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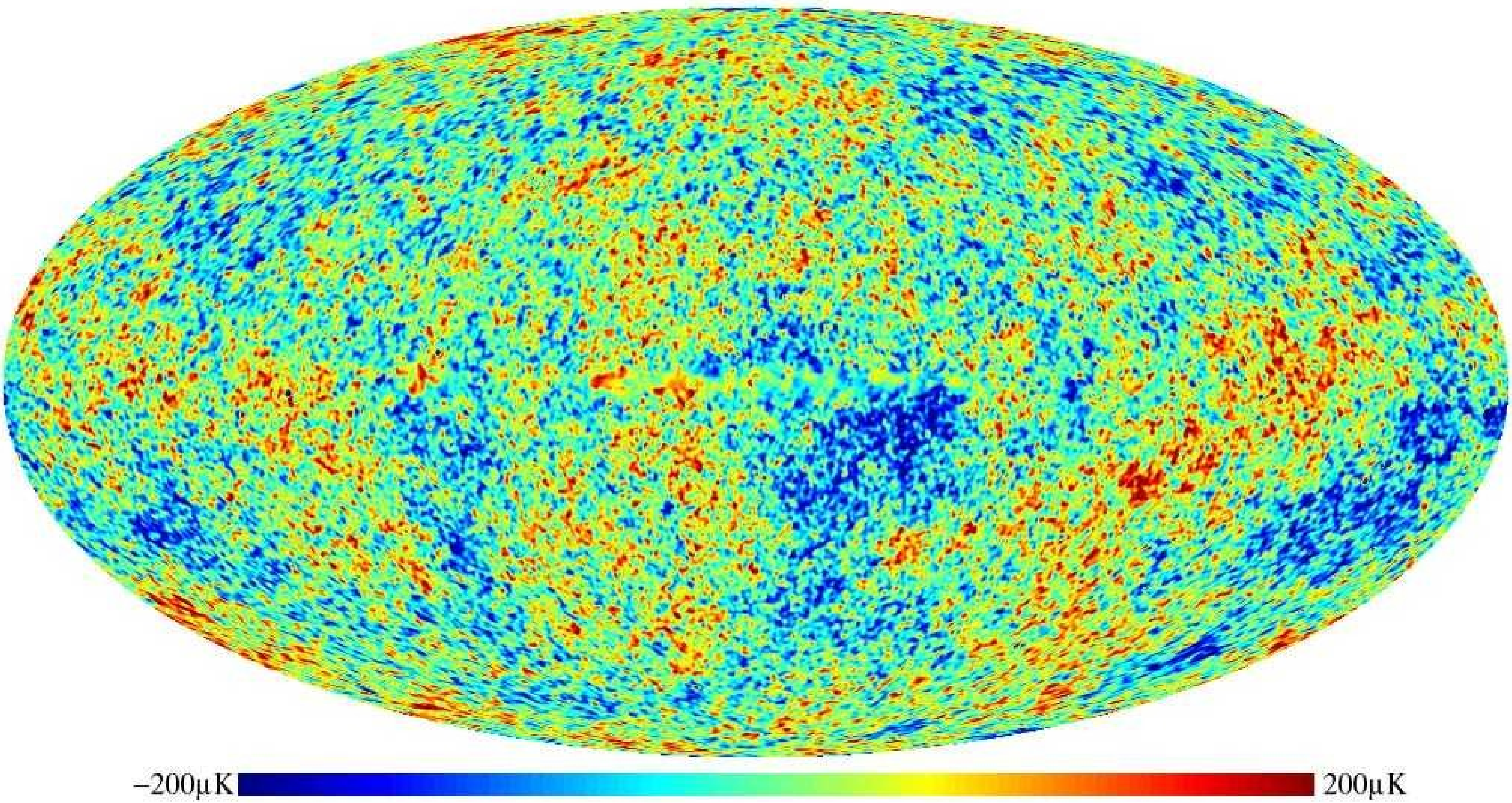
Limits on non-Gaussianities from WMAP 3 yr data

astro-ph/0610600

with P. Creminelli, M. Tegmark,
and M. Zaldarriaga

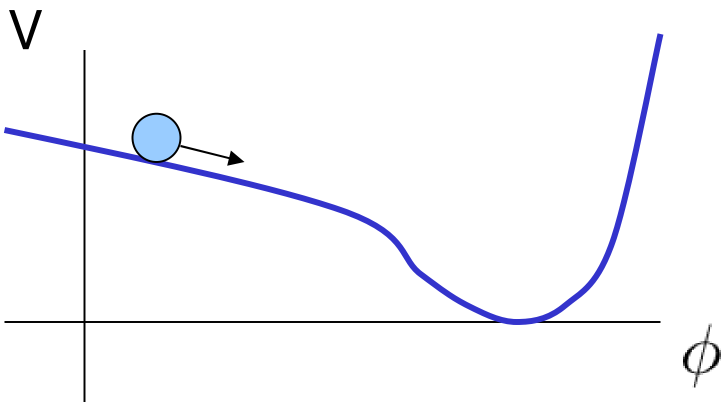
Is there any correlation among modes?



OUTLINE

- Standard slow roll inflation predicts very small NG: NG < 10^{-6}
- NG as smoking gun for "non-standard" inflation
- Models with detectable NG
  - Local models
  - Equilateral models
- Different predictions for the "shape" of the 3-point function
- Data analysis of 3 year WMAP data
- No detection (sigh!). The tightest limits on NG.

Slow-roll = weak coupling



$$\ddot{\phi} + \underline{\underline{3H\dot{\phi}}} + V'(\phi) = 0$$

Friction is dominant

To have ~ dS space the potential must be very flat:

$$\epsilon, \eta \ll 1$$

$$\epsilon = \frac{1}{2} M_P^2 \left(\frac{V'}{V}\right)^2 \qquad \eta = M_P^2 \frac{V''}{V}$$

The inflaton is extremely weakly coupled. Leading NG from gravity.

$$\frac{\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle}{\langle \zeta_{\vec{k}} \zeta_{-\vec{k}} \rangle^{3/2}} \sim \epsilon \frac{H}{\sqrt{\epsilon} M_P} \ll 10^{-5}$$

Completely model independent as it comes from gravity

Unobservable (?). To see any deviation you need > 10^{12} data. WMAP ~ 2 x 10^6

Maldacena, **JHEP 0305:013,2003,** Acquaviva etal **Nucl.Phys.B667:119-148,2003**

Smoking gun for "new physics"

Any signal would be a clear signal of something non-minimal

- Any modification enhances NG
  - Modify inflaton Lagrangian. Higher derivative terms, ghost inflation, DBI inflation…
  - Additional light fields during inflation. Curvaton, variable decay width…

- Potential wealth of information

Translation invariance: $$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 \delta(\sum_i \vec{k}_i) F(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

Scale invariance: $$F(\lambda\, \vec{k}_1, \lambda\, \vec{k}_2, \lambda\, \vec{k}_3) = \lambda^{-6} F(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

F contains information about the source of NG

Note. We are only considering primordial NGs. Neglect non-linear relation with observables. Good until primordial NG > 10^{-5}.

Higher derivative terms

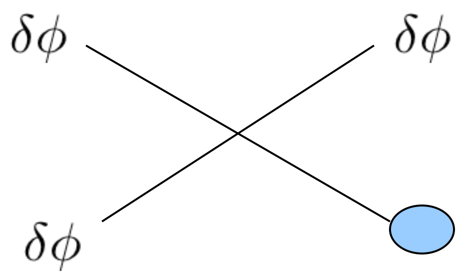
Creminelli **JCAP 0310:003,2003**

Change inflaton dynamics and thus density perturbations

Potential terms are strongly constrained by slow-roll.
Impose shift symmetry: $\phi \to \phi + \mathrm{const}$

Most relevant operator: $\frac{1}{8\Lambda^4}(\nabla\phi)^2(\nabla\phi)^2$

3 point function:



$$\frac{\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle}{\langle \zeta_{\vec{k}}\zeta_{-\vec{k}}\rangle^{3/2}} \sim \frac{\dot{\phi}^2}{\Lambda^4}\frac{H}{\sqrt{\epsilon}M_P}$$

In EFT regime NG < 10^{-5}
Difficult to observe

We get big NG only if h. d. terms are important also for the classical dynamics

DBI inflation: $\mathcal{L} = \phi^4\sqrt{1-\lambda\frac{\dot{\phi}^2}{\phi^4}}$

Speed limit in AdS

Alishahiha, Silverstein and Tong, **Phys.Rev.D70:123505,2004**

Ghost inflation

Arkani-Hamed, Creminelli, Mukoyama and Zaldarriaga,
JCAP 0404:001,2004
Leonardo Senatore
Phys. Rev. D71:043512,2005

Ghost condensation: $\underline{\underline{-(\partial\phi)^2}} + \frac{1}{M^4}(\partial\phi)^4 + \dots$

WRONG SIGN

- Spontaneous breaking of Lorentz symmetry: $\langle\dot{\phi}\rangle = M^2$

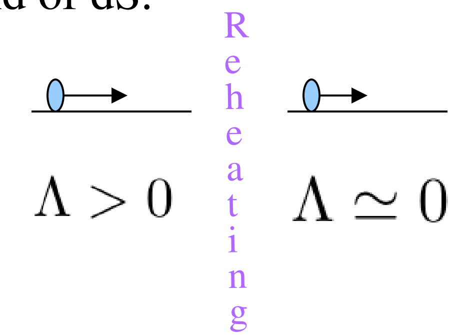
- Consistent derivative expansion: $\phi = M^2 t + \pi$

$$S = \int d^4x \, \frac{1}{2}\dot{\pi}^2 - \frac{\alpha^2}{2M^2}(\nabla^2\pi)^2 - \frac{\beta}{2M^2}\dot{\pi}(\nabla\pi)^2 + \dots$$

- Non Lorentz-invariant action, standard spatial kinetic term NOT allowed

Big non-Gaussianities

- Use the "ghost" field as the inflaton. It triggers the end of dS.



- Non Lorentz-invariant scaling

$$\frac{1}{2}\dot{\pi}^2 - \frac{\alpha^2}{2M^2}(\nabla^2\pi)^2 \longrightarrow [\pi] = \frac{1}{4}$$

$$\delta\pi_H \simeq M\left(\frac{H}{M}\right)^{1/4} \gg H$$

- The leading non-linear operator:

$$-\frac{\beta}{2M^2}\dot{\pi}(\nabla\pi)^2$$ has dimension 1/4

$$NG \simeq \left(\frac{H}{M}\right)^{1/4} \simeq 10^{-3}$$ Quite big. Close to exp. bound.

Derivative interactions are enhanced wrt standard case by NR relativistic scaling

General approach

X. Chen, M.x.Huang, S.Kachru and G.Shiu, hep-th/0605045
P. Creminelli, C. Chung L. Fitzpatrick, J.Kaplan, L.Senatore, in progr.

We want to calculate $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle$ for a generic: $\mathcal{L} = P(X, \phi)$ $\quad X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$

Let us rephrase the calculation in a way which emphasis symmetries: EFT around FRW

EFT around a given FRW background $a(t)$, $\psi(t)$. Focus on the perturbation

$$\delta\psi_m(x) \equiv \psi_m(t + \pi(x)) - \psi_m(t) \,,$$

Goldstone of broken time-translation

- This mode can be reabsorbed in the metric going to unitary gauge
- In this gauge, only gravity with time-dependent spatial diff. : $x^i \to x^i + \xi^i(\vec{x}, t)$
- ADM variables are useful: $N \equiv 1/\sqrt{g^{00}}$ $\quad N_i \equiv g_{0i}$ $\quad \hat{g}_{ij}$
- The Goldstone π can be reintroduced with Stuckelberg trick
- Most generic action invariant under spatial diff. + reintroduce the Goldstone

At 0th order in derivative I have only: $\frac{1}{N^2} = g^{00} \mapsto \frac{1}{N^2} + 2\,\partial_0 \xi^0 - (\partial_\mu \xi^0)^2$

Tadpole terms to make the given a(t) a solution: $\delta N \equiv N - 1, \; \Lambda$

Coefficients can be time dependent: $S_{\rm matter} = \int d^4x \, \sqrt{-g} \left[c(t) \frac{1}{N^2} - \Lambda(t) \right]$

E.g. this is all for a standard scalar field:

$$S_\phi = \int d^4x \sqrt{-g}\left[-\frac{1}{2}(\partial\phi)^2 - V(\phi)\right] = \int d^4x \sqrt{-g}\left[\frac{1}{2N^2}\dot{\phi}^2 - V(\phi(t))\right]$$

Fix the coefficients from the background:

$$S_{\rm matter} = \int d^4x \sqrt{-g}\left[-M_{\rm Pl}^2\dot{H}\frac{1}{N^2} - M_{\rm Pl}^2(3H^2+\dot{H}) + \cdots\right]$$

Reintroducing π

$$S_{\rm matter} \to S_\pi = \int d^4x \sqrt{-g}\,(M_{\rm Pl}^2\dot{H})(\partial\pi)^2$$

At 0^{th} derivative level up to 3^{rd} order in pert:

$$+\int d^4x\sqrt{-g}\left[\frac{1}{4}M^4(t)(g^{00}-1)^2 + Q^4(t)(g^{00}-1)^3\right]$$

Only two operators are allowed by symmetries: 2 contributions (up to slow-roll)
The first operator changes the speed of sound

$$c_s^2 = \frac{M_P^2|\dot{H}|}{M_P^2|\dot{H}| + M^4}$$

$M^4 > 0$ to avoid superluminality

Contribution to NGs:

$$f_{\rm NL} \sim \left(\frac{1}{c_s^2} - 1\right)$$

See P.Creminelli., M.Luty, A.Nicolis, L.Senatore, hep-th/0606090

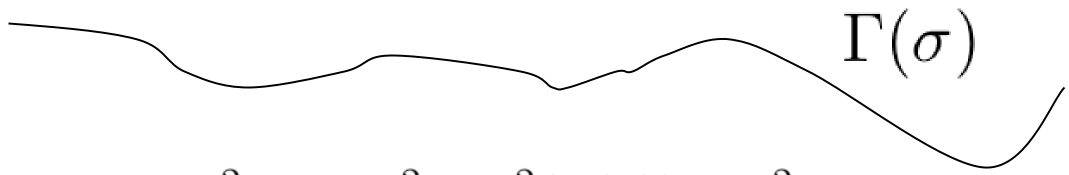
NG in variable decay scenario (~curvaton)

Dvali, Gruzinov and Zaldarriaga **Phys.Rev.D69:023505,20**

- Fluctuation of the decay width of the inflaton gives $\delta\rho/\rho$

$$\Gamma = m_I g^2 K(\sigma)$$

- Parallel Universes:



Final RD metric: $ds^2 = -dt^2 + g^2(\Gamma(x))t\, dx^2$

- Many sources of NG:

$$\delta\sigma \quad \longrightarrow \quad \delta\Gamma \quad \longrightarrow \quad \zeta$$

Every step gives non-gaussianity. E.g. $\frac{\delta\Gamma}{\Gamma} \gg \frac{\delta\rho}{\rho} \quad \rightarrow \quad \left(\frac{\delta\Gamma}{\Gamma}\right)^2$ is big

- In general: NG > 10^{-5}, but model dependent.

The shape of non-Gaussianities

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 \delta(\sum_i \vec{k}_i) F(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

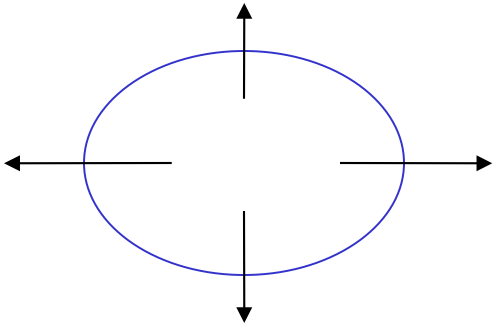
Babich, Creminelli, Zaldarriaga, **JCAP 0408:009,2004**

- LOCAL DISTRIBUTION $\zeta(x) = \zeta_g(x) - \frac{3}{5} f_{\rm NL} (\zeta_g(x)^2 - \langle \zeta_g^2 \rangle)$

$$F(k_1, k_2, k_3) = -f_{\rm NL}^{\rm local} \cdot \frac{6}{5} \Delta_\zeta^2 \cdot \left(\frac{1}{k_1^3 k_2^3} + \frac{1}{k_1^3 k_3^3} + \frac{1}{k_2^3 k_3^3} \right)$$

Typical for NG produced outside the horizon

- EQUILATERAL DISTRIBUTIONS



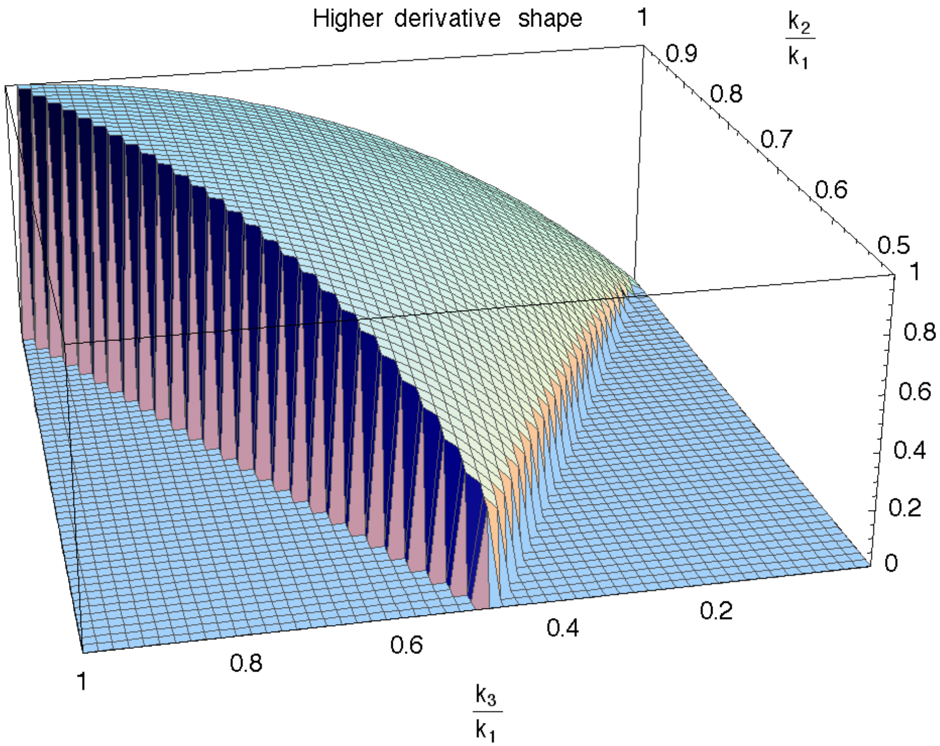
Derivative interactions irrelevant after crossing.
Correlation among modes of comparable λ.

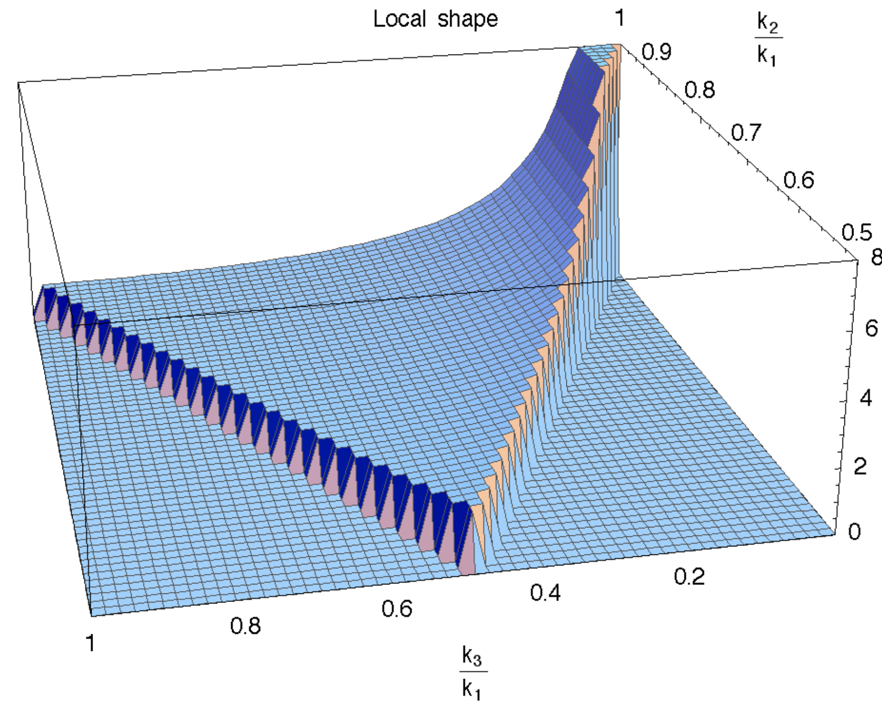
F is quite complicated in the various models. But in general

$$F \sim k_1^{-1} \quad \text{for} \quad k_1 \to 0$$

Quite similar in different models

Shape comparison





The NG signal is concentrated on different configurations.

- They can be easily distinguished (once NG is detected!)

- They need a dedicated analysis

Consistency relation for 3-p.f.

J. Maldacena, **JHEP 0305:013,2003**
P. Creminelli. + M. Zaldarriaga, **JCAP 0410:006, 2004**

Under the usual "adiabatic" assumption (a single field is relevant),
INDEPENDENTLY of the inflaton Lagrangian

$$\lim_{k_1 \to 0} \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = -(2\pi)^3 \delta^3 (\sum_i \vec{k}_i) P_{k_1} P_{k_3} \left[\frac{d \log(k_3^3 P_{k_3})}{d \log k_3} + \mathcal{O}(\frac{k_1}{k_3}) \right]$$

$$ds^2 = -dt^2 + e^{2\zeta(x)} a^2(t) dx_i dx^i$$

The long wavelength mode is a frozen background for the other two: it redefines spatial coordinates.

$n_s - 1 \ll 1$ In the squeezed limit the 3pf is small and probably undetectable

- Models with a second field have a large 3pf in this limit.

Violation of this relation is a **clear, model independent evidence** for a second field (same implications as detecting isocurvature).

- This is experimentally achievable if NG is detected.

Analysis of WMAP 1st year data

Creminelli, Nicolis, Senatore and Zaldarriaga,
JCAP 0605:004,2006

WMAP alone gives almost all we know about NG. Large data sample + simple.

Not completely straightforward!

$$\mathcal{E} = \frac{1}{N} \cdot \sum_{l_i m_i} \frac{\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle}{C_{l_1} C_{l_2} C_{l_3}} \; a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}$$

$$\mathcal{E} = \frac{1}{N} \cdot \sum_{l_i m_i} \int d^2\hat{n}\; Y_{l_1 m_1}(\hat{n}) Y_{l_2 m_2}(\hat{n}) Y_{l_3 m_3}(\hat{n}) \int_0^\infty r^2 dr\; j_{l_1}(k_1 r) j_{l_2}(k_2 r) j_{l_3}(k_3 r)\; C_{l_1}^{-1} C_{l_2}^{-1} C_{l_3}^{-1}$$

$$\int \frac{2k_1^2 dk_1}{\pi} \frac{2k_2^2 dk_2}{\pi} \frac{2k_3^2 dk_3}{\pi} F(k_1, k_2, k_3) \Delta_{l_1}^T(k_1) \Delta_{l_2}^T(k_2) \Delta_{l_3}^T(k_3)\; a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}$$

It scales like $N_{\text{pixels}}^{5/2} \sim 10^{16}$ for WMAP!!! Too much…

But if F is “factorizable” the computation time scales as $N_{\text{pixels}}^{3/2} \sim 10^9$. Doable!

Use a fact. shape with equilateral properties

$$F(k_1, k_2, k_3) = f_{\rm NL}^{\rm equil.} \cdot 6 \Delta_\Phi^2 \cdot \left(-\frac{1}{k_1^3 k_2^3} - \frac{1}{k_1^3 k_3^3} - \frac{1}{k_2^3 k_3^3} - \frac{2}{k_1^2 k_2^2 k_3^2} + \frac{1}{k_1 k_2^2 k_3^3} + 5\; perm. \right)$$

Real space VS Fourier space

CMB signal diagonal in Fourier space (without NG!!). Foreground and noise in real space.

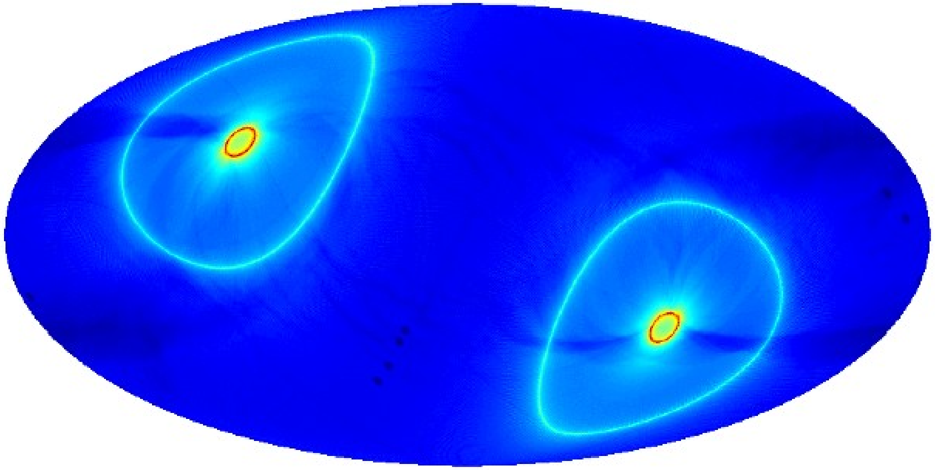
Non-diagonal error matrix + linear term in the estimator

Minimum variance estimator:

$$\mathcal{E}_{\rm lin}(a) = \frac{1}{N}\sum_{l_i m_i}\Big(\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}\rangle_1\, C^{-1}_{l_1m_1,l_4m_4}C^{-1}_{l_2m_2,l_5m_5}C^{-1}_{l_3m_3,l_6m_6}a_{l_4m_4}a_{l_5m_5}a_{l_6m_6} - 3\,\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}\rangle_1\, C^{-1}_{l_1m_1,l_2m_2}C^{-1}_{l_3m_3,l_4m_4}a_{l_4m_4}\Big)$$

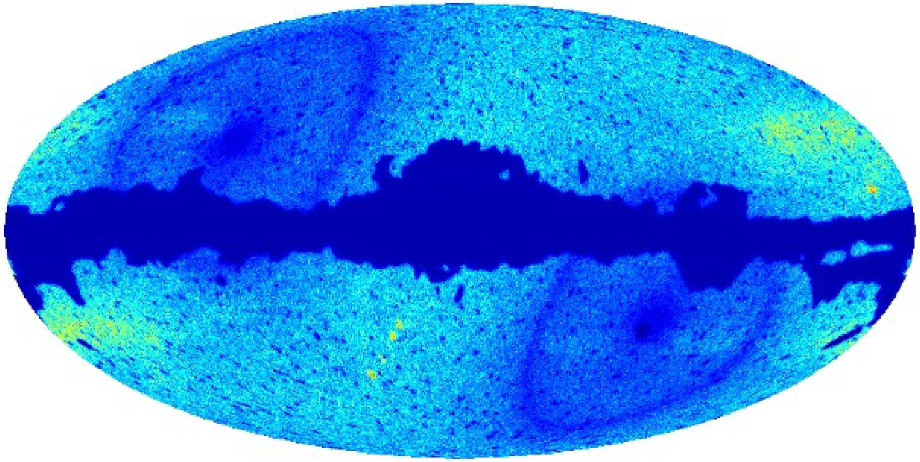
It saturates Cramers-Rao inequality. Reduces variance wrt WMAP coll. analysis.

Correction for anisotropic noise



N_obs varies across the sky. Smaller power in more observed regions.

On a given realization it looks like a NG signal. Bigger variance.



Linear term of the estimator. Subtracts this effect. Reduces variance.

The Optimal Estimator for $f_{\rm NL}$

Creminelli, Senatore and Zaldarriaga,
astro-ph/0606001

- Specialize to Local case (in flat sky, with unit transfer function)

$$\Phi_{\vec{\theta}} = f(g_{\vec{\theta}}) = g_{\vec{\theta}} + f_{\rm NL}\left(g^2_{\vec{\theta}} - \sigma^2\right)$$

- Find the Log-Likelihood:

$$\mathcal{L}_g = \frac{1}{2}\sum_{l_1 l_2} C^{-1}_{l_1 l_2} g_{l_1} g_{l_2}$$

$$\mathcal{L} = \frac{1}{2}\sum_{l_1 l_2} C^{-1}_{l_1 l_2}\left(f^{-1}(\Phi)\right)_{l_1}\left(f^{-1}(\Phi)\right)_{l_2} - {\rm Tr}\ln\left({\rm J}\right)$$

$$\begin{aligned}\mathcal{L} \;=\;& \frac{1}{2}\sum_l\left(\frac{1}{\Omega C_l}\left(\Phi_l\Phi_{-l} - 2\tilde{f}_{\rm NL}\chi_l\Phi_{-l} + \tilde{f}^2_{\rm NL}\left(\chi_l\chi_{-l} + 4\Phi_l\eta_{-l}\right)\right)\right) \\ & +2\tilde{f}_{\rm NL}\frac{\rm N_{pix}}{\Omega}\Phi_{l=0} - 4\tilde{f}^2_{\rm NL}\frac{\rm N_{pix}}{\Omega}\chi_{l=0} - 2\tilde{f}^2_{\rm NL}{\rm N_{pix}}\sigma^2,\end{aligned}$$

The Optimal Estimator for $f_{\rm NL}$

- Our estimator is ~equivalent to the full Likelihood:
  - (With a slight modification in case of detection), it saturates to the Cramer Rao bound $\langle \frac{\partial^2 \mathcal{L}}{\partial \tilde{f}_{\rm NL}^2} \rangle^{-1}$
  - While the error on $f_{\rm NL}$ from the Likelihood, on a given realization:

$$\frac{\partial^2 \mathcal{L}}{\partial \tilde{f}_{\rm NL}^2} \longrightarrow \langle \frac{\partial^2 \mathcal{L}}{\partial \tilde{f}_{\rm NL}^2} \rangle^{-1} \text{ as } \frac{1}{\log N_{\rm pix}}$$

- Other methods can not help on $f_{\rm NL}$:
  - 4-point function
  - Minkowski functionals
  - Wavelets
  - etc

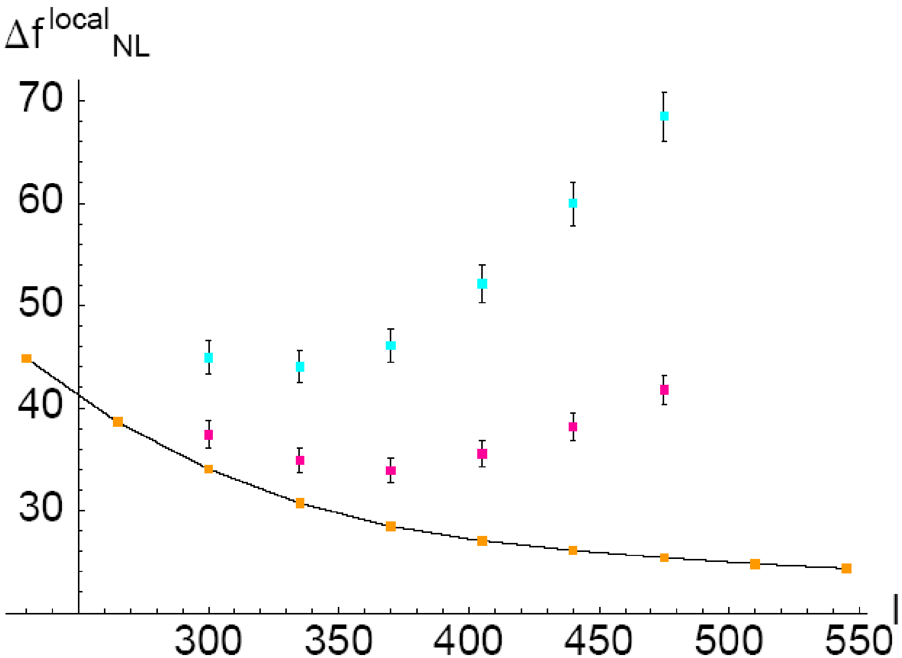
Let us do it!

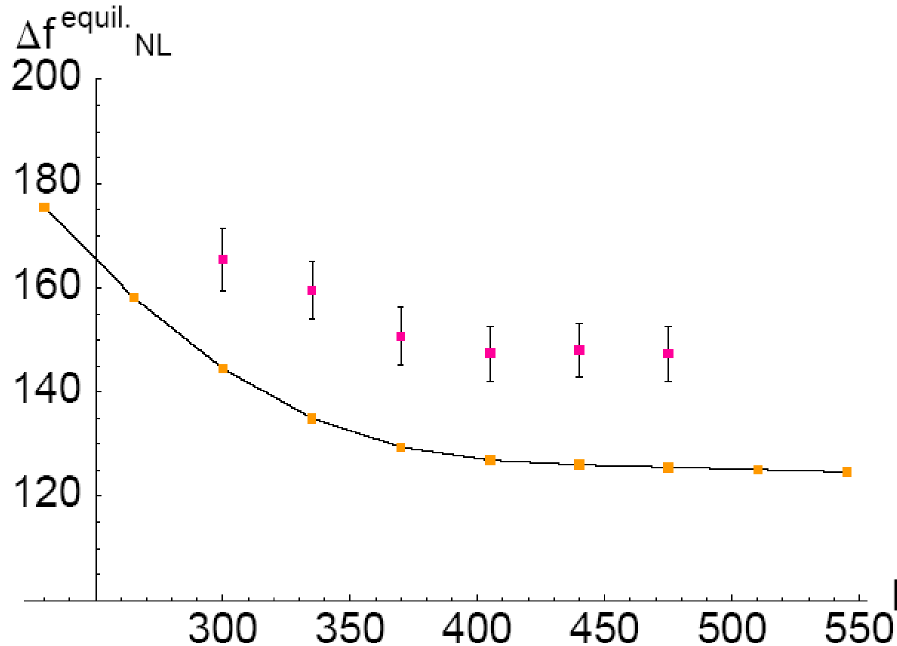
- Close to WMAP collaboration analysis to cross check.
- Fix best fit cosmological parameters and produce MonteCarlos with HEALpix.
- Smooth maps with 8 different beams corresponding to Q1, Q2, V1, V2, W1, W2, W3, W4
- Add independent noise realization (each pixel).
- Combine maps and mask the (would be) Galaxy (kp0 mask: 76.8% sky).
- Calculate the estimator on each realization for both shapes: f_{NL}^{local} and $f_{NL}^{equil.}$. It needs an integral over the distance to LSS. Hundreds of FFTs.
- Every MonteCarlo 100 minutes on a 2 GHz, 2 GB Opteron processor.
- You need tens of machines (thanks to Sauron cluster at CfA).
- Apply the very same procedure on the real data (foreground subtraction applied).

Differences wrt 1yr Analysis

- Introduction of the tilt in the shapes
- Improved Combination of the Maps (l-dependent)
- Variation of the Cosmological Parameters
    - Reionization: from $\tau = 0.17$ to $\tau = 0.092$ worse limits ~8%
    - Red tilt: better limits on $f_{\rm NL}^{\rm local}$ ~8%; worse for $f_{\rm NL}^{\rm equil.}$ ~5%
    - Higher number of signal dominates multipoles: better ~20%

Error Bars





- For the local shape the linear piece helps at high l's (irrelevant for equil. shape)
- In both cases we are not far from the theoretical limit (~20%)
- Full inversion of the covariance matrix
- Limits: Improve on $f_{\rm NL}^{\rm local}$ ~10%; on $f_{\rm NL}^{\rm equil.}$ ~3%

☹ No detection ☹

WMAP data (after foreground template corrections) are compatible with Gaussianity

We have the best limits on NG for the two shapes

$$-36 < f_{NL}^{local} < 100 \quad \text{at } 95\% \text{ C.L.}$$

$$-256 < f_{NL}^{equil.} < 336 \quad \text{at } 95\% \text{ C.L.}$$

- Slight (20%) improvement wrt to WMAP analysis for the local shape.
- Limits on equil. shape are not weaker: different normalization.

Conclusions

- Non-Gaussianities as probe of something non-minimal going on
- Two classes of models
  1) Non minimal inflaton Lagrangian
  2) Additional light fields during inflation
- Equilateral shape VS local shape
- WMAP data analysis for the two shapes
  1) Factorizable equil. shape
  2) Linear piece in the estimator
- No detection! Tightest limit on NG parameters

  $-36 < f_{NL}^{local} < 100$ at 95% C.L.

  $-256 < f_{NL}^{equil} < 332$ at 95% C.L.

- Future

  WMAP 8 yrs: 20% improvement

  PLANCK: factor of 4 (additional factor 1.6 from polarization)
- Non-minimal models will be strongly constained