ¼ BPS and 1/8 BPS geometries worked out

A. Donos
hep-th/0606199, hep-th/0610259

N. Kim,
hep-th/0511029

But here boundary conditions (possible bubbling picture) missing !
(hard to get explicit solutions)

Gravity picture that has emerged :

1/2 BPS

$$ds^2_{1/2} = -h^{-2}(dt+V)^2 + h^2(dy^2 + dx_1^2 + dx_2^2) + ye^{G} d\Omega_3^2 + ye^{-G} d\tilde{\Omega}_3^2$$

BC's here (droplets on 2D plane)

$4D \times S^3 \times S^3$

1/4 BPS

$$ds^2_{1/4} = -h^{-2}(dt+W)^2 + h^2 dy^2 + \frac{1}{ye^{G}} ds_4^2 + ye^{G} d\Omega_3^2 + ye^{-G} d\psi^2$$

Expect BC's here

$6D \times S^3 \times S^1$

1/8 BPS

$$ds^2_{1/8} = -e^{2\alpha}(dt + w)^2 + e^{-2\alpha} ds_6^2 + e^{2\alpha} d\Omega_3^2$$

$7D \times S^3$

For smoothness look at collapsing spheres !

NICE PICTURE

- $S^3 \times 7D$ — 1/8 BPS
- $S^3 \times S^1 \times 6D$ — 1/4 BPS
- $S^3 \times S^3 \times 4D$ — 1/2 BPS

¼ BPS solution STILL depends on only one function z (as in ½ BPS case) :

$$ds^2_{1/4} = -h^{-2}(dt+W)^2 + h^2 dy^2 + \frac{1}{ye^G} ds^2_4 + ye^G d\Omega_3^2 + ye^{-G} d\psi^2$$

BUT VERY COMPLICATED

$$z = -2y\partial_y\left(\frac{1}{y}\partial_y K\right). \qquad \left(g^{4D}_{m\,n} = \partial_m\partial_n K\right)$$

$$\begin{vmatrix} \partial_z\partial_{\bar z}K & \partial_w\partial_{\bar z}K \\ \partial_z\partial_{\bar w}K & \partial_w\partial_{\bar w}K \end{vmatrix} = y\frac{e^{\frac{2}{y}\partial_y K}}{2}\left(-2y\partial_y\left(\frac{1}{y}\partial_y K\right)+1\right)$$

Difficulties with ¼ BPS construction :

- solving equation for K is challenging (explicit solutions?)
- even hard to reproduce simple ½ BPS states

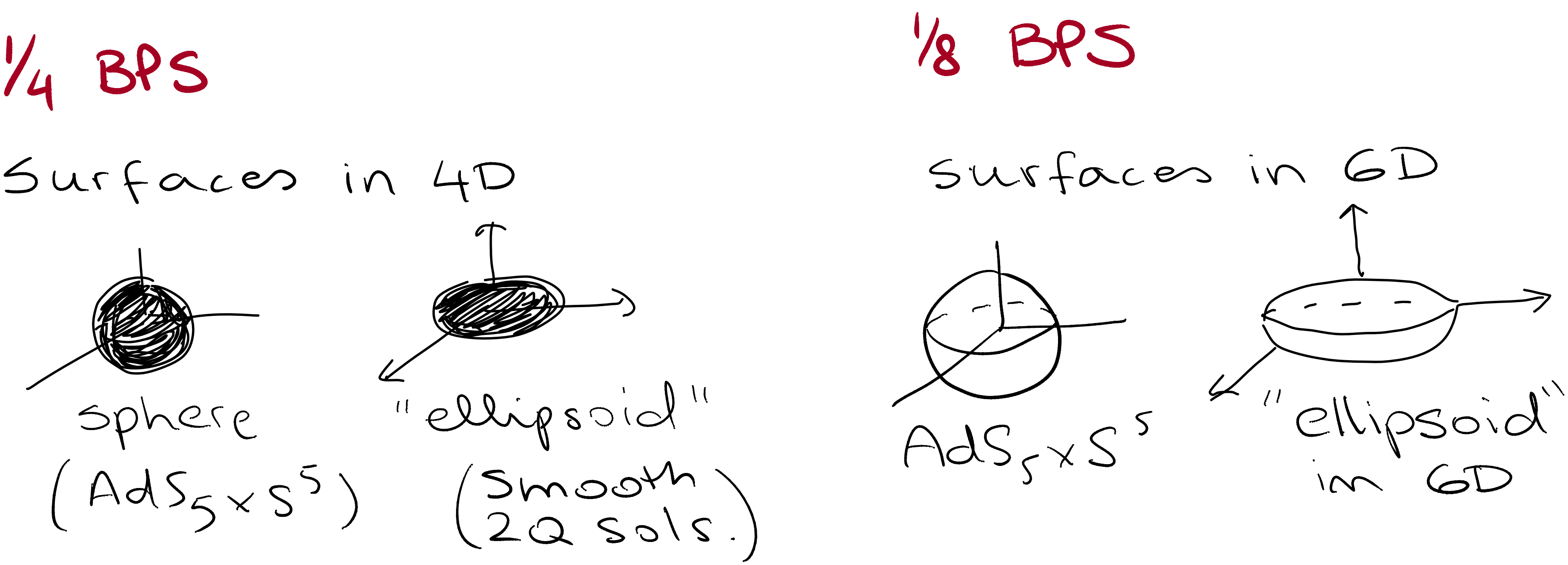
Natural Question:

Is there an analog of the "special" 2D plane of the ½ BPS solutions, on which **boundary conditions** (for regularity) would be defined?

The answer is **yes** for examples worked out:

- We **embedded many known SUGRA solutions** into ¼ and 1/8 BPS general geometries
- Found some new solutions for simplifying assumptions on K
- Found **relevant boundary conditions**

Picture:



Questions still open:

- Can you draw any shape in these 4D and 6D spaces and get a UNIQUE geometry?
- What do we have on gauge theory side? Fermions?

How can this be useful?

- Push forward AdS/CFT duality with **less SUSY** (and more general geometries)
- Can we understand **more realistic** gravity/gauge theory dualities starting from the more "controlled" setting of AdS/CFT? (dS/CFT?)
- Can we learn anything about **time dependence**? Hard question
- Black hole applications

Future Applications ?

We saw LLM describes many vacua of the theory:

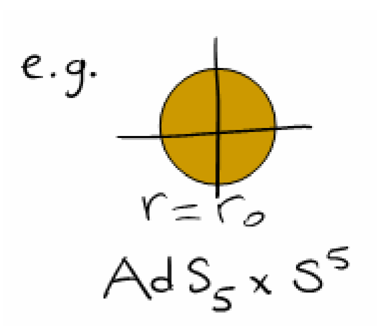
- **Instanton solutions** interpolating between different LLM vacua?

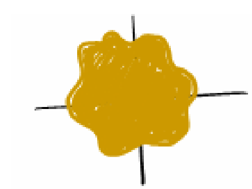
(recent work by H. Lin)

- Bubbles merging or separating (**topology change**)?

Can we make **bubbles fluctuate in time**?

Yes, but small fluctuations not necessarily new (just spectrum)





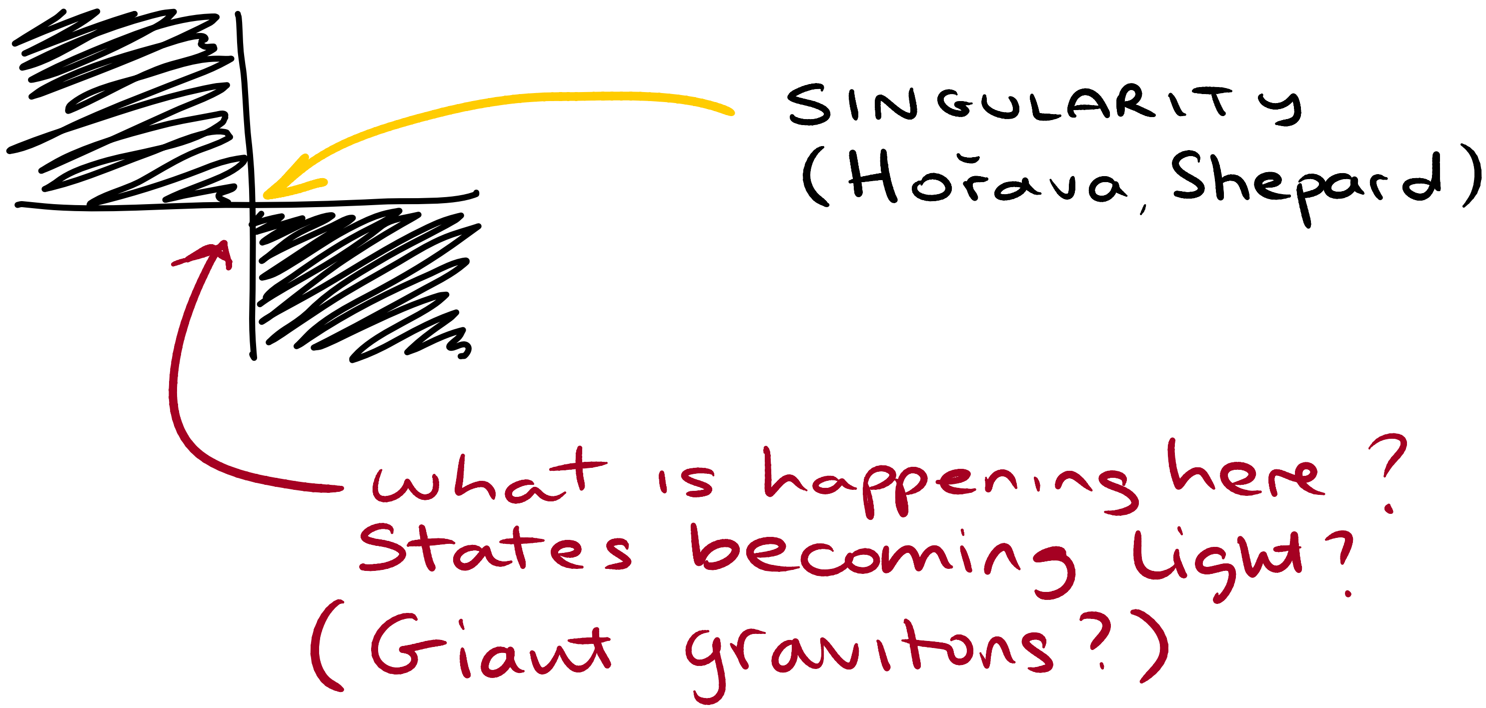
$$r(\phi,t) = \sum_n a_n(t)\, e^{in\phi}$$

⇓

Known spectrum of $AdS_5 \times S^5$ (Kim et al.)

Topology change: can bubbles split or recombine?

If yes, at transition they would locally look like:



Can we resolve the singularity? Can we describe topology change?

Conclusions…

- Although AdS/CFT is still a conjecture, much progress recently
- **Nice fermion** (bubbling) **picture** for ½ BPS SUGRA solutions
- **Interactions** in bulk are challenging, but **symmetries** may help (they give useful identities for generating **multi-matrix** interactions)
- **Bubbling** picture may survive with less SUSY
- Future directions:
  - Can we make any progress **beyond static solutions**? Connect with cosmology work?
  - Can we understand whether bubbles can merge and separate?
  - Can we go **from one vacuum to the other** (possibly relevant for cosmology)?