2HDM Benchmarks for LHC Higgs Studies

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April 22, 2013
@ UC Davis

HEFTI Workshop: "The LHC Higgs Signal: Characterization, Interpretation and BSM Model Implications"

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Theoretical structure of the 2HDM

Start with the 2HDM scalar doublet, hypercharge-one fields, Φ_1 and Φ_2, in a generic basis, where $\langle\Phi_i\rangle = v_i$, and $v^2 \equiv |v_1|^2 + |v_2|^2 = (246~\text{GeV})^2$. It is convenient to define new Higgs doublet fields:

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} \equiv \frac{v_1^*\Phi_1 + v_2^*\Phi_2}{v}\,, \qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \equiv \frac{-v_2\Phi_1 + v_1\Phi_2}{v}\,.$$

It follows that $\langle H_1^0\rangle = v/\sqrt{2}$ and $\langle H_2^0\rangle = 0$. This is the *Higgs basis*, which is uniquely defined up to an overall rephasing, $H_2 \to e^{i\chi}H_2$. In the Higgs basis, the scalar potential is given by:

$$\begin{aligned}\mathcal{V} = {}& Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2}Z_1(H_1^\dagger H_1)^2 \\ &+ \tfrac{1}{2}Z_2(H_2^\dagger H_2)^2 + Z_3(H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4(H_1^\dagger H_2)(H_2^\dagger H_1) \\ &+ \left\{\tfrac{1}{2}Z_5(H_1^\dagger H_2)^2 + \big[Z_6(H_1^\dagger H_1) + Z_7(H_2^\dagger H_2)\big]H_1^\dagger H_2 + \text{h.c.}\right\}\,,\end{aligned}$$

where Y_1, Y_2 and $Z_1,\ldots,Z_4$ are real and uniquely defined, whereas Y_3, Z_5, Z_6 and Z_7 are complex and transform under the rephasing of H_2,

$$[Y_3, Z_6, Z_7] \to e^{-i\chi}[Y_3, Z_6, Z_7] \quad \text{and} \quad Z_5 \to e^{-2i\chi}Z_5\,.$$

Motivation for a general analysis

- If Φ_1 and Φ_2 are intistinguishable fields, then observables can only depend on combinations of Higgs basis parameters that are independent of χ.

- Symmetries (such as discrete symmetries or supersymmetry) can distinguish between Φ_1 and Φ_2, and select out a specific basis for the scalar fields, which yield additional observables (such as tan β).

- Such symmetries are typically broken symmetries, so that below the symmetry-breaking energy scale, the effective 2HDM is generic, and all possible scalar potential terms can appear.

The Higgs mass-eigenstate basis

The physical charged Higgs boson is the charged component of the Higgs-basis doublet H_2, and its mass is given by $m^2_{H^\pm} = Y_2 + \frac{1}{2} Z_3 v^2$.

The three physical neutral Higgs boson mass-eigenstates are determined by diagonalizing a 3×3 real symmetric squared-mass matrix that is defined in the Higgs basis

$$\mathcal{M}^2 = v^2 \begin{pmatrix} Z_1 & \mathrm{Re}(Z_6) & -\mathrm{Im}(Z_6) \\ \mathrm{Re}(Z_6) & \frac{1}{2} Z_{345} + Y_2/v^2 & -\frac{1}{2}\mathrm{Im}(Z_5) \\ -\mathrm{Im}(Z_6) & -\frac{1}{2}\mathrm{Im}(Z_5) & \frac{1}{2} Z_{345} - \mathrm{Re}(Z_5) + Y_2/v^2 \end{pmatrix},$$

where $Z_{345} \equiv Z_3 + Z_4 + \mathrm{Re}(Z_5)$.The diagonalizing matrix is a 3×3 real orthogonal matrix that depends on three angles: θ_{12}, θ_{13} and θ_{23}. The corresponding neutral Higgs masses will be denoted: m_1, m_2 and m_3. Under the rephasing $H_2 \to e^{i\chi} H_2$,

$$\theta_{12}\,,\ \theta_{13} \text{ are invariant, and } \quad \theta_{23} \to \theta_{23} - \chi\,.$$

A first step toward 2HDM benchmarks

- The general 2HDM scalar potential has too many parameters

After minimizing the scalar potential, $Y_1 = -\frac{1}{2}Z_1 v^2$ and $Y_3 = -\frac{1}{2}Z_6 v^2$. This leaves 11 free parameters: 1 vev, 8 real parameters, Y_2, $Z_{1,2,3,4}$, $|Z_{5,6,7}|$, and two relative phases.

- The LHC Higgs data suggests that the Higgs boson at 126 GeV is Standard Model (SM)-like. Thus, we can treat the deviations from SM-like behavior as a perturbation to simplify the general analysis.

- A SM-like Higgs boson is naturally achieved in the decoupling limit in which $Y_2 >> v$. In this limit,

$$s_{12} \equiv \sin\theta_{12} \simeq \frac{\mathrm{Re}(Z_6 e^{-i\theta_{23}})v^2}{m_2^2} \ll 1\,,$$

$$s_{13} \equiv \sin\theta_{13} \simeq -\frac{\mathrm{Im}(Z_6 e^{-i\theta_{23}})v^2}{m_3^2} \ll 1\,,$$

$$\mathrm{Im}(Z_5 e^{-2i\theta_{23}}) \simeq \frac{2m_2^2 \sin\theta_{12}\sin\theta_{13}}{v^2} \simeq -\frac{\mathrm{Im}(Z_6^2 e^{-2i\theta_{23}})v^2}{m_3^2} \ll 1\,,$$

where $m_1^2 \simeq Z_1 v^2 \ll m_2^2, m_3^2$ and $m_3^2 - m_2^2 \simeq \mathcal{O}(v^2)$. For example, the h_1VV coupling ($V = W$ or Z) relative to the SM is given by $c_{12}c_{13} \simeq 1 - \frac{1}{2}(s_{12}^2 + s_{13}^2)$, where $c_{12} \equiv \cos\theta_{12}$ and $c_{13} \equiv \cos\theta_{13}$.

Distinguishing between two types of decoupling

- Large-mass decoupling

Below the scale of the heavy Higgs bosons of mass $m_{2,3}$, the effective scalar theory is that of a single Higgs doublet.

- Weak-coupling decoupling

In the limit of $Z_6 \to 0$, the tree-level couplings of h_1 are precisely those of the SM Higgs boson since $s_{12} = s_{13} = \mathrm{Im}(Z_5 e^{-2i\theta_{23}}) = 0$, independent of values of m_1, m_2 and m_3. (In this case, it is possible to have decays such as $h_1 \to h_2 h_2$ if kinematically allowed.)

Strategy for choosing 2HDM benchmark points

1. Identify h_1 with the observed Higgs boson, with $m_1 \simeq 126$ GeV.
2. Choose input parameters s_{12} and s_{13} small to give SM-like h_1VV couplings.
3. Scan in the couplings Z_4, $\mathrm{Re}(Z_5 e^{-2i\theta_{23}})$ and $\mathrm{Re}(Z_6 e^{-i\theta_{23}})$ [where these couplings are bounded by unitarity constraints]. These quantities determine the masses m_2, m_3 and $m_{H^\pm}$, and the CP-violating quantity $\mathrm{Im}(Z_5^* Z_6^2)$, which governs the CP-mixing in h_2 and h_3.

$$m_2^2 = m_1^2 + \frac{\mathrm{Re}(Z_6 e^{-i\theta_{23}})v^2}{c_{13}s_{12}c_{12}}\,,$$

$$m_3^2 = m_2^2 + \frac{v^2}{c_{13}^2}\left\{\frac{c_{13}(c_{12}^2 s_{13}^2 - s_{12}^2)}{s_{12}c_{12}}\mathrm{Re}(Z_6 e^{-i\theta_{23}}) - \mathrm{Re}(Z_5 e^{-2i\theta_{23}})\right\},$$

$$m_{H^\pm}^2 = m_3^2 c_{13}^2 - m_1^2 s_{13}^2 c_{12}^2 - m_2^2 s_{12}^2 s_{13}^2 - \tfrac{1}{2}\left[Z_4 - \mathrm{Re}(Z_5 e^{-2i\theta_{23}})\right]v^2\,,$$

$$\mathrm{Im}(Z_5^* Z_6^2)v^6 = 2s_{13}c_{13}^2 s_{12}c_{12}(m_2^2 - m_1^2)(m_3^2 - m_1^2)(m_3^2 - m_2^2)\,.$$

No approximations have been made in obtaining the above formulae.

A further simplification: The CP-conserving limit:

One can choose Higgs field phases such that $Z_{5,6,7}$ are real and $Z_6 > 0$. Then, we identify

$$\begin{aligned} c_{12} &= \sin(\beta - \alpha)\,, \\ s_{12} &= -\cos(\beta - \alpha)\,, \\ \theta_{13} &= \theta_{23} = 0\,, \end{aligned}$$

where β and α refers to some generic basis which has no special meaning, but $\beta - \alpha$ is an observable. Note that $m_2 > m_1$ implies that $\sin 2(\beta - \alpha) < 0$.

Notation: $c_{\beta-\alpha} \equiv \cos(\beta - \alpha)$ and $s_{\beta-\alpha} \equiv \sin(\beta - \alpha)$.

CP-conserving 2HDM benchmarks

1. Identify h with the observed Higgs boson, with $m_h \simeq 126$ GeV.

2. Choose $c_{\beta-\alpha}$ to give SM-like hVV couplings.

3. Scan in the couplings Z_4, Z_5 and Z_6 [where $Z_6 > 0$ and $s_{\beta-\alpha}c_{\beta-\alpha} < 0$ by convention]. These quantities determine the masses m_H, m_A and $m_{H^\pm}$,

$$m_H^2 = m_h^2 - \frac{Z_6 v^2}{s_{\beta-\alpha}c_{\beta-\alpha}}\,,$$

$$m_A^2 = m_H^2 + \left[\frac{c_{\beta-\alpha}}{s_{\beta-\alpha}}Z_6 - Z_5\right]v^2\,,$$

$$m_{H^\pm}^2 = m_A^2 - \tfrac{1}{2}(Z_4 - Z_5)v^2\,.$$

The case of $Z_6 = s_{\beta-\alpha} = 0$ will be treated separately.

Decoupling limit without heavy Higgs masses

The case of $Z_6=0$ is special (since it forbids H_1–H_2 mixing). It leads to one scalar state h with exact SM tree-level couplings. The other Higgs states have independent masses,

$$m^2_{H,A} = Y_2 + \tfrac{1}{2}\left[Z_3 + Z_4 \pm \mathrm{Re}(Z_5 e^{-2i\theta_{23}})\right]v^2\,,$$
$$m^2_{H^\pm} = Y_2 + \tfrac{1}{2}Z_3 v^2\,.$$

In light of the scalar potential minimum conditions, we have $Y_3= Z_6=0$. This condition is not natural unless $Z_7=0$ as well, in which case we have a $\mathbf{Z_2}$ symmetry in the Higgs basis. The 2HDM with $Y_3= Z_6=Z_7=0$ is called the inert 2HDM. In this model, the Higgs potential is CP-conserving, and the Higgs basis field H_1 is identical to the SM Higgs boson. The lightest neutral scalar inside H_2 is absolutely stable (and provides a possible candidate for dark matter).

However, even in the inert 2HDM, there are some clues to distinguish h from the SM Higgs boson. In particular the hH^+H^-, hAA and hHH couplings are nonzero (H.E. Haber and D. O'Neil):

$$\begin{aligned} g_{hH^+H^-} &= Z_3 v\,, \\ g_{hAA} &= \big[Z_3 + Z_4 - \mathrm{Re}(Z_5 e^{-2i\theta_{23}})\big] v\,, \\ g_{hHH} &= \big[Z_3 + Z_4 + \mathrm{Re}(Z_5 e^{-2i\theta_{23}})\big] v\,. \end{aligned}$$

Hence, even without detecting the non-minimal Higgs states, the properties of h can be shifted:

- The tri-linear Higgs couplings can introduce new radiative corrections. For example, a charged Higgs loop would (slightly) modify the rate for $h \to \gamma\gamma$.
- If any of the non-minimal Higgs states were lighter than $\frac{1}{2}m_h$, then new h decay channels would open up. In the inert 2HDM, this would lead to invisible Higgs decays (e.g. $h \to AA$).

Higgs Yukawa couplings in the 2HDM

In the Higgs basis, $\kappa^{U,D}$ and $\rho^{U,D}$, are the 3×3 Yukawa coupling matrices,

$$-\mathscr{L}_{\rm Y} = \overline{U}_L(\kappa^U H_1^{0\,\dagger} + \rho^U H_2^{0\,\dagger})U_R - \overline{D}_L K^\dagger(\kappa^U H_1^- + \rho^U H_2^-)U_R$$
$$+\overline{U}_L K(\kappa^{D\,\dagger} H_1^+ + \rho^{D\,\dagger} H_2^+)D_R + \overline{D}_L(\kappa^{D\,\dagger} H_1^0 + \rho^{D\,\dagger} H_2^0)D_R + \text{h.c.},$$

where $U=(u,c,t)$ and $D=(d,s,b)$ are the physical quark fields and K is the CKM mixing matrix. (Repeat for the leptons.)

By setting $H_1^0 = v/\sqrt{2}$ and $H_2^0 = 0$, one obtains the quark mass terms. Hence, κ^U and κ^D are proportional to the diagonal quark mass matrices M_U and M_D, respectively,

$$M_U = \frac{v}{\sqrt{2}}\kappa^U = \text{diag}(m_u\,,\,m_c\,,\,m_t)\,, \qquad M_D = \frac{v}{\sqrt{2}}\kappa^{D\,\dagger} = \text{diag}(m_d\,,\,m_s\,,\,m_b)\,.$$

Note that $\rho^Q \to e^{-i\chi}\rho^Q$ under the rephasing $H_2 \to e^{i\chi}H_2$, (for $Q=U,D$).

The Yukawa couplings of the mass-eigenstate Higgs bosons to the quarks are:

$$
\begin{aligned}
-\mathscr{L}_Y = {} & \frac{1}{v}\overline{D}\sum_k \Big\{ M_D(q_{k1}P_R + q^*_{k1}P_L) + \frac{v}{\sqrt{2}}\left[q_{k2}\,[e^{i\theta_{23}}\rho^D]^\dagger P_R + q^*_{k2}\,e^{i\theta_{23}}\rho^D P_L\right]\Big\} D h_k \\
& + \frac{1}{v}\overline{U}\sum_k \Big\{ M_U(q_{k1}P_L + q^*_{k1}P_R) + \frac{v}{\sqrt{2}}\left[q^*_{k2}\,e^{i\theta_{23}}\rho^U P_R + q_{k2}\,[e^{i\theta_{23}}\rho^U]^\dagger P_L\right]\Big\} U h_k \\
& + \Big\{ \overline{U}\left[K[e^{i\theta_{23}}\rho^D]^\dagger P_R - [e^{i\theta_{23}}\rho^U]^\dagger K P_L\right] D H^+ + \text{h.c.}\Big\},
\end{aligned}
$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ are left and right-handed projection operators, K is the CKM matrix, and the q_{ki} are invariant combinations of mixing angles:

k	q_{k1}	q_{k2}
1	$c_{12}c_{13}$	$-s_{12} - ic_{12}s_{13}$
2	$s_{12}c_{13}$	$c_{12} - is_{12}s_{13}$
3	s_{13}	ic_{13}

- The combinations $e^{i\theta_{23}}\rho^U$ and $e^{i\theta_{23}}\rho^U$ that appear in the interactions above are invariant under the rephasing of the Higgs basis field H_2.
- Note that no $\tan\beta$ parameter appears above! This is because $\tan\beta$ is an unphysical parameter in the general 2HDM.

In general ρ^Q is a complex non-digaonal matrix. As a result, the most general 2HDM exhibits tree-level Higgs-mediated FCNCs and new sources of CP-violation in the interactions of the neutral Higgs bosons.

In the decoupling limit where $m_1 \ll m_{2,3}$, CP-violating and tree-level Higgs-mediated FCNCs are suppreseed by factors of $\mathcal{O}(v^2/m_{2,3}^2)$. In contrast, the interactions of the heavy neutral Higgs bosons (h_2 and h_3) and the charge Higgs bosons ($H^{\pm}$) in the decoupling limit can exhibit both CP-violating and quark flavor non-diagonal couplings (proportional to ρ^Q).

Special case: a CP-conserving Higgs potential

In the generic 2HDM, new sources of CP-violation arise due to the fact that

- $Z_{5,6,7}$ are complex, and cannot be made real by rephasing $H_2 \to e^{i\chi} H_2$.
- CP-violating neutral Higgs–fermion couplings due to complex ρ^U and ρ^D.

Imposing CP-violation in the neutral Higgs sector, we take ρ^U and ρ^D to be real 3×3 matrices, and the q_{ki} are given by:

k	q_{k1}	q_{k2}
1	$s_{\beta-\alpha}$	$c_{\beta-\alpha}$
2	$-c_{\beta-\alpha}$	$s_{\beta-\alpha}$
3	0	i

The resulting Higgs-fermion Yukawa couplings are:

$$
\begin{aligned}
-\mathscr{L}_Y = {} & \overline{D}\left[\frac{M_D}{v}s_{\beta-\alpha} + \frac{\rho^D}{\sqrt{2}}c_{\beta-\alpha}\right]Dh^0 \\
& + \overline{D}\left[\frac{M_D}{v}c_{\beta-\alpha} - \frac{\rho^D}{\sqrt{2}}s_{\beta-\alpha}\right]DH^0 + \frac{i}{\sqrt{2}}\rho^D\overline{D}\gamma_5 DA^0 \\
& + \overline{U}\left[\frac{M_U}{v}s_{\beta-\alpha} + \frac{\rho^U}{\sqrt{2}}c_{\beta-\alpha}\right]Uh^0 \\
& + \overline{U}\left[\frac{M_U}{v}c_{\beta-\alpha} - \frac{\rho^U}{\sqrt{2}}s_{\beta-\alpha}\right]UH^0 - \frac{i}{\sqrt{2}}\rho^U\overline{U}\gamma_5 UA^0 \\
& + \left\{\overline{U}\left[K\rho^D P_R - \rho^U K P_L\right]DH^+ + \text{h.c.}\right\} ,
\end{aligned}
$$

which exhibits Higgs-mediated FCNCs since ρ^D and ρ^U are generically non-diagonal matrices.

Special Case: Higgs bosons couple to only one generation of fermions

To sidestep the problem of Higgs-mediated FCNCs, suppose that the Higgs bosons couple primarily to third-generation quarks and leptons.

$$
\begin{aligned}
g_{hqq} &= \frac{m_q}{v} s_{\beta-\alpha} + \frac{1}{\sqrt{2}}\big(S_q + i\gamma_5 P_q\big) c_{\beta-\alpha}\,, \\
g_{Hqq} &= \frac{m_q}{v} c_{\beta-\alpha} - \frac{1}{\sqrt{2}}\big(S_q + i\gamma_5 P_q\big) s_{\beta-\alpha}\,, \\
g_{Att} &= -\frac{1}{\sqrt{2}}\big(iS_u\gamma_5 - P_u\big)\,, \\
g_{Abb} &= \quad \frac{1}{\sqrt{2}}\big(iS_d\gamma_5 - P_d\big)\,, \\
g_{H^+bt} &= \tfrac{1}{2}\big[\rho^D(1+\gamma_5) - \rho^{U\,*}(1-\gamma_5)\big]\,,
\end{aligned}
$$

where $q = (t, b)$ and

$$
S_q \equiv \mathrm{Re}\ \rho^Q\,, \qquad\qquad P_q \equiv \mathrm{Im}\ \rho^Q\,.
$$

Note that If Im $\rho^Q \neq 0$, then there is a new source of CP-violation in the neutral Higgs-fermion interactions.

How to avoid tree-level Higgs-mediated FCNCs

- Arbitrarily declare ρ^U and ρ^D to be diagonal matrices. This is an unnaturally fine-tuned solution.

- Impose a discrete symmetry or supersymmetry (e.g., "Type-I" or "Type-II" Higgs-fermion interactions), which selects out a special basis of the 2HDM scalar fields. In this case, ρ^Q is automatically proportional to M_Q (for $Q = U, D, L$), and is hence diagonal.

- Impose alignment without a symmetry principle: $\rho^Q = \alpha^Q \kappa^Q$, $(Q = U, D, L)$, where the α^Q are complex scalar parameters [e.g. see Pich and Tuzon (2009)].

- Impose the heavy Higgs mass decoupling limit. Tree-level Higgs-mediated FCNCs will be suppressed by factors of squared-masses of heavy Higgs states. (How heavy is sufficient?)

Example: Type-II 2HDM Higgs-fermion couplings

$$\rho^D = -\frac{\sqrt{2}m_b}{v}\tan\beta\,, \qquad \rho^U = \frac{\sqrt{2}m_t}{v}\cot\beta\,, \qquad \text{(Type-II)}\,,$$

are real quantities, which yields:

$$\begin{aligned}
g_{hbb} &= -\frac{m_b}{v}\frac{\sin\alpha}{\cos\beta} = \frac{m_b}{v}\left(s_{\beta-\alpha} - \tan\beta\, c_{\beta-\alpha}\right)\,,\\
g_{htt} &= \frac{m_t}{v}\frac{\cos\alpha}{\sin\beta} = \frac{m_t}{v}\left(s_{\beta-\alpha} + \cot\beta\, c_{\beta-\alpha}\right)\,,\\
g_{Hbb} &= \frac{m_b}{v}\frac{\cos\alpha}{\cos\beta} = \frac{m_b}{v}\left(c_{\beta-\alpha} + \tan\beta\, s_{\beta-\alpha}\right)\,,\\
g_{Htt} &= \frac{m_t}{v}\frac{\sin\alpha}{\sin\beta} = \frac{m_t}{v}\left(c_{\beta-\alpha} - \cot\beta\, s_{\beta-\alpha}\right)\,.
\end{aligned}$$

Contrast this case with Type-I 2HDM Higgs-fermion couplings, where

$$\rho^D = \frac{\sqrt{2}m_b}{v}\cot\beta\,, \qquad \rho^U = \frac{\sqrt{2}m_t}{v}\cot\beta\,, \qquad \text{(Type-I)}\,.$$

Conclusions

1. I propose a strategy for developing 2HDM benchmarks that are based on the current trend in the Higgs data
   - The observed Higgs boson is identified with the lightest CP-even Higgs boson h of the 2HDM
   - The coupling of the observed Higgs boson to vector boson pairs is close to the Standard Model expectation.
2. We can therefore identify small parameters for which the 2HDM is amenable to a perturbative analysis.
3. A set of scanning parameters are identified that yield a set of heavier Higgs states, whose masses are internally consistent.
4. The procedure is exhibited for both the generic model (with CP-violation) and a special case where the Higgs potential is CP-conserving.
5. The to-do list:
   - Choose a set of practical benchmark points.
   - Consider benchmarks where the observed Higgs boson is H.
   - Consider benchmarks where the observed Higgs signal is due to mass-degenerate states.

Backup slides

The CP-conserving 2HDM with Type I or II Yukawa couplings

The scalar potential exhibits a $\mathbb{Z}_2$ symmetry that is at most softly broken,

$$V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right) + \tfrac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \tfrac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2$$
$$+ \lambda_3 \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2 + \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 + \left[\tfrac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right] ,$$

where m_{12}^2 and λ_5 are real. The most general Yukawa Lagrangian, in terms of the quark mass-eigenstate fields, is:

$$-\mathscr{L}_{\rm Y} = \overline{U}_L \widetilde{\Phi}_a^0 \eta_a^U U_R + \overline{D}_L K^\dagger \widetilde{\Phi}_a^- \eta_a^U U_R + \overline{U}_L K \Phi_a^+ \eta_a^{D\,\dagger} D_R + \overline{D}_L \Phi_a^0 \eta_a^{D\,\dagger} D_R + \text{h.c.} ,$$

where $a = 1, 2$, $\widetilde{\Phi}_a \equiv (\widetilde{\Phi}^0 , \widetilde{\Phi}^-) = i\sigma_2 \Phi_a^*$ and K is the CKM mixing matrix. The $\eta^{U,D}$ are 3×3 Yukawa coupling matrices.

Type-I Yukawa couplings: $\eta_1^U = \eta_1^D = 0$.

	h^0	A^0	H^0
up-type quarks	$\cos\alpha/\sin\beta$	$\cot\beta$	$\sin\alpha/\sin\beta$
down-type quarks and leptons	$\cos\alpha/\sin\beta$	$-\cot\beta$	$\sin\alpha/\sin\beta$

Type-II Yukawa couplings: $\eta_1^U = \eta_2^D = 0$ [employed by the MSSM].

	h^0	A^0	H^0
up-type quarks	$\cos\alpha/\sin\beta$	$\cot\beta$	$\sin\alpha/\sin\beta$
down-type quarks and leptons	$-\sin\alpha/\cos\beta$	$\tan\beta$	$\cos\alpha/\cos\beta$

Here, α is the CP-even Higgs mixing angle and $\tan\beta = v_u/v_d$. The h^0 and H^0 are CP-even neutral Higgs bosons with $m_{h^0} \leq m_{H^0}$ and A^0 is a CP-odd neutral Higgs boson.

Example: decoupling of the non-minimal Higgs bosons of the MSSM Higgs sector (tree-level analysis)

The MSSM employs a type-II Higgs-fermion Yukawa coupling scheme. In addition, supersymmetry restricts the Higgs basis potential parameters,

$$Z_1 = Z_2 = \tfrac{1}{4}(g^2 + g'^2)\cos^2 2\beta\,, \qquad Z_3 = Z_5 + \tfrac{1}{4}(g^2 - g'^2)\,, \qquad Z_4 = Z_5 - \tfrac{1}{2}g^2\,,$$
$$Z_5 = \tfrac{1}{4}(g^2 + g'^2)\sin^2 2\beta\,, \qquad Z_7 = -Z_6 = \tfrac{1}{4}(g^2 + g'^2)\sin 2\beta\cos 2\beta\,.$$

In the limit of $m_A \gg m_Z$, the tree-level expressions for the MSSM Higgs masses and mixings are:

$$m_h^2 \simeq m_Z^2\cos^2 2\beta\,, \qquad m_H^2 \simeq m_A^2 + m_Z^2\sin^2 2\beta\,,$$
$$m_{H^\pm}^2 = m_A^2 + m_W^2\,, \qquad \cos^2(\beta-\alpha) \simeq \frac{m_Z^4\sin^2 4\beta}{4m_A^4}\,.$$

Indeed, $\cos(\beta-\alpha) = \mathcal{O}(m_Z^2/m_A^2)$ and $m_A \simeq m_H \simeq m_{H^\pm}$, up to corrections of $\mathcal{O}(m_Z^2/m_A)$, as expected. This is the decoupling limit of the MSSM Higgs sector.

In general, in the limit of $\cos(\beta-\alpha)\to 0$, all the h^0 couplings to SM particles approach their SM limits. In particular, if λ_V is a Higgs coupling to vector bosons and λ_f is a Higgs couplings to fermions, then

$$\frac{\lambda_V}{[\lambda_V]_{\rm SM}} = \sin(\beta-\alpha) = 1+\mathcal{O}\left(m_Z^4/m_A^4\right)\,, \qquad\qquad \frac{\lambda_f}{[\lambda_f]_{\rm SM}} = 1+\mathcal{O}\left(m_Z^2/m_A^2\right)\,.$$

The behavior of the $h^0 ff$ coupling is:

$$h^0 b\bar{b} \quad (\text{or } h^0\tau^+\tau^-): \qquad -\frac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha)\,,$$

$$h^0 t\bar{t}: \qquad \frac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha)\,.$$

Note the extra $\tan\beta$ enhancement in the deviation of λ_{hbb} from $[\lambda_{hbb}]_{\rm SM}$.

Thus, the approach to decoupling is fastest for the h^0VV couplings, and slowest for the couplings of h^0 to down-type quarks and leptons (if $\tan\beta$ is large).