The Cavity Shift of Electron $g - 2$

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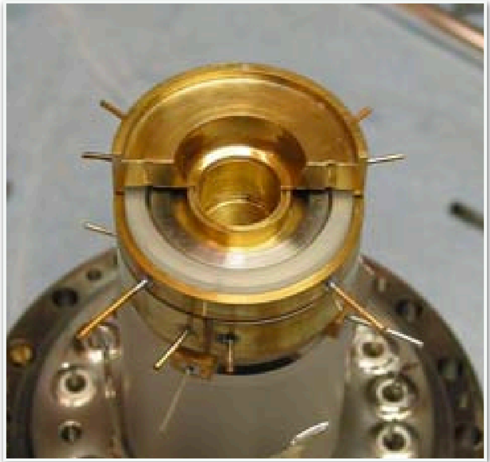
arXiv:2511.xxxxx, with Hannah Day, Roni Harnik, Yonatan Kahn, Shashin Pavaskar

The “Other” $g-2$

$g_\mu - 2$	$g_e - 2$
team of 200, costs $\$10^8$	team of 4, costs $\$10^6$
$10^{11}\ \mu^-$ orbit in $L \sim 10\,\mathrm{m}$ storage ring	$1\ e^-$ in a trap of size $L \sim 0.01\,\mathrm{m}$

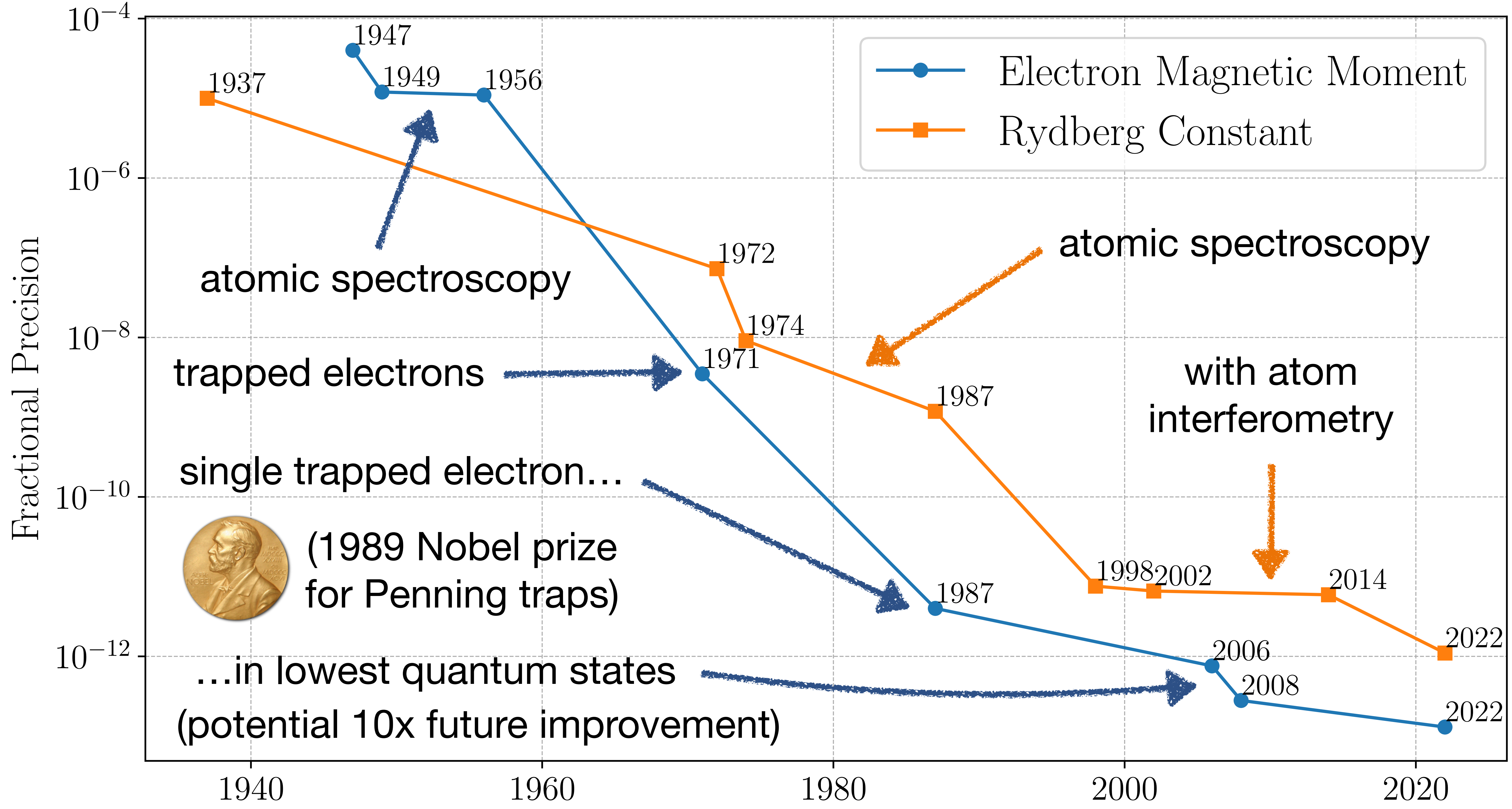
achieves $\Delta g_\mu / g_\mu \sim 10^{-10}$



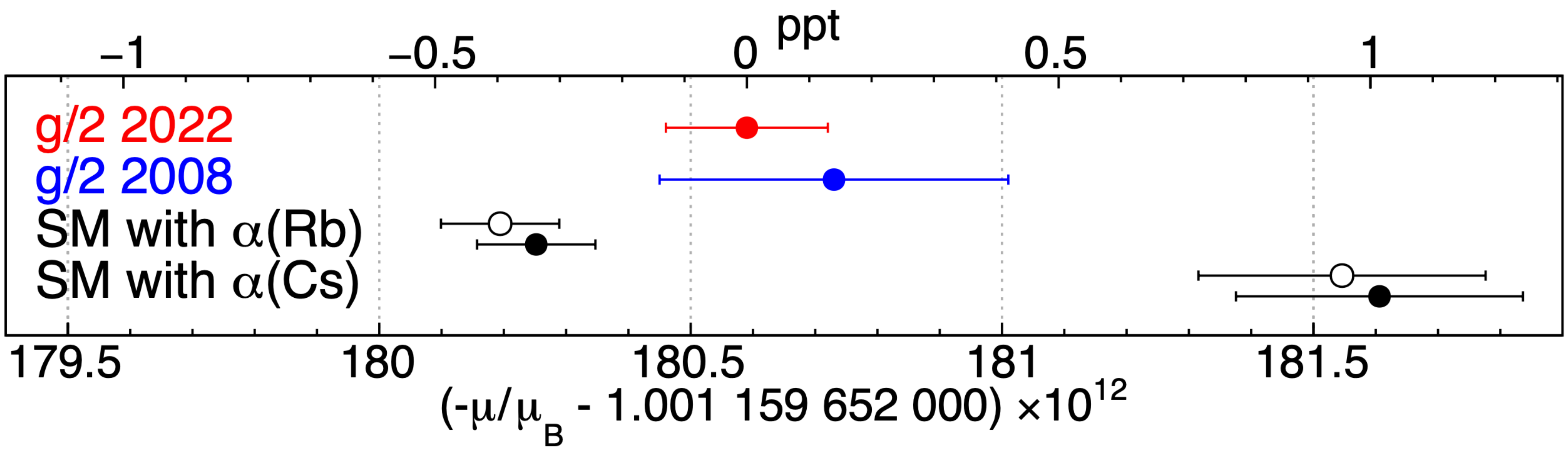
achieves $\Delta g_e / g_e \sim 10^{-13}$

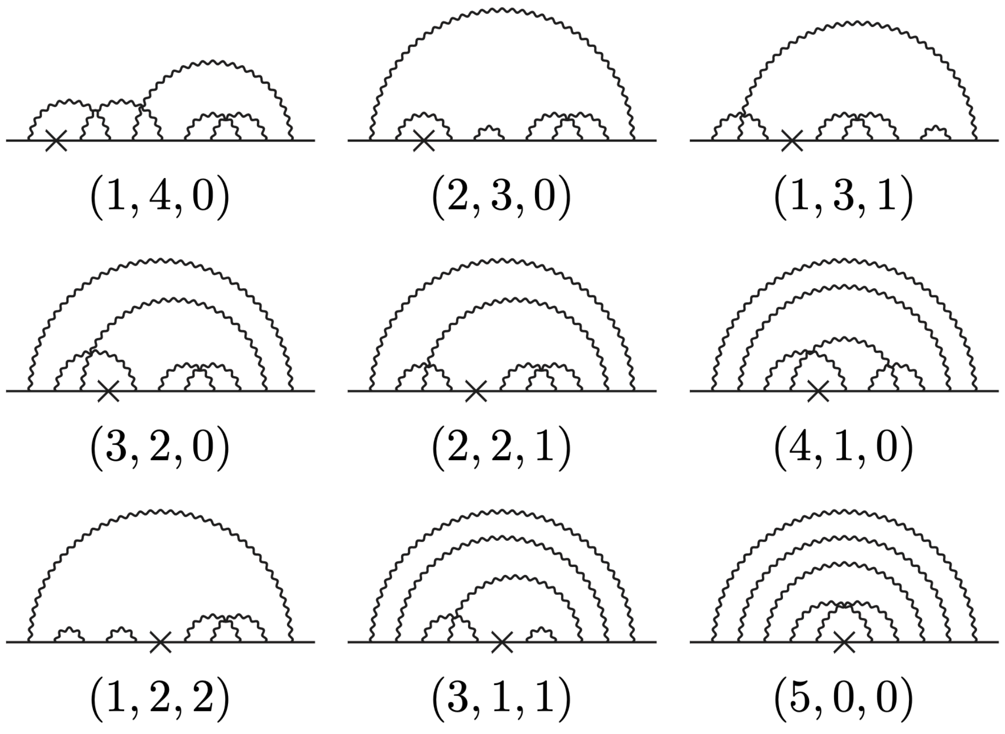
Since $(m_\mu/m_e)^2 \sim 10^4$, “generic” BSM sensitivity of $g_\mu - 2$ higher,
but $g_e - 2$ can scale up, presents different theory questions

A Brief History of Measuring QED Couplings



Precision QED Today





Current measurements of electron $g - 2$ are sensitive to 5-loop QED corrections, and ~TeV scale electron compositeness

Comparison to SM limited by theory uncertainty (but not hadronic), discrepancies in measurements of α

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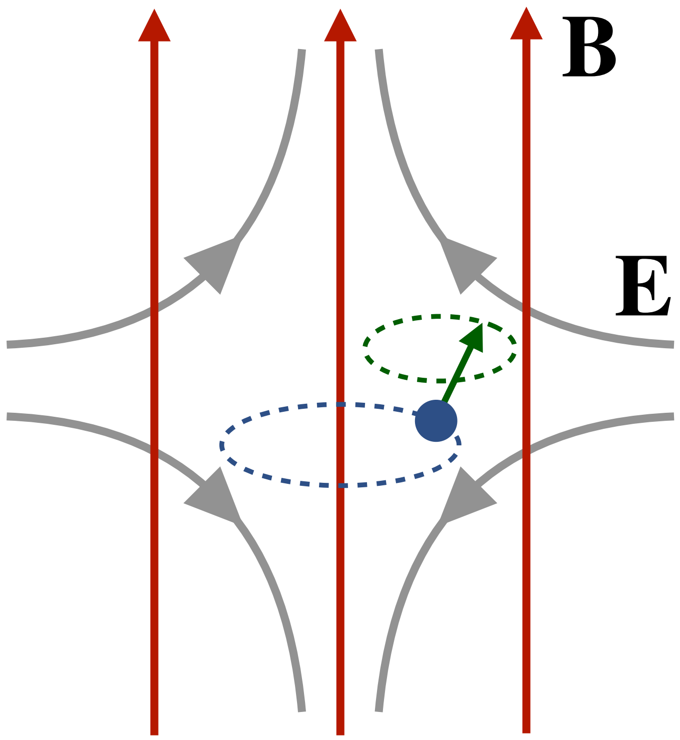
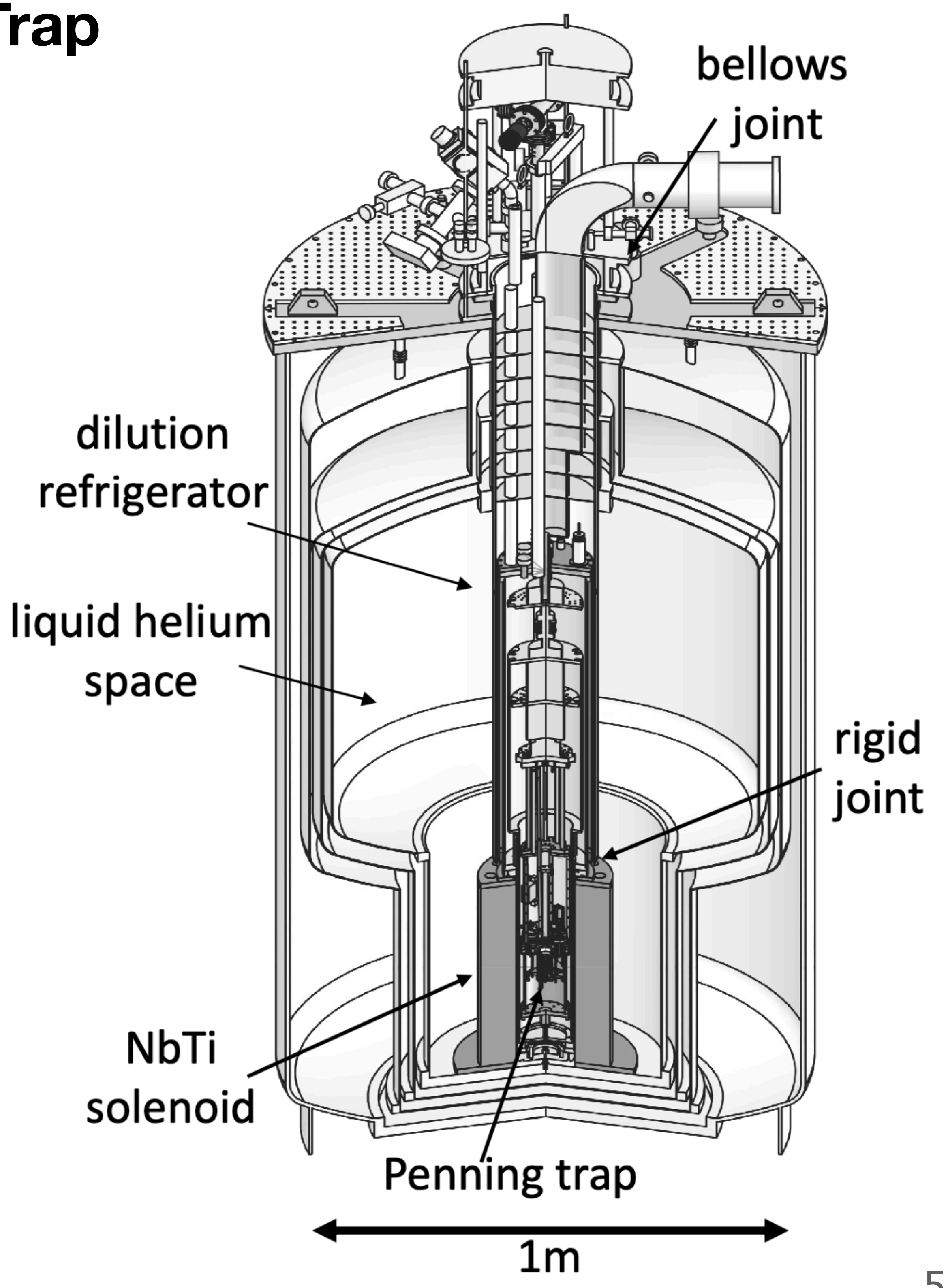
Electron in a Penning Trap



Electron placed in uniform, stable magnetic field

Confined in axial direction by electric field

Cooled to ground state with dilution fridge



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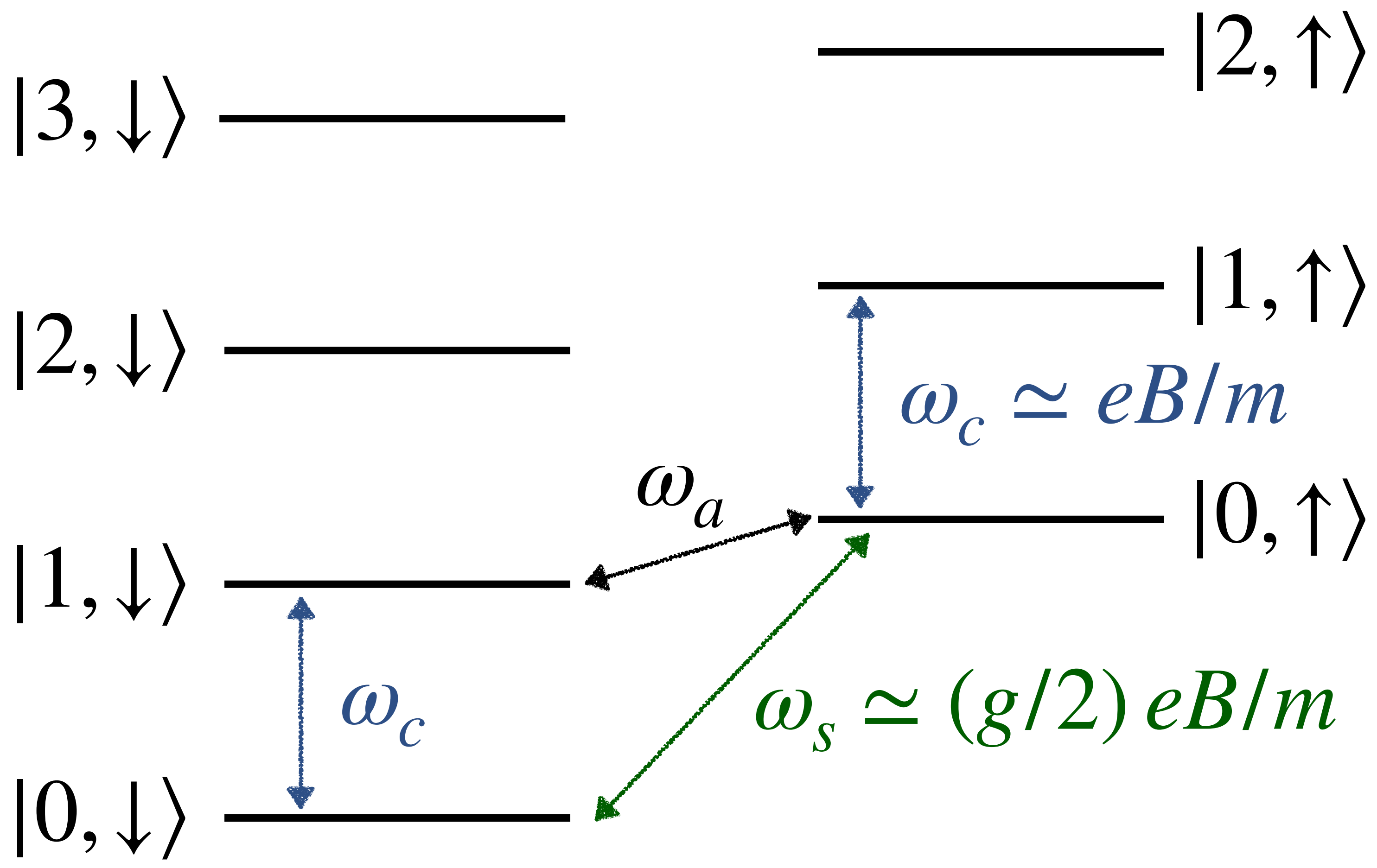
Measuring the g-factor

Electron g determines spacing ω_s between spin states

Can cause transitions by applying oscillating driving fields



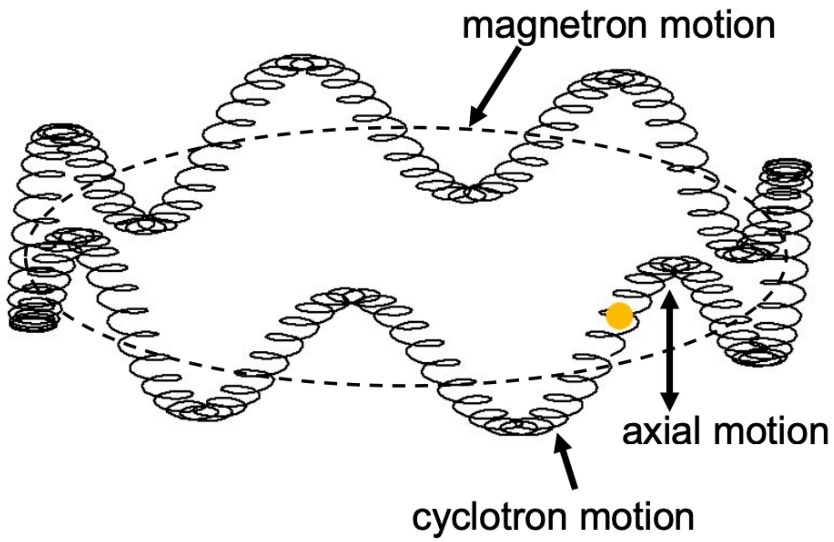
Trick: measure the "anomaly" ω_a

$$\frac{g-2}{2} = \frac{\omega_a}{\omega_c} \sim 10^{-3}$$

Cancels dependence on B and m_e,
precision on g multiplied by 10^3

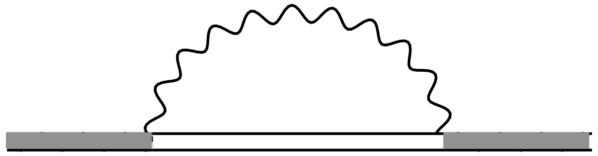
Corrections to Ideal Result

Current measurements must account for any effect that shifts ω_c or ω_s at 10^{-13} level



Slow axial motion: $\dfrac{\Delta\omega_c}{\omega_c} \sim \dfrac{\omega_z^2}{\omega_c^2} \sim 10^{-6}$

Relativistic effects: $\dfrac{\Delta\omega_c}{\omega_c} \sim \dfrac{\text{kinetic energy}}{\text{mass energy}} \sim \dfrac{\omega_c}{m} \sim \dfrac{\text{meV}}{\text{MeV}} \sim 10^{-9}$



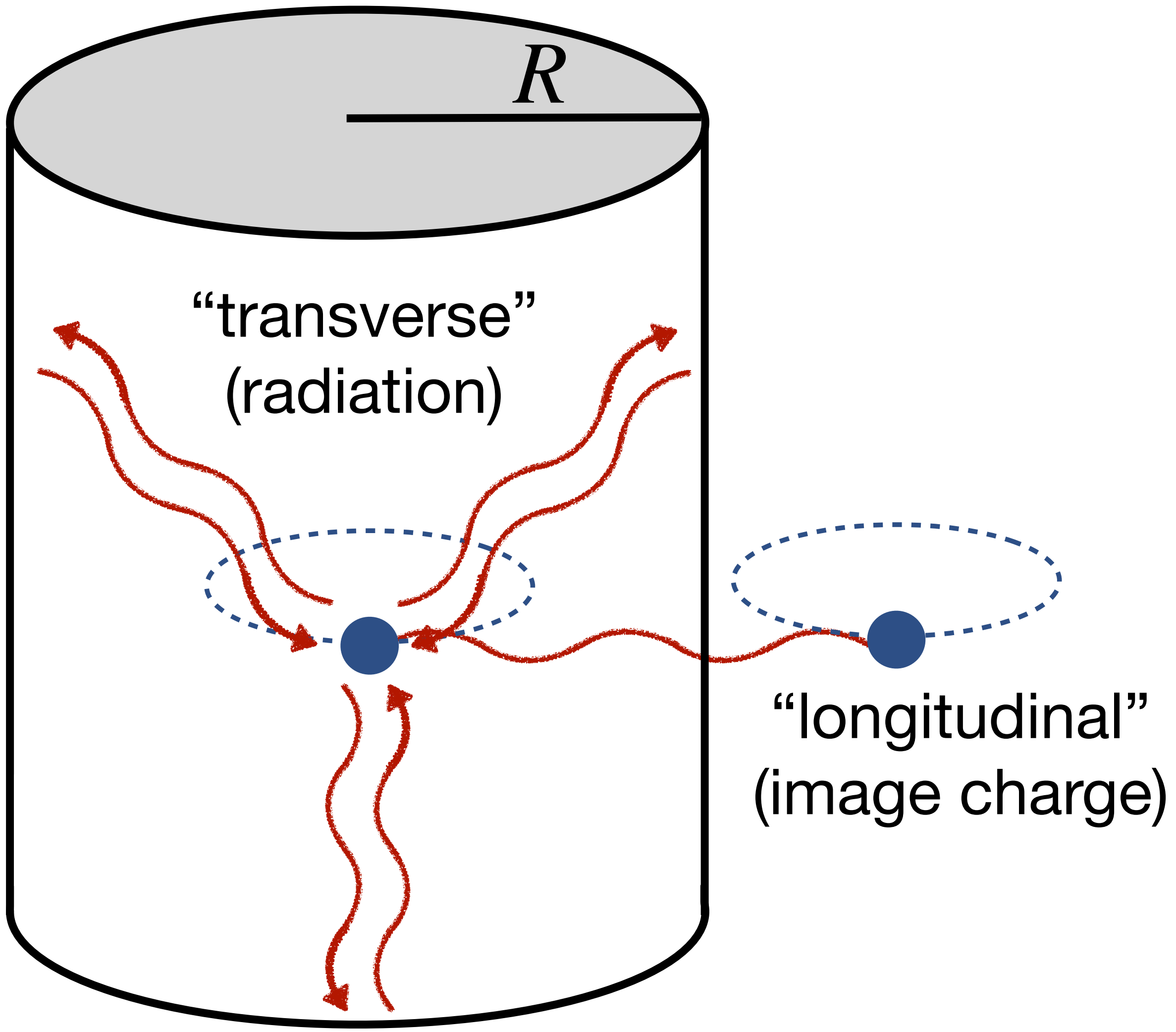
Self-energy in B: $\dfrac{\Delta\omega_c}{\omega_c} \sim \alpha \left(\dfrac{\omega_c}{m}\right)^2 \log(m/\omega_c) \sim 10^{-19}$

Finite lifetime from radiation emission: $\dfrac{\delta\omega_c}{\omega_c} \sim \dfrac{\Gamma}{\omega_c} \sim \dfrac{\alpha\omega_c}{m} \sim 10^{-11}$

uncertainty, not shift!

Effects of the Cavity

Enclosing electron in a cavity increases lifetime if ω_c between cavity modes, but it also yields a "cavity shift" of ω_c itself!



Estimating the longitudinal effect ($\omega_c R \sim 10$):

$$\frac{\Delta\omega_c}{\omega_c} \sim \frac{F_{\text{image}}}{F} \sim \frac{q^2 r_e/(4\pi R^3)}{m\omega_c^2 r_e} \sim \frac{\alpha}{m\omega_c^2 R^3} \sim 10^{-14}$$

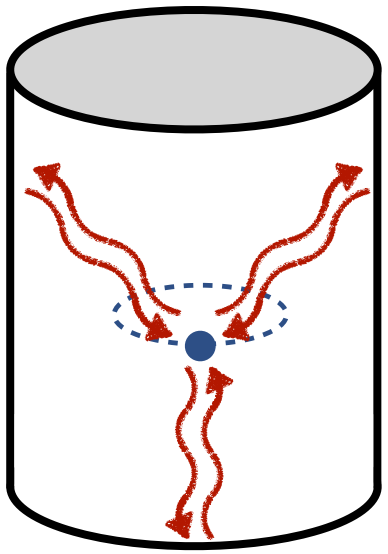
Negligible, though dominant for ion traps

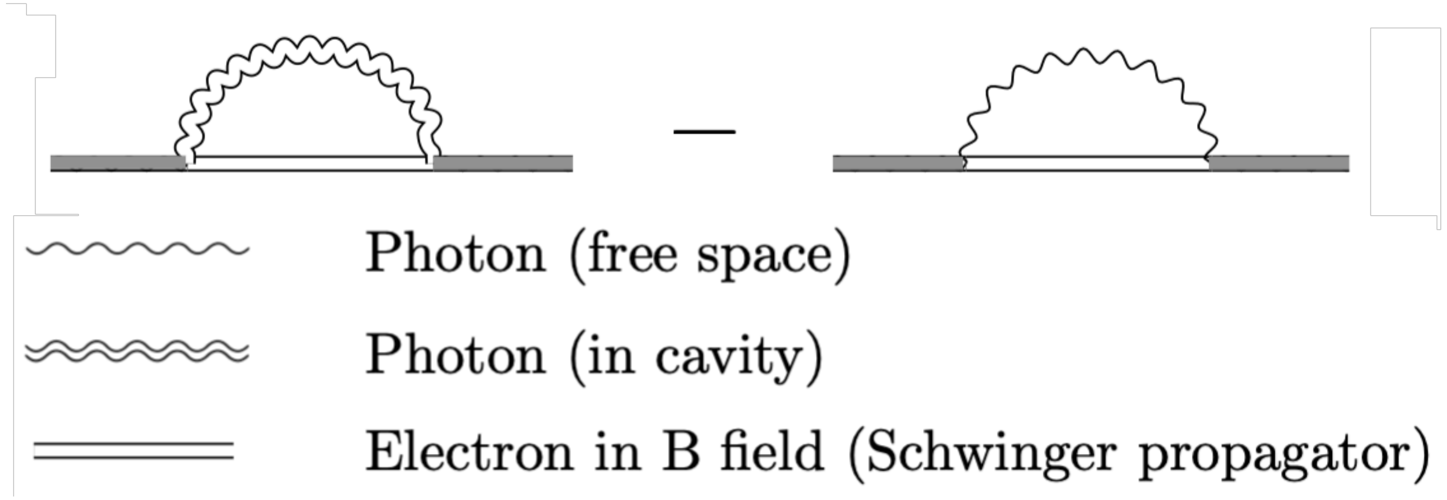
Estimating the transverse effect:

$$\frac{\Delta\omega_c}{\omega_c} \sim \frac{F_{\text{rad}}}{F} \sim \frac{q^2 a_e Q/(4\pi R)}{m a_e} \sim \frac{Q\alpha}{mR} \sim 10^{-12}\, Q$$

Important, and can be resonantly enhanced!

Computing the Cavity Shift





classical: radiation self-field

must remove divergent self-field

done for plates, spherical and cylindrical cavities in 1980s

matches quantum result, at least for plates

hard to generalize to realistic cavities

quantum: modification of self-energy

must renormalize divergent loop diagram

QED between conducting plates: Corrections to radiative mass and $g-2$

Magnetic effects in non-relativistic quantum electrodynamics: image corrections to the electron moment

QUANTUM ELECTROMAGNETICS OF AN ELECTRON NEAR MIRRORS

Mass and magnetic moment of localised electrons near conductors

The interaction of an atom with electromagnetic vacuum fluctuations in the presence of a pair of perfectly conducting plates

ON THE APPARATUS DEPENDENCE OF THE ANOMALOUS MAGNETIC MOMENT OF THE ELECTRON

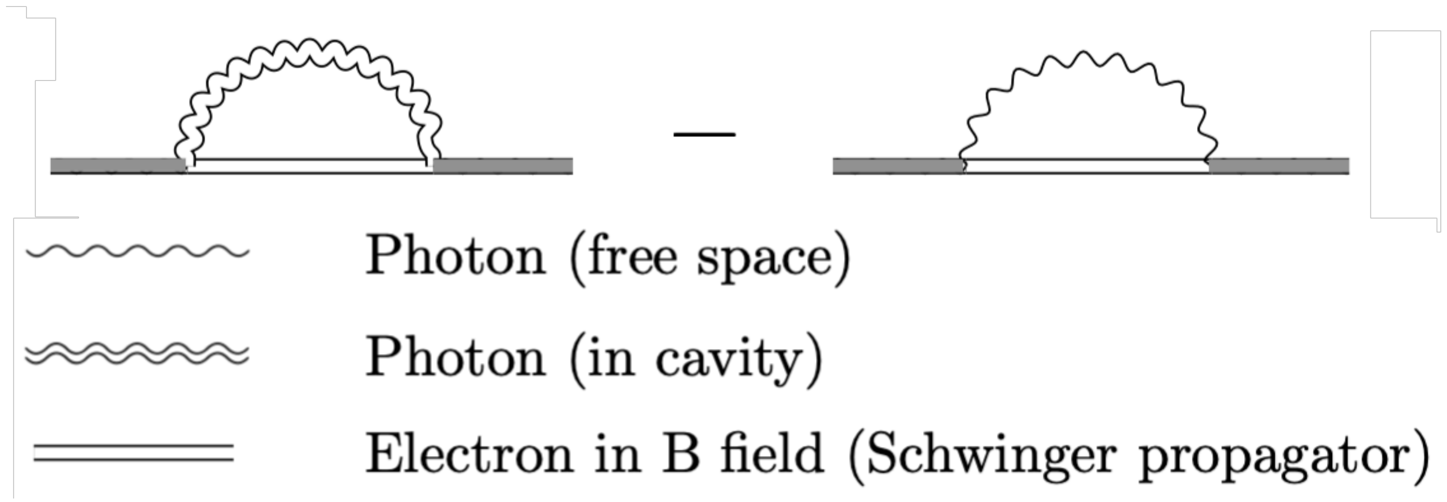
Apparatus-dependent contributions to $g-2$ and other phenomena

Mass and Anomalous Magnetic Moment of an Electron between Two Conducting Parallel Plates

subtle: many papers initially disagreed for plate

never attempted for closed cavity — too hard?

Starting Over From Relativistic QED



Electron energy levels can be extracted from poles of self-energy diagrams

(usual $g - 2$ can also be found this way)

$$\delta E_n = \int d\mathbf{x}\, d\mathbf{x}'\, \overline{u}_n(\mathbf{x}) \Sigma_A(\mathbf{x}, \mathbf{x}') u_n(\mathbf{x}') \qquad \Sigma_A(\mathbf{x}, \mathbf{x}'; E) \sim e^2 \int d\tau\, \gamma^\mu S_A(\mathbf{x}, \mathbf{x}'; \tau) \gamma^\nu D_{\mu\nu}(\mathbf{x}, \mathbf{x}'; \tau) e^{iE\tau},$$

electron state

Schwinger's exact propagator in external $\mathbf{B}$

photon propagator

$$D^{ij}(\omega, \mathbf{k}) \sim \sum_s u_s^i(\mathbf{0})\, u_s^j(\mathbf{0})^* \frac{\delta(\mathbf{k})}{\omega^2 - \omega_s^2 + i\epsilon}.$$

Finite after subtracting off free space self-energy, **and** the $B = 0$ self-energy

approximates electron as localized

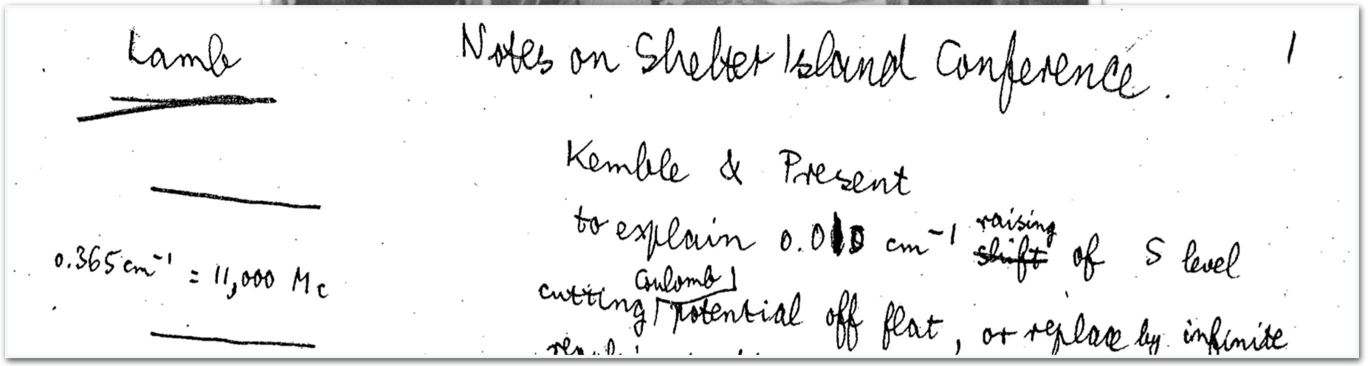
sum over cavity modes s

The Nonrelativistic Limit

Delicately taking the nonrelativistic limit recovers the answer one would get in nonrelativistic quantum mechanics

$$\delta E_n \simeq \frac{e^2}{m^2} \sum_{s,n'} \frac{|\langle n', 1_s | \mathbf{A} \cdot \pi | n, 0 \rangle|^2}{E_n - (E_{n'} + \omega_s)}$$

Result is linearly divergent, but convergent when cavity and free space subtracted



Lamb

Notes on Shelter Island Conference. 1

Kemble & Present

to explain 0.0[illegible] cm^{-1} ~~shift~~ raising of S level

0.365 cm^{-1} = 11,000 Mc

cutting Coulomb potential off flat, or replace by infinite

Not a foregone conclusion!

Lamb shift is log divergent in the nonrelativistic theory, needs UV matching to get quantitatively right

Example: Spherical Cavity of Radius a

$$\delta E_n \simeq \frac{e^2}{m^2} \sum_{s,n'} \frac{|\langle n', 1_s | \mathbf{A} \cdot \pi | n, 0 \rangle|^2}{E_n - (E_{n'} + \omega_s)}$$

$$\Delta\omega_c = \delta E_{n+1} - \delta E_n$$

In cavity: only the TM_{11p} modes are nonzero at cavity center

$$\frac{\Delta\omega_c^{\mathrm{cav}}}{\omega_c} = -\frac{8}{3\pi}\frac{\alpha}{ma} S$$

$$S = \sum_{p=1}^{\infty} f(p) = \sum_{p=1}^{\infty} \frac{c_p^3}{c_p^2 - 2} \frac{1}{J_{3/2}^2(c_p)} \frac{1}{c_p^2 - z^2}$$

In free space: integrate over plane waves $(\omega, \mathbf{k})$

$$\frac{\Delta\omega_c^{\mathrm{free}}}{\omega_c} = -\frac{8}{3\pi}\frac{\alpha}{ma} I \qquad I = \frac{1}{2}\int_0^{\infty} dc \, \frac{c^2}{c^2 - z^2}$$

where $c_p = \omega_p a$, $z = \omega_c a$, $c = \omega a$

Smooth regulator should treat c_p and c the same way

How can we subtract the divergent sum and integral analytically?

Subtracting the Sum and Integral

$$S = \sum_{p=1}^{\infty} f(p) = \sum_{p=1}^{\infty} \frac{c_p^3}{c_p^2 - 2} \frac{1}{J_{3/2}^2(c_p)} \frac{1}{c_p^2 - z^2} \qquad I = \frac{1}{2}\int_0^\infty dc \, \frac{c^2}{c^2 - z^2} \qquad \begin{aligned} c_p &= \omega_p a \\ c &= \omega\, a \end{aligned}$$

First trick: define a continuous analogue of the spherical Bessel root c_p

$$\frac{d}{dx}\left(\frac{\sin x}{x} - \cos x\right)_{x=c_p} = 0 \quad \longleftrightarrow \quad c(p) + \arctan\left(\frac{c(p)}{c(p)^2 - 1}\right) = \pi p$$

after Bessel identities: $f(p) = \frac{1}{2}\frac{dc}{dp}\frac{c^2}{c^2 - z^2}$

Second trick: relate both the regulated sum and integral to the same contour integral

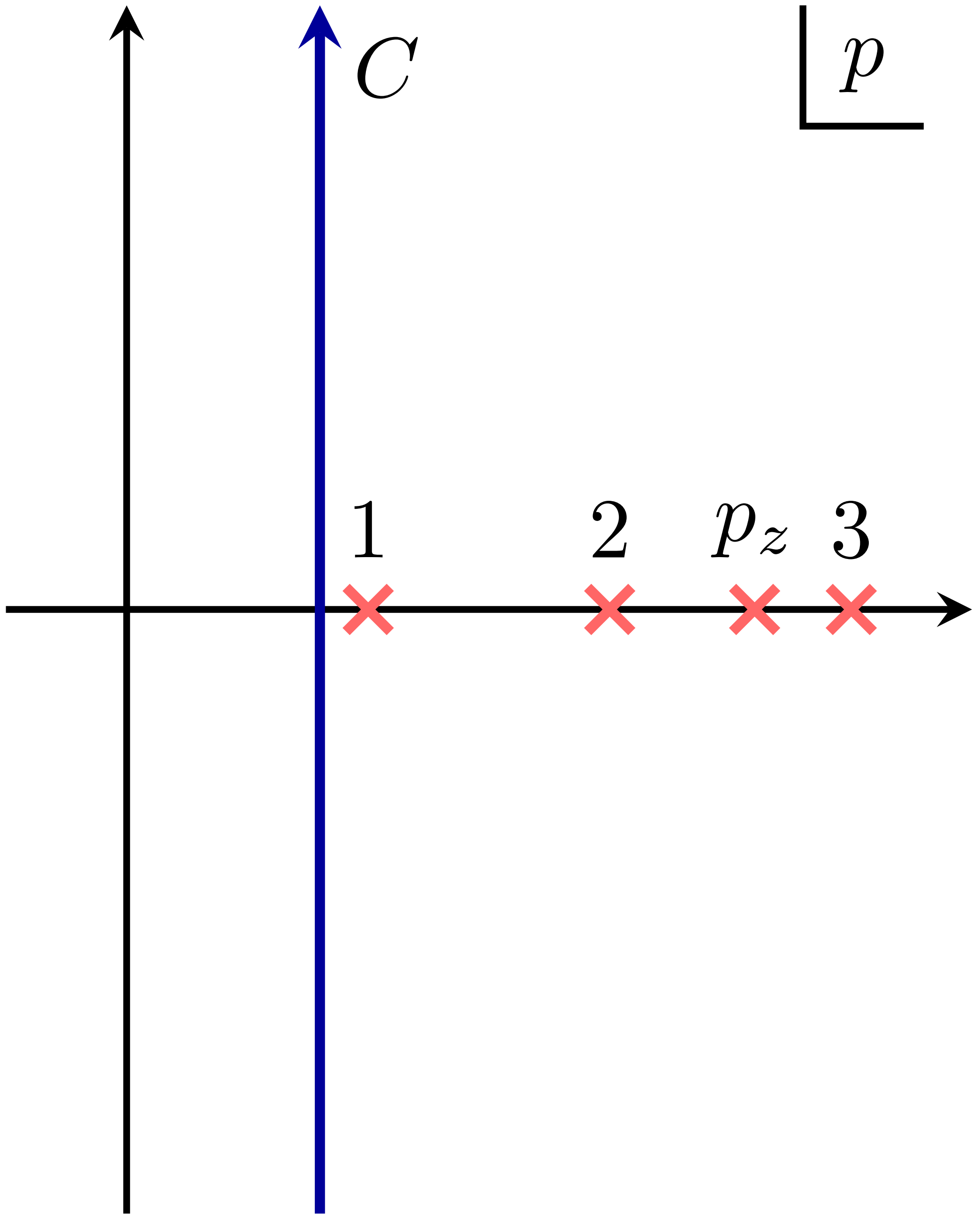
Will keep the regulator (damping at large c) implicit

Physical difference arises at low c, regulator independent

Contour Integration for the Spherical Cavity

Consider contour integral $A = \int_C \frac{f(p)}{e^{-2\pi i p} - 1} \, dp$

Analytic regulator lets us close at infinity, $A = S + (\text{pole})$



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Consider contour integral $A = \int_C \frac{f(p)}{e^{-2\pi i p} - 1}\, dp$

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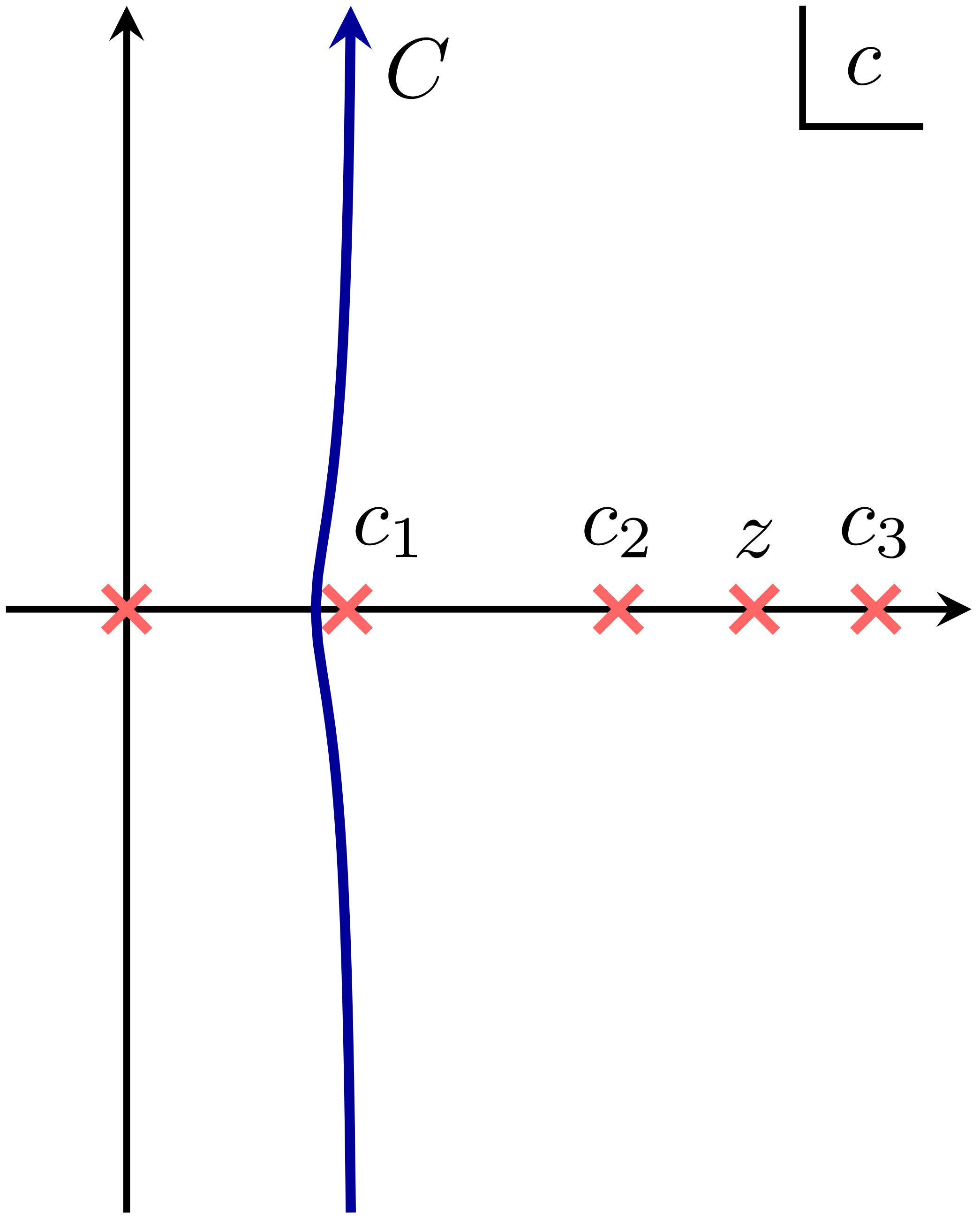
Changing variables to c gives $A = \int_C \frac{1}{2} \frac{c^2}{c^2 - z^2}\, g(c)\, dc$

$$g(c) = \frac{1}{2}\left(-1 + \frac{(c^2 - 1)\,\cos c - c\,\sin c}{c\,\cos c + (c^2 - 1)\,\sin c}\, i \right)$$



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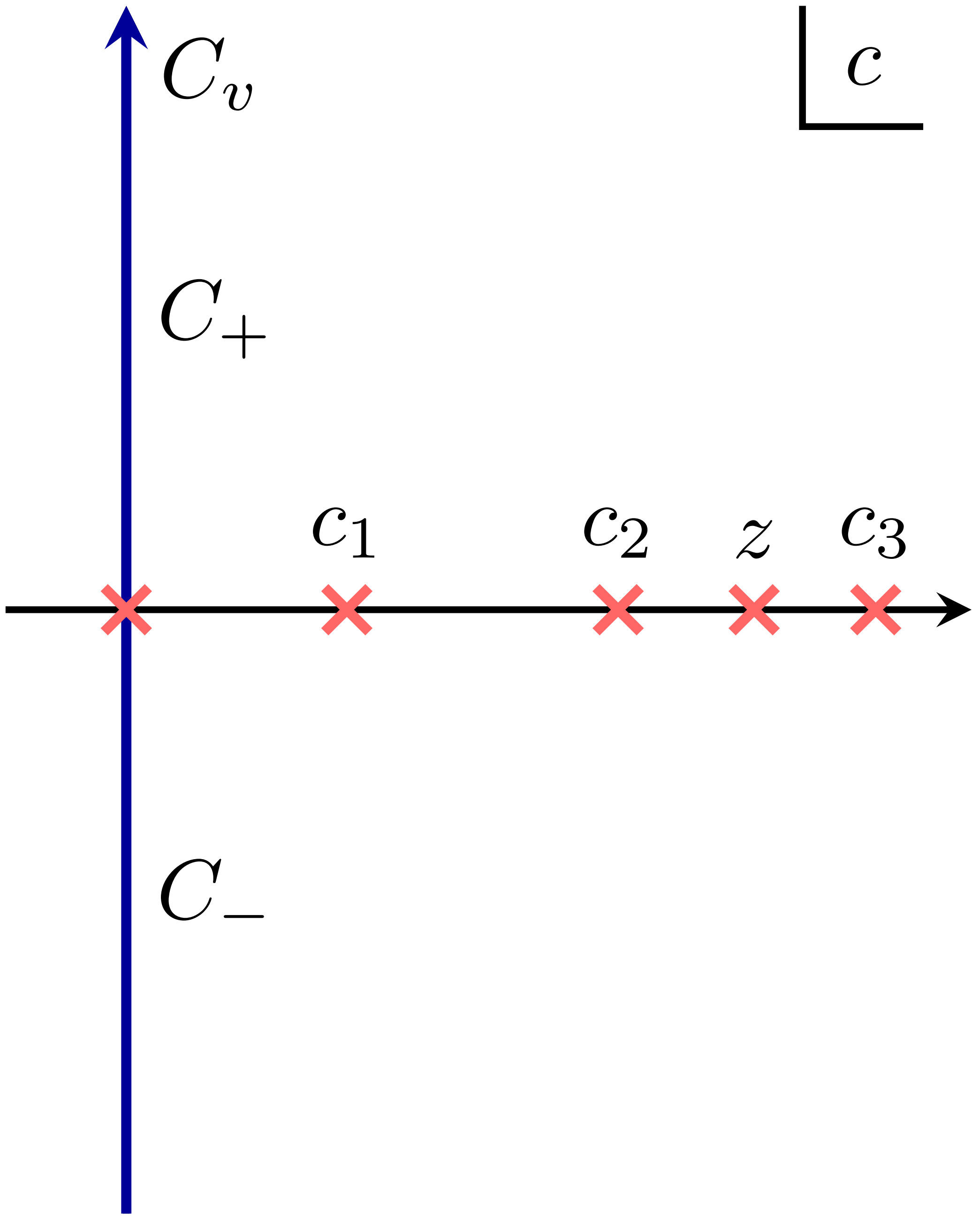
Push contour to imaginary axis, $g(-iy) + 1 = -\, g(iy)$

Pull out $+1$ and cancel, $A = \text{pole} - \int_{C_-} \frac{1}{2} \frac{c^2}{c^2 - z^2}\, dc$



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Combining results:

$$I - S = \frac{\pi z}{4}$$

$$\times \left(\frac{(1 - z^2)\cos z + z\sin z}{(1 - z^2)\sin z - z\cos z} + \frac{3}{2z^3} \right)$$

perfectly matches classical answer!

New Features for the Cylindrical Cavity

Real cavity is approximately cylindrical, more complicated

TE_{1np} and TM_{1np} modes contribute, giving two double sums

Sum over p can be performed exactly

Separate out part of sum that depends on aspect ratio L/a

Subtract against integral of plane waves in cylindrical coordinates

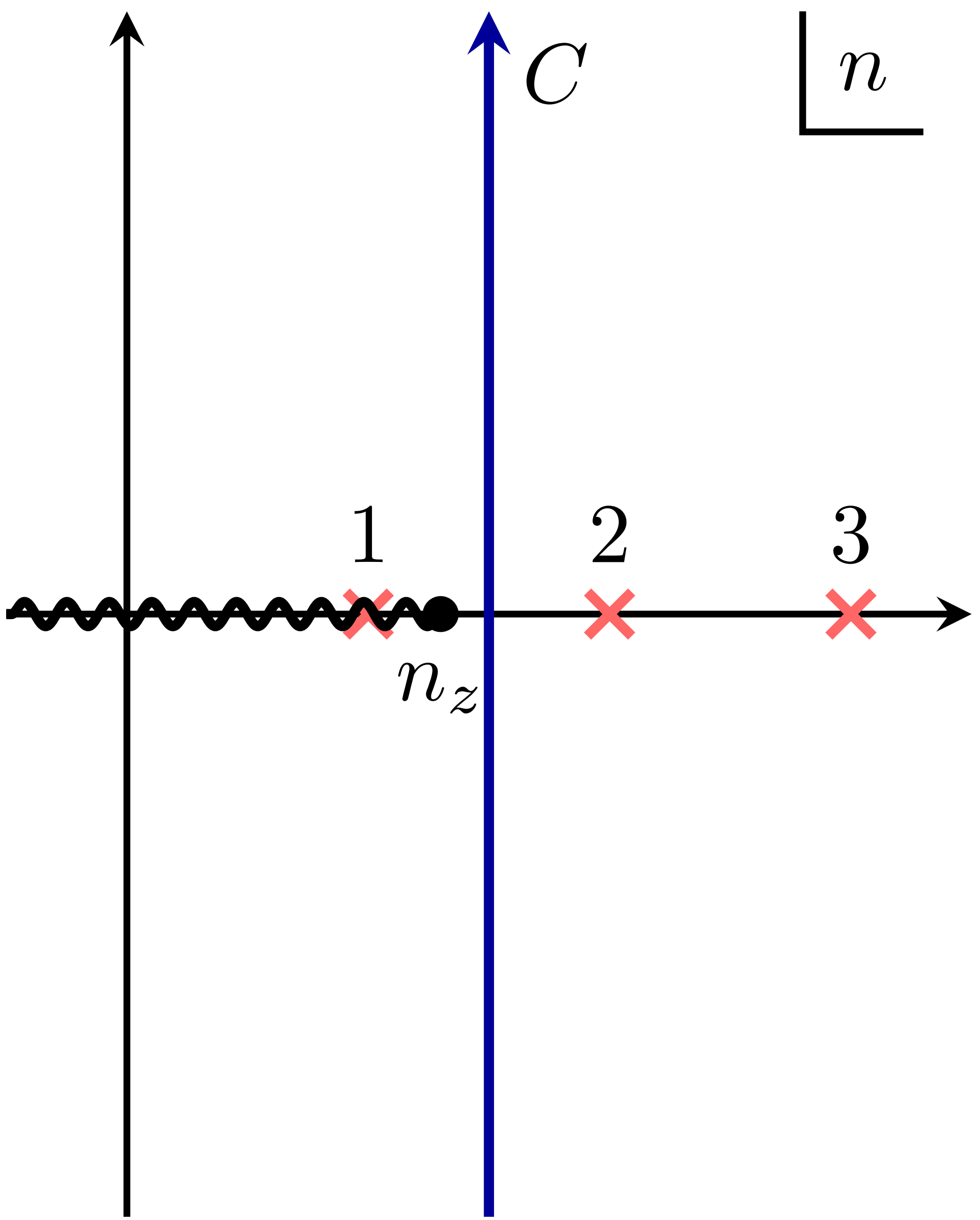
Make summand analytic using Bessel identities and

$$\tan(\pi n) + \frac{J_1(c_n)}{Y_1(c_n)} = 0 \qquad \tan(\pi n) + \frac{J_1'(\bar{c}_n)}{Y_1'(\bar{c}_n)} = 0$$

Sum and integral contain square roots, giving branch cuts

Sketch: Contour Integration for the Cylindrical Cavity

For TE modes, let $A_{\text{TE}} = \int_C \frac{f_{\text{TE}}(n)}{e^{-2\pi i n} - 1}\, dp = S_{\text{TE}} - (\text{poles})$



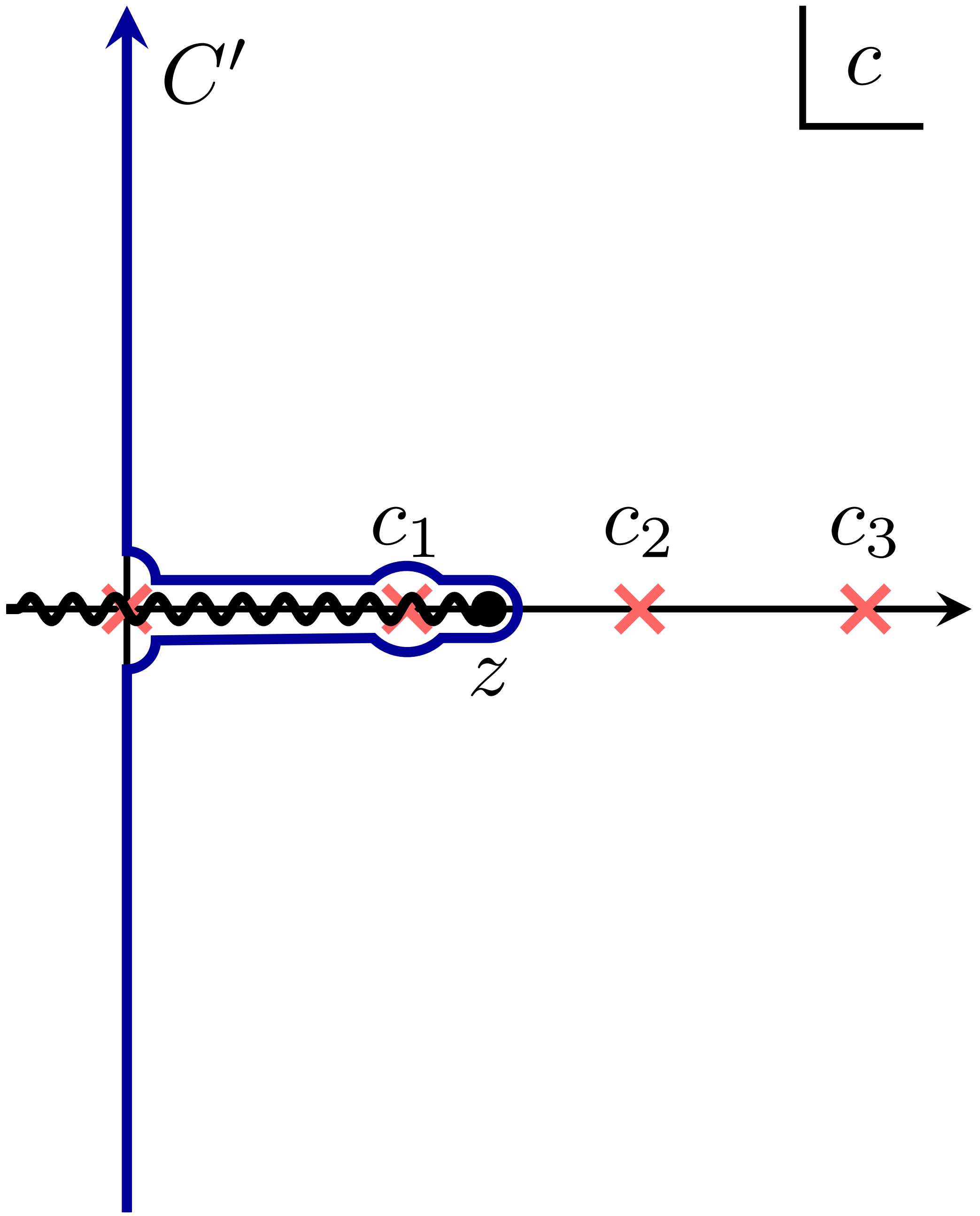
Sketch: Contour Integration for the Cylindrical Cavity

For TE modes, let $A_{\text{TE}} = \displaystyle\int_C \frac{f_{\text{TE}}(n)}{e^{-2\pi i n} - 1}\, dp = S_{\text{TE}} - (\text{poles})$

Changing variables to c gives $A_{\text{TE}} = \dfrac{1}{2}\displaystyle\int_C \frac{c}{\sqrt{c^2 - z^2}}\, g(c)\, dc$

$$g(c) = -\frac{1}{2}\left(1 + \frac{Y_1'(c)}{J_1'(c)}\, i\right) \qquad g(-iy) + 1 = g(iy)$$

Deform contour left, extract $+1$, use symmetry



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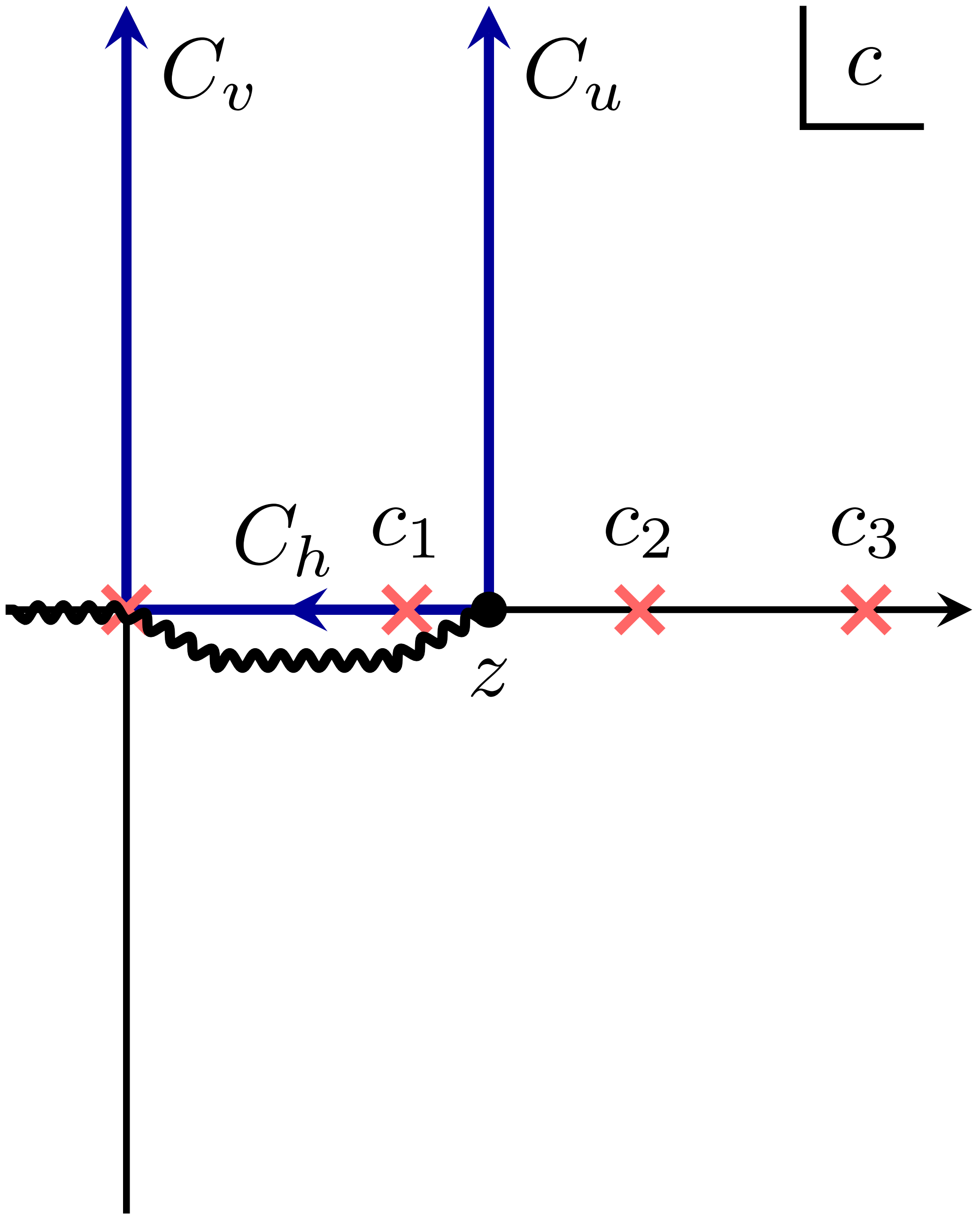
Deform contour left, extract $+1$, use symmetry

Point contour up for exponentially damped remainder

$$A_{\mathrm{TE}} = I_{\mathrm{TE}} + \mathrm{Re} \int_{C_u} \frac{c}{\sqrt{c^2 - z^2}}\, g(c)\, dc$$



Final Results for the Cylindrical Cavity

New quantum result

$$\frac{\Delta\omega_c}{\omega_c} = -\frac{\alpha}{ma}\left(\Delta S + J\right)$$

$$J = \frac{1}{\pi}\,\mathrm{Re}\int_0^\infty \frac{(y-iz)\sqrt{y^2-2iyz}}{z^2}\frac{K_1(y-iz)}{I_1(y-iz)} - \frac{K_1(y)}{I_1(y)}\frac{y^2}{z^2} + \frac{y-iz}{\sqrt{y^2-2iyz}}\frac{K_1'(y-iz)}{I_1'(y-iz)}\,dy$$

exponentially damped

$$\Delta S = \sum_{n=\bar{n}^*+1}^{\infty}\left(\tanh\left(\frac{L\sqrt{\bar{c}_n^2-z^2}}{2a}\right)-1\right)f_{\mathrm{TE}}(n) + \sum_{n=1}^{\bar{n}^*}\tan\left(\frac{L\sqrt{z^2-\bar{c}_n^2}}{2a}\right) i f_{\mathrm{TE}}(n) + (\mathrm{TM\ terms})$$

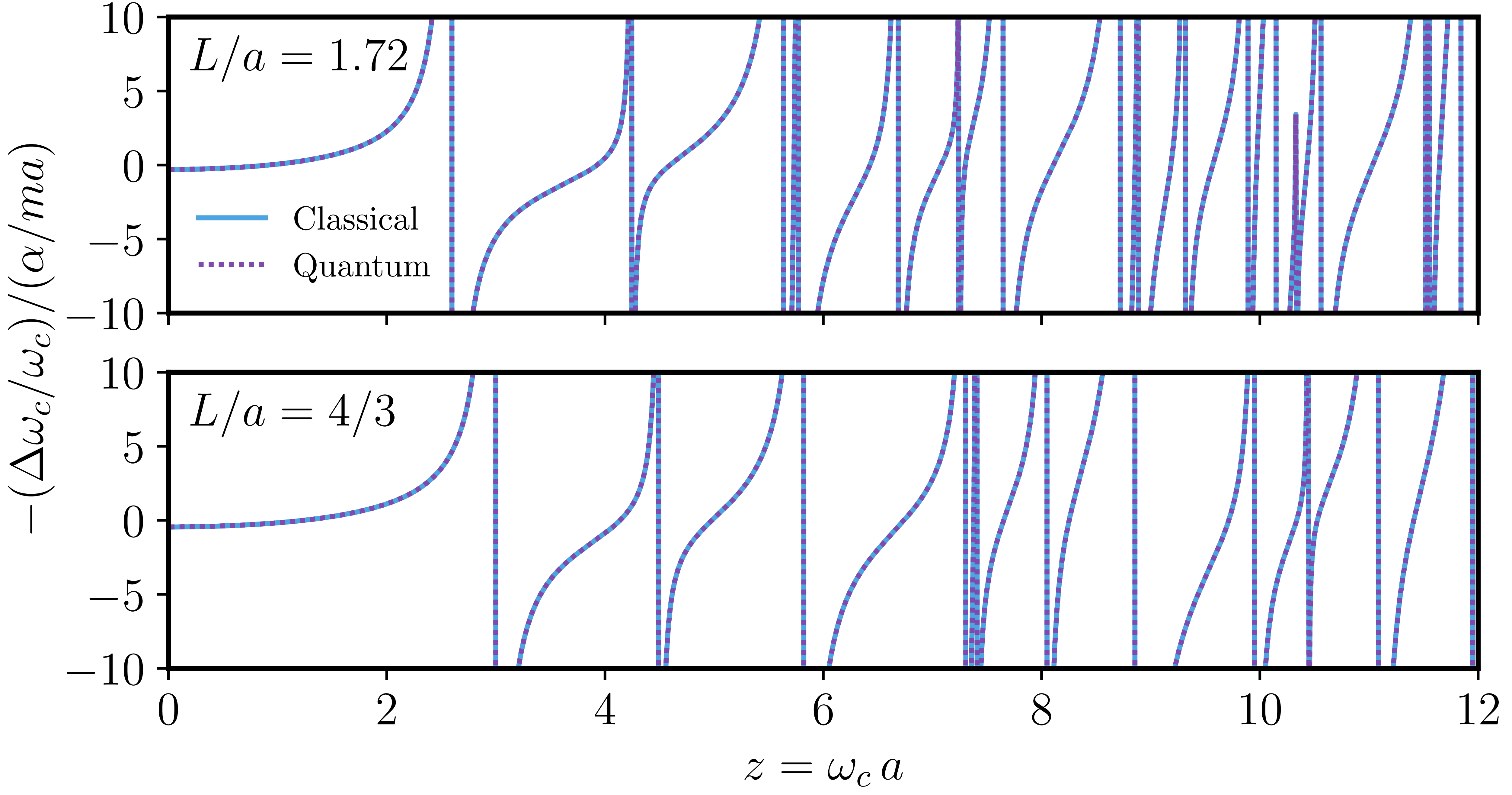
finite + damped

Existing classical result

$$\frac{\Delta\omega_c}{\omega_c} = \frac{2\alpha}{mL}\left[\frac{1}{2}\log(4\cos^2(\xi/2)) + \sum_{n=1}^{\infty}(-1)^n\left(\frac{\sin(n\xi)}{n^2\xi} + \frac{\cos(n\xi)-1}{n^3\xi^2}\right) - \mathrm{Re}\sum_{p=0}^{\infty}\left(\frac{K_1'(\mu_p a)}{I_1'(\mu_p a)} + \frac{k_p^2}{\omega_c^2}\left(\frac{K_1(\mu_p a)}{I_1(\mu_p a)} - \frac{K_1(k_p a)}{I_1(k_p a)}\right)\right)\right]$$

$$\xi = \omega_c L \qquad k_p = (2p+1)\pi/L \qquad \mu_p = \sqrt{k_p^2-\omega_c^2}$$

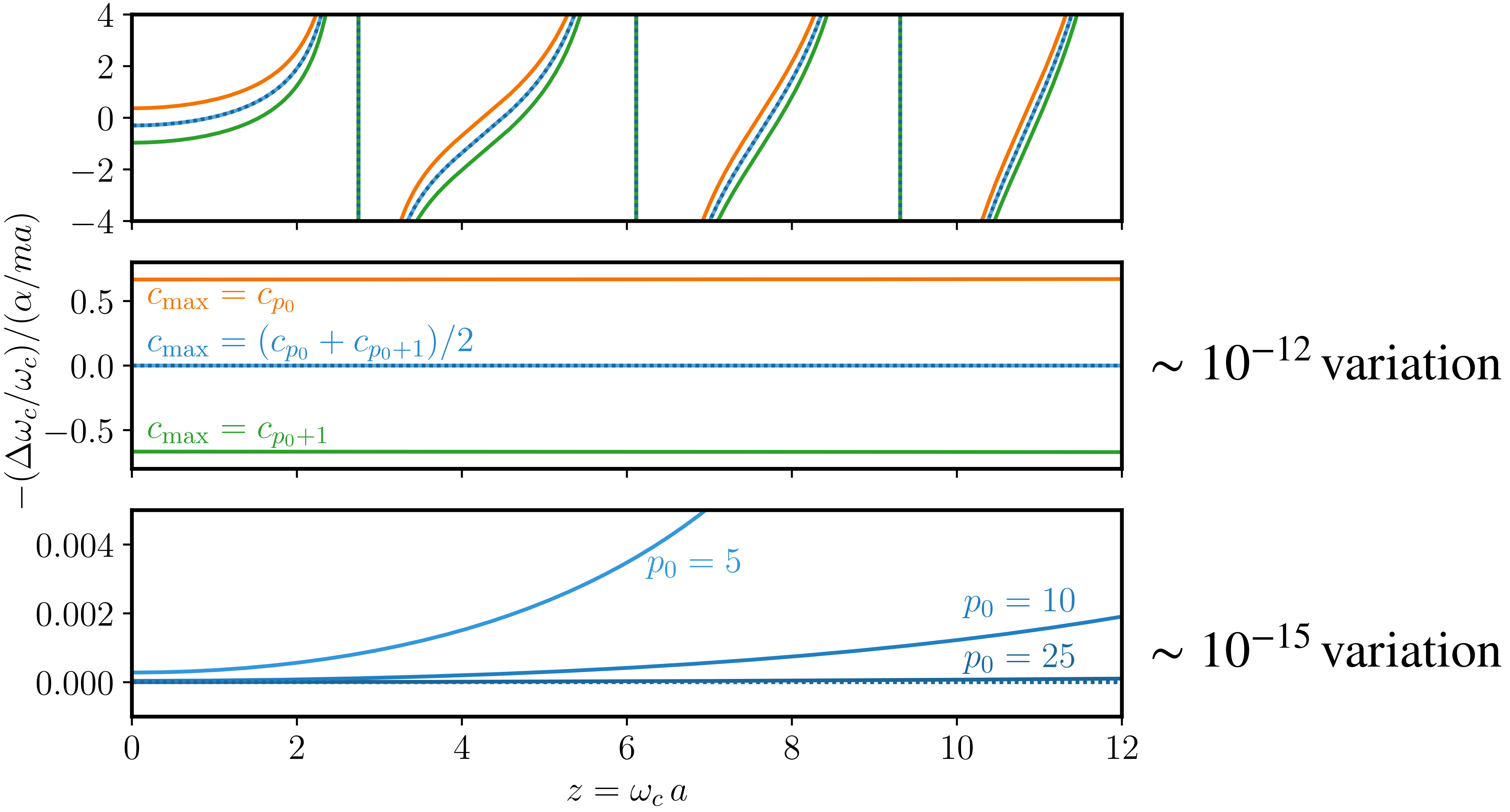
Quantum and Classical Comparison



Analytic match possible in special cases; in general, perfect numeric match

Sphere with Concrete Cutoffs

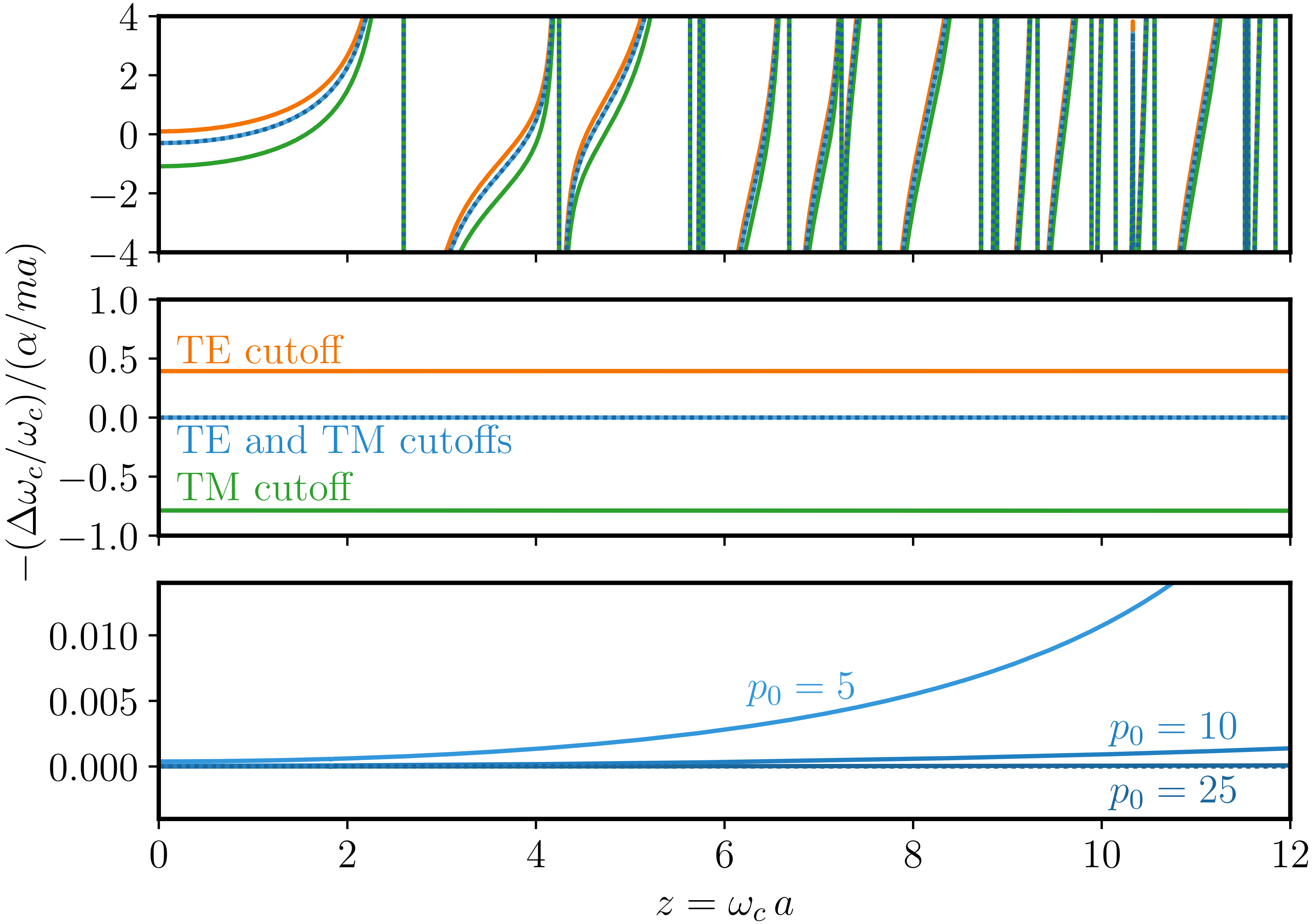
Now that we know the quantum answer (with frequency-dependent cutoff) matches the classical one, we can efficiently evaluate it with specific cutoffs



For sphere, hard cutoff in the right place gives highly accurate result with few terms

Cylinder with Concrete Cutoffs

For the cylinder, need to cut off parts of the integral differently, depending on if they correspond to TE or TM modes



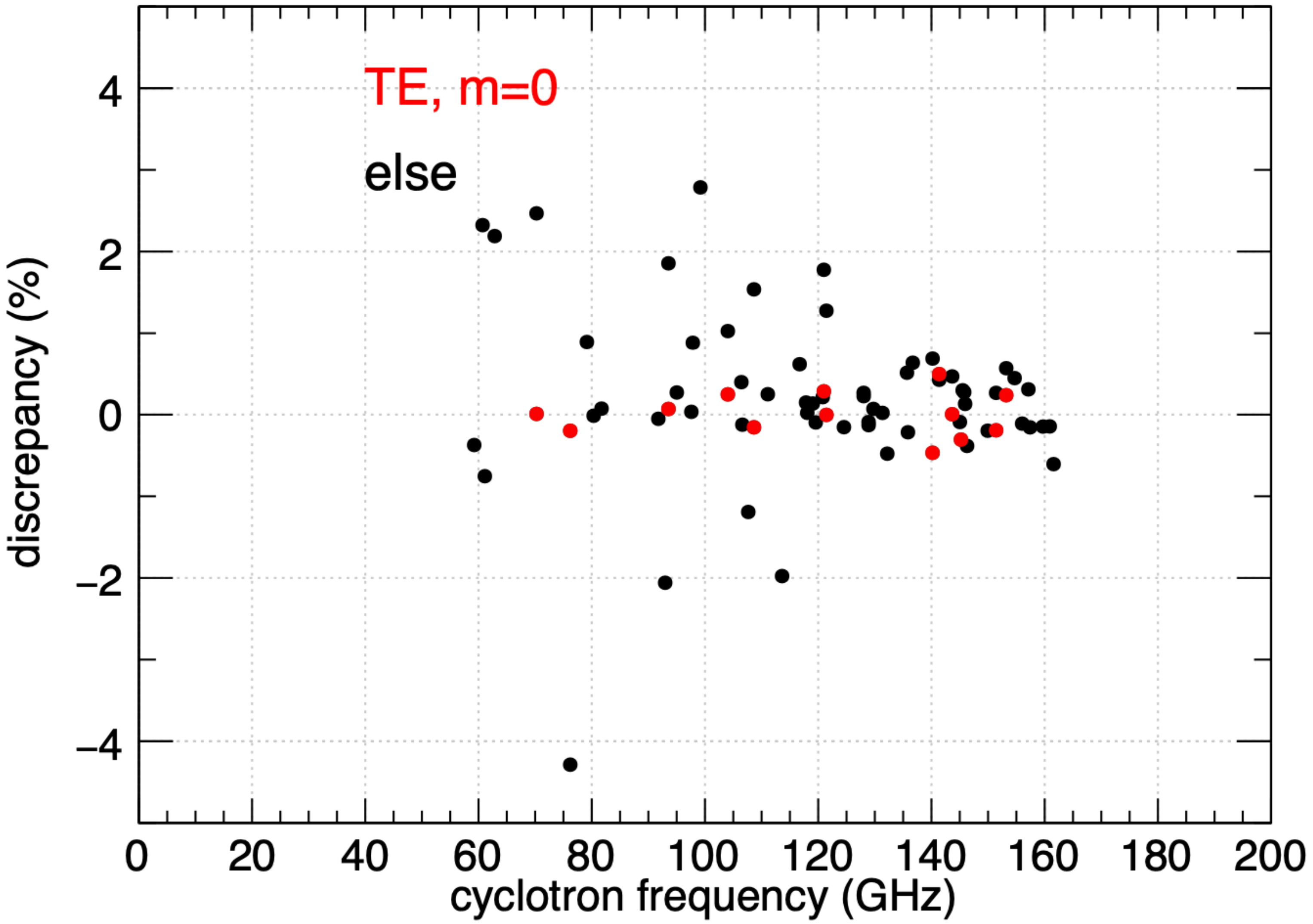
$\sim 10^{-12}$ variation

$\sim 10^{-15}$ variation

Again, highly accurate results with small number of terms

Next Steps

Results with low cutoff are still very accurate, because the exact $S - I$ only depends on exponentially damped quantities



Xing Fan, PhD thesis (2022)

Opens door to treating the real cavity, which is **not** an ideal cylinder

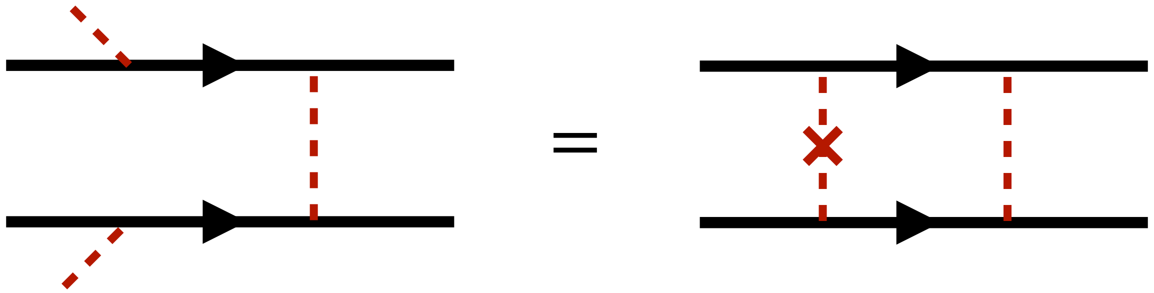
Mode frequencies and coupling strengths deviate by few %, and quality factors vary

Already one of the most important systematics

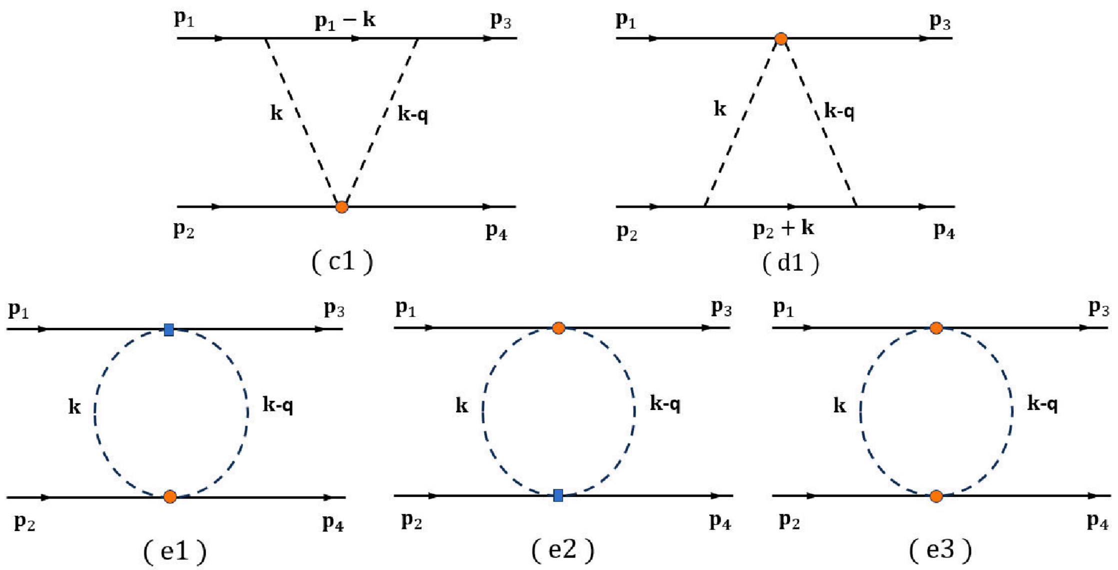
Mode-based calculation naturally accommodates these features!

Dark Matter "Background-Induced" Forces are Also Classical

Tree-level coupling to external axions is formally a background-corrected loop diagram



Recent claim: axion-mediated potential can become $1/r$, spin independent



Looks quantum, but isn't: short classical calculation gives exact same result!

$$(\partial^2 + m_a^2)\, a = J$$

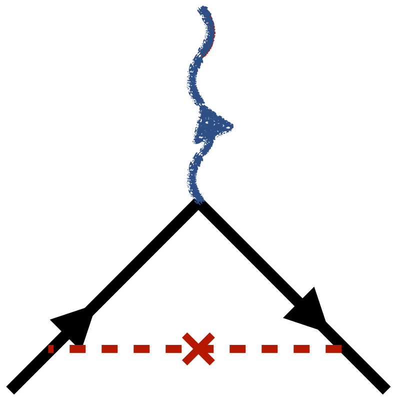
$$J(\mathbf{x}, t) = \partial_t(\mathbf{v} \cdot \hat{\mathbf{s}})\, \delta^{(3)}(\mathbf{x} - \mathbf{r}(t))\,.$$

$$\frac{d\mathbf{v}}{dt} = \frac{\mathbf{F}}{m} = \frac{g\, \ddot{a}\, \hat{\mathbf{s}}}{m}$$

Classical analogues for ultralight DM effects essentially always exist

Dark Matter “Background-Induced” $g-2$ Shifts are Also Classical

Claim: DM background yields very strong shifts of electron $g-2$ and EDMs



$$\Delta g_e \sim \frac{\rho_{\rm DM}}{m_{\rm DM}^2 m_e^2}\, g^2$$

2302.08746, PRL (2024)
2308.05375, JHEP (2025)
2410.10715
2412.14664
2509.12869

Sold as intrinsically quantum, but again can be derived classically!

KZ, JHEP (2025)

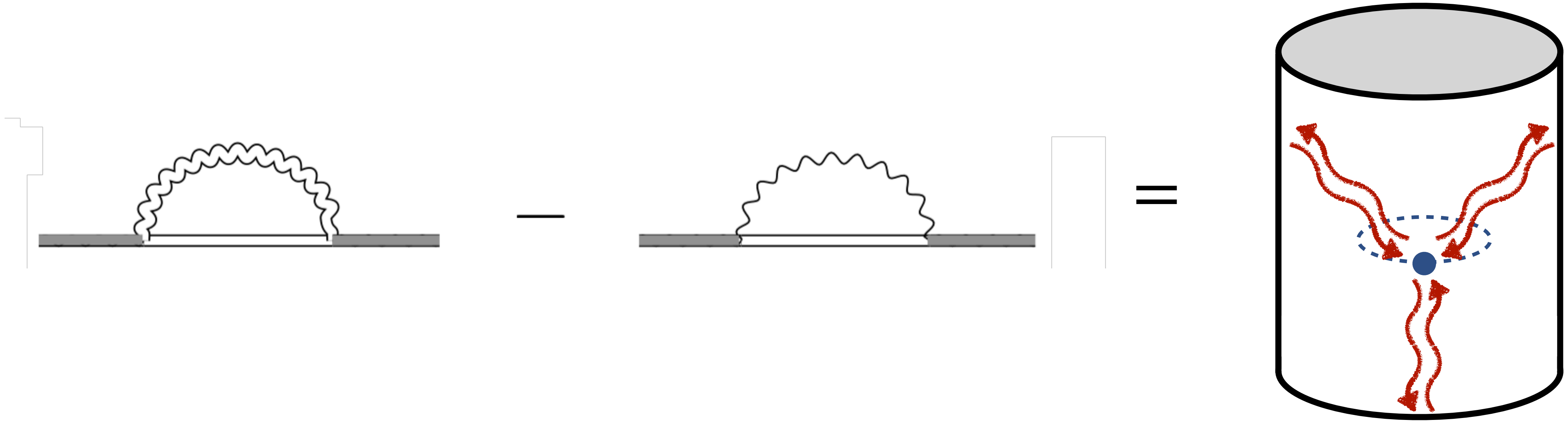
$$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad \frac{d\mathbf{S}}{dt} = \mathbf{S} \times \left(\frac{q g_e}{2m_e} \left(\mathbf{B} - \frac{\gamma}{\gamma+1}(\mathbf{v}\cdot\mathbf{B})\mathbf{v} - \mathbf{v}\times\mathbf{E} \right) + \frac{\gamma^2}{\gamma+1}\mathbf{v}\times\mathbf{a} \right).$$

Like the ponderomotive force or gravitational wave memory,
derived by carefully solving for classical motion at second order

Classical derivation also reveals IR cutoff, making effect negligible in practice

Conclusion

Physics works: QED knows about subtle long-distance phenomena



Classical physics works better than many think, especially in the infrared

But derivations with “quantum” language can have benefits as well

Future work can model cavity and trap imperfections,
improving systematic uncertainty for future measurements

From the Lamb shift to the cavity shift: 75 years of the rich physics of $g - 2$