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Stable violation of the null energy condition and non-standard cosmologies

hep-th/0606090

with M. Luty, A. Nicolis and L. Senatore

What is the NEC?

Energy conditions:
- Singularity theorems
- Entropy bounds

For every null vector *n*: $T_{\mu\nu} n^\mu n^\nu \geq 0$

If $\Lambda \neq 0$ this is the only tenable energy condition

E.g. Strong energy condition.
For every unit time-like vector: $T_{\mu\nu}\xi^\mu\xi^\nu \geq -\frac{1}{2}T$ $\quad \left(\Rightarrow R_{\mu\nu}\xi^\mu\xi^\nu \geq 0 \right)$

Is violated by a positive c.c.: $T_{\mu\nu} = -\Lambda g_{\mu\nu}$

Cosmologically: $$T_{\mu\nu} = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$$

It forbids inflation!

Strong energy condition: $\rho \geq -\frac{1}{2}(-\rho + 3p) \quad \Rightarrow \quad \rho + 3p \geq 0 \quad \Rightarrow \quad \ddot{a} \leq 0$

Cosmological implications of NEC

In a FRW Universe: $T_{\mu\nu}n^\mu n^\nu \geq 0 \quad \Rightarrow \quad \rho + p \geq 0$ $\qquad w \equiv \frac{p}{\rho} \qquad w \geq -1$

$\nabla_\mu T^{\mu 0} = 0 \quad \Rightarrow \quad \dot\rho = -3H(\rho + p)$

In a spatially flat Universe: $\text{NEC} \quad \Rightarrow \quad \dot H \leq 0$

NEC says the energy density (and therefore H) decreases while the Universe expands

If ~~NEC~~ :

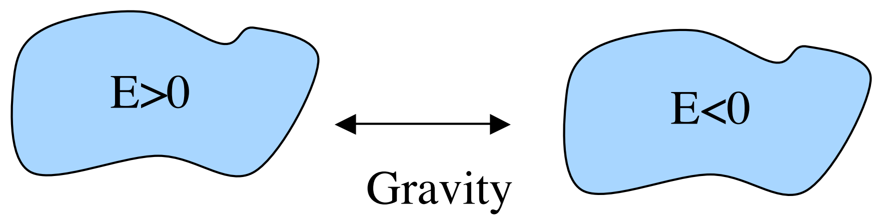
- No need for a Big Bang.
- One can even have $H \to 0$ in the far past: we can start the Universe !
- Bouncing cosmologies. H must flip from negative to positive: $\dot H > 0$
- Observation of w < -1 in the present acceleration (present bound w > -1.1)
  (Einstein frame metric + not a fake super-acceleration)

What is wrong with NEC?

Typically a ~~NEC~~ theory suffers from instabilities.

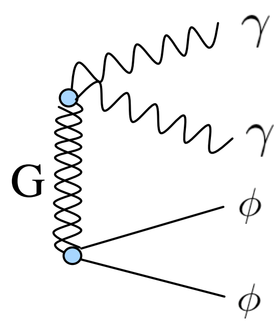
E.g. States with negative energy (ghosts) will violate it.

Classical instability:



Can we make sense of it as an EFT with a cut-off?

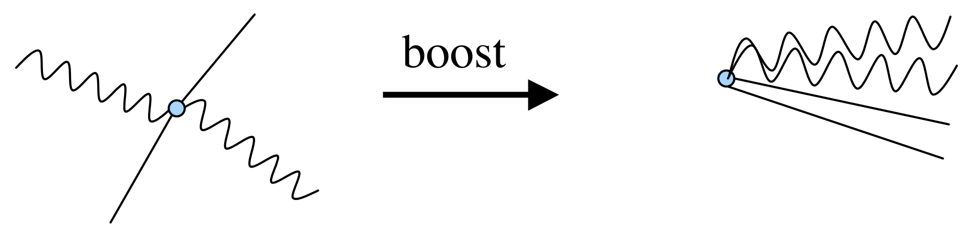
Vacuum decay:



By dim. analysis: $\Gamma \sim \frac{\Lambda^8}{M_P^4}$

Exp. limit: $\Lambda < 3\ \mathrm{MeV}$

The cut-off cannot be Lorentz invariant:



Rather general argument

Dubovsky, Gregoire, Nicolis, Rattazzi, hep-th/0512260

Consider a scalar Lagrangian: $\mathcal{L} = \Lambda^4 F\left(\epsilon\frac{\Phi_I}{\Lambda}, \frac{\partial^\mu \Phi_I \partial_\mu \Phi_J}{\Lambda^4}, \frac{\partial^2 \Phi_I \partial^2 \Phi_J}{\Lambda^6} \ldots\right)$

Around a background: $g^B_{\mu\nu}$, Φ^B with: $|\partial_\mu| \ll \Lambda$

In the regime: $\frac{1}{\Lambda} \ll r \ll r_G = \frac{M_P}{\Lambda^2}$; $\frac{1}{\epsilon\Lambda}$

- Flat space limit: neglect mixing with gravity
- Neglect higher derivative and potential
- Linearized background: $\Phi^B_I = A^\mu_I x_\mu$

One <u>always</u> gets either gradient or ghost instabilities,
in the Goldstones of broken space-time translations,
i.e. sound waves.

(assuming no super-luminality)

NEC and modification of gravity

Usually mixing with gravity can be neglected if I concentrate on a ~ flat region

E.g. Conventional scalar field

Mixing: $$h_{\mu\nu}\delta T^{\mu\nu} \sim h\,\dot{\phi}^B \partial\varphi \sim M_P H h \partial\varphi$$

We do not say we modified gravity: mixing is only relevant when space is curved

Modification of gravity: non-trivial background with $T_{\mu\nu} \simeq 0$

$\Longleftrightarrow$ Background at the edge of violating NEC

$\Longleftrightarrow$ Degenerate dispersion relations (see Pauli-Fierz massive gravity, ghost condensation…)

$\Longrightarrow$ (Healthy) theories of massive gravity are a good starting point for ~~NEC~~

Ghost condensation

Arkani-Hamed, Cheng, Luty, Mukohyama, hep-th/0312099

Scalar with shift symmetry: $\mathcal{L} = \sqrt{-g}\, M^4 P(X)\ ,\qquad X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

In an expanding Universe, we expect $\dot\phi \to 0$.

Another possibility: $\phi = c\, t \qquad P'(c^2) = 0 \qquad (\text{with}\ \ P'' > 0)$

Stress energy tensor is the one of c.c. : background without curvature. Edge of NEC.

Perturbations around background: $\phi(t,\vec{x}) = t + \pi(t,\vec{x})$

No standard spatial kinetic term. Degenerate dispersion relation.

I need to consider higher order terms: e.g. $(\Box\phi)^2$

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M^4\dot\pi^2 - \frac{1}{2}\bar{M}^2(\nabla^2\pi)^2 - \frac{1}{2}M^4\dot\pi(\nabla\pi)^2 - \Lambda + \cdots\right]$$

… and its deformation

Softly break the shift symmetry with a linear potential

$$V = V'\phi \; , \qquad V' = \text{const.}$$

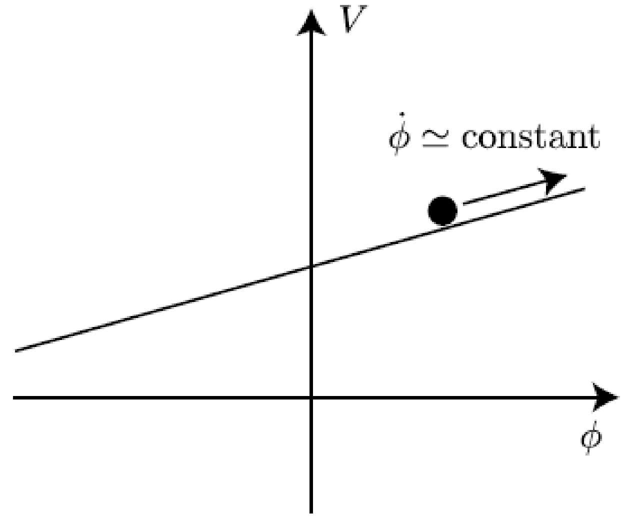
Slightly change the background solution:

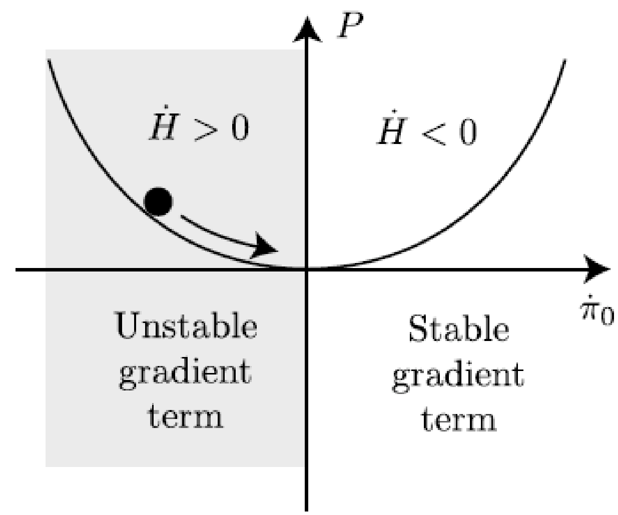
$$\ddot{\pi}_0 + 3H\dot{\pi}_0 + \frac{1}{M^4}\frac{dV}{d\phi} = 0$$

For $\dot{H} \lesssim H^2$ $\qquad \dot{\pi}_0 = -\frac{V'}{3M^4 H}$

Asymptotically: $H^2 \simeq \frac{V(\phi)}{3M_P^2} \propto t$

$$\dot{\pi}_0 \simeq -\frac{V'}{3H(t)M^4} \propto \frac{1}{\sqrt{t}}$$





From the general argument we expect pathologies:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M^4 \dot{\pi}^2 + \underline{\underline{\dot{H} M_P^2 (\nabla \pi)^2}} - \frac{1}{2} \bar{M}^2 (\nabla^2 \pi)^2 \right]$$

Stability of the system

Instabilities are relevant only if their rate is faster than Hubble.

Neglecting mixing with gravity: $\omega^2 + \frac{2\dot{H} M_P^2}{M^4} k^2 - \frac{\bar{M}^2}{M^4} k^4 = 0$

$$\omega^2_{\rm grad} \sim - \left(\frac{\dot{H} M_P^2}{\bar{M} M^2} \right)^2 \Longrightarrow \frac{\dot{H}}{H} \lesssim \frac{\bar{M} M^2}{M_P^2}$$

Mixing with gravity gives a second instability. ~ Jeans instability.

$$\frac{1}{2} M^4 (\Phi - \dot{\pi})^2 \qquad \omega^2 + \frac{\bar{M}^2}{2 M_P^2} k^2 - \frac{\bar{M}^2}{M^4} k^4 = 0$$

$$\omega^2_{\rm Jeans} \sim - \left(\frac{\bar{M} M^2}{M_P^2} \right)^2 \Longrightarrow \frac{\bar{M} M^2}{M_P^2} \lesssim H$$

The two instabilities push in opposite direction:

$$\omega_{\rm grad}\, \omega_{\rm Jeans} \sim \dot{H} \qquad \frac{\dot{H}}{H} \lesssim \frac{\bar{M} M^2}{M_P^2} \lesssim H$$

Parametric window only for $\dot{H} \ll H^2$

A systematic approach

EFT around a given FRW background $a(t)$, $\psi(t)$. Focus on the perturbation

$$\delta\psi_m(x) \equiv \psi_m(t+\pi(x)) - \psi_m(t) \ ,$$

Goldstone of broken time-translation

- This mode can be reabsorbed in the metric going to unitary gauge
- In this gauge, only gravity with time-dependent spatial diff. : $x^i \to x^i + \xi^i(\vec{x},t)$
- The Goldstone π can be reintroduced with Stuckelberg trick
- ADM variables are useful: $N \equiv 1/\sqrt{g^{00}}$ $\quad N_i \equiv g_{0i}$ $\quad \hat{g}_{ij}$

$$ds^2 = -N^2 dt^2 + \hat{g}_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$S_{\rm EH} = \frac{1}{2} M_{\rm Pl}^2 \int d^4x \sqrt{-g}\, R = \frac{1}{2} M_{\rm Pl}^2 \int d^3x\, dt\, \sqrt{\hat{g}}\, [N R^{(3)} + \frac{1}{N}(E^{ij}E_{ij} - E^i{}_i{}^2)]$$

$$E_{ij} \equiv N K_{ij} = \frac{1}{2}[\partial_t \hat{g}_{ij} - \hat{\nabla}_i N_j - \hat{\nabla}_j N_i]$$

- Most generic action invariant under spatial diff. + reintroduce the Goldstone

E.g. $\quad \dfrac{1}{N^2} = g^{00} \mapsto \dfrac{1}{N^2} + 2\,\partial_0 \xi^0 - (\partial_\mu \xi^0)^2$

Leading order in derivative

Tadpole terms to make the given a(t) a solution: $\delta N \equiv N - 1,\ \Lambda$

Coefficients can be time dependent: $S_{\rm matter} = \int d^4x\, \sqrt{-g}\left[c(t)\frac{1}{N^2} - \Lambda(t)\right]$

E.g. this is all for a standard scalar field: $S_\phi = \int d^4x\, \sqrt{-g}\left[-\frac{1}{2}(\partial\phi)^2 - V(\phi)\right] = \int d^4x\, \sqrt{-g}\left[\frac{1}{2N^2}\dot\phi^2 - V(\phi(t))\right]$

Fix the coefficients from the background:

$$S_{\rm matter} = \int d^4x\, \sqrt{-g}\left[-M_{\rm Pl}^2\dot H\frac{1}{N^2} - M_{\rm Pl}^2(3H^2+\dot H) + \cdots\right]$$

Reintroducing π

$$S_{\rm matter} \to S_\pi = \int d^4x\, \sqrt{-g}\,(M_{\rm Pl}^2\dot H)(\partial\pi)^2$$

For ~~NEC~~ we get a ghost!

At 0th derivative level we can only add: $\int d^4x\, \sqrt{-g}\,\frac{1}{2}M^4(t)\,(\delta N)^2 \implies S_\pi \to S_\pi + \int d^4x\, \sqrt{-g}\,\frac{1}{2}M^4\,\dot\pi^2$

Ghost instability fixed, but gradient term is still wrong!

Higher derivative terms

Extrinsic curvature: $\delta E_{ij} \equiv E_{ij} - E_{ij}^{(0)}$

$$\int d^4x \sqrt{-g} \left[-\frac{1}{2}\tilde{M}^2 \, \delta E^i{}_i{}^2 - \frac{1}{2}\tilde{M}'^2 \, \delta E^{ij} \delta E_{ij} \right] \to \int d^4x \sqrt{-g} \left[-\frac{1}{2}\overline{M}^2 \, \frac{1}{a^4}(\partial_i^2 \pi)^2 \right]$$

$$S_\pi = \int d^4x \sqrt{-g} \left[\left(\frac{M^4}{2} - M_{\rm Pl}^2 \dot{H} \right) \dot{\pi}^2 + (M_{\rm Pl}^2 \dot{H}) \, \frac{1}{a^2} (\partial_i \pi)^2 - \frac{\overline{M}^2}{2} \, \frac{1}{a^4} (\partial_i^2 \pi)^2 \right]$$

- This is the action we had in the deformed ghost condensation example
- Gradient instability gets smaller and smaller for large $\bar{M}$
- Jeans instability gets enhanced by $\bar{M}$. Mixing is unrelated to curvature (usually Jeans time ~ H^{-1})

The two instabilities push in opposite direction:

$$\omega_{\rm grad} \, \omega_{\rm Jeans} \sim \dot{H} \qquad \frac{\dot{H}}{H} \lesssim \frac{\bar{M} M^2}{M_P^2} \lesssim H$$

Parametric window only for $\dot{H} \ll H^2$

Everything is explicit in a full gravitational analysis

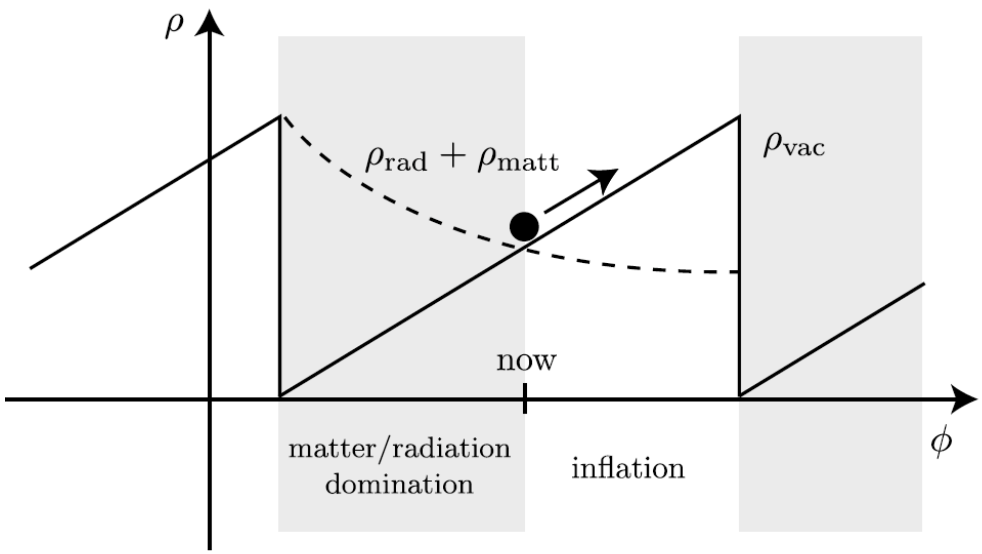
Applications (~ everything is allowed!)

- Today's acceleration with w < -1. Long wavelength instability?

- Start the Universe. It approaches flatness in the asymptotic past: $H \to 0$ for $t \to -\infty$

- Eternally expanding Cyclic Universe.
  Our present acceleration is the same as primordial inflation!

  a) w now related to the tilt of the spectrum
  b) Blue GWs spectrum.
  Direct evidence of $\dot{H} > 0$.



- Bouncing cosmology. As H=0 at the bounce one has to check for instability. Ok for a sufficiently fast bounce.

One can build a cyclic/ekpyrotic scenario in which the bounce is under control.
(and check that the spectrum of density perturbation is wrong)

Conclusions and questions

- NEC can be violated without catastrophic instabilities
- Tension between gradient and Jeans instabilities: $\dot{H} \lesssim H^2$
- Completely new cosmological evolutions are possible

- Is is possible to avoid the tension $\dot{H} \lesssim H^2$?
- Has the theory a UV completion in field theory?
- Can string theory violate NEC? Or these theories lie in the swampland?