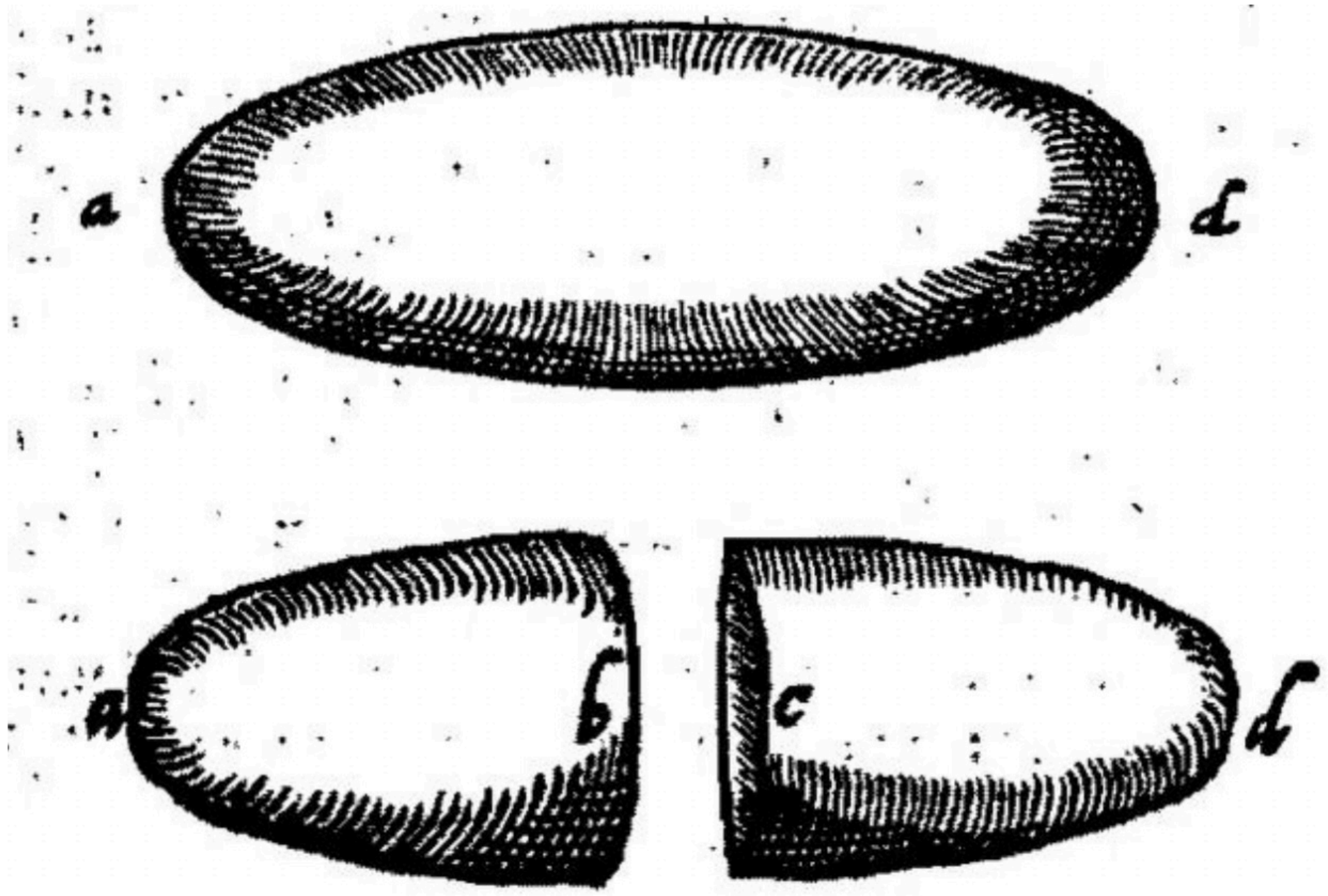
Phenomenology of Dark Magnetic Monopole

Hsing-Yi Lai

Outline

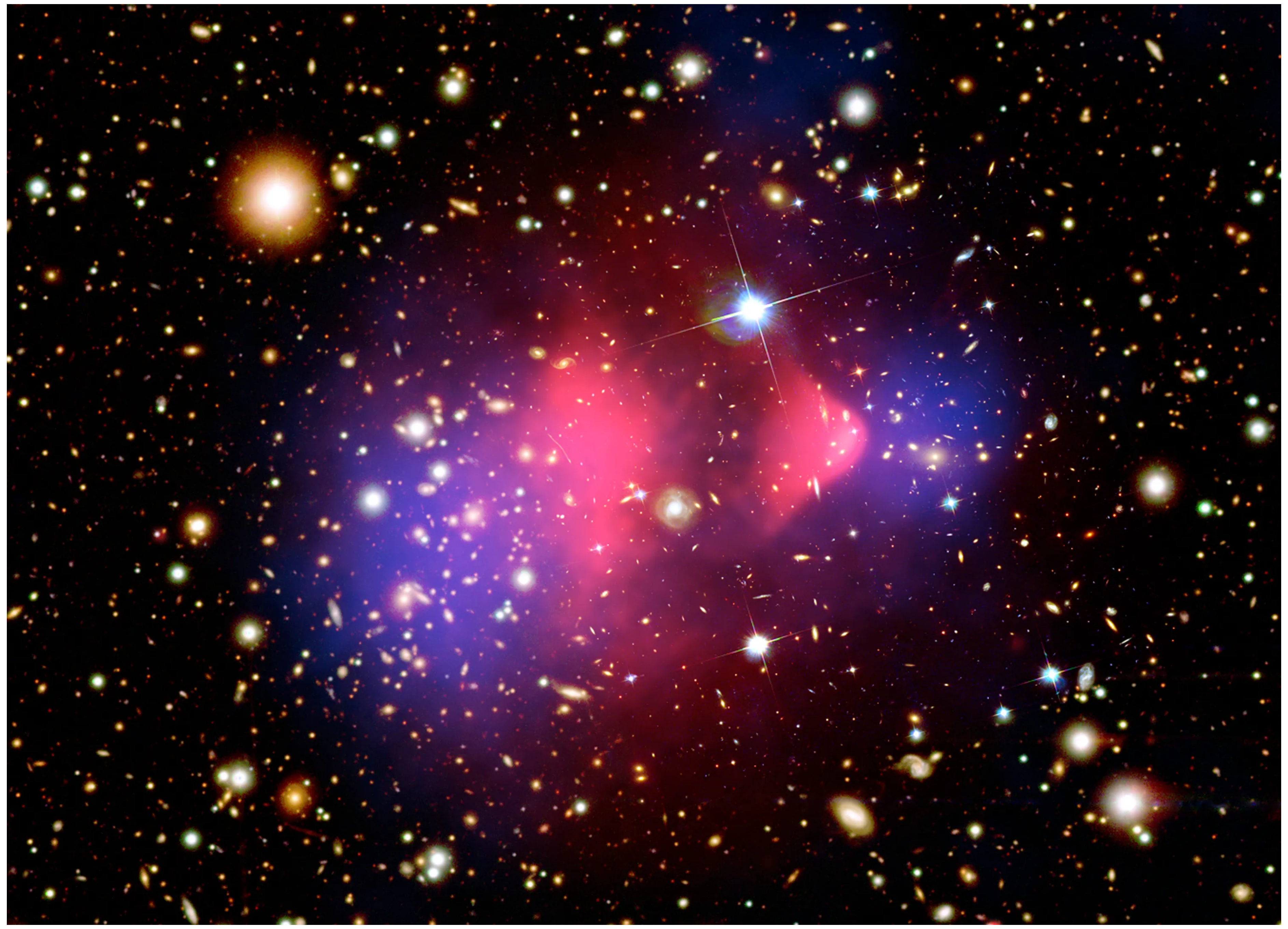
- Dark Magnetic Monopole
- Constructive Method
- Single photon production of dark monopole pair
- Total cross-section at LHC
- Summary

William Gilbert (1600)



Everyone predict it,
no one seen it

Observed 1930, not
recognized until 1970



No one asked for it, but it’s there

Dark Magnetic Monopole!

What is Magnetic Charge?

$$\vec{\nabla} \cdot \vec{E} = \rho_e$$

$$\vec{\nabla} \cdot \vec{B} = \rho_m$$

magnetic charge

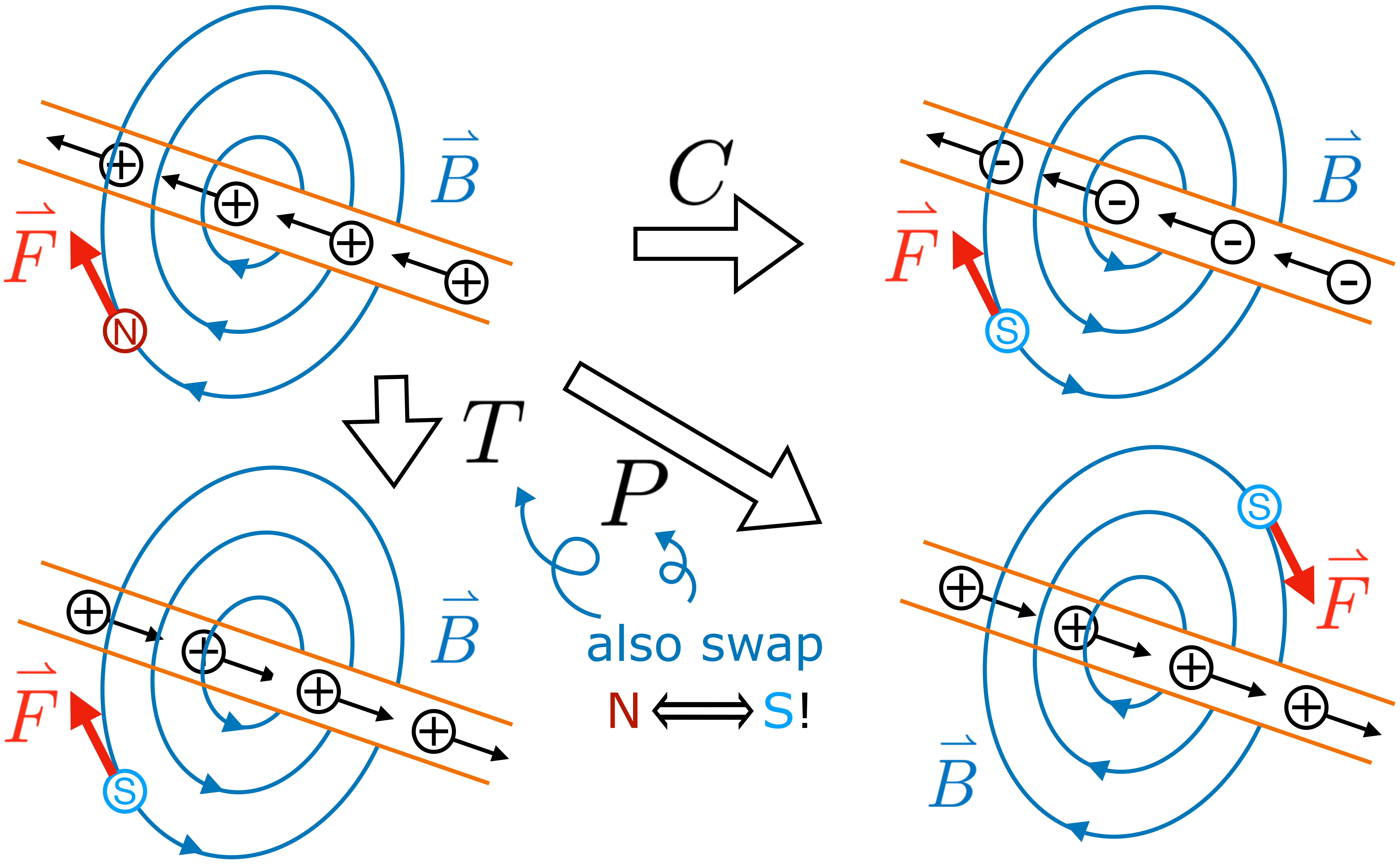
$$\vec{\nabla} \times \vec{E} = -\vec{K} - \frac{\partial \vec{B}}{\partial t}$$

magnetic current

$$\vec{\nabla} \times \vec{B} = \vec{J} + \frac{\partial \vec{E}}{\partial t}$$

$$\vec{F} = q_e(\vec{E} + \vec{v} \times \vec{B}) + q_m(\vec{B} - \vec{v} \times \vec{E})$$

What is Magnetic Charge?



What is Magnetic Charge?

Coupling to photon:

$$\mathcal{L}_{\text{int},e} = -A^\mu J_\mu \qquad \mathcal{L}_{\text{int},m} = -B^\mu K_\mu$$

Under parity:

swap of N ⟺ S

$$PJ_\mu P^{-1} = (-1)J_\mu \qquad PK_\mu P^{-1} = (-1)(-1)K_\mu = +K_\mu$$

$$PA^\mu P^{-1} = (-1)A^\mu \qquad PB^\mu P^{-1} = +B^\mu$$

$$A^\mu = A^\mu_+ + A^\mu_- \qquad B^\mu = i(A^\mu_+ - A^\mu_-)$$

photon helicity

magnetic charge has extra phase difference

Weinberg Phys. Rev. **138**. B988 (1965)

Dark Magnetic Monopole

Dark matter that transforms like magnetic charge under C, P, T

Lagrangian involving magnetic charge (Zwanziger 1971):

constant vector

$$\mathcal{L}_{\text{vis}} = -\frac{n^\alpha}{2n^2}\left[n^\mu g^{\beta\nu}\left(F^A_{\alpha\beta}F^A_{\mu\nu} + F^B_{\alpha\beta}F^B_{\mu\nu}\right) - \frac{n_\mu}{2}\varepsilon^{\mu\nu\gamma\delta}\left(F^B_{\alpha\nu}F^A_{\gamma\delta} - F^A_{\alpha\nu}F^B_{\gamma\delta}\right)\right]$$

$$- eJ_\mu A^\mu - \frac{4\pi}{e}K_\mu B^\mu$$

$$\mathcal{L} = \mathcal{L}_{\text{vis}} + \mathcal{L}_D + \mathcal{L}_\epsilon$$

$$\mathcal{L}_\epsilon = \frac{\epsilon e e_D}{2}F_{\mu\nu}F^{\mu\nu}_D \qquad F^{\mu\nu} = \frac{n^\alpha}{n^2}\left(n_\mu F^A_{\alpha\nu} - n_\nu F^A_{\alpha\mu} - \varepsilon^\beta_{\mu\nu\alpha}n^\gamma F^B_{\gamma\beta}\right)$$

dark photon

Dark Magnetic Monopole

After diagonalizing $F^{\mu\nu}, F_D^{\mu\nu}$

$$\mathcal{L} \supset -A^\mu J_\mu - B^\mu (K_\mu - \epsilon e^2 K_{D\mu}) - A_D^\mu (J_{D\mu} + \epsilon e^2 J_\mu) - B_D^\mu K_{D\mu}$$

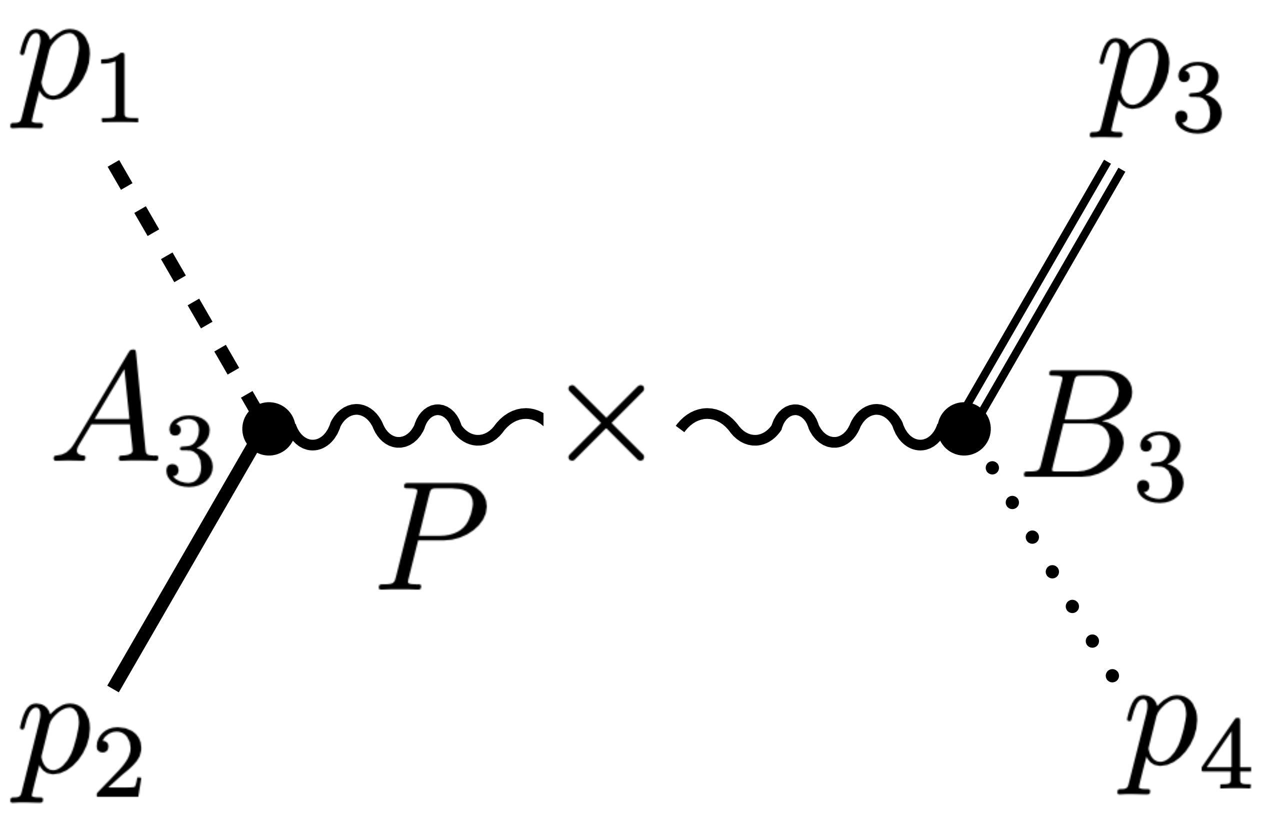
phase different between photon and dark photon!

Terning & Verhaaren hep-th/1809.05102

Still, the n^μ vector cause a lot of trouble. Terning & Verhaaren hep-th/2010.02232

Constructive Method

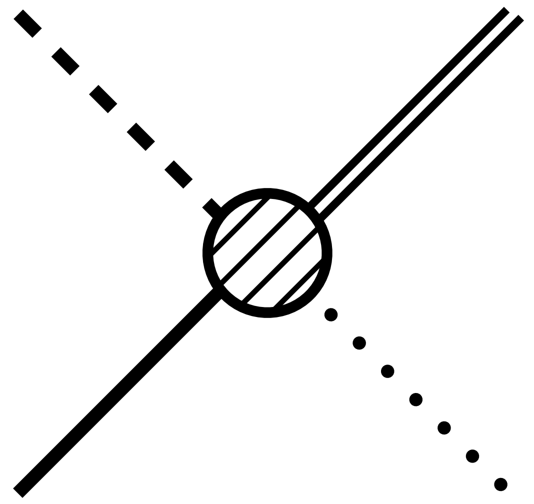
On-shell particles with complex momenta $P^2 = m^2$, four-momentum conserved.



$$= A_3(p_1, p_2, P) \times B_3(p_3, p_4, P) \to R(p_1, p_2, p_3, p_4)$$

back to real momenta

$\mathcal{M}_4 =$



$$= \frac{R(p_1, p_2, p_3, p_4)}{P^2 - m^2}$$

real $P^2 \neq m^2$

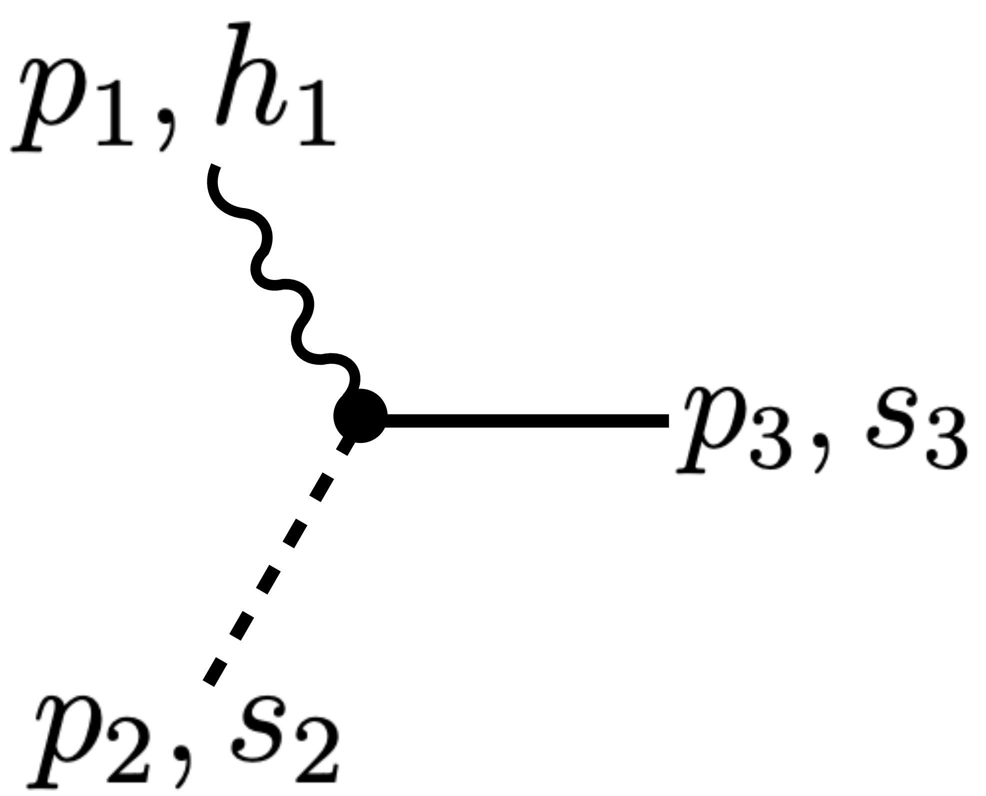
Use 3-point amplitudes as building blocks, bypass field theory.

Arkani-Hamed, Huang & Huang hep-th/1709.04891

Little Group Weight & Spinor-Helicity Variables

$$p^\mu(\sigma_\mu)_{a\dot{a}} = |p^I\rangle_a [p_I|_{\dot{a}} = \varepsilon_{IJ} |p^I\rangle_a [p^J|_{\dot{a}} \qquad |p^I\rangle [p_I| = |p^I\rangle (U_I^{\dagger J})(U_J^K)[p_K|$$

Massless: $p^\mu(\sigma_\mu)_{a\dot{a}} = |p\rangle_a [p|_{\dot{a}}$ $\qquad |p\rangle [p| = |p\rangle \frac{1}{w} w [p|$



$|i\rangle \equiv |p_i\rangle \qquad |\mathbf{i}\rangle |\mathbf{i}] \equiv |i^I\rangle |i^J] + |i^J\rangle |i^I]$ symmetrized

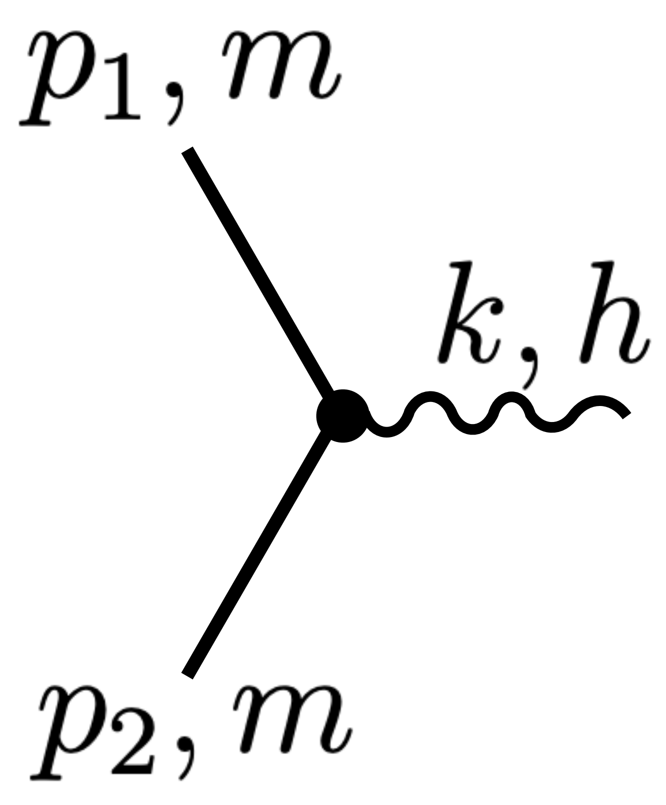
$$\sim |1\rangle^{-h_1+\frac{n}{2}} |1]^{h_1+\frac{n}{2}} |\mathbf{2}\rangle^{n_2} |\mathbf{2}]^{2s_2-n_2} |\mathbf{3}\rangle^{n_3} |\mathbf{3}]^{2s_3-n_3}$$

$$\varepsilon^{ab} |i\rangle_b |j\rangle_a \equiv \langle ij\rangle \qquad \langle ij\rangle, [ij], [i|p|\mathbf{j}\rangle \ ✓$$

$$[ij\rangle \ ✗$$

Arkani-Hamed, Huang & Huang hep-th/1709.04891

Equal Mass, x-factor

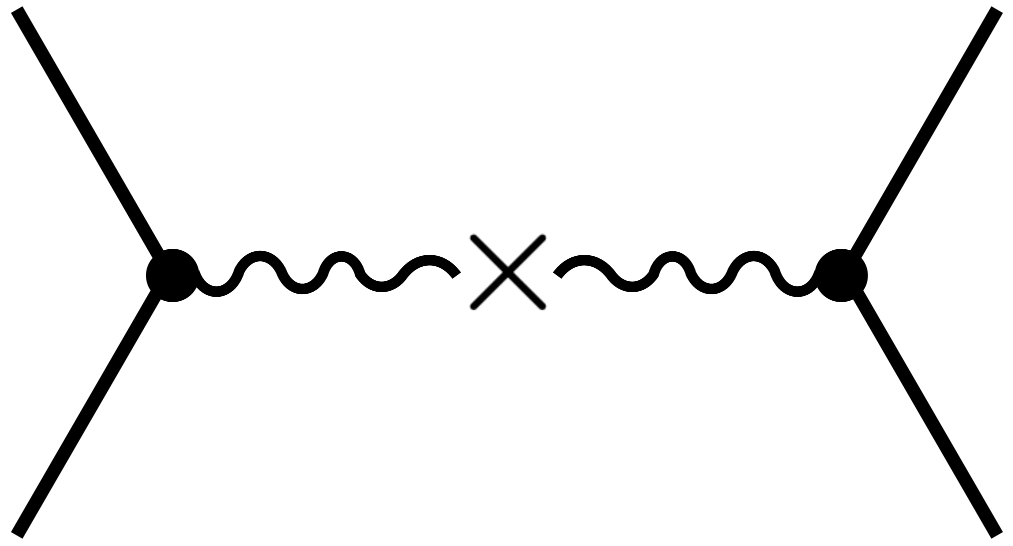


$|k\rangle \propto p_2|k] = -p_1|k]$ are not independent

$$x_{12} \equiv \frac{\langle q|\, p_2|k]}{m\,\langle qk\rangle} = -\frac{\langle q|\, p_1|k]}{m\,\langle qk\rangle} \qquad \mathcal{M}_3 \sim x^h$$

arbitrary reference spinor, becomes n^μ in Zwanziger's Lagrangian

$$\tilde{x}_{12} \equiv \frac{[\tilde{q}|p_2\,|k\rangle}{m[\tilde{q}k]} = \frac{1}{x_{12}}$$

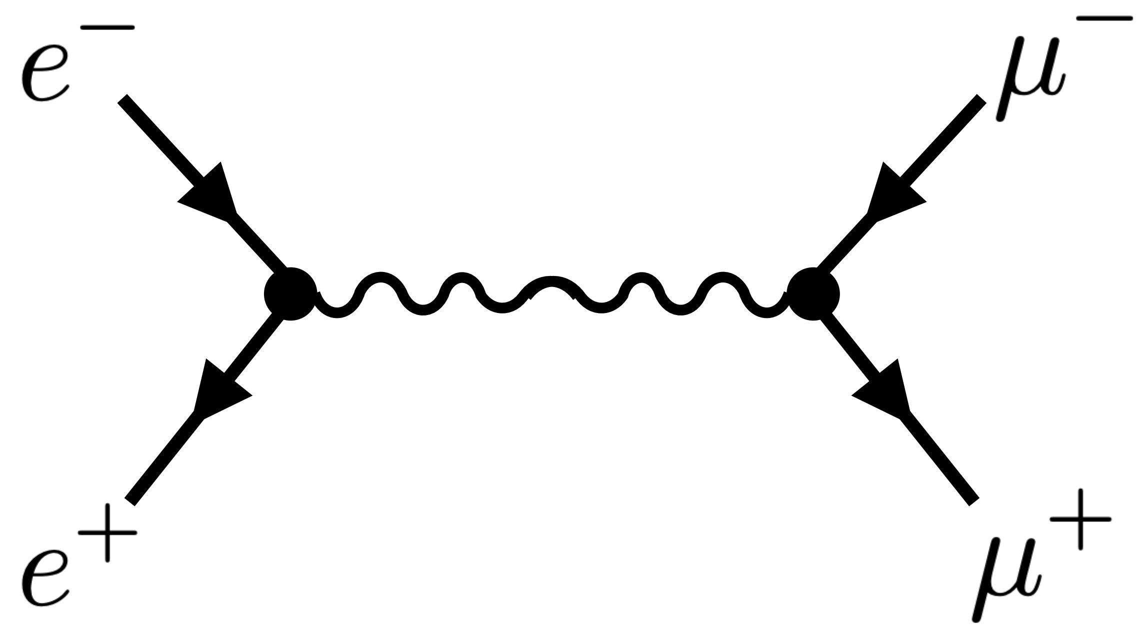


gluing requires removing q and k from the expression

Arkani-Hamed, Huang & Huang hep-th/1709.04891

Challenge with Internal Photon

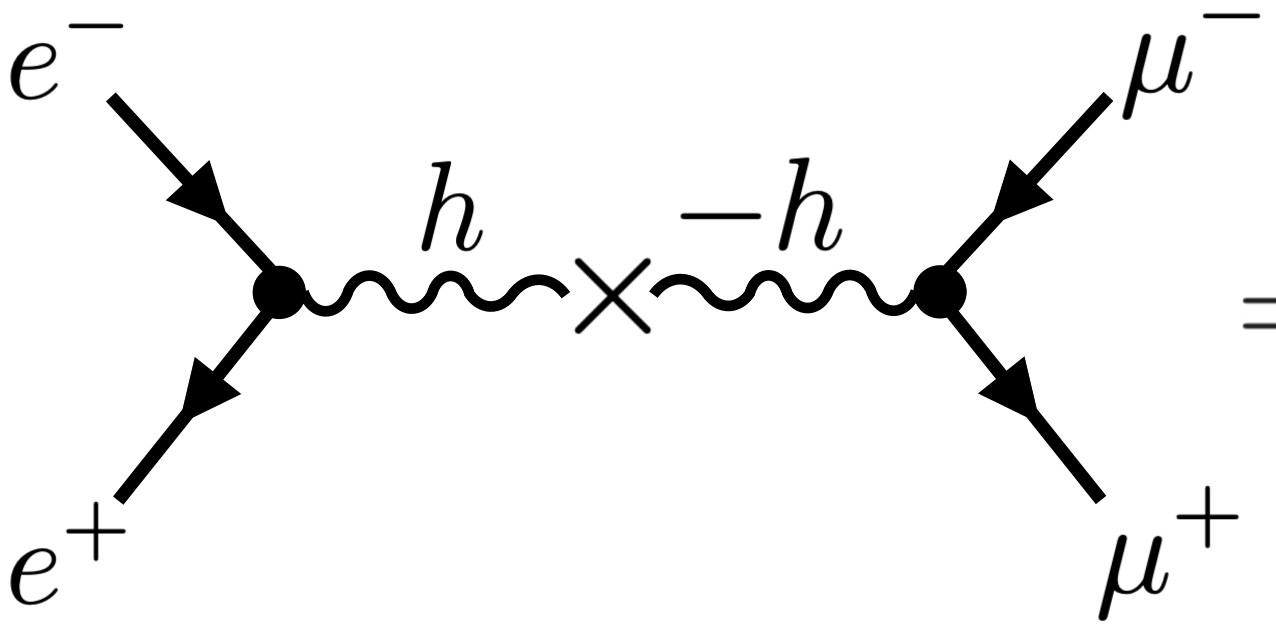
Using Feynman Rule:



$$= \frac{e^2}{s} \left(\langle \mathbf{13} \rangle [\mathbf{24}] + [\mathbf{13}] \langle \mathbf{24} \rangle + [\mathbf{14}] \langle \mathbf{23} \rangle + \langle \mathbf{14} \rangle [\mathbf{23}] \right)$$

Constructive Method:

$$\sum_{h=\pm}$$



$$= \frac{e^2}{s} \left(x_{34} \tilde{x}_{12} [\mathbf{12}] \langle \mathbf{34} \rangle + x_{12} \tilde{x}_{34} \langle \mathbf{12} \rangle [\mathbf{34}] \right)$$

Christensen et al. hep-ph/2209.15018

Challenge with Internal Photon

Christensen et al. hep-ph/2209.15018

$$x_{34}\tilde{x}_{12}[\mathbf{12}]\langle\mathbf{34}\rangle + x_{12}\tilde{x}_{34}\langle\mathbf{12}\rangle[\mathbf{34}]$$

$$= \frac{1}{2m_e m_\mu}\Big[(u - t + 2m_e^2 + 2m_\mu^2)[\mathbf{12}][\mathbf{34}] + 2\big([\mathbf{12}][\mathbf{3}|p_2 p_1|\mathbf{4}] + [\mathbf{1}|p_4 p_3|\mathbf{2}][\mathbf{34}]\big)\Big]$$

Feynman Rule result

$$= [\mathbf{13}]\,\langle\mathbf{24}\rangle + [\mathbf{14}]\,\langle\mathbf{23}\rangle + [\mathbf{23}]\,\langle\mathbf{14}\rangle + [\mathbf{24}]\,\langle\mathbf{13}\rangle + \frac{s}{2m_e m_\mu}\big([\mathbf{12}][\mathbf{34}] - 2[\mathbf{14}][\mathbf{23}]\big)$$

with OFPT Lai, Liu & Terning hep-ph/2312.11621

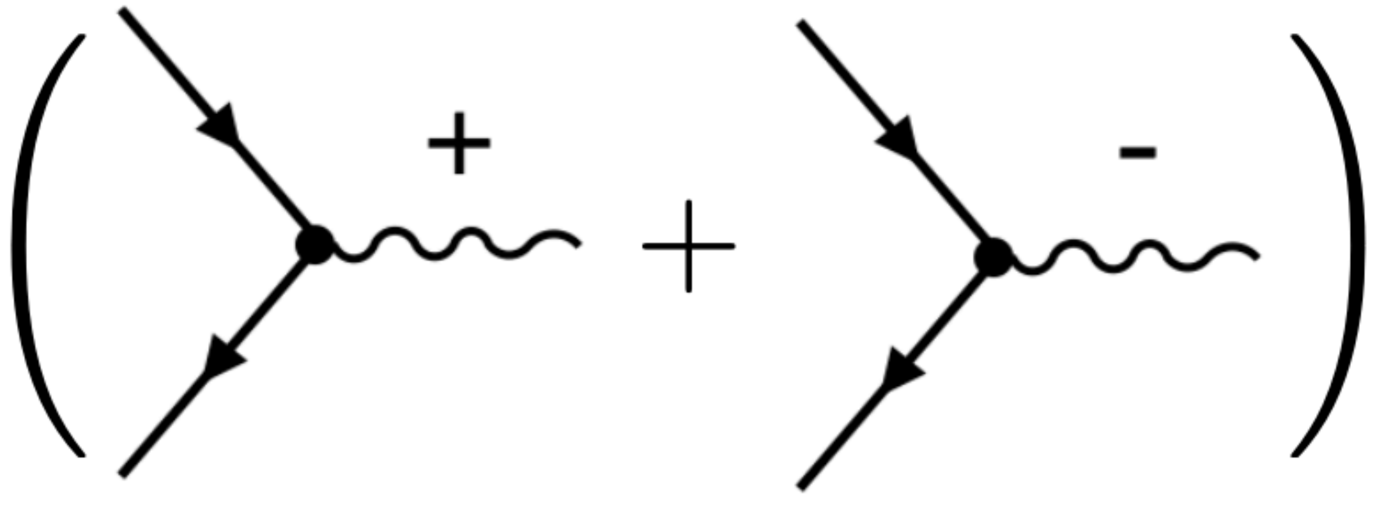
On-shell $s = (p_1 + p_2)^2 = k^2 = 0$, should drop the $\mathcal{O}(s)$ term.

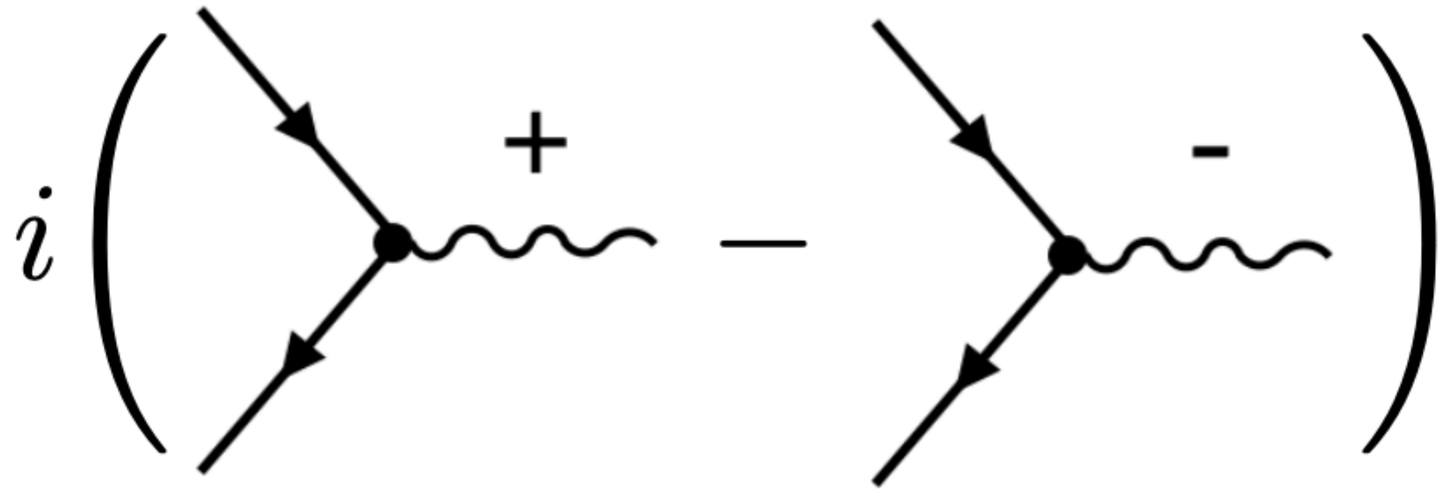
⇨ Constructive Method works!

Single Photon Production of DMM Pair

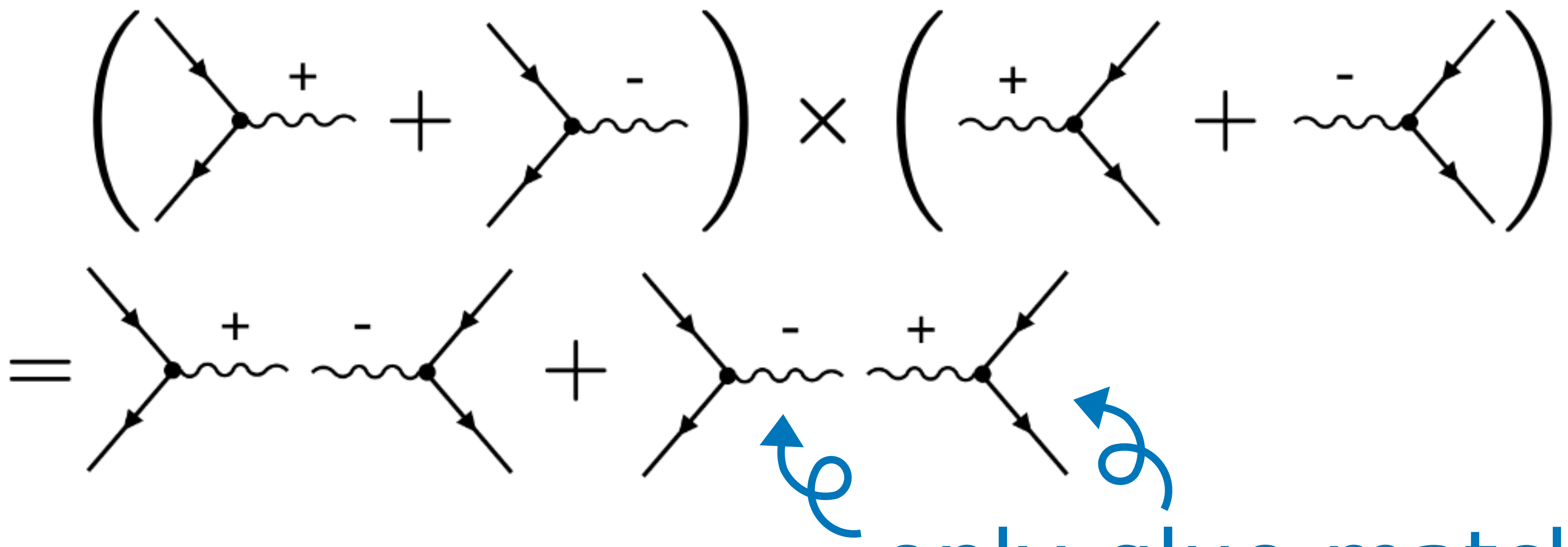
Intrinsic quantum number of photon: $J^{PC} = 1^{--}$

Construct 3-point amplitudes with parity eigenstates of photon

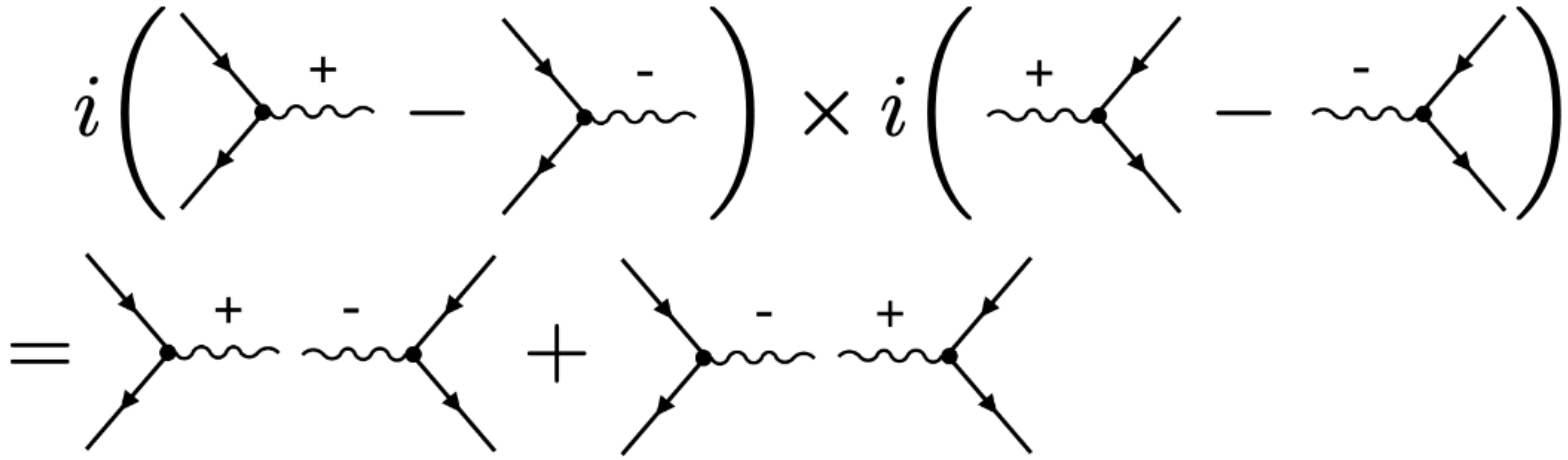
P-odd:


P-even: i


electric $\to$ electric: P-odd $\times$ P-odd



magnetic $\to$ magnetic: P-even $\times$ P-even



only glue matching angular momentum

Single Photon Production of DMM Pair

For two particle bound state:

${}^{d}L_J$	Fermionic J^{PC} electric	Fermionic J^{PC} magnetic
${}^{1}S_0$	0^{-+}	0^{-+}
${}^{3}S_1$	1^{--}	1^{+-}
${}^{1}P_1$	1^{+-}	1^{--}
${}^{3}P_0$	0^{++}	0^{++}
${}^{3}P_1$	1^{++}	1^{++}
${}^{3}P_2$	2^{++}	2^{++}
${}^{3}D_1$	1^{--}	1^{+-}

${}^{d}L_J$	Scalar J^{PC} electric	Scalar J^{PC} magnetic
${}^{1}S_0$	0^{++}	0^{++}
${}^{1}P_1$	1^{--}	1^{+-}
${}^{1}D_2$	2^{++}	2^{++}

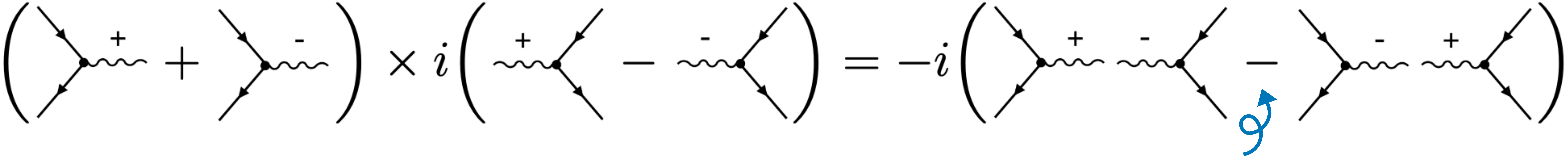
Only glue matching J^{PC}

For electric → magnetic, one side must have fermionic singlet.

Terning & Verhaaren hep-th/2010.02232

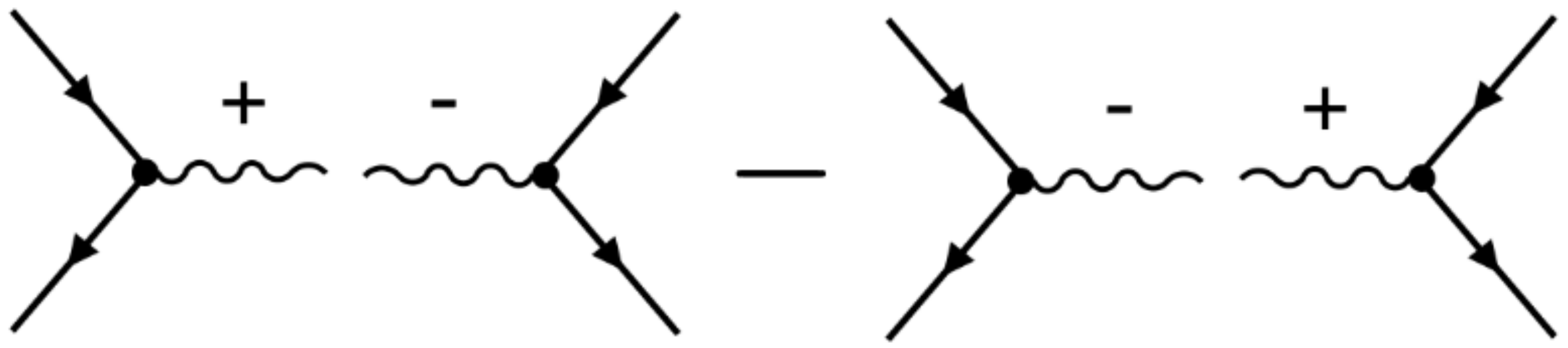
Single Photon Production of DMM Pair

electric $\to$ magnetic: P-odd $\times$ P-even



relative minus sign between different helicity, agrees with Weinberg!

For electric fermion singlet → magnetic scalar,



$$= \varepsilon_{IJ}\left(m_4 x_{12}\tilde{x}_{34}\,\langle 1^I 2^J\rangle - m_4\tilde{x}_{12}x_{34}[1^I 2^J]\right)$$

Single Photon Production of DMM Pair

$$x_{12}\tilde{x}_{34}\langle\mathbf{12}\rangle - \tilde{x}_{12}x_{34}[\mathbf{12}]$$

$$=2i\varepsilon_{\mu\nu\alpha\beta}p_4^{\mu}q^{\nu}(\langle\mathbf{2}|\,\sigma^{\alpha}|\mathbf{1}] + \langle\mathbf{1}|\,\sigma^{\alpha}|\mathbf{2}])k^{\beta}$$

$$=2i\varepsilon_{\mu\nu\alpha\beta}p_4^{\mu}q^{\nu}\bar{u}_2\gamma^{\alpha}v_1k^{\beta}$$ agrees with Zwanziger's Lagrangian.

Terning & Verhaaren hep-th/2010.02232

$$\varepsilon_{IJ}(\langle 2^J|\,\sigma^{\mu}|1^I] + \langle 1^I|\,\sigma^{\mu}|2^J]) = \mathrm{Tr}\left(|1_I]\,\langle 2^I|\,\sigma^{\mu} + |2_I]\,\langle 1^I|\,\sigma^{\mu}\right)$$

In the C.M. frame, only $k^0 \neq 0$

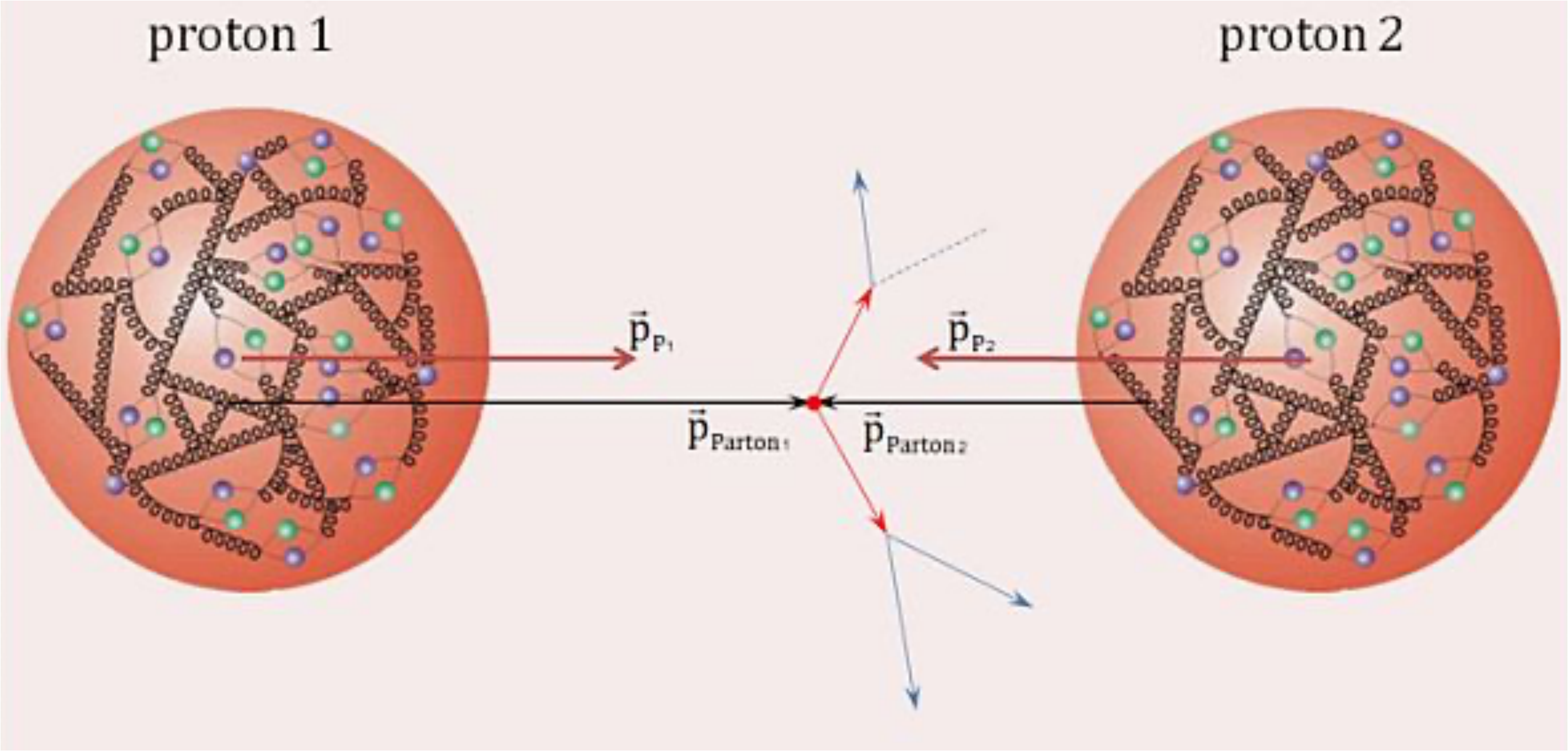
$$\mathrm{Tr}\left(|1_I]\,\langle 2^I|\,\vec{\sigma}\right) = \mathrm{Tr}\left(|2_I\rangle\,\langle 2^I|\,\vec{\sigma}\right) = m_2\,\mathrm{Tr}\left(\mathbb{1}\vec{\sigma}\right) = 0$$

All single photon residue vanish!

Total Cross-Section of DMM Pair Production at LHC

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 f_a(x_1, Q^2) f_b(x_2, Q^2) \times \hat{\sigma}_{ab\to X}(x_1, x_2, Q^2)$$

parton distribution function (PDF)

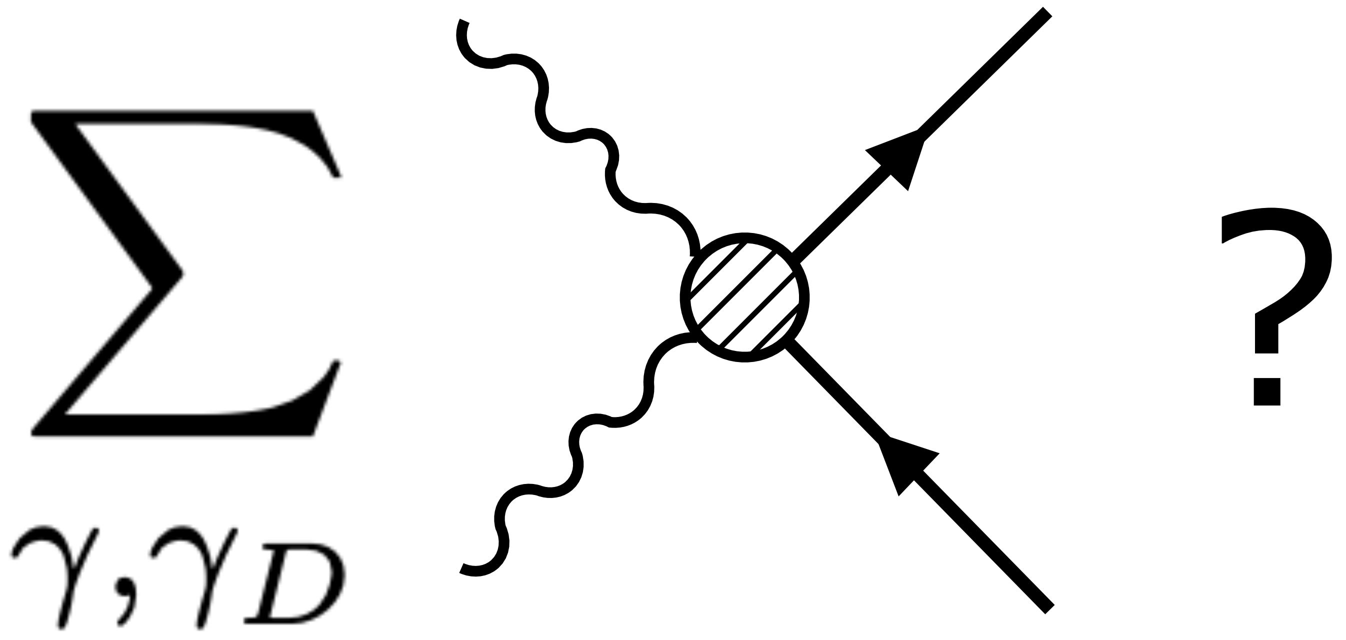


Photon PDF:

Manohar et al. hep-ph/1607.04266

Dark Photon PDF:

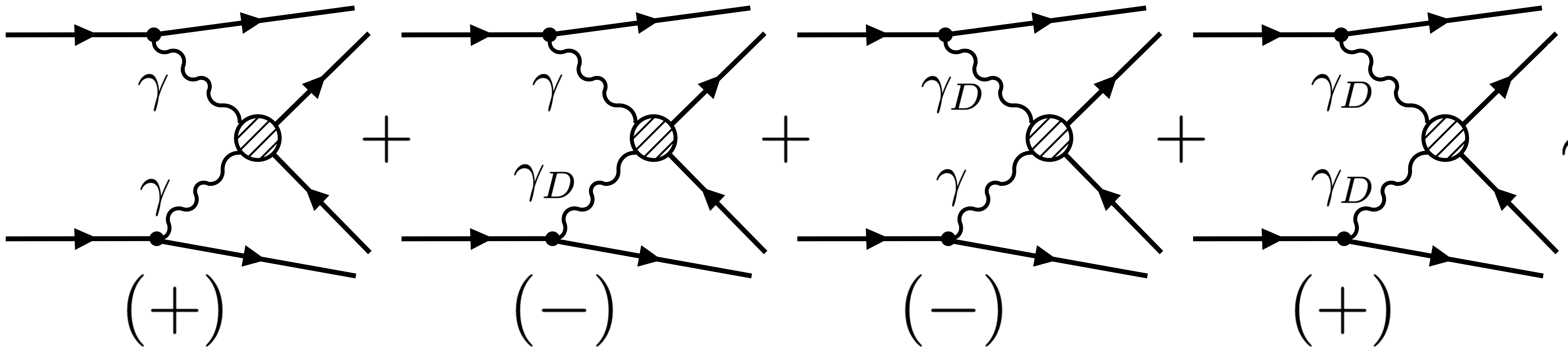
McCullough, Moore & Ubiali hep-ph/2203.12628



Total Cross-Section of DMM Pair Production at LHC

$$\mathcal{L} \supset -A^\mu J_\mu - B^\mu (K_\mu - \epsilon e^2 K_{D\mu}) - A_D^\mu (J_{D\mu} + \epsilon e^2 J_\mu) - B_D^\mu K_{D\mu}$$

phase different between photon and dark photon!



$$\sim \frac{m_{\gamma_D}^4}{Q^6}$$

Interference between photon and dark photon matters, cannot just use photon and dark photon PDF.

Total Cross-Section of DMM Pair Production at LHC

$$\mathcal{M}_{\gamma\gamma_D} = -\mathcal{M}_{\gamma\gamma}\left(\frac{k^2}{k^2 - m_{\gamma_D}^2}\right) \approx -\mathcal{M}_{\gamma\gamma}\left(1 + \frac{m_{\gamma_D}^2}{Q^2}\right)$$

valid for $M \gg m_{\gamma_D}$

DMM mass

$$\left|\mathcal{M}_{\gamma\gamma} + \mathcal{M}_{\gamma\gamma_D} + \mathcal{M}_{\gamma_D\gamma} + \mathcal{M}_{\gamma_D\gamma_D}\right|^2 \approx 4\left(\frac{m_{\gamma_D}}{Q}\right)^4 \left|\mathcal{M}_{\gamma\gamma}\right|^2$$

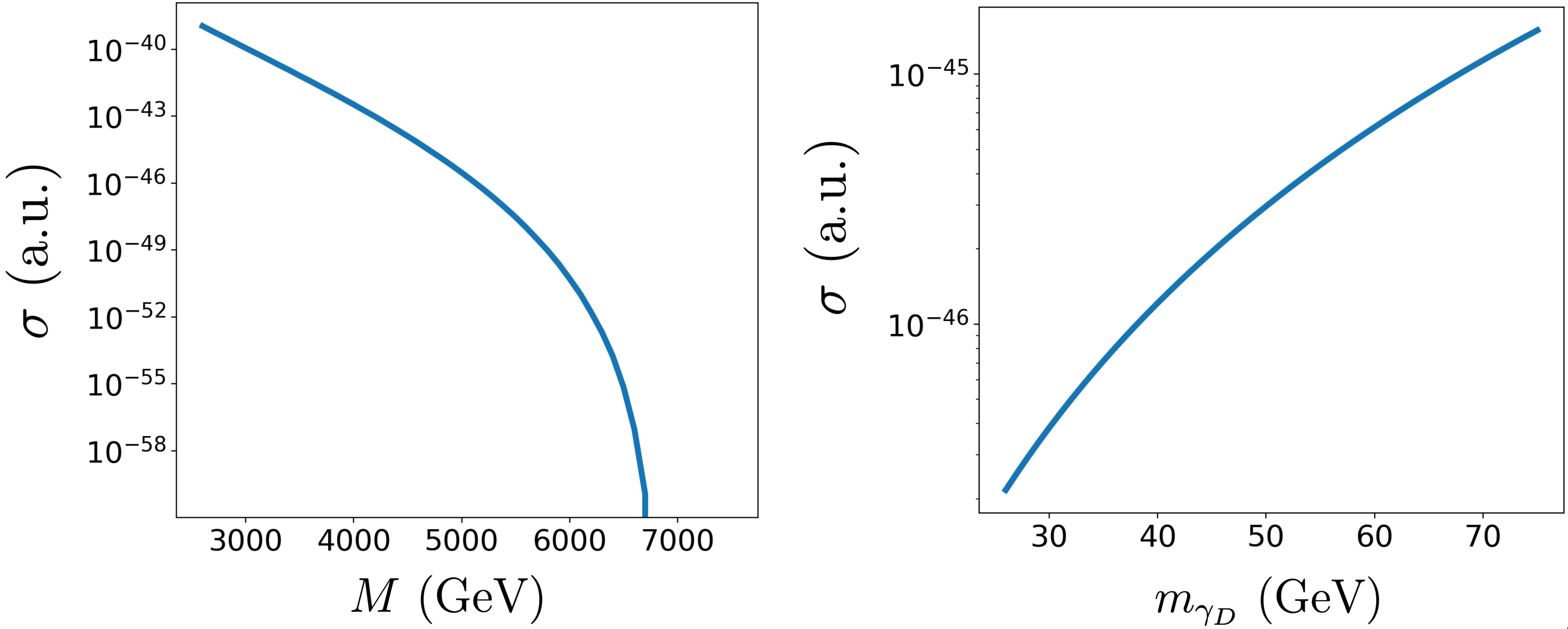
Only need photon PDF and four-point amplitude!

$$\sum_{\text{spin}} |\mathcal{M}|^2 = 16g^4 \frac{p^8 + 8M^2p^6 - 32M^4p^4 - 128M^2p^2\,(k\cdot q)^2 - 256\,(k\cdot q)^4}{\left[p^4 - 16\,(k\cdot q)^2\right]^2}$$

Terning & Verhaaren hep-th/2010.02232

Total Cross-Section of DMM Pair Production at LHC

$m_{\gamma_D} = 50$ (GeV) $E_{\text{beam}} = 6800$ (GeV) $M = 5000$ (GeV)



Summary

- Constructive method can give electric $\times$ magnetic amplitude.
- DMM pair production at LHC can be calculated using photon PDF for $M \gg m_{\gamma_D}$.
- There's still a lot more to be done for DMM phenomenology.

Old Fashioned Perturbation Theory

Particles on-shell, spatial momentum conserved, energy not conserved.

A_3 B_3

$$\langle f|\, S\, |i\rangle = \langle f|\, H_{\text{int}}\, |i\rangle + \sum_n \frac{\langle f|\, H_{\text{int}}\, |n\rangle \langle n|\, H_{\text{int}}\, |i\rangle}{E_i - E_n} + \ldots$$

$P^2 - m^2$ After summing over time-ordering

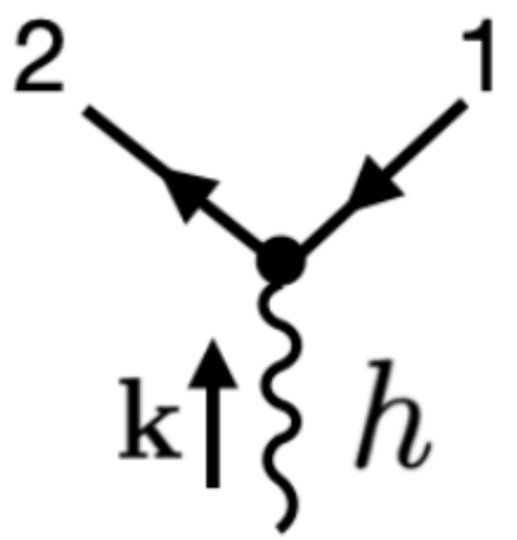
Equivalent to Feynman Rule Dyson Phys. Rev. **75** (1949) 486

For QED in Coulomb gauge, $H_{\text{int}} = H_T + H_{\text{Coul}}$

$$H_T = -\int d^3\mathbf{x}\, \mathbf{J}\cdot\mathbf{A} \qquad H_{\text{Coul}} = \frac{1}{2}\int d^3\mathbf{x} d^3\mathbf{y}\, \frac{J^0(\mathbf{x})J^0(\mathbf{y})}{4\pi|\mathbf{x}-\mathbf{y}|}$$

Does not contribute to residue when $s = 0$

Old Fashioned Perturbation Theory

$H_T \sim$ 

$= \frac{e}{\sqrt{2\omega_{\mathbf{k}}}} \bar{v}_2 \not{\epsilon}_h u_1 \xrightarrow{s=0} \begin{cases} \frac{e}{\sqrt{\omega_{\mathbf{k}}}\langle k q_+\rangle}\left(\langle \mathbf{2} q_+\rangle [k\mathbf{1}] + [\mathbf{2}k]\langle q_+ \mathbf{1}\rangle\right), & h=+ \\ \frac{e}{\sqrt{\omega_{\mathbf{k}}}[k q_-]}\left(\langle \mathbf{2} k\rangle [q_-\mathbf{1}] + [\mathbf{2}q_-]\langle k \mathbf{1}\rangle\right), & h=- \end{cases}$

$s=0$

$$-\frac{\langle \mathbf{2}q\rangle [k\mathbf{1}] + [\mathbf{2}k]\langle q\mathbf{1}\rangle}{\langle qk\rangle} + \frac{\langle \mathbf{2}q\rangle \langle \mathbf{1}| k |k]}{m_e \langle qk\rangle}$$

Schouten identity

$|i\rangle \langle jk\rangle + |j\rangle \langle ki\rangle = |k\rangle \langle ji\rangle$

$$=\left(\langle \mathbf{2}q\rangle \langle \mathbf{1}| + \langle q\mathbf{1}\rangle \langle \mathbf{2}|\right) \frac{p_2|k]}{m_e \langle qk\rangle}$$

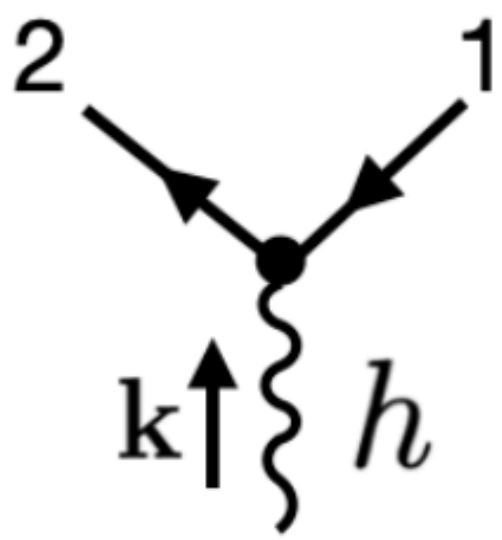
$$=\frac{\langle q| p_2 |k]}{m_e \langle qk\rangle} \langle \mathbf{12}\rangle = x_{12} \langle \mathbf{12}\rangle$$

Constructive Method Amplitude!

Liu & Yin hep-th/2204.13119

Old Fashioned Perturbation Theory

$H_T \sim$



$$= \frac{e}{\sqrt{2\omega_{\mathbf{k}}}}\bar{v}_2 \not{\epsilon}_h u_1 \xrightarrow{s=0} \begin{cases} \dfrac{e}{\sqrt{\omega_{\mathbf{k}}}\langle k q_+\rangle}\left(\langle \mathbf{2} q_+\rangle [k\mathbf{1}] + [\mathbf{2}k]\langle q_+ \mathbf{1}\rangle\right), & h = + \\ \dfrac{e}{\sqrt{\omega_{\mathbf{k}}}[k q_-]}\left(\langle \mathbf{2} k\rangle [q_-\mathbf{1}] + [\mathbf{2}q_-]\langle k \mathbf{1}\rangle\right), & h = - \end{cases}$$

$s = 0$

$$-\frac{\langle \mathbf{2}q\rangle [k\mathbf{1}] + [\mathbf{2}k]\langle q\mathbf{1}\rangle}{\langle qk\rangle} + \frac{\langle \mathbf{2}q\rangle \langle \mathbf{1}| k |k]}{m_e \langle qk\rangle}$$

Schouten identity

$$|i\rangle \langle jk\rangle + |j\rangle \langle ki\rangle = |k\rangle \langle ji\rangle$$

$$= \left(\langle \mathbf{2}q\rangle \langle \mathbf{1}| + \langle q\mathbf{1}\rangle \langle \mathbf{2}|\right) \frac{p_2|k]}{m_e \langle qk\rangle}$$

$$= \frac{\langle q| p_2 |k]}{m_e \langle qk\rangle} \langle \mathbf{12}\rangle = x_{12} \langle \mathbf{12}\rangle$$

Constructive Method Amplitude!

Liu & Yin hep-th/2204.13119