Cosmology with Warped String Compactification

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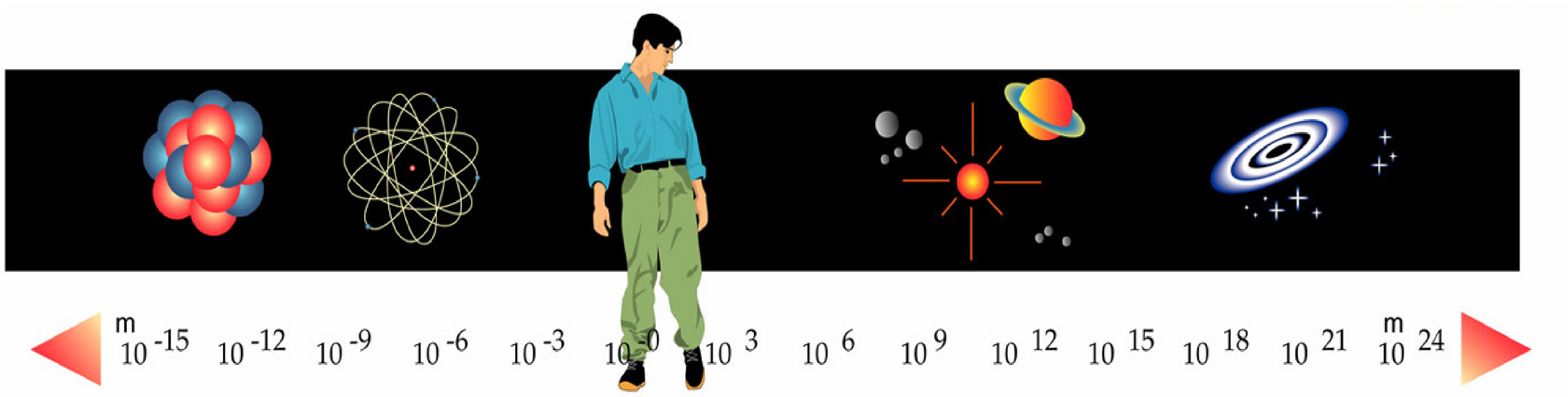
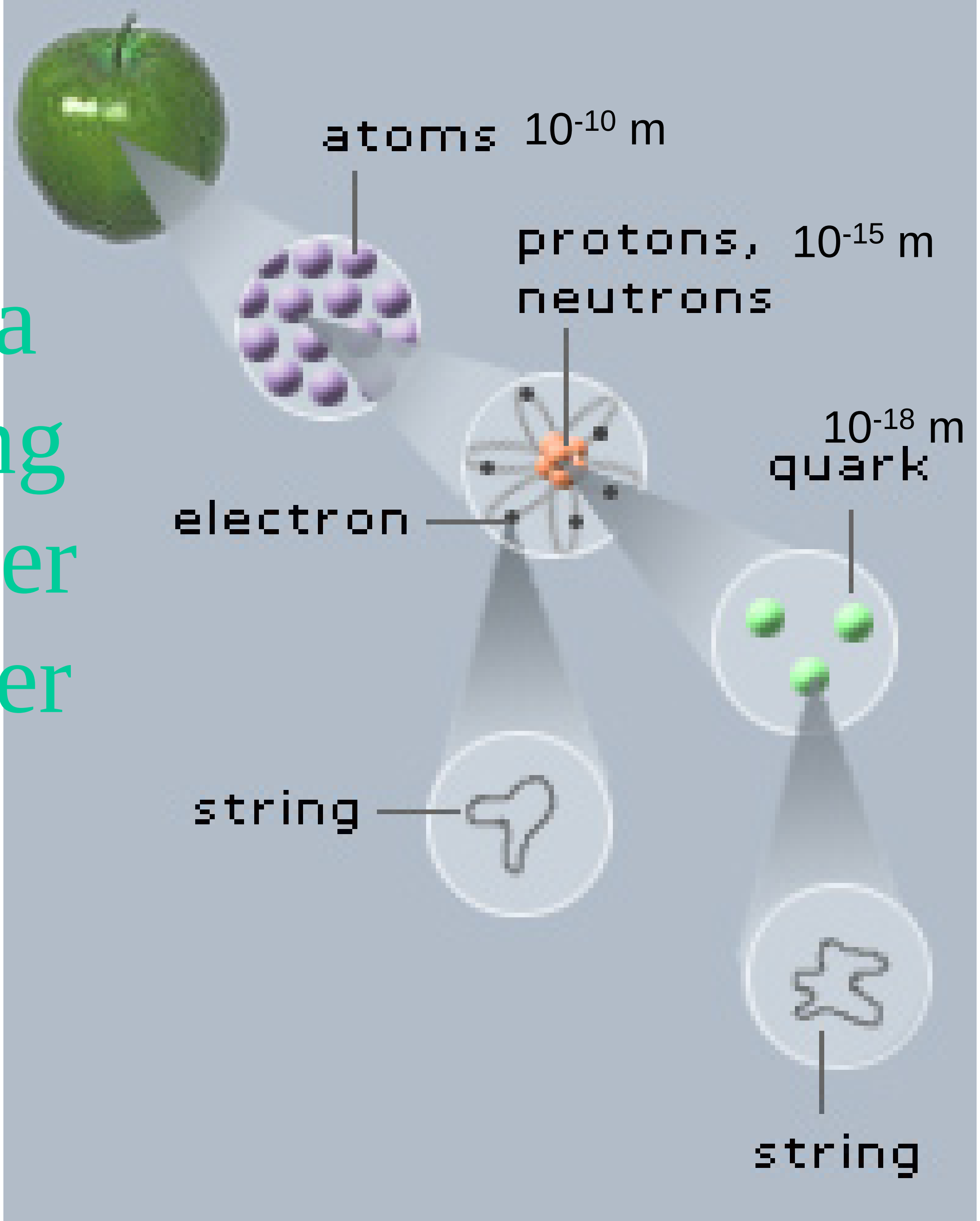
There are Frontiers in Physics:



at Short and Long Scales

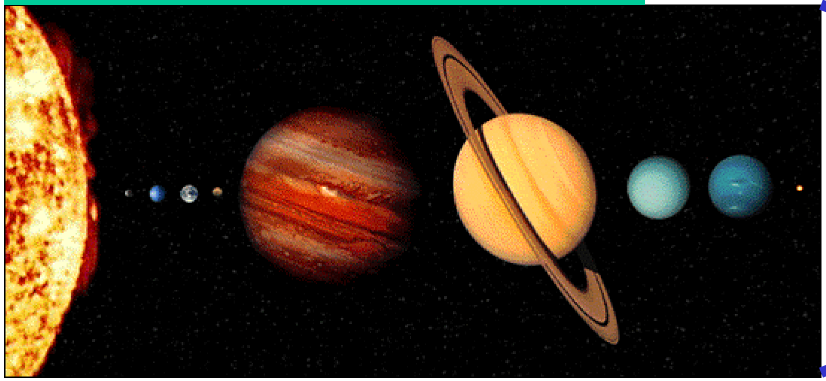
There is a story going into smaller and smaller scales.



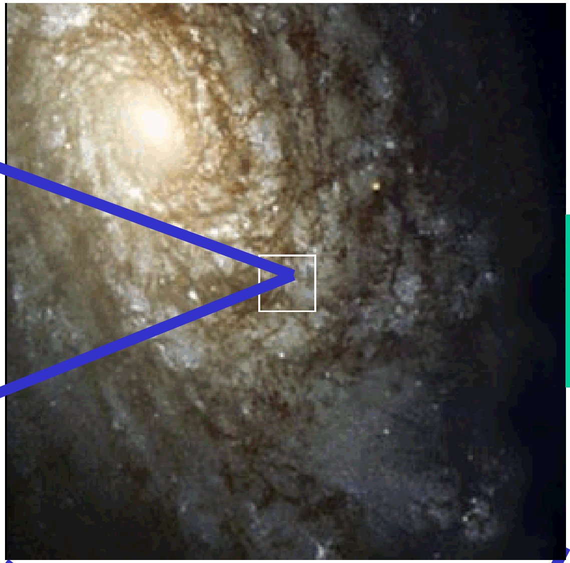
Also toward Larger scales

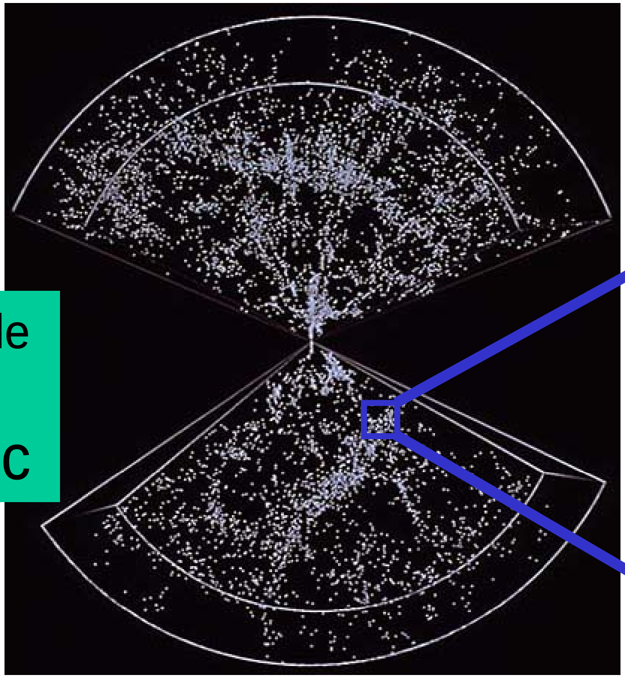
(pc = 3.3 light year=3.1 × 10^{18} cm)

Solar system 10^{15}cm

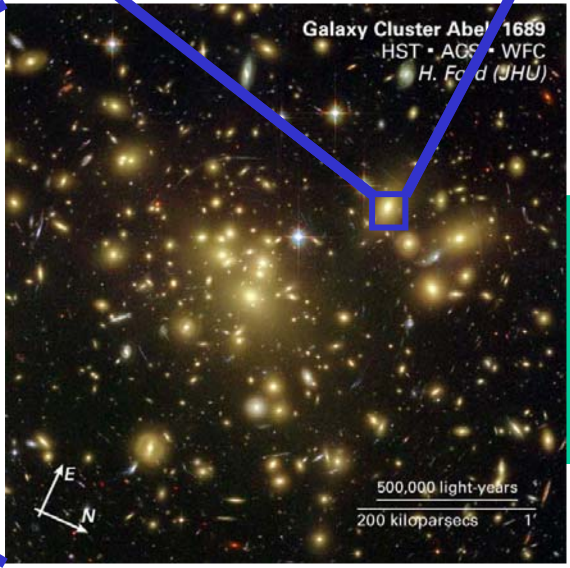


Galaxy
10 kpc





Large scale structure
100 Mpc

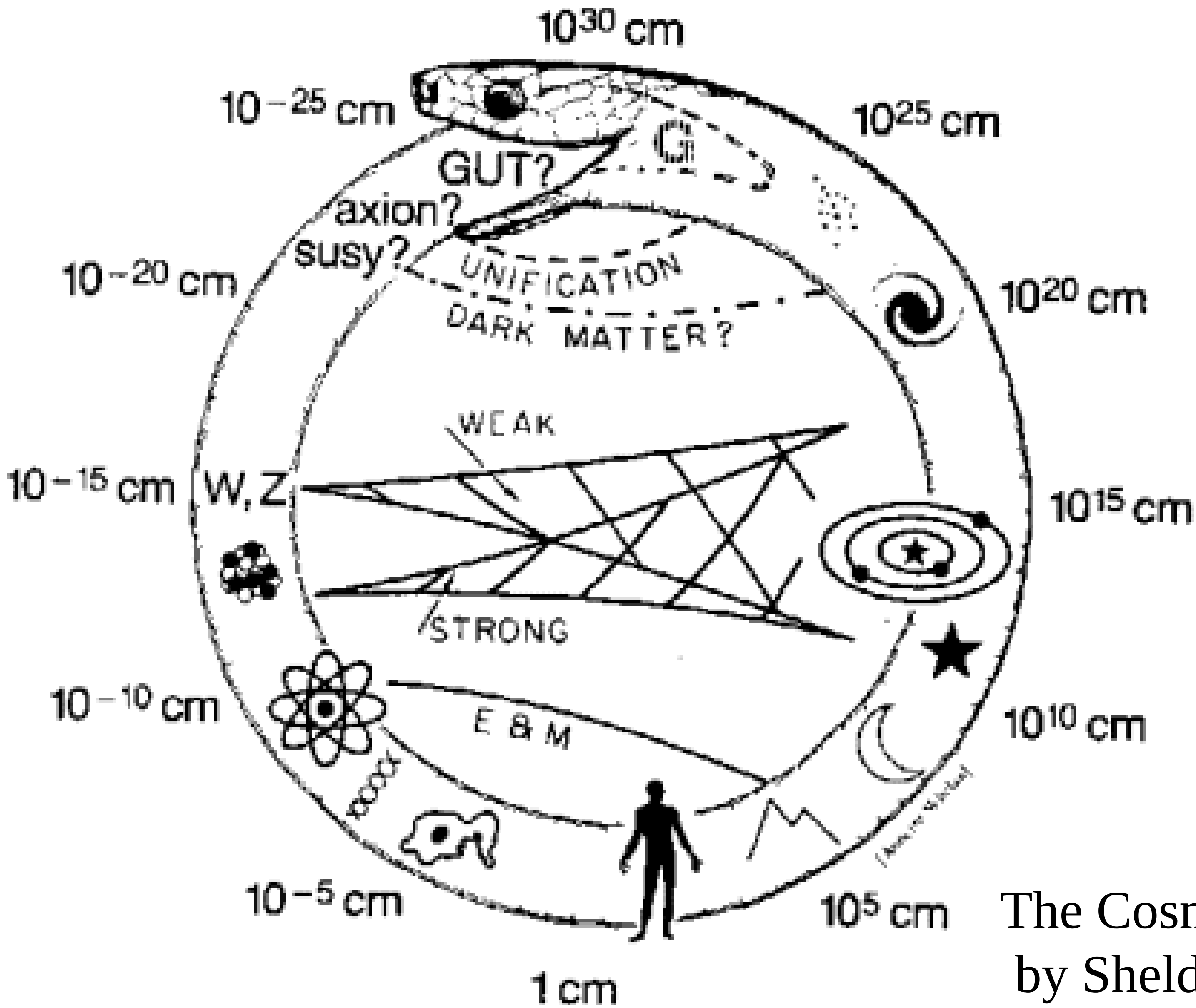


Cluster of galaxies
Mpc

At largest scale:

Cosmology

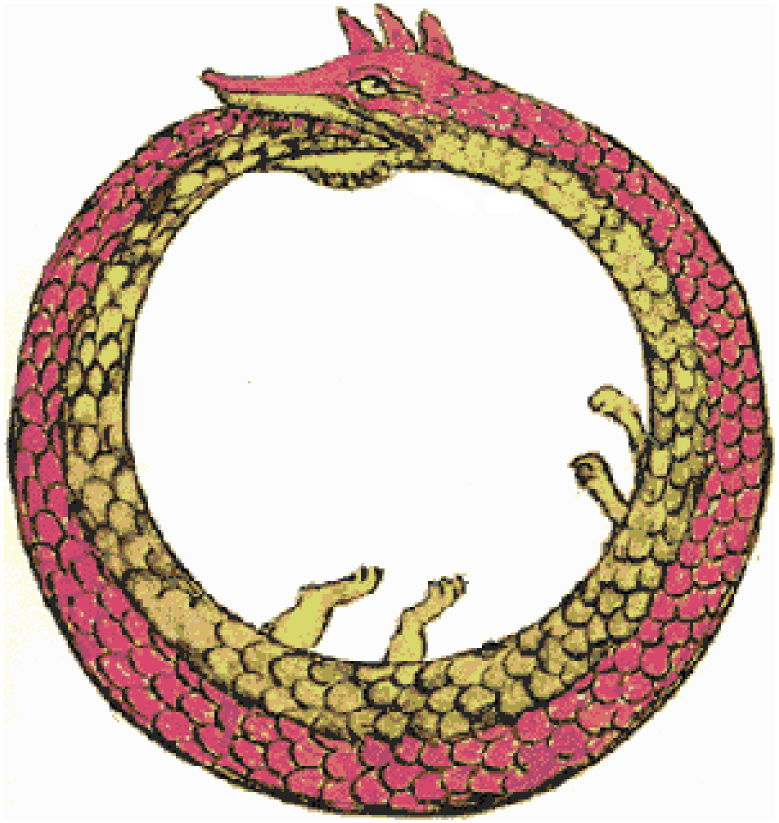
But...the largest scale and shortest scale must be connected!



The Cosmic Uroboros
by Sheldon Glashow

The Uroboros

http://fact-archive.com/encyclopedia/Uroboros



Snake growing by swallowing its tail !

The Ouroboros

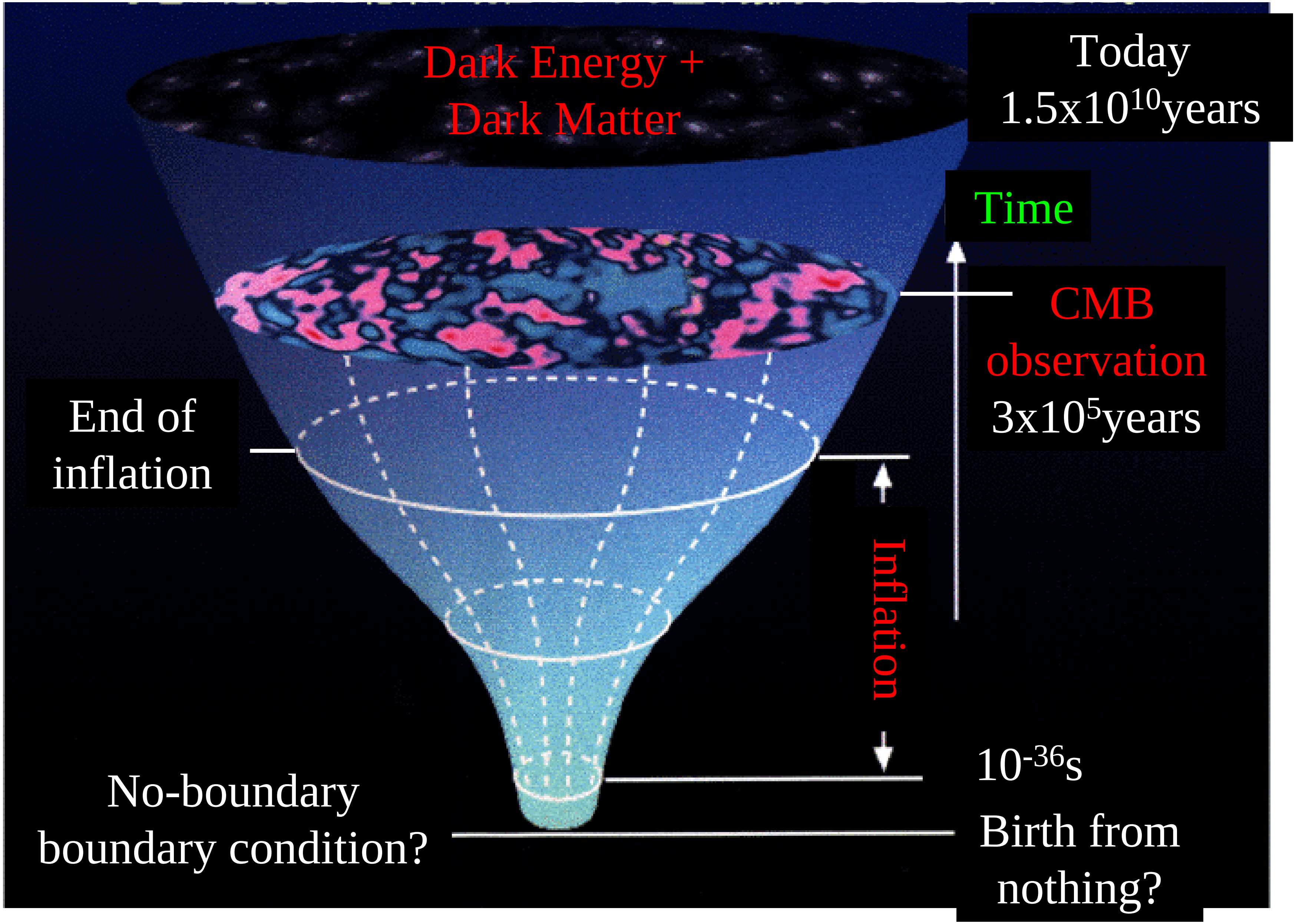
Alternate spellings: Oroborus, Uroboros, Uroborus

The **Ouroboros** is an ancient symbol depicting a snake or dragon swallowing its tail, constantly creating itself and forming a circle. It is associated with alchemy, Gnosticism, and Hermeticism. It represents the cyclical nature of things, eternal return, and other things perceived as cycles that begin anew as soon as they end. In some representations the serpent is shown as half light and half dark, echoing the dichotomy of other similar symbols such as the Yin Yang. The ouroboros is an example of tail recursion and self-reference, though not in a programming context.

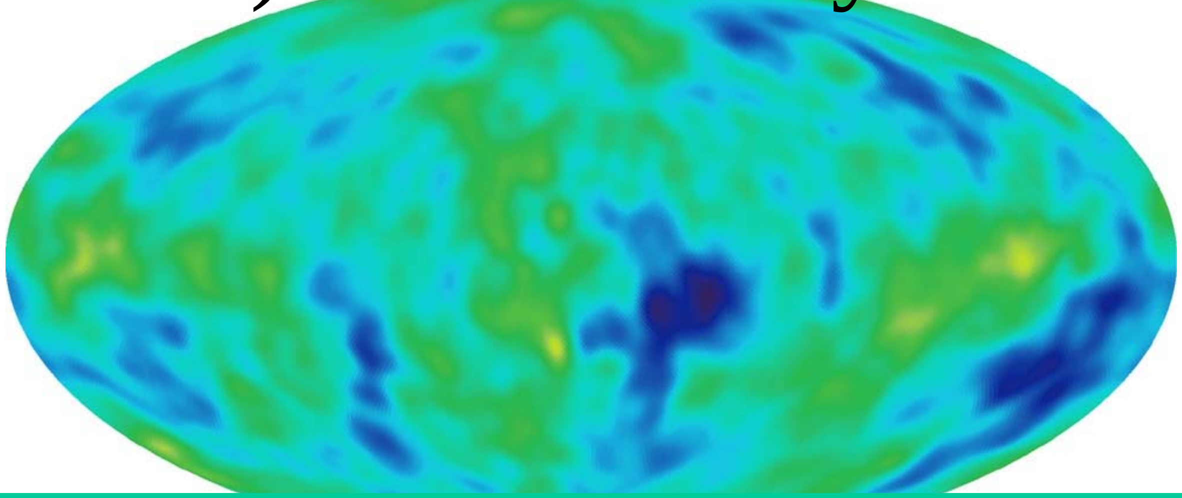
In alchemy, the ouroboros symbolises the circular nature of the alchemist's opus which unites the opposites: the conscious and unconscious mind.

It is believed to have been inspired by the Milky Way, as some ancient texts refer to a serpent of light residing in the heavens.

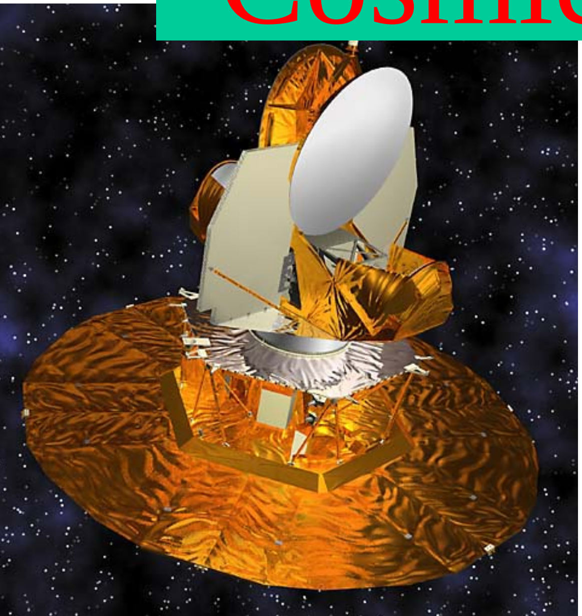
History of the Universe

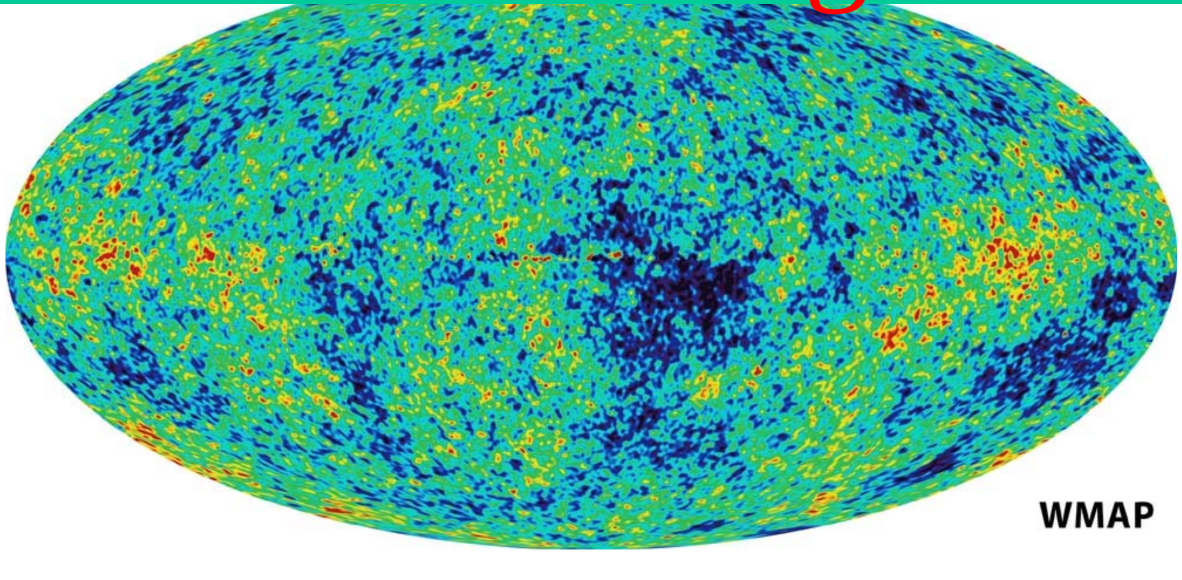


Inflation, dark energy and dark matter are (almost) confirmed by



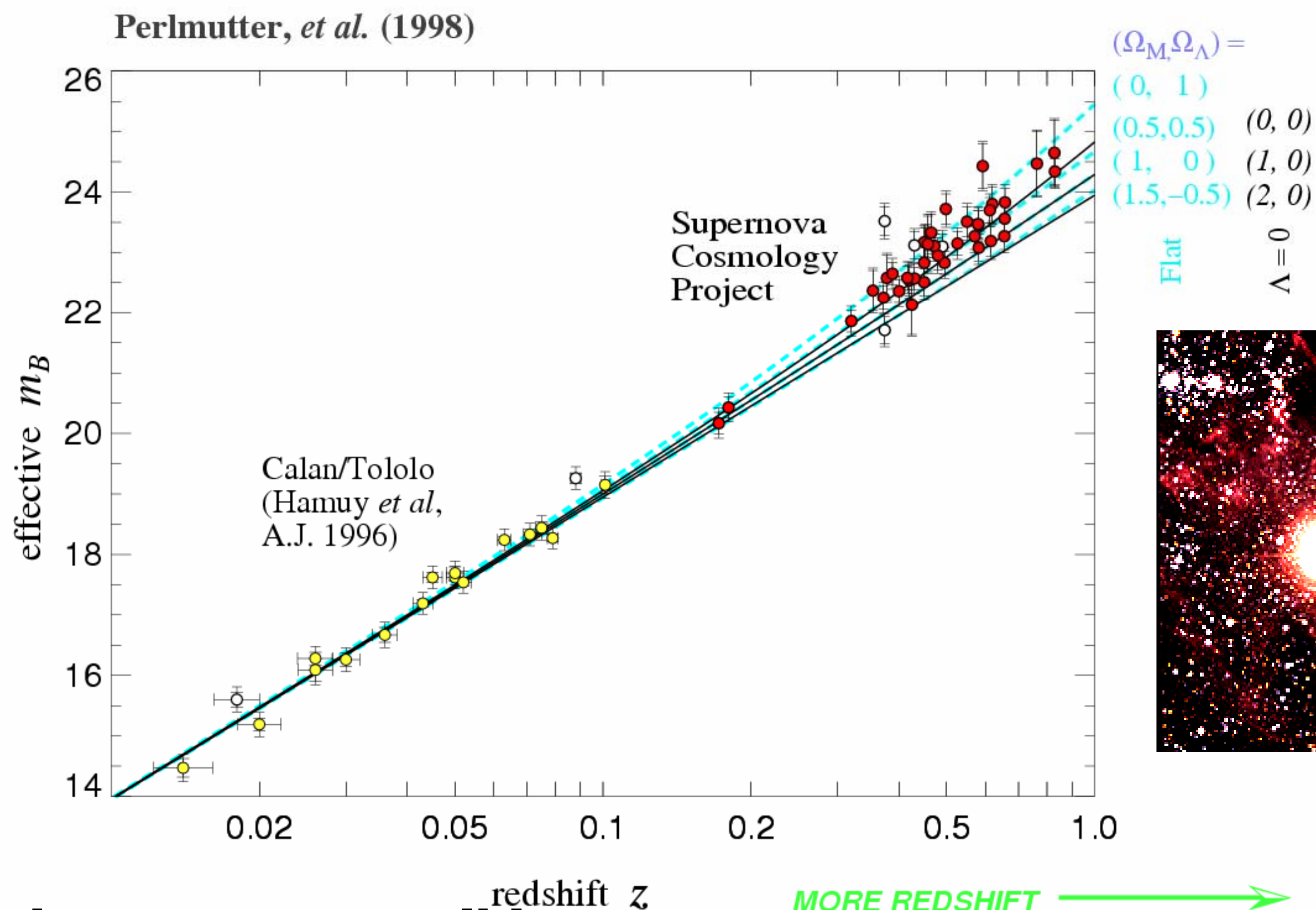
Cosmic microwave background

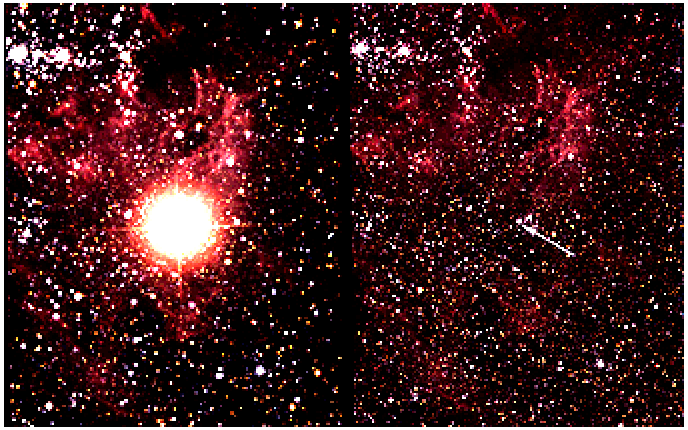




& Supernava observation

FAINTER
(Farther)
(Further back in time)





http://supernova.lbl.gov/

MORE REDSHIFT
(More total expansion of universe since the supernova explosion)

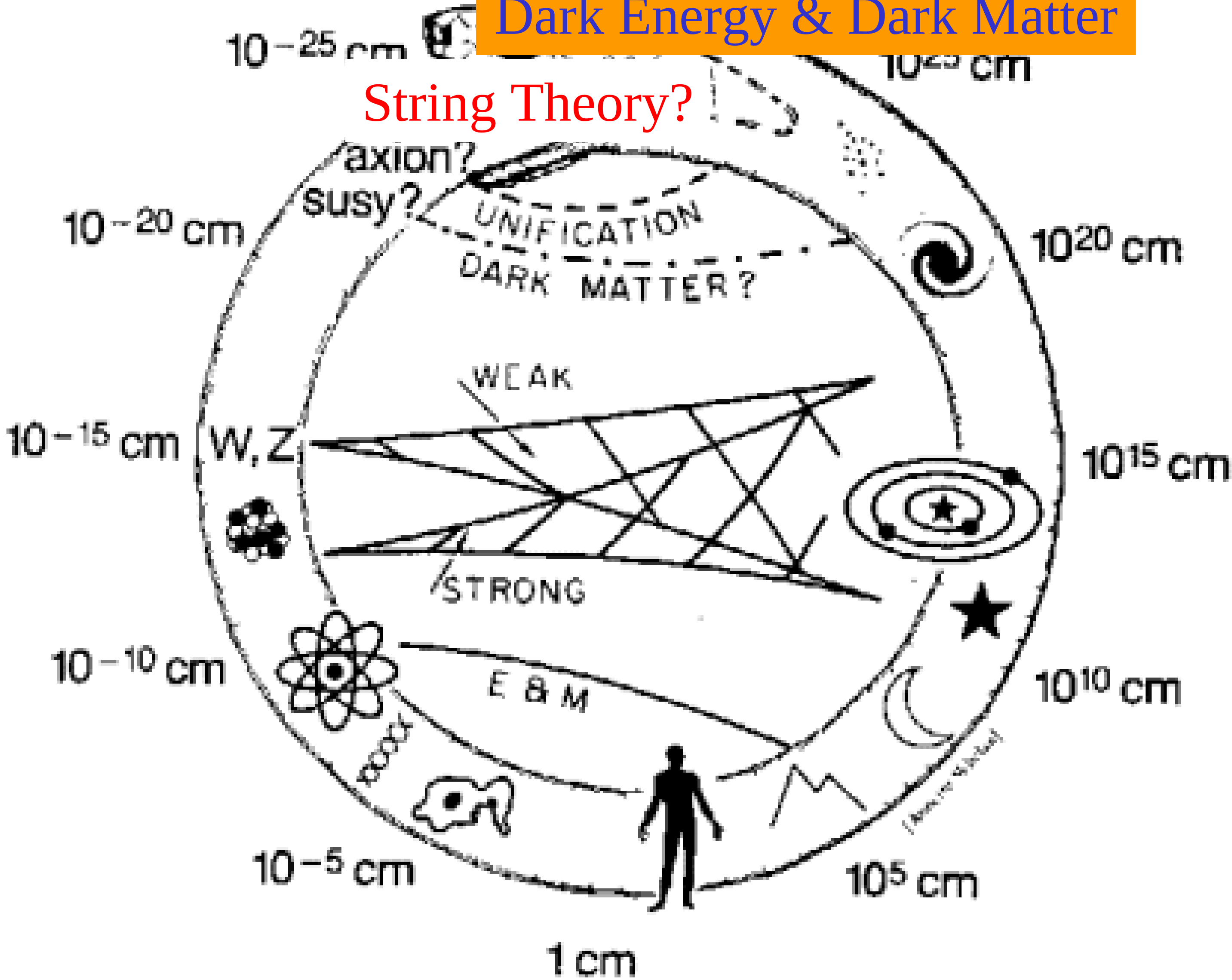
Three major mysteries in modern cosmlogy

- Early Universe
  Inflation
- Universe Today
  Dark Energy & Dark Matter

Two major (quasi-) de Sitter phases

We know they are (or were) there...
But, we don't know what they are.

Three mysteries: Inflation, Dark Energy & Dark Matter



The Cosmic Uroboros by Sheldon Glashow

Unified Theory (Candidate): String Theory

Good things

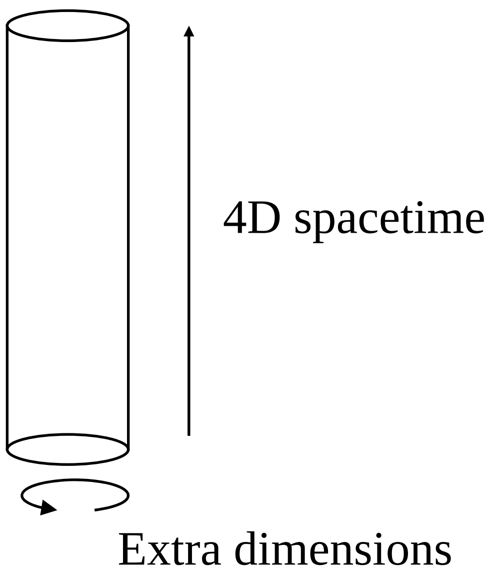
- Different particles = different oscillation modes of a string: possibility to explain complicated and diverse phenomena by LESS ELEMENTS.
- Unified theory candidate including GRAVITY
- GOOD CONTROL of quantum corrections (at least perturbatively, partly non-perturbatively)

String theory

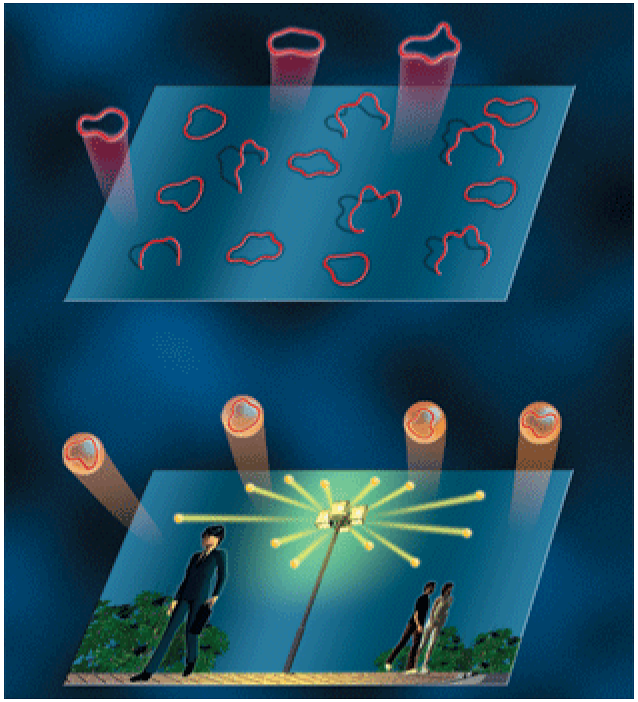
Something unusual

- Spacetime is 10 or 11 dimensional
- But, we know how to make those extra 6 or 7 dimensions invisible at low energy

Compactification



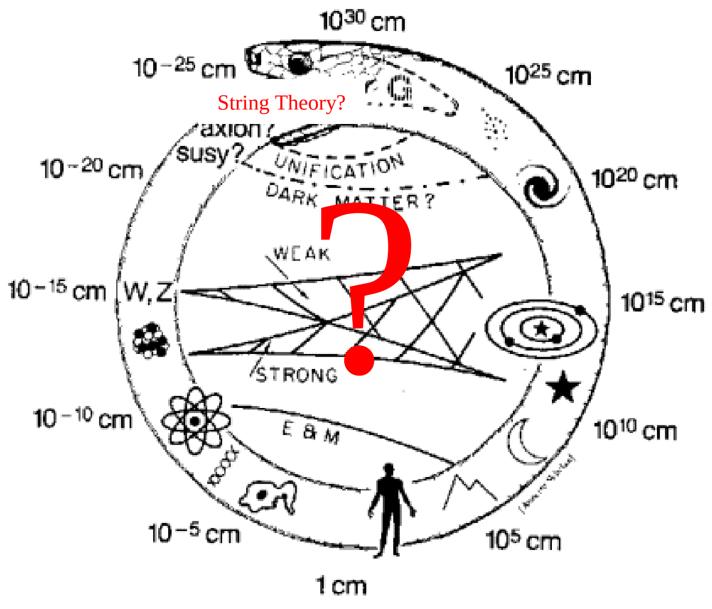
Brane world



String theory until 2002

Bad thing

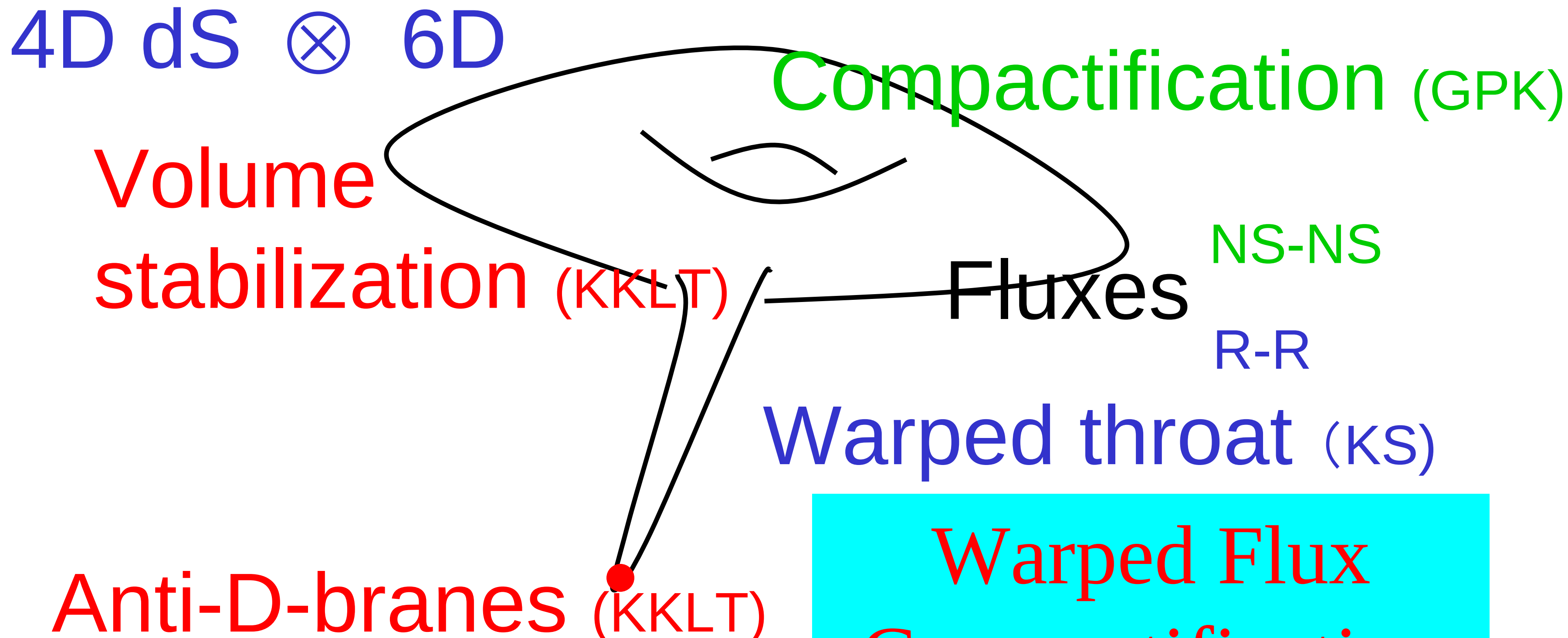
- 4-dimensional de Sitter solution with stabilized moduli had not been found.
- Even a no-go theorem had been proved!
- Contradict with inflation and dark energy?
- No way to reconcile with cosmology???



The Cosmic Uroboros does not close?

Recent Progress

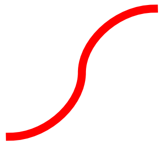
- In 2003, a 4-dimensional de Sitter solution was finally found! Kachru, Kallosh, Linde and Trivedi (KKLT)
- In the previous no-go theorem, branes (extended objects like membranes) were not taken into account.



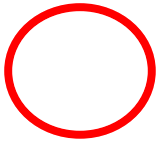
Warped Flux Compactification

What is anti-D-brane?

- There are two kinds of strings:



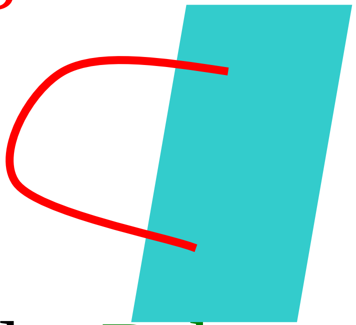
Open string
(Neumann b.c.)



Closed string
(Periodic b.c.)

- Actually, open strings can end on a surface:

Open string
(Dirichlet b.c.)



- This surface is called a D-brane and couples to a (p+1)-form potential, where p is the spacelike dimension of the surface.
- Anti-D-brane is also a surface on which open strings can end, but has the opposite charge.

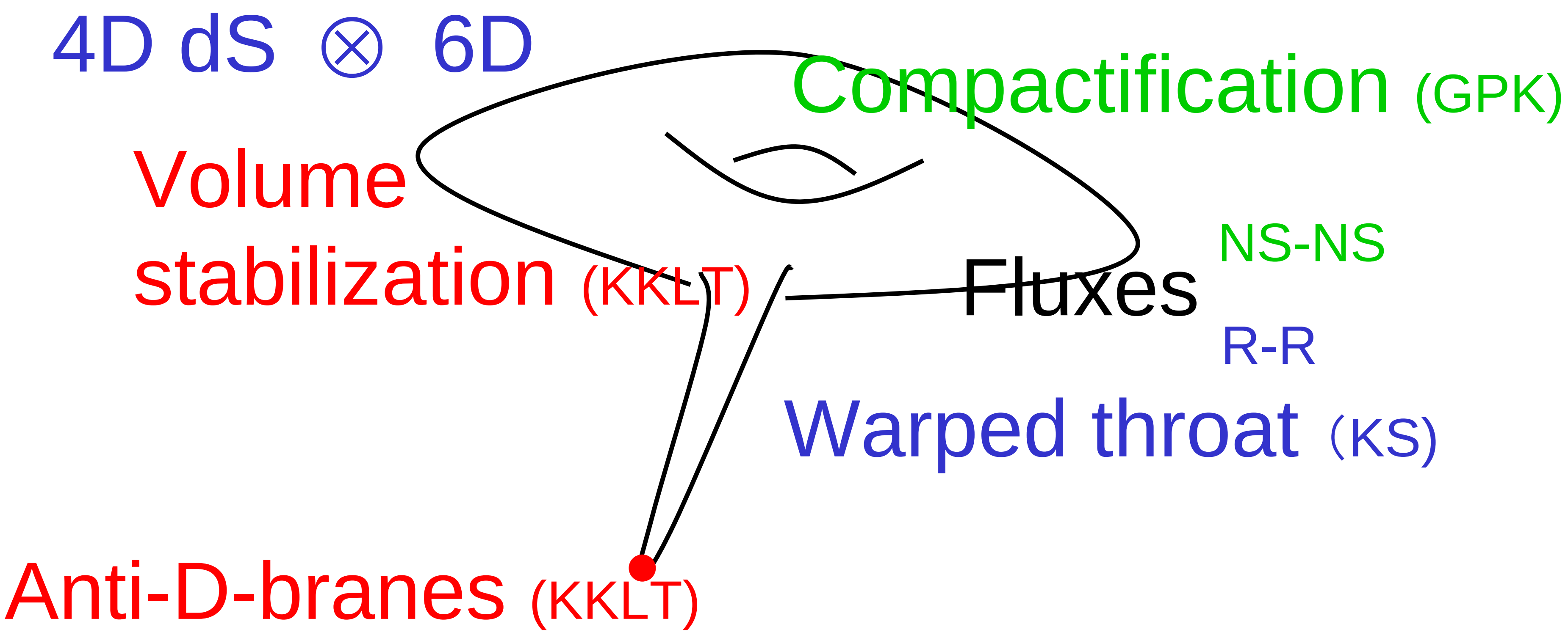
KKLT 4-dimensional de Sitter “solution”

- After stabilizing all moduli, anti-D-branes were introduced.
- Anti-D-branes are indispensable!
- Without anti-D-branes, 4-dimensional cosmological constant would be negative and completely contradicts with cosmology.

Anti-D-branes as Dark Matter

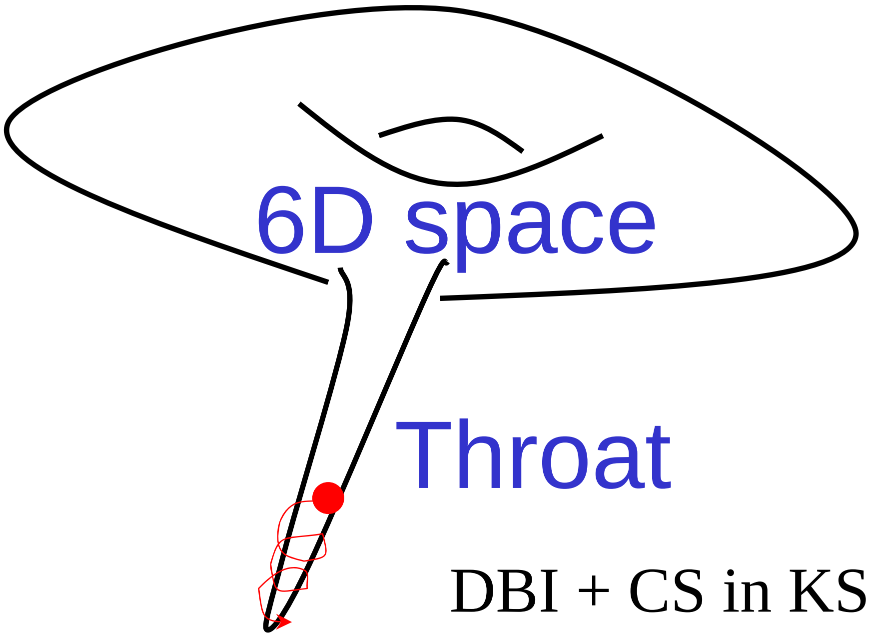
- **S.Mukohyama, "Anti-D-brane as Dark Matter in Warped String Compactification", Phys.Rev.D72, 061901 (2005) [hep-th/0505042].**

- What happens if anti-D-branes move in the extra 6 dimensions?



Anti-D-branes as Dark Matter

- Falls toward the bottom of the throat, with rotation in the extra 5 dimensions.
- Behaves as DARK MATTER, from 4-dimensional viewpoint.



$\otimes$ 4D universe

DBI + CS in KS

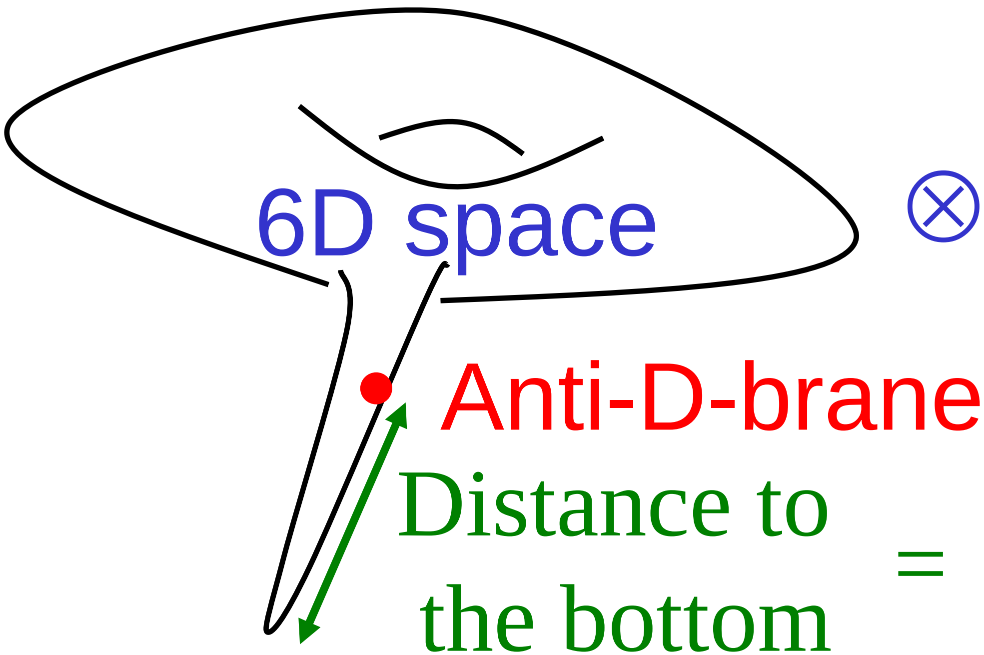
$$\rho \propto a^{-3}$$

Anti-D-brane motion = Dark Matter

Chaotic Inflation driven by brane motion

In progress, with Kallosh, Kofman and Linde

- Large motion of anti-D-brane
- In 4D, $V \sim \lambda \phi^4$



$\otimes$ 4D universe

= Inflaton

Anti-D3-brane action in KS geometry

DBI + CS

$$S_{\bar{D}3} = -T_3 \int d^4\xi e^{-\phi} \sqrt{-\det(G_{\alpha\beta} - B_{\alpha\beta})} - T_3 \int d^4\xi C_4$$

KS background + **Non-rel approximation**

$$\begin{aligned} S_{\bar{D}3} &= -T_3 \int d^4\xi \sqrt{-g^{(4)}} \left[\frac{\epsilon^{4/3}}{12K^2(\tau)} g^{(4)\alpha\beta} \partial_\alpha \tau \partial_\beta \tau + 2h^{-1}(\tau) \right] \\ &= -\int d^4\xi \sqrt{-g^{(4)}} \left[\frac{1}{2} g^{(4)\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi + V_{\bar{D}3}(\varphi) \right], \end{aligned}$$

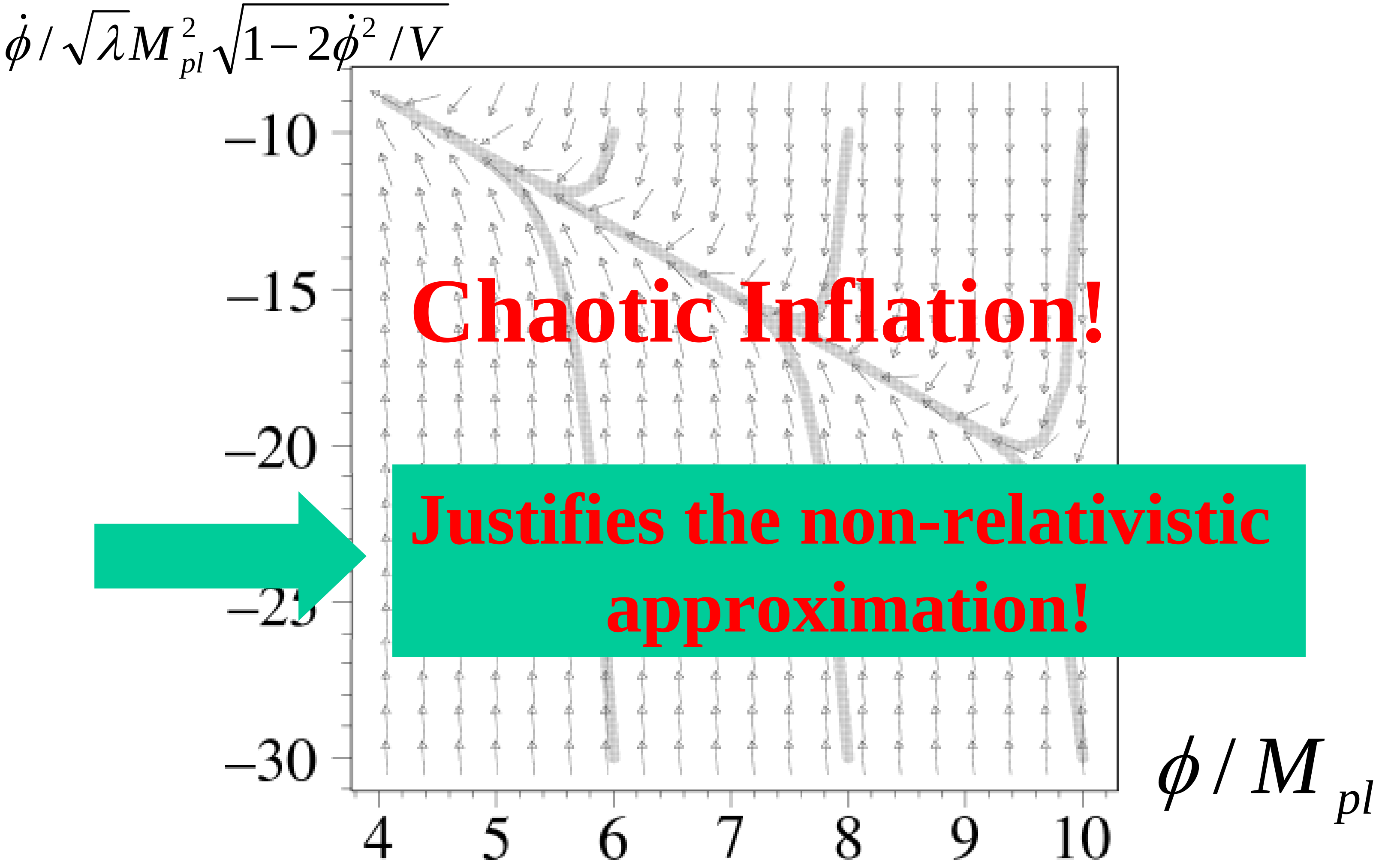
$$\varphi \equiv \epsilon^{2/3} \sqrt{\frac{T_3}{6}} \int_0^\tau \frac{d\tau'}{K(\tau')}, \quad V_{\bar{D}3}(\varphi) \equiv \frac{2T_3}{h(\tau)}.$$

Potential with non-rel. approx.

$$V(\varphi) \simeq \frac{(2/3)^4}{(g_s M \alpha')^2 T_3} \cdot \frac{\varphi^4}{\ln(\varphi/\varphi_0)} = \frac{\lambda_{\bar{D}3}\varphi^4}{1 + C_{\bar{D}3}\ln(\varphi/M_{Pl})}$$

$$\begin{aligned}
\varphi_0 &= \frac{\epsilon^{2/3}\sqrt{3T_3}}{2^{5/6}} = \frac{\epsilon^{2/3}\sqrt{3}}{2^{5/6}(2\pi)^{3/2}\alpha' g_s^{1/2}} \simeq \frac{3^{1/2} I^{1/4}(0)}{2^{2/3}(2\pi)^{3/2}}\sqrt{\frac{M}{\alpha'}}\exp\left(-\frac{2\pi K}{3 g_s M}\right), \\
\lambda_{\bar{D}3} &= \frac{(2/3)^4 (2\pi)^3 C_{\bar{D}3}}{g_s M^2} \simeq \frac{64\pi^2}{27MK}\left\{1 + \frac{3 g_s M}{4\pi K}\ln\left[\frac{2^{7/3} V_6}{3(2\pi)^4 I^{1/2}(0) g_s^2 M \alpha'^3}\right]\right\}^{-1}, \\
C_{\bar{D}3} &= \frac{1}{\ln(M_{Pl}/\varphi_0)} \simeq \frac{3 g_s M}{2\pi K}\left\{1 + \frac{3 g_s M}{4\pi K}\ln\left[\frac{2^{7/3} V_6}{3(2\pi)^4 I^{1/2}(0) g_s^2 M \alpha'^3}\right]\right\}^{-1}.
\end{aligned}$$

Phase portrait for an anti-D3-brane without non-rel. approximation



Chaotic Inflation!

Justifies the non-relativistic approximation!

Value of coupling constant λ？

- CMB $\delta\rho/\rho \sim 10^{-5}$ requires $\lambda \sim 10^{-13}$
- For anti-D3-brane, $\lambda \sim 64\pi^2/27MK$, where M and K are values of fluxes (integers)... It seems difficult to make λ small enough... [c.f. $KM \sim \chi/24$. The known maximum value of $\chi = 1820448$.]
- If we consider D7-brane wrapped around a 4-cycle, $\lambda \sim 32\pi g_s/27K^4[\ln(2\pi K/g_s M)+4\ln 2+2]$. Much better! [DBI & CS almost cancel, but the former (gravity) slightly wins because of the NS flux within the brane world-volume.]

Anti-D3 vs D7

- Different branes ~ different physics ~ various values of λ (coupling constant)
- Anti-D3-brane: RR charge is opposite to the background (KS geometry).
  Will be attracted towards the bottom of the throat.
- D7-brane: the sign of RR charge is the same as the background. But, gravity and RR field do not cancel exactly because of the wrapping along NS-NS flux. Gravity slightly wins and induces a small λ.

Examples of D7 potential

- D7-brane wrapped over a 4-cycle:
  $\lambda \sim 32\pi g_s/27K^4[\ln(2\pi K/g_s M)+4\ln 2+2]$
  λ can be as small as $\sim 10^{-13}$!
- Example 1:
  $g_s=0.2$, $M=25$, $K=3034$ ($\chi=1820400$)
  $\lambda \sim 1.1*10^{-13}$, $\phi_{max}^2/M_{Pl}^2 \sim 10^3$
- Example 2:
  $g_s=0.4$, $M=21$, $K=3612$ ($\chi=1820448$)
  $\lambda \sim 1.1*10^{-13}$, $\phi_{max}^2/M_{Pl}^2 \sim 2*10^3$

Open issues

- Effects of volume moduli stabilization
- Coupling to curvature
- $H_{max} \sim m_{3/2}$
- Reheating
- e.t.c.

Summary so far...

- Anti-D-branes are indispensable in the KKLT construction of 4D de Sitter solution in string theory.
- Motion of anti-D-branes may be the origin of Dark Matter in the universe.
- Motion of anti-D-branes or/and D-branes may be the inflaton(s) of chaotic inflation.

Toy model of warped flux compactification

Mukohyama, Sendouda, Yoshiguchi and Kinoshita, "Warped Flux Compactification and Brane Gravity", JCAP 0507, 013 (2005) [hep-th/0506050].

- Minimal setup of warped flux compactification
- Includes

1. Warped extra-dimension(s)
2. Magnetic flux of anti-symmetric field
3. Brane(s)

6D Warped Flux Compactification

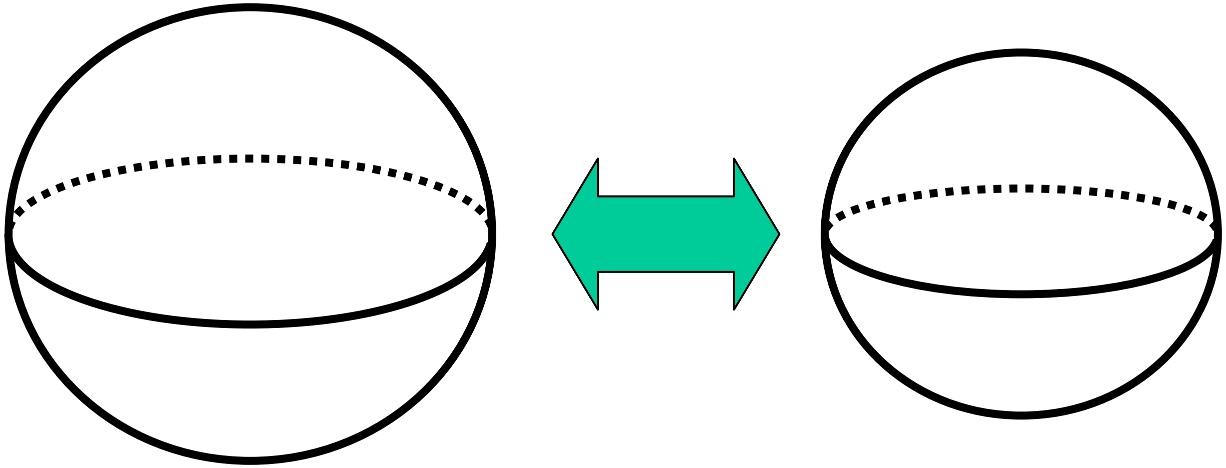
6D Einstein-Maxwell

$$S = \frac{M_6^4}{2} \int d^6 x \sqrt{-g} \left(R - 2\Lambda_6 - \frac{1}{2} F^{MN} F_{MN} \right)$$

Symmetry : 4D de Sitter × Axisymmetry in extra dimensions

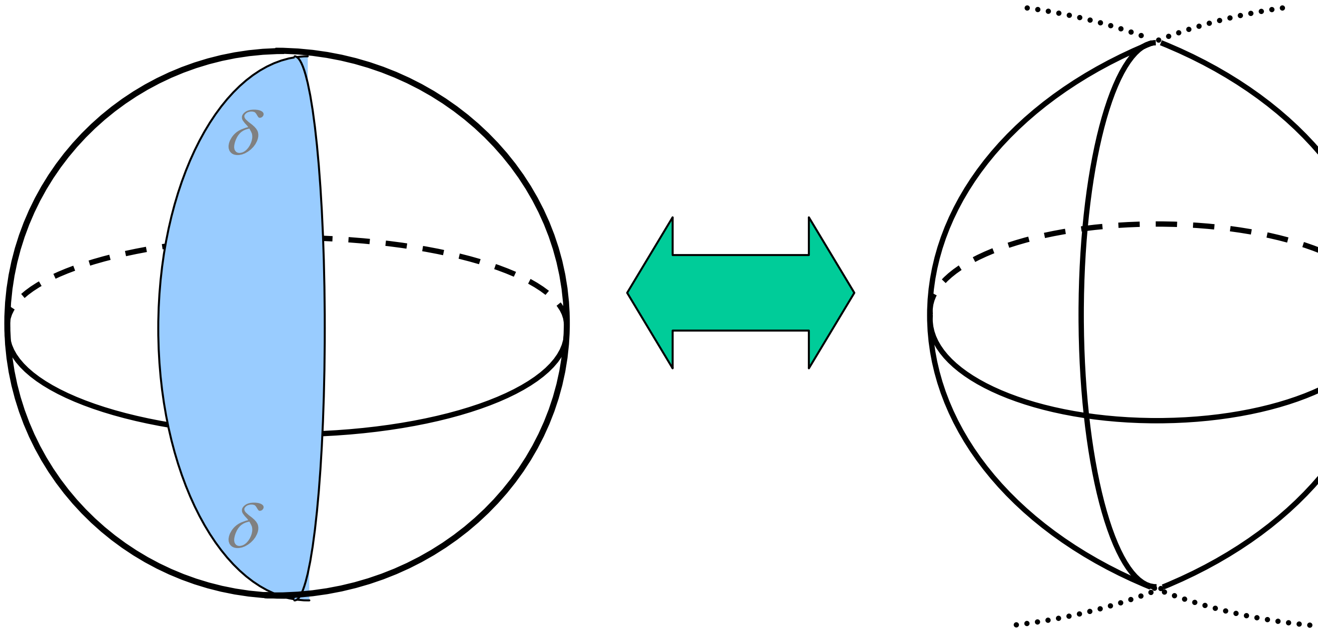
Deformation of extra dimensions: at least 3 degrees of freedom

(1) Volume



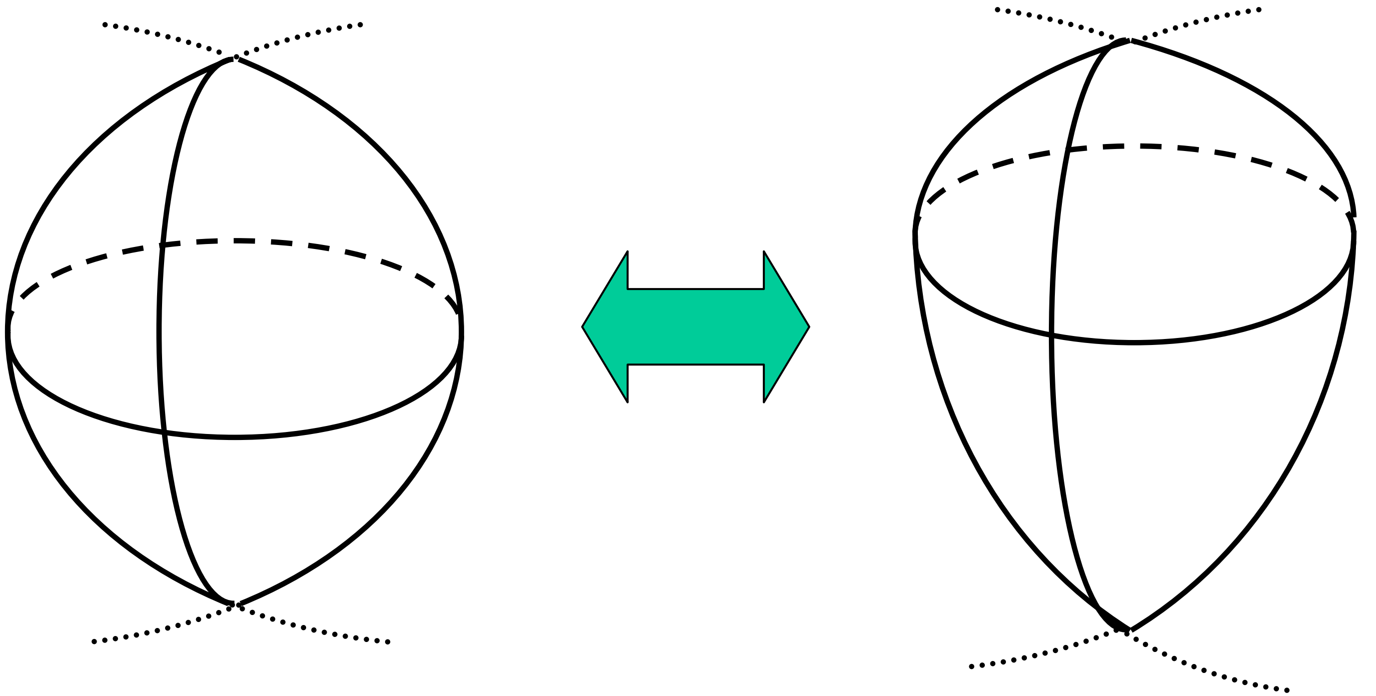
6D Warped Flux Compactification

(2) Asymmetry between longitudinal and lateral directions



6D Warped Flux Compactification

(3) Asymmetry between north and south poles



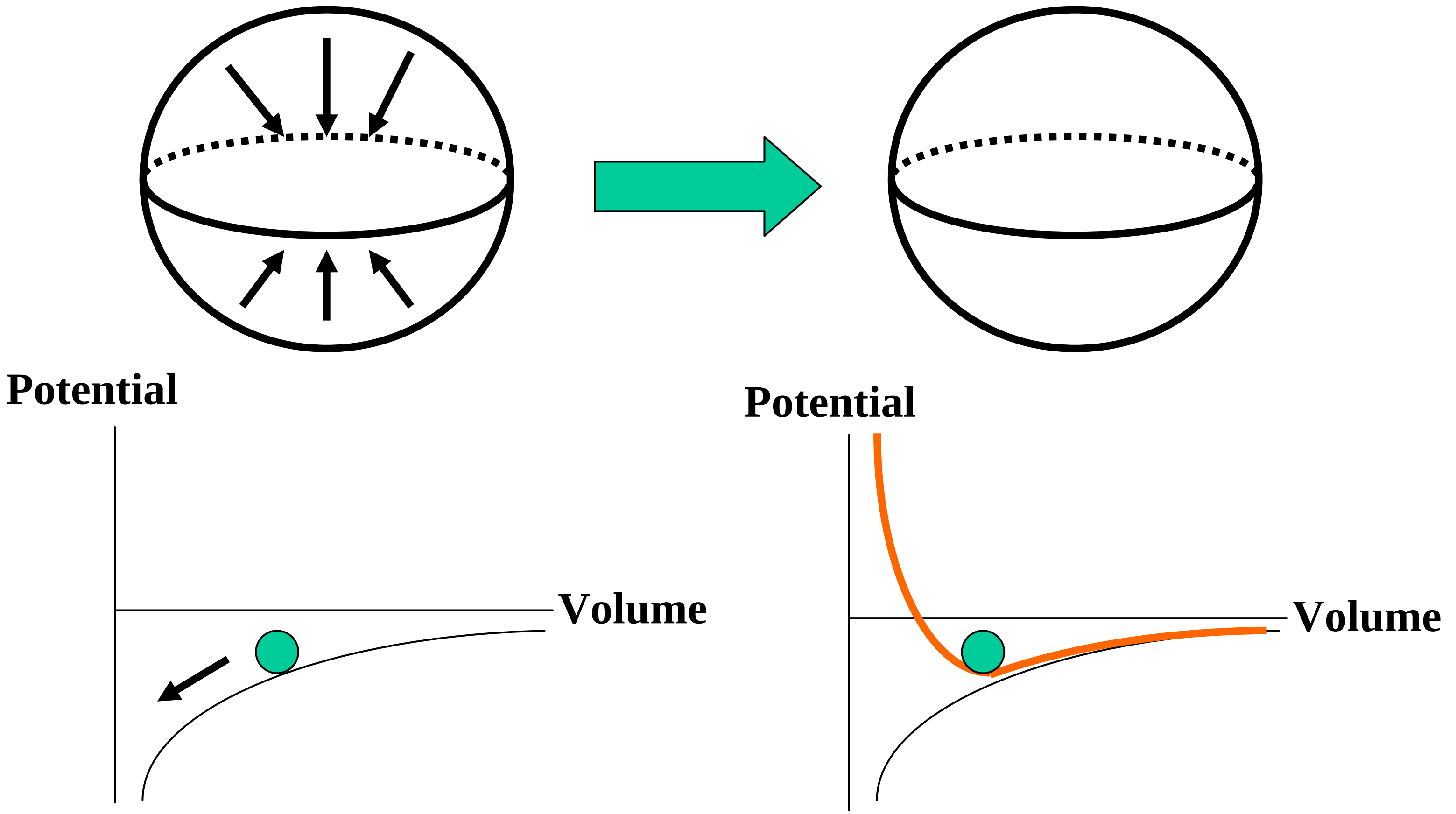
6D Warped Flux Compactification

How to stabilize (fix) each degrees of freedom?

(1) Volume

2D curvature → collapse

Stabilize by magnetic flux

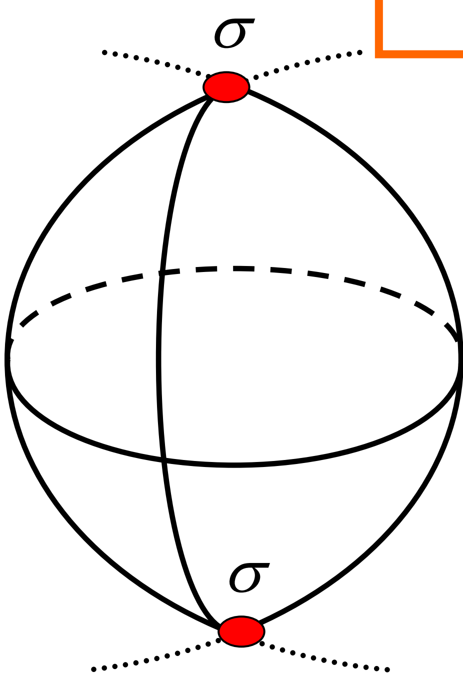


6D Warped Flux Compactification

(2) Asymmetry between longitudinal and lateral directions

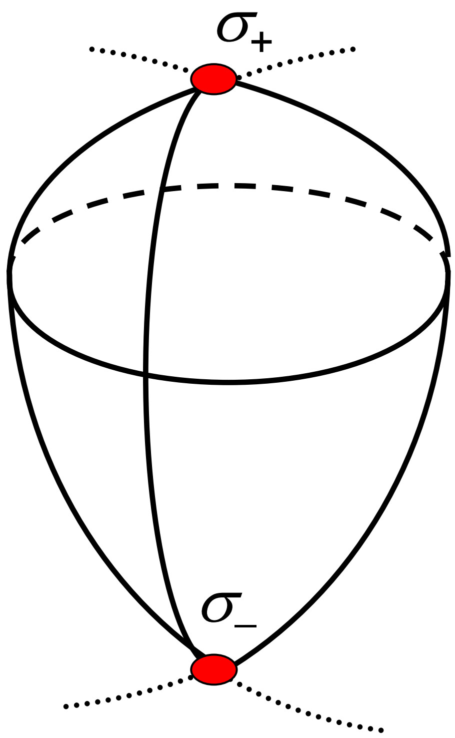
Brane tension

$$\boxed{\delta = \frac{\sigma}{M_6^4}}$$



(3) Asymmetry between north and south poles

Difference between branes



6D Warped Flux Compactification

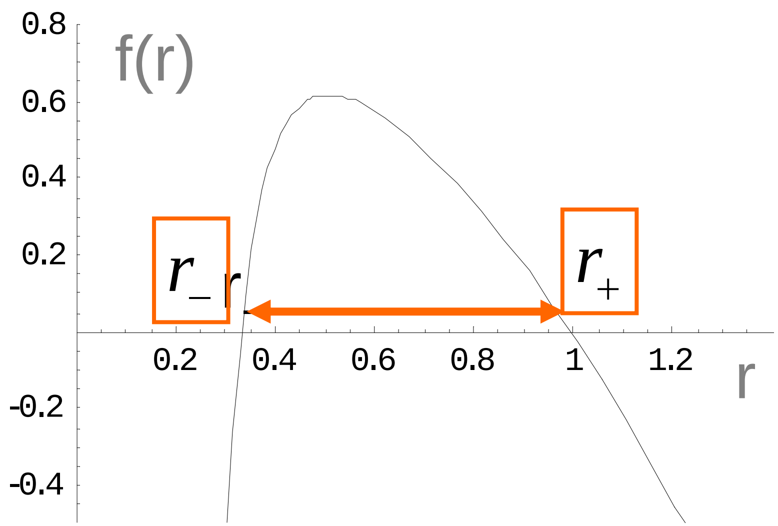
Exact solution

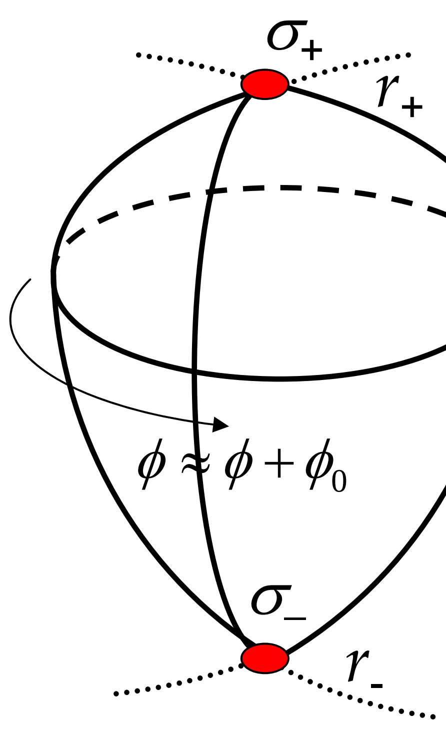
$$\boxed{ds^2 = r^2 ds^2_{dS_4} + \frac{dr^2}{f(r)} + f(r)d\phi^2}$$

Warp factor

U(1) potential $\boxed{A_\phi = \dfrac{b}{3r^3}}$

$$f(r) = 1 - \frac{\Lambda_6}{10}r^2 - \frac{\mu}{r^3} - \frac{b^2}{12r^6}$$





$$H_\pm = \frac{1}{r_\pm}$$

Bulk geometry

(r_+, r_-, ϕ_0)

input

$(\Phi, \sigma_+, \sigma_-)$

Expectation from 4D effective theory

$$ds^2_{4+n} = r^2(y) g^{(4)}_{\mu\nu}(x) dx^\mu dx^\nu + \gamma_{ij}(y) dy^i dy^j$$

Physical metric on the brane at $y^i = y_0^i$:

$$g^{(phys)}_{\mu\nu} = r^2(y_0) g^{(4)}_{\mu\nu}$$: induced metric

Moduli stabilization ⇒ Integrate massive modes out

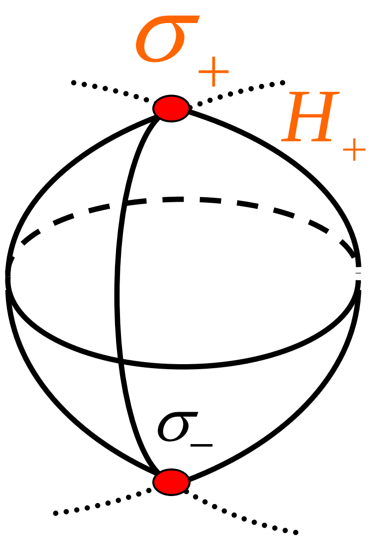
$$(M_{4+n})^{2+n} \int d^4x d^n y \sqrt{-g^{(4)}} R^{(4+n)}$$

can be dropped

$$= \boxed{(M_{4+n})^{2+n} \int d^n y \sqrt{\gamma} \frac{r^2(y)}{r^2(y_0)}} \times \int d^4x \sqrt{-g^{(phys)}} R^{(phys)} + \cdots$$

$$= \frac{1}{8\pi G_N}$$

Higher-dimensional viewpoint



When σ_+ changes,

Deficit angle δ_+ changes.

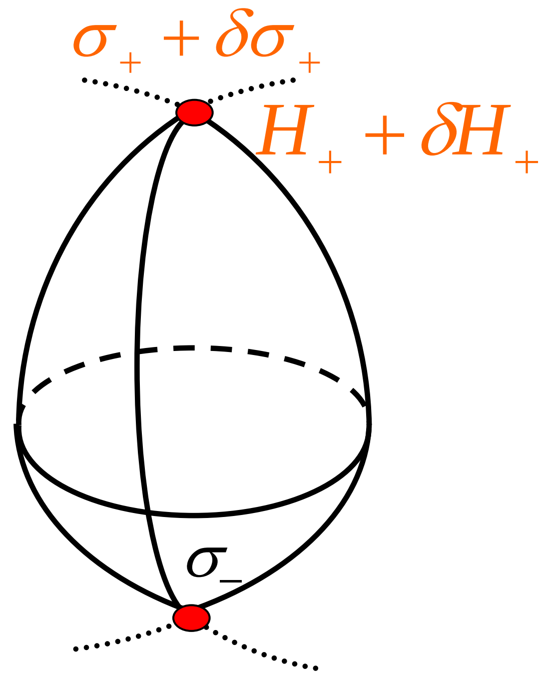
・δ_- does not change

$$\Delta\phi_0 = \Delta\left[\frac{2\pi-\delta_+}{\kappa_+}\right] = \Delta\left[\frac{2\pi-\delta_-}{\kappa_-}\right]$$

If dynamically stable

・Conservation of magnetic flux Φ

Bulk geometry changes under these conditions



$\boxed{(\Delta r_+, \Delta r_-, \Delta\phi_0)}$

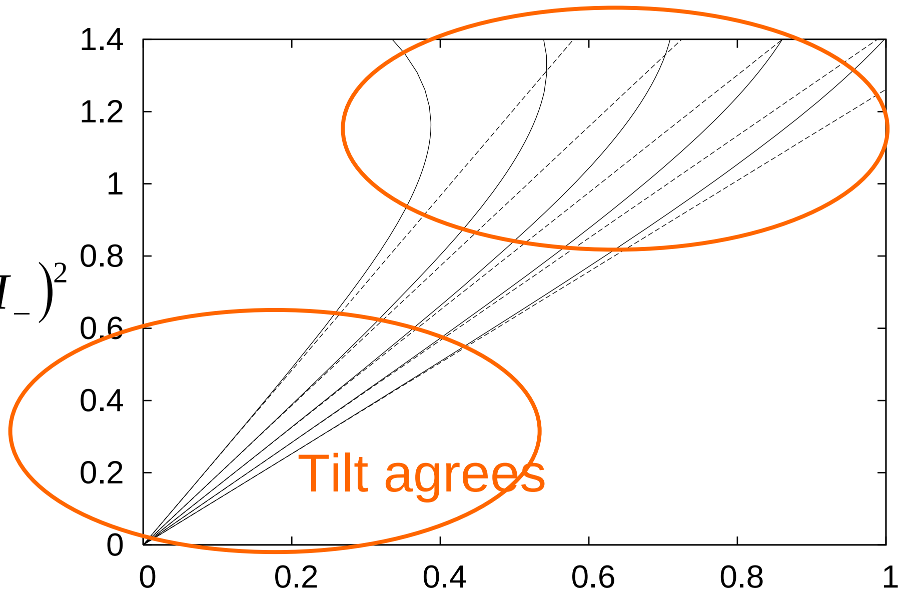
As a consequence, the induced geometry on the brane changes.
Does this agree with what we expect from the 4D effective theory?

Recovery of “Friedmann equation”

Tension σ vs Hubble H^2 (with the flux Φ fixed)

Brane at r_-

$(LH_-)^2$



$$\eta_-^{(0)} - \eta_- \left(\propto \sigma_- - \sigma_-^{(0)} \right)$$

$$L \equiv \sqrt{\frac{10}{\Lambda_6}} \qquad \left(\eta_- \equiv \frac{2\pi - \delta_-}{2\pi - \delta_+} \right)$$

Straight lines:

$$H_-^2 = \frac{8\pi G_{N-}}{3} \left(\sigma_- - \sigma_-^{(0)} \right)$$

$$\left(\frac{1}{8\pi G_{N-}} = (M_{4+n})^{2+n} \int d^n y \sqrt{\gamma} \left[\frac{r(y)}{r_-} \right]^2 \right)$$

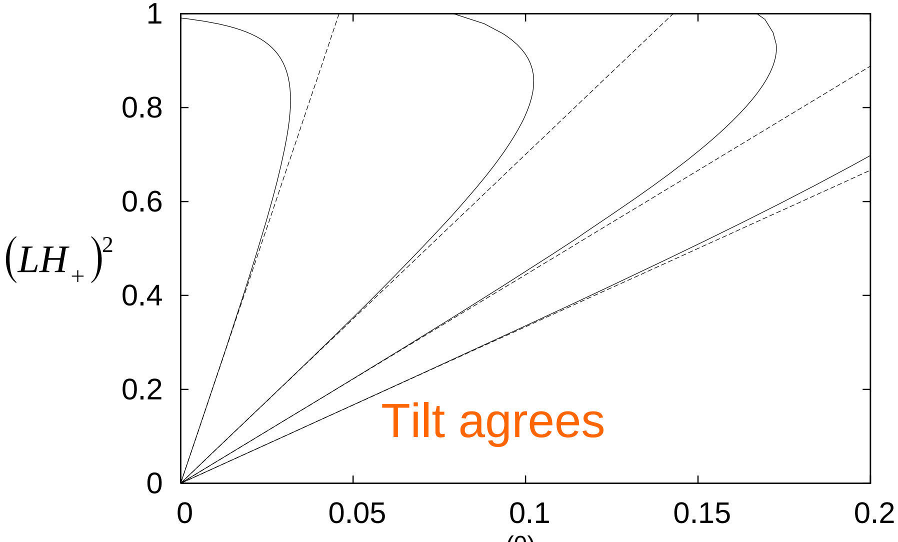
- Agrees with 4D effective theory at low E

- High E corrections

Recovery of “Friedmann equation”

Tension σ vs Hubble H^2 (with the flux Φ fixed)

Brane at r_+



$$\eta_+^{(0)} - \eta_+ \left(\propto \sigma_+ - \sigma_+^{(0)}\right)$$

$$\left(\eta_+ \equiv \frac{2\pi - \delta_+}{2\pi - \delta_-}\right)$$

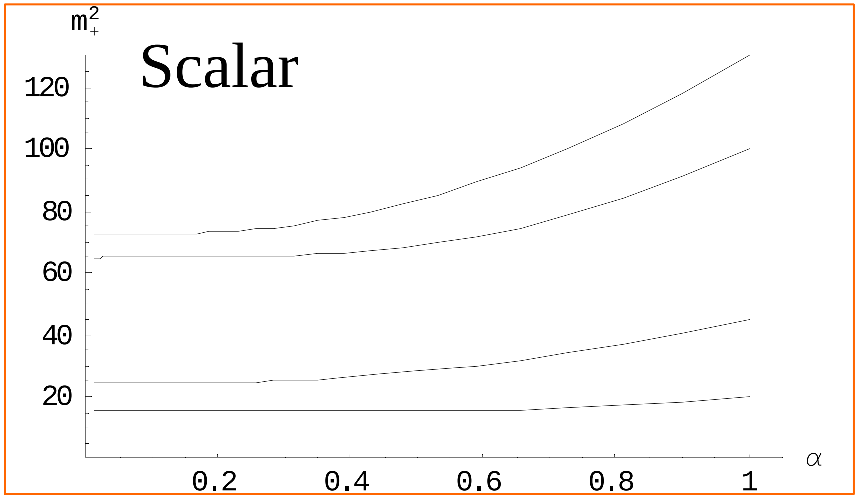
Straight lines:

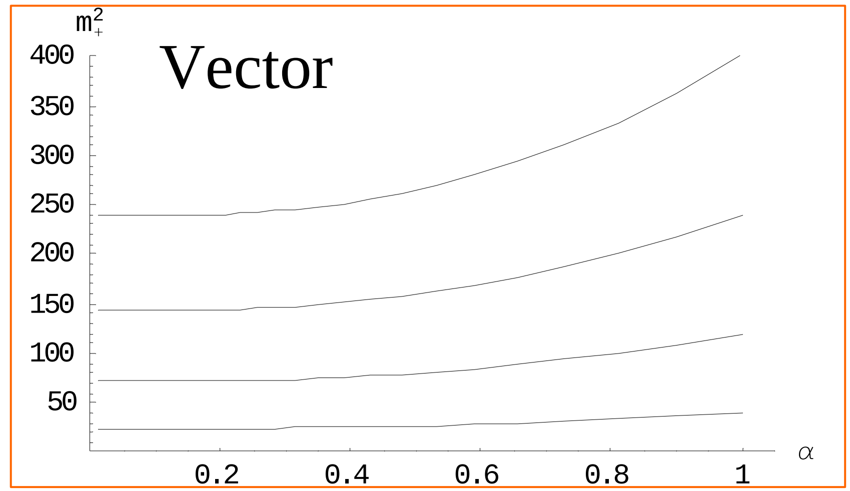
$$H_+^2 = \frac{8\pi G_{N+}}{3}\left(\sigma_+ - \sigma_+^{(0)}\right)$$

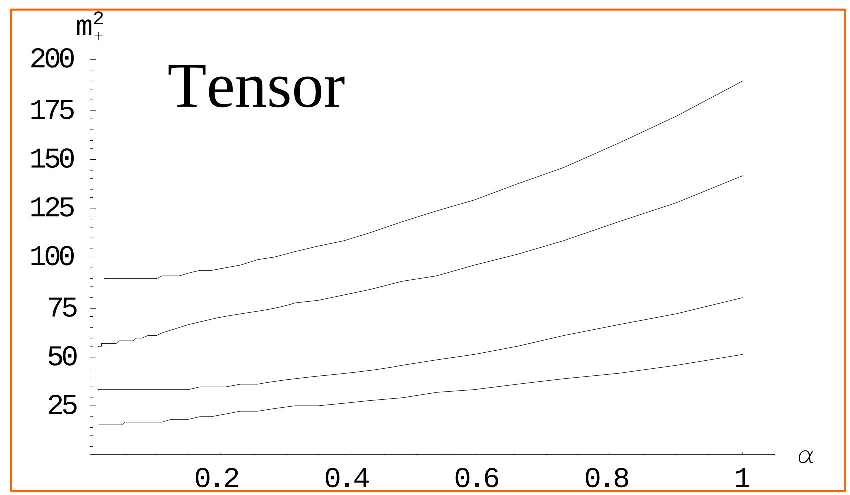
$$\left(\frac{1}{8\pi G_{N+}} = \left(M_{4+n}\right)^{2+n}\int d^n y \sqrt{\gamma}\left[\frac{r(y)}{r_+}\right]^2\right)$$

Stability & KK spectrum

Yoshiguchi, Sendouda, Kinoshita and Mukohyama, "Stability of 6D warped flux compactification", hep-th/0512212.







$\alpha = r_-/r_+$: hierarchy

m_+: KK mass at $r=r_+$

Summary

- Motion of (anti-)D-branes may be the origin of Dark Matter in the universe.
- (Anti-)D-branes may be the inflaton of chaotic inflation.
- 6D toy model of warped flux compactification

Summary

- It seems that we can really enjoy cosmology in the framework of string theory, not just a string-inspired cosmology.
- Keywords:
  Extra dimensions
  Warped Flux Compactification
  Branes
- A lot of interesting subjects are still remaining!

Thank you very much

for your listening!