Breaking statistical isotropy

Marco Peloso, University of Minnesota

Gumrukcuoglu, Contaldi, M.P, JCAP 0711:005,2007

Gumrukcuoglu, Kofman, MP, PRD 78:103525,2008

Himmetoglu, Contaldi, M.P, PRL 102:111301,2009

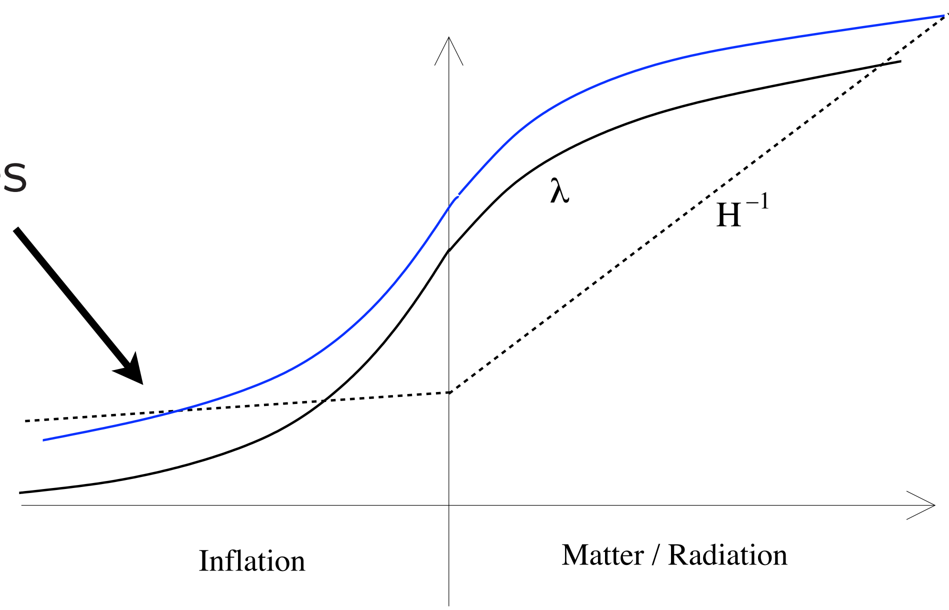
Himmetoglu, Contaldi, M.P, PRD 79:063517,2009

For FRW $\langle a^*_{\ell m}\, a_{\ell' m'}\rangle = C_\ell\, \delta_{\ell\ell'}\, \delta mm'$

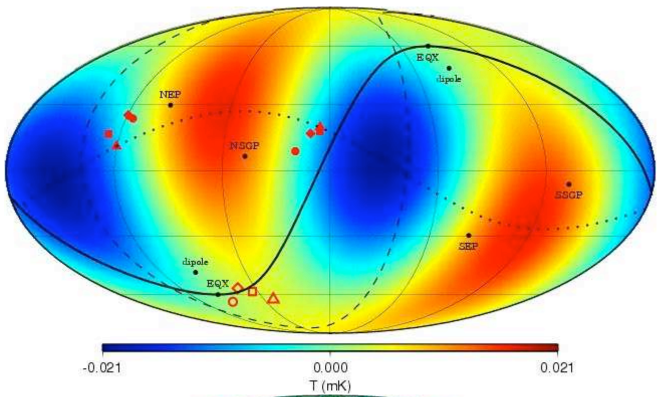
Claims of violation of statistical isotropy of the CMB perturbations. A number of "$2-3\,\sigma$ effects", significance susceptible to statistics used. Some of these claims concern the largest scale modes, for which additional problems due to galaxy contamination

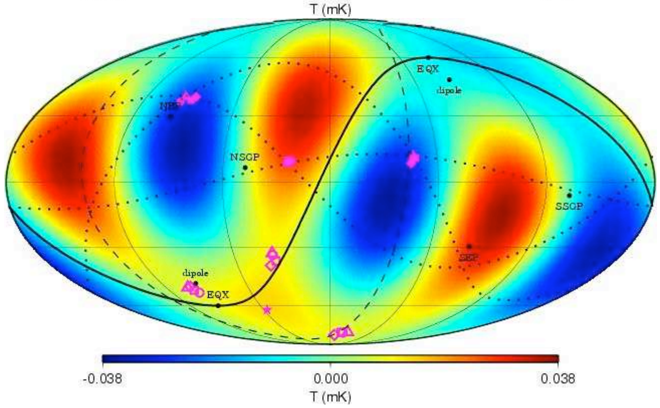
Larger modes (low ℓ) probe

earlier inflationary stages



Cleaned CMB map by Tegmark et al' 03. WMAP1 $\neq$ channels combined to eliminate foregrounds (depending on ℓ and latitude)





Axis from $\quad \max_{\hat{n}} \sum_m m^2 \, |a_{\ell m}(\hat{n})|^2$

Low quadrupole	$\sim 1/20$
Planar octupole	$\sim 1/20$
Alignment, $\Delta\theta \simeq 10^0$	$\sim 1/62$

Plane of alignment, 30^0 apart from galactic plane

Anomalous lack of power at large scales outside galaxy
Slightly greater quadrupole than WMAP1 (using galaxy cut)

Land, Magueijo, '05, $\ell = 2-5$ aligned $\max_{m,\hat{n}} |a_{\ell m}(\hat{n})|^2$

WMAP1 and WMAP+ data agree at large scales. $\neq$ treatment

Land, Magueijo, '06

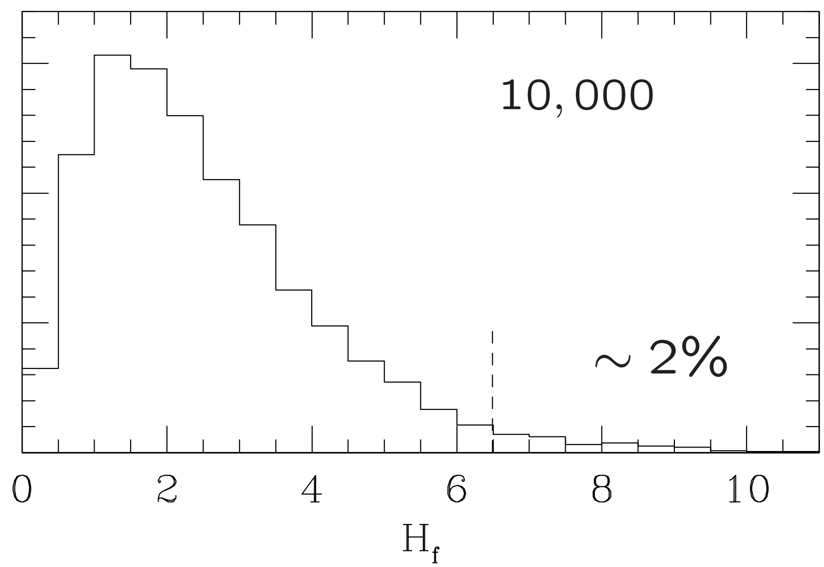
	$\ell=2$		$\ell=3$		$\ell=4$		$\ell=5$		Mean
Map	(b,l)	m	(b,l)	m	(b,l)	m	(b,l)	m	inter-θ
LILC1	(0.9, 156.7)	0	(63.0, -126.9)	3	(56.7, -163.7)	2	(48.6, -94.7)	3	51.4
TOH1	(58.5, -102.9)	2	(62.1, -120.6)	3	(57.6, -163.3)	2	(48.6, -93.4)	3	22.4
TOH3	(76.5, -134.0)	2	(27.0, 51.9)	1	(35.1, -130.6)	1	(47.7, -94.7)	3	53.8
WMAP3	(2.7, -26.5)	0	(62.1, -122.6)	3	(34.2, -131.2)	1	(47.7, -96.0)	3	53.7

← 0.1% (expect 57^0)

Model comparison (Bayesian)

$$\text{S. M. vs.} \quad |a_{\ell m}|^2(\hat{n})|^2 = \begin{cases} C_\ell & \text{if } |m| = \ell \\ \epsilon\, C_\ell & \text{if } |m| \neq \ell \end{cases}$$

Data-set	$(b$	$l)$	ϵ	H_f	H^{AIC}	H^{BIC}	$\ln B$
LILC1	63	-120	.042	6.51	2.01	2.78	1.36
TOH1	61	-113	.032	7.48	2.98	3.75	1.85
TOH3	74	-129	.018	6.97	2.47	3.24	1.27
WMAP3	64	-123	.043	6.49	1.99	2.76	1.32

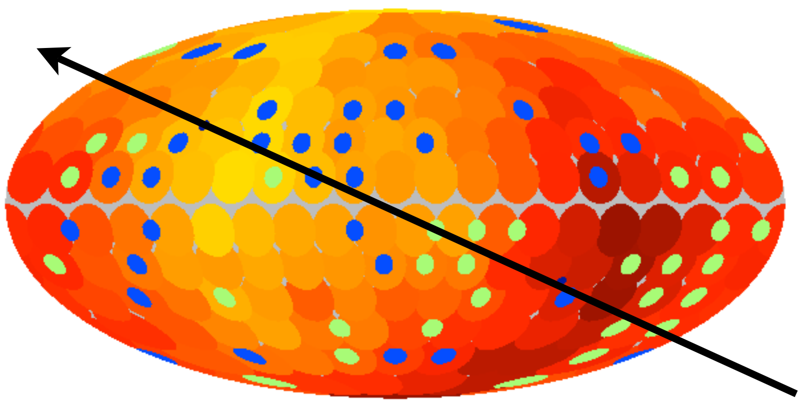


North-south asymmetry

Eriksen et al' 04

WMAP1, Q, V, W bands with Kp0 (-25%)

Ecliptic axis



164 circles, draw 82 axis, and compute power in the separate hemispheres, in $C_\ell = 2 - 63$

Dark = high $C_{\ell,N}/C_{\ell,S}$ when north points inside big circle

- Power in that disk $>$ in 80% of isotropic simulations
- Power in that disk $<$ in 20% of isotropic simulations

In the frame in which is maximal, asymmetry $>$ than for 99.7% of isotropic realizations

Hoftuft et al '09, model comparison SM vs. $[1 + A\,\hat{n} \cdot \hat{v}]\, s\,(\hat{n})$

dipole modulation
Gordon et al '05

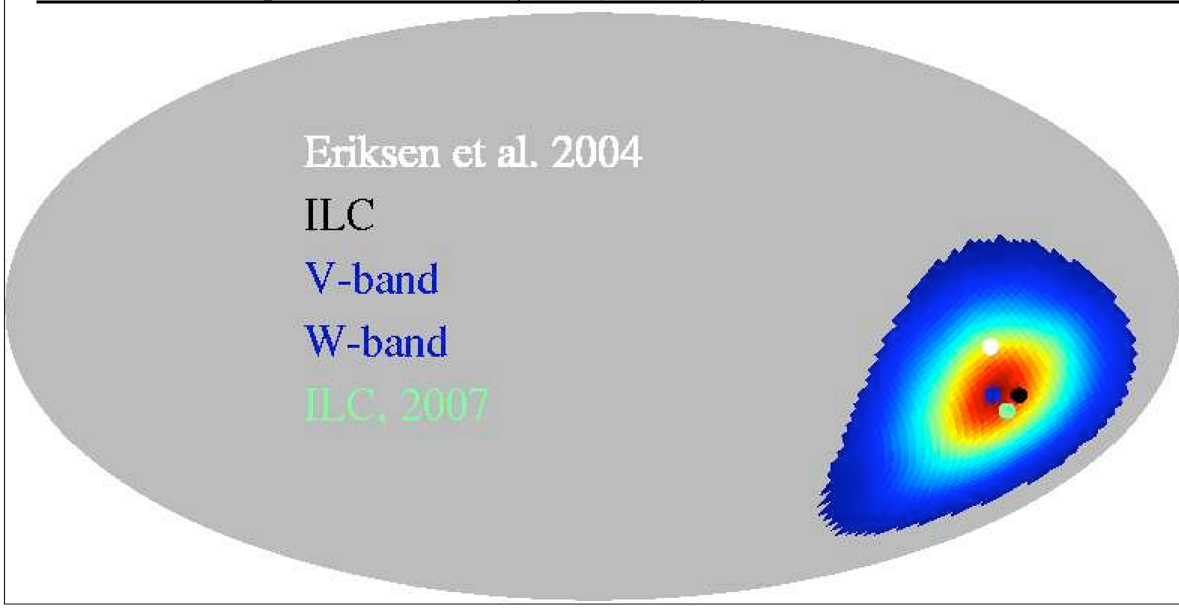
random gaussian

WMAP5 downgraded to 4.5^0

KQ85 (−16.3%); KQ85e (−26.9%)

Data	Mask	ℓ_{mod}	(l_{bf}, b_{bf})	A_{bf}	Significance (σ)	$\Delta \log \mathcal{L}$	$\Delta \log E$
ILC	KQ85	64	$(224°, -22°) \pm 24°$	0.072 ± 0.022	3.3	7.3	2.6
V-band	KQ85	64	$(232°, -22°) \pm 23°$	0.080 ± 0.021	3.8	...	...
V-band	KQ85	40	$(224°, -22°) \pm 24°$	0.119 ± 0.034	3.5	...	...
V-band	KQ85	80	$(235°, -17°) \pm 22°$	0.070 ± 0.019	3.7	...	...
W-band	KQ85	64	$(232°, -22°) \pm 24°$	0.074 ± 0.021	3.5	...	...
ILC	KQ85e	64	$(215°, -19°) \pm 28°$	0.066 ± 0.025	2.6	...	...
Q-band	KQ85e	64	$(245°, -21°) \pm 23°$	0.088 ± 0.022	3.9	...	...
V-band	KQ85e	64	$(228°, -18°) \pm 28°$	0.067 ± 0.025	2.7	...	...
W-band	KQ85e	64	$(226°, -19°) \pm 31°$	0.061 ± 0.025	2.5	...	...
ILC[a]	Kp2	~ 40	$(225°, -27°)$	0.11 ± 0.04	2.8	6.1	1.8

WMAP3
1/2 resolution



Groeneboom and Eriksen '08 studied the "ACW model"

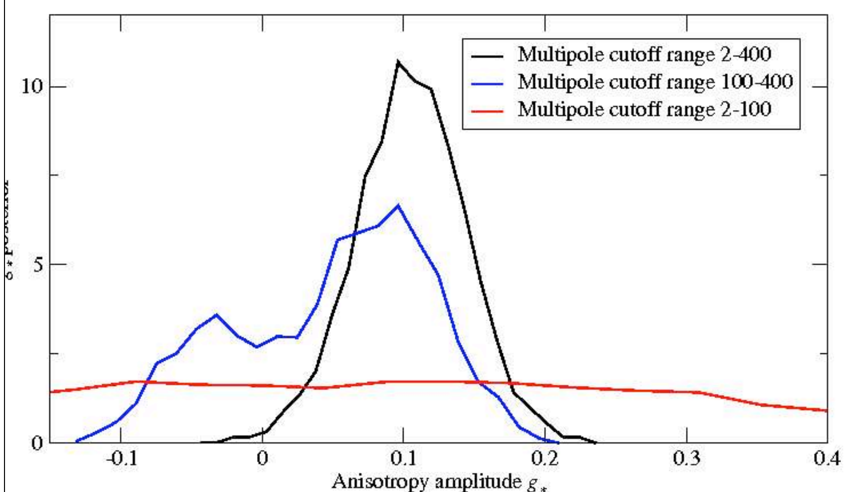
Ackerman, Carroll, Wise '07

$$P\left(\vec{k}\right) = P\left(k\right)\left[1 + g_*\left(\vec{k}\cdot\vec{v}\right)^2\right]$$

Note: "Taylor expansion" that may be expected for axis $(x - y)$

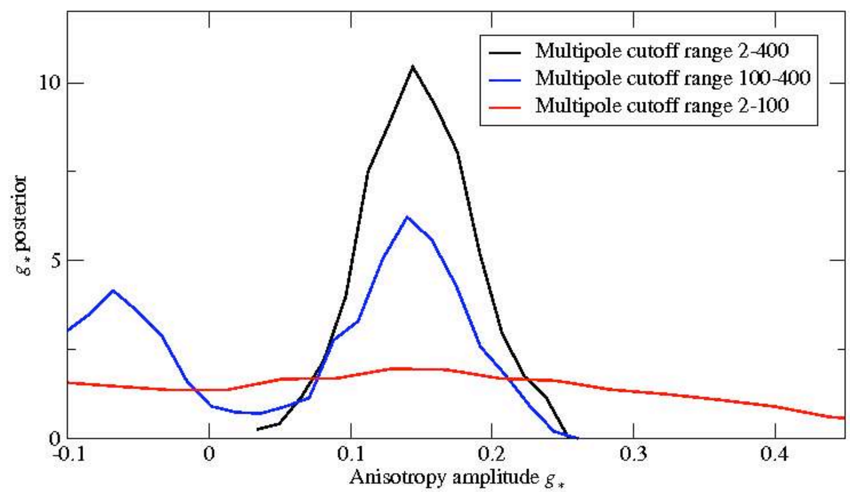
& planar $(z \leftrightarrow -z)$ symmetric geometry. ACW model is unstable

V-band



$g_* = 0.10 \pm 0.04$

W-band



$g_* = 0.15 \pm 0.04$ (3.8σ)

Increase of significance with ℓ Pullen, Kamionkowski '07

Models ?

Likely, accepted only if it explains $N > 1$ effects

Perhaps we should learn to compute cosmological perturbations beyond FRW

Simplest: Bianchi-I with residual 2d isotropy

$$ds^2 = -dt^2 + a(t)^2 \, dx^2 + b(t)^2 \left[dy^2 + dz^2 \right]$$

Standard formalism for FRW

Bardeen '80; Mukhanov '85

$$\delta g_{\mu\nu} \, , \, \delta\phi \;\; \leftrightarrow \;\; v \, , \, h_+ \, , \, h_\times$$

Anisotropic background

How many physical modes ?	Still 3 $\quad \delta g_{\mu\nu}\ ,\ \delta\phi \quad +11$ modes Gen. coord. transf. $\quad -4$ $\delta g_{0\mu}$ non dynamical $\quad -4$
How do they behave ?	Coupled to each other already at the linearized level (due to less symmetric background) Decoupled – UV regime – limit of isotropic background
Signatures ?	$\langle a_{\ell m} a^*_{\ell' m'} \rangle \not\propto \delta_{\ell\ell'}\,\delta_{mm'}$ Nonstandard amplitude for gravity waves ($\neq$ results for the 2 polarizations)

$$g_{\mu\nu} = \begin{pmatrix} -(1+2\Phi) & a\,\chi & b\left(\partial_i B + B_i^T\right) \\ & a^2\,(1-2\Psi) & a\,b\left(\partial_i \tilde{B} + \tilde{B}_i^T\right) \\ & & b^2\left[(1-2\Sigma)\,\delta_{ij} + \partial_i\partial_j E + \partial_{(i} E^T{}_{j)}\right] \end{pmatrix}$$

seven 2d scalars + three 2d vectors, decoupled at linearized level

Choose a gauge preserving all $\delta g_{0\mu}$, since they are the nondynamical fields (ADM formalism)

- Harder to indentify nondynamical modes in standard gauges
- Can be promoted to gauge invariant formulation

$$\left(\hat{\Phi} \equiv \Phi + \left(\frac{\Sigma}{H_b}\right)^{\bullet} \; ; \; \hat{B} = B - \frac{\Sigma}{b\,H_b} + b\,\dot{E} \; ; \; \ldots \right)$$

Dynamical Y_i and nondynamical N_i fields

$$S = \int d^3k\, dt\, \Big[a_{ij}\, \dot{Y}_i^*\, \dot{Y}_j + \Big(b_{ij}\, N_i^*\, \dot{Y}_j + \text{h.c.} \Big) + c_{ij}\, N_i^*\, N_j + \Big(d_{ij}\, \dot{Y}_i^*\, Y_j + \text{h.c.} \Big) + e_{ij}\, Y_i^*\, Y_j + \left(f_{ij}\, N_i^*\, Y_j + \text{h.c.} \right) \Big]$$

Coefficients background-dependent

Solving for N, $\quad \dfrac{\delta S}{\delta N_i^*} = 0 \;\Rightarrow\; c_{ij}\, N_j = -b_{ij}\, \dot{Y}_j - f_{ij}\, Y_j$

Action for the dynamical (propagating) modes

$$S \;\rightarrow\; \int d^3k\, dt\, \Big[\dot{Y}_i^*\, K_{ij}\, \dot{Y}_j + \Big(\dot{Y}_i^*\, \Lambda_{ij}\, Y_j + \text{h.c.} \Big) - Y_i^*\, \Omega_{ij}^2\, Y_j \Big]$$

$$K_{ij} \equiv a_{ij} - \left(b^\dagger \right)_{ik} \left(c^{-1} \right)_{km} b_{mj}$$

$$\Lambda_{ij} \equiv d_{ij} - \left(b^\dagger \right)_{ik} \left(c^{-1} \right)_{km} f_{mj}$$

$$\Omega_{ij}^2 \equiv e_{ij} - \left(f^\dagger \right)_{ik} \left(c^{-1} \right)_{km} f_{mj}$$

← Eigenvalues indicate the nature of a mode

$$\frac{\delta S}{\delta Y_i^*} = 0 \;\rightarrow\; K_{ij}\, \ddot{Y}_j + \Big[\dot{K}_{ij} + \left(\Lambda_{ij} + \text{h.c.} \right) \Big]\, \dot{Y}_j + \Big(\dot{\Lambda}_{ij} + \Omega_{ij}^2 \Big)\, Y_j = 0$$

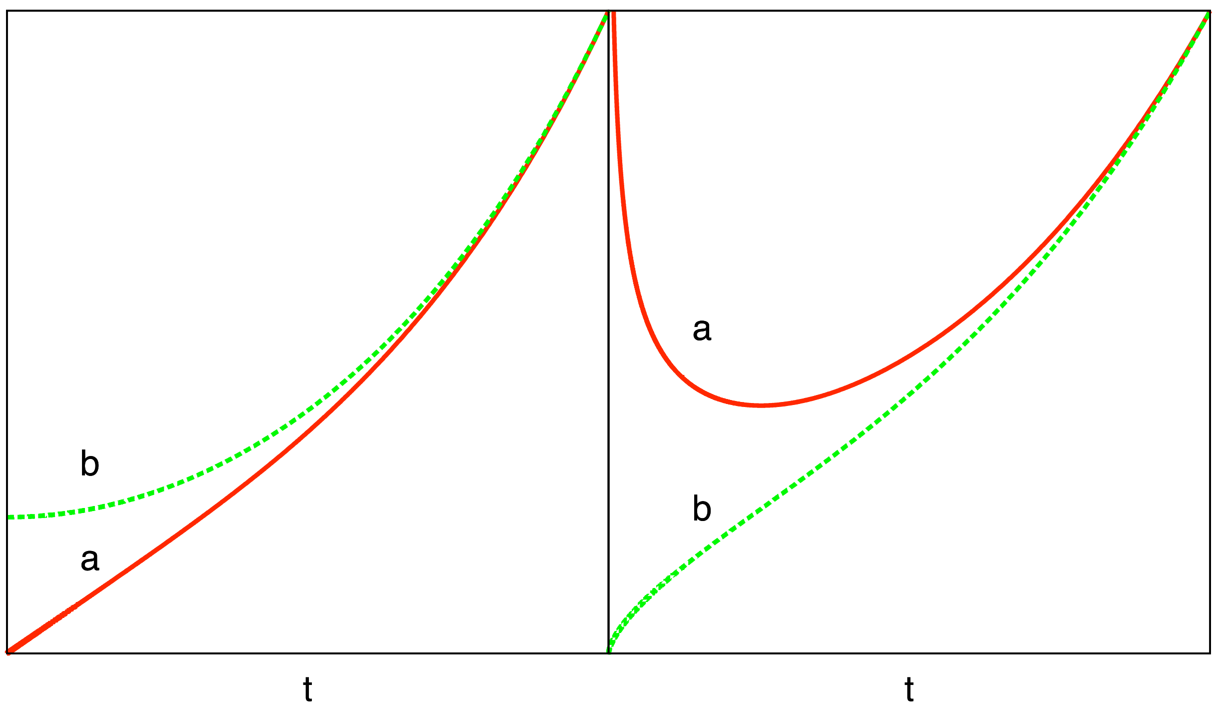
Expect divergency if K is noninvertible

Simplest example $\mathcal{L} = \frac{M_p^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi)$

Asymptotic Kasner (vacuum) solution in the past

$$ds^2 = -dt^2 + t^{2\alpha} dx^2 + t^{2\beta} \left(dy^2 + dz^2\right)$$

$$\alpha + 2\beta = 1 \ , \ \ \alpha^2 + 2\,\beta^2 = 1$$



$$t_{\text{iso}} \sim \frac{M_p}{\sqrt{V_{\text{in}}}}$$

$\alpha = 1 \ , \ \ \beta = 0$ $\alpha = -\frac{1}{3} \ , \ \ \beta = \frac{2}{3}$

$$S_{(2)} = \frac{1}{2}\int d\eta\, d^3k \left[|H'_\times|^2 - \omega_\times^2 |H_\times|^2 + |H'_+|^2 + |V|^2 - \left(H_+^*, V^*\right) \Omega^2 \begin{pmatrix} H_+ \\ V \end{pmatrix} \right]$$

GW polarizations — Scalar (density) mode

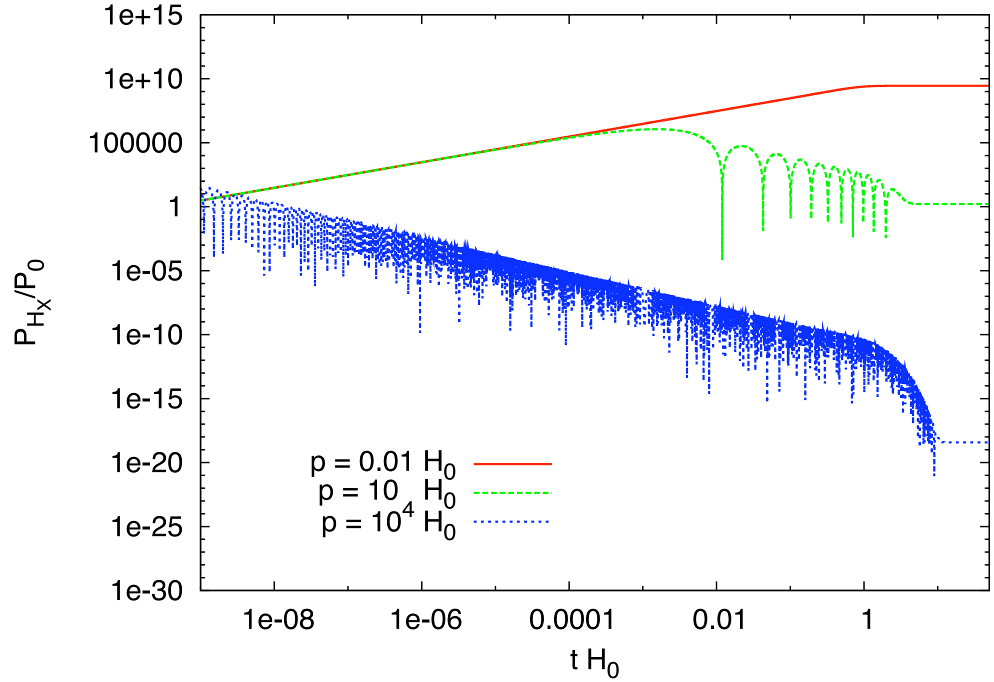
Initial conditions from early time frequencies ?

Pbm: $\omega_\times^2, \Omega^2 \sim p^2 + f\left(H^2\right)$, and $H \sim \frac{1}{t}$, $p_x \sim t^{1/3}$, $p_{y,z} \sim \frac{1}{t^{2/3}}$

$H \gg p$ in the asymptotic past (mode in long wavelength regime)

$$\omega_\times^2 \to a^2 \left[-\frac{5}{9t^2} + p_y^2 + p_z^2 \right] \quad , \quad \Omega_{ij}^2 \to a^2 \left[\frac{4}{9t^2} + p_y^2 + p_z^2 \right] \delta_{ij}$$

No adiabatic evolution; $\omega_\times^2 < 0$



$P_H \propto p^3 \, |H_\times|^2$

Large growth

Strong scale dependency

- Analogous to instability of contracting Kasner

Belinsky, Khalatnikov, Lifshitz '70, '82

- Potentially detectable GW, even if $V_0^{1/4} < 10^{16}$ GeV
- Tuned duration of inflation. If $N \gg 60$, effect blown away

Search for a longer / controllable anisotropic stage

(contrast Wald's theorem on isotropization of Bianchi spaces)

- Higer curvature terms Barrow, Hervik '05
- Kalb–Ramond axion Kaloper '91
- Vector field, $\langle A_z \rangle \neq 0$

$\longrightarrow$ Potential term $V\left(A_\mu A^\mu\right)$ Ford '89

$\longrightarrow$ Fixed norm Ackerman, Carroll, Wise '07

$\longrightarrow$ Slow roll due to $A_\mu A^\mu R$

Golovnev, Mukhanov, Vanchurin '08

Kanno, Kirma, Soda, Yokoyama '08

Yokoyama, Soda '08 Chiba '08 Kovisto, Mota '08

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{F^2}{4} + \frac{\xi}{2} R A^2 \right]$$

Nonminimal coupling

$$g_{\mu\nu} = \text{diag}\left(-1,\, a^2,\, b^2,\, b^2\right)$$

$$H = \frac{1}{3}\left[H_a + 2\, H_b\right]$$

$$h = \frac{1}{3}\left[H_b - H_a\right]$$

$$A_\mu = (0,\, a\, B\, M_p,\, 0,\, 0) \;\rightarrow\; A^2 = M_p^2\, B^2$$

$$\ddot{B} + 3\, H\, \dot{B} + \left\{ -2\, H\, h - 5\, h^2 - 2\dot{h} + (1 - 6\,\xi)\left(2H^2 + h^2 + \dot{H}\right)\right\} B = 0$$

$\xi = 1/6$ used for

Primordial magnetic fields — Turner, Widrow '88

Vector inflation — Golovnev, Mukhanov, Vanchurin '08

Vector curvaton — Dimopoulos, Lyth, Rodriguez '08

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{1}{12}\, R\, A^2 = -\frac{1}{4}F^2 + H^2\, A^2$$

$+$ sign leads to a ghost (not a tachyon !)

Stückelberg: $$A_\mu \longrightarrow B_\mu^T + \frac{1}{H}\,\partial_\mu \phi$$

$$H^2 A^2 \;\longrightarrow\; H^2\, B_\mu^T\, B^{\mu T} + \partial_\mu \phi\, \partial^\mu \phi$$ (signature $-+++$)

Does not require anisotropy !

Cf. PF mass $-m^2\, h_{\mu\nu} h^{\mu\nu} + m^2\, \left(h_\mu^\mu\right)^2$ to avoid ghost

Alternatively, $\mathcal{L} = -\frac{1}{4}F^2 - \frac{M^2}{2}A^2 = \frac{1}{2}\,A^\mu P^{-1}_{\mu\nu}\,A^\nu$

$$P_{\mu\nu} = -\frac{\eta_{\mu\nu} + k_\mu\,k_\nu/M^2}{k^2 + M^2}$$

- $M^2 > 0$. Go in the rest frame, $k^\mu = -k_\mu = \left(\sqrt{M^2},\, 0,\, 0,\, 0\right)$

$-\left(\eta_{\mu\nu} + k_\mu\,k_\nu/M^2\right) = \text{diag}\,(0,\, -1,\, -1,\, -1)$

- $M^2 < 0$. Frame with no energy, $k^\mu = k_\mu = \left(0,\, 0,\, 0,\, \sqrt{-M^2}\right)$

$-\left(\eta_{\mu\nu} + k_\mu\,k_\nu/M^2\right) = \text{diag}\,(1,\, -1,\, -1,\, 0)$

Exhaustive computation if $A\mu$ has no VEV
(no $\delta A_\mu \leftrightarrow \delta g_{\mu\nu}$ linearized mixing)

Vector inflation

Golovnev, Mukhanov, Vanchurin '08

Kanno, Kimura, Soda, Yokoyama '08

$$\mathcal{L} = \sum_a -\frac{1}{4} F_{\mu\nu}^{(a)} F^{(a)\mu\nu} - \frac{1}{2}\left(m^2 - \frac{R}{6}\right) A_\mu^{(a)} A^{(a)\mu}$$

$$\vec{A}^{(a)} = a\, M_p\, \vec{B}^{(a)} \quad \rightarrow \quad \ddot{B} + 3\, H\, \dot{B} + m^2\, B = 0 \ , \quad \frac{h}{H} \sim \frac{1}{\sqrt{N}}$$

$$\begin{cases} \delta g_{\mu\nu} \rightarrow 10 - 4 = 2\,\text{dyn} + 4\,\text{non dyn} \\ \delta A_\mu^{(a)} \rightarrow 4\,N = 3N\,\text{dyn} + N\,\text{non dyn} \end{cases}$$

Simplest case, 3 mutually orthogonal vectors with equal vev

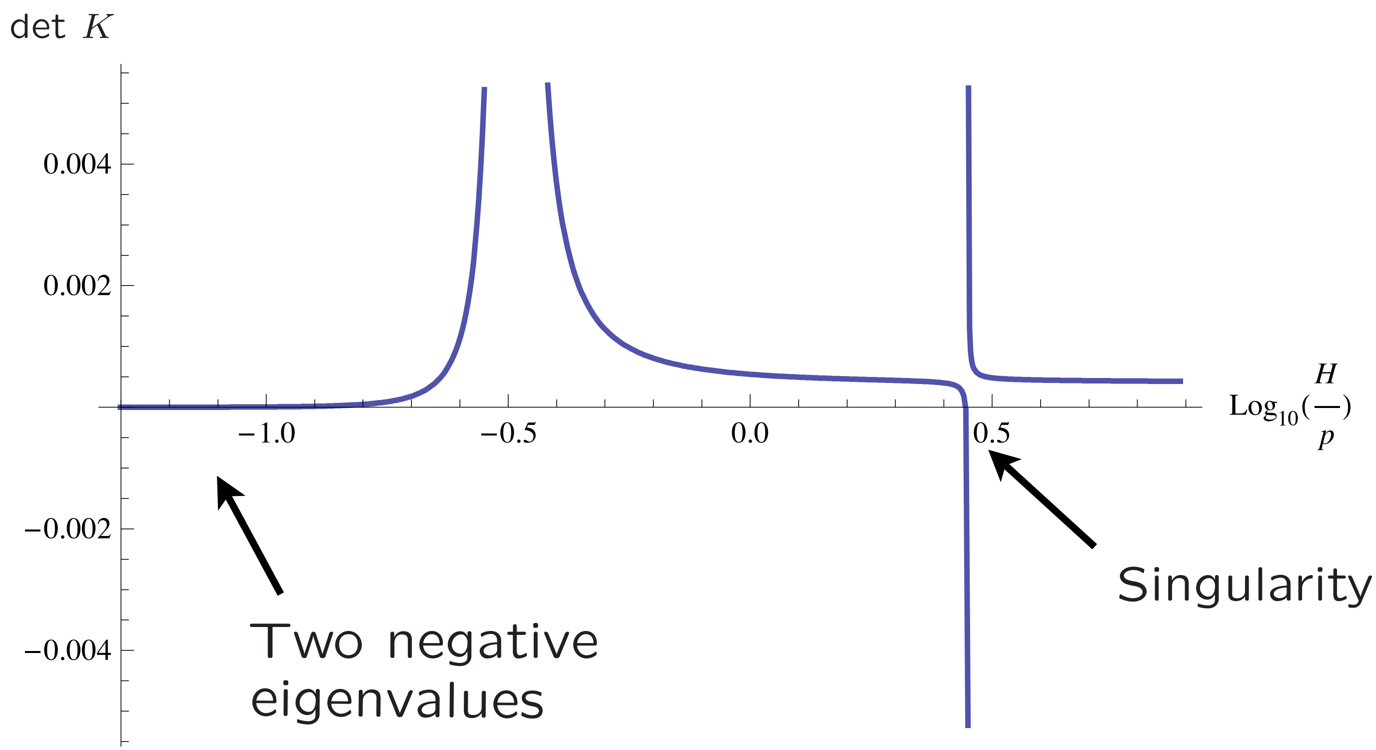
$\rightarrow$ 18 coupled modes

$$\delta_2 S \supset a^2\, M_p \left[\left(\dot{B} + H\, B\right) \delta F_{0j}^{(i)} + \left(m^2 - 2H^2 - \dot{H}\right) B\, \delta A_j^{(i)}\right] h_{ij}^{TT}$$

18 coupled modes, 11 dynamical and 7 non dynamical

We computed the kinetic matrix for the dynamical modes

$$\delta_2 S = \int d^3k\, dt\, \left[\dot{Y}_i^*\, K_{ij}\, \dot{Y}_j + \ldots\right]$$



$k_1 : k_2 : k_3 = 100 : 80 : 60$

Simplified computation: concentrate on one vector field

Collective effect of the remaining ones $\equiv$ cosmological constant

$$\mathcal{L} = \frac{M_p^2}{2} R - V_0 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \left(m^2 - \frac{R}{6} \right) A_\mu A^\mu$$

$$A_\mu = (0,\, a\, B\, M_p,\, 0,\, 0) + \delta A_\mu \quad \rightarrow \quad H = \frac{\sqrt{V_0}}{\sqrt{3}\, M_p} + O\left(B^2\right) \quad , \quad h = \frac{H}{3} B^2 + O\left(B^4\right)$$

B slowly rolling for $m \ll H$

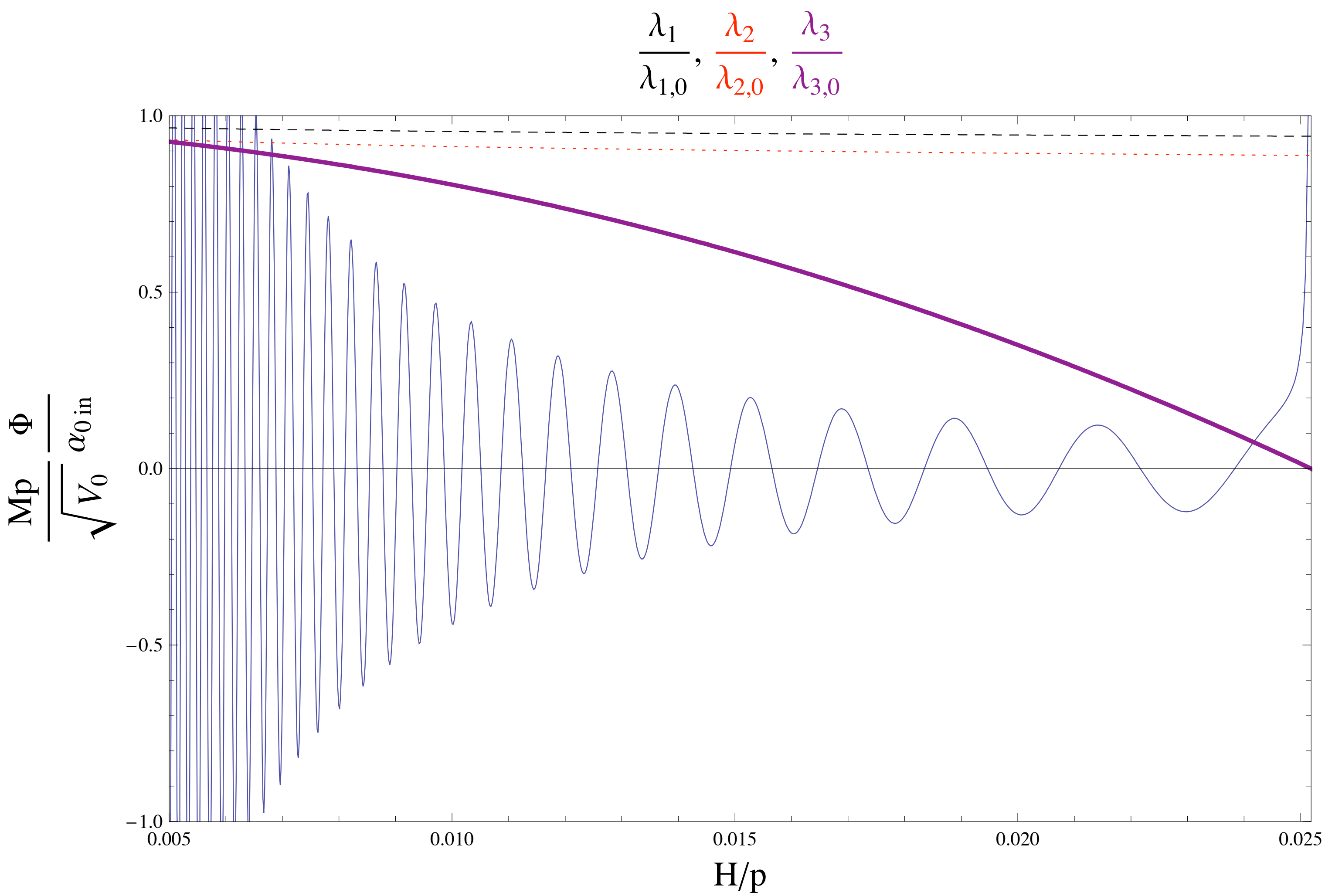
vev along x only, can do 2d decomposition

$$\delta g_{\mu\nu} \begin{cases} 1\,\text{dyn} + 3\,\text{non dyn 2ds} \\ 1\,\text{dyn} + 1\,\text{non dyn 2dv} \end{cases} \qquad \delta A_\mu \begin{cases} 2\,\text{dyn} + 1\,\text{non dyn 2ds} \\ 1\,\text{dyn 2dv} \end{cases}$$

7 coupled modes rather than 18

Parametrically, $\det K = B^2 - \frac{H^2}{p^2} + \ldots$

Perturbations diverge when it vanishes



Fixed norm vector fields

Kostelecky, Samuel '89; Jacobson, Mattingly '04; Carroll, Lim '04

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \lambda \left(A^2 - m^2 \right) - V_0 \right]$$

Ackerman, Carroll, Wise '07

$$ds^2 = -dt^2 + \mathrm{e}^{2 H_a t} dx^2 + \mathrm{e}^{2 H_b t} \left(dy^2 + dz^2 \right)$$

$$\langle A_x \rangle = m \quad \Rightarrow \quad H_b = \left(1 + \frac{m^2}{M_p^2} \right) H_b$$

Test field χ

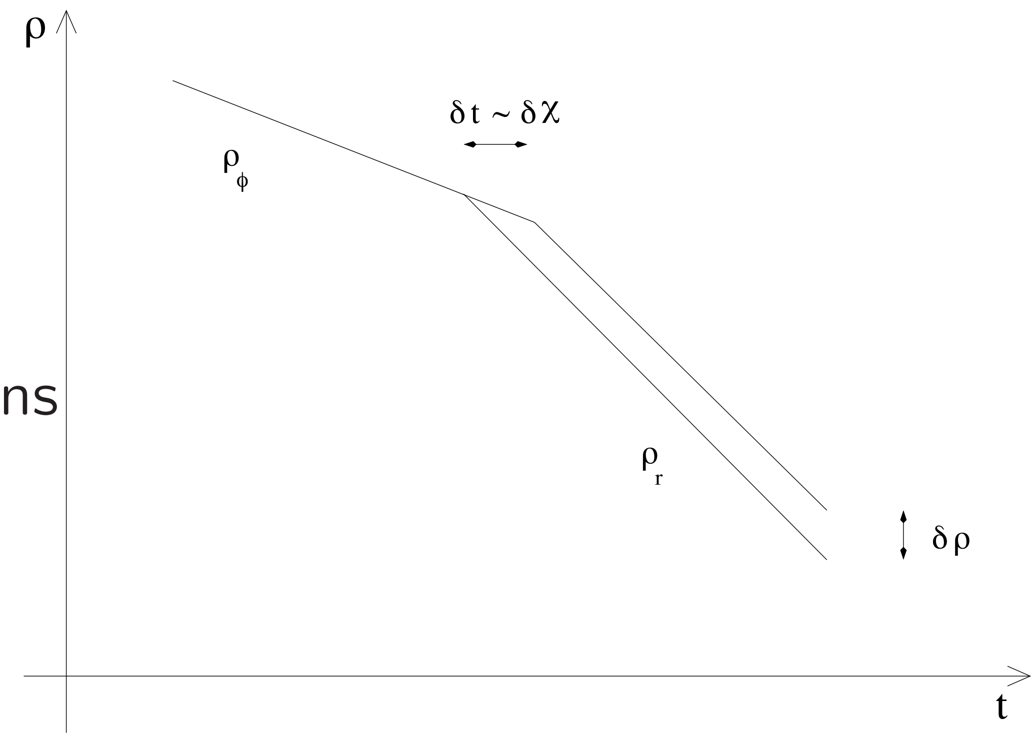
$$P_{\delta\chi} = P\left(|\vec{k}| \right) \left(1 + g_* \, k_x^2 \right)$$

Assumed $\delta\chi \rightarrow \delta g_{\mu\nu}$

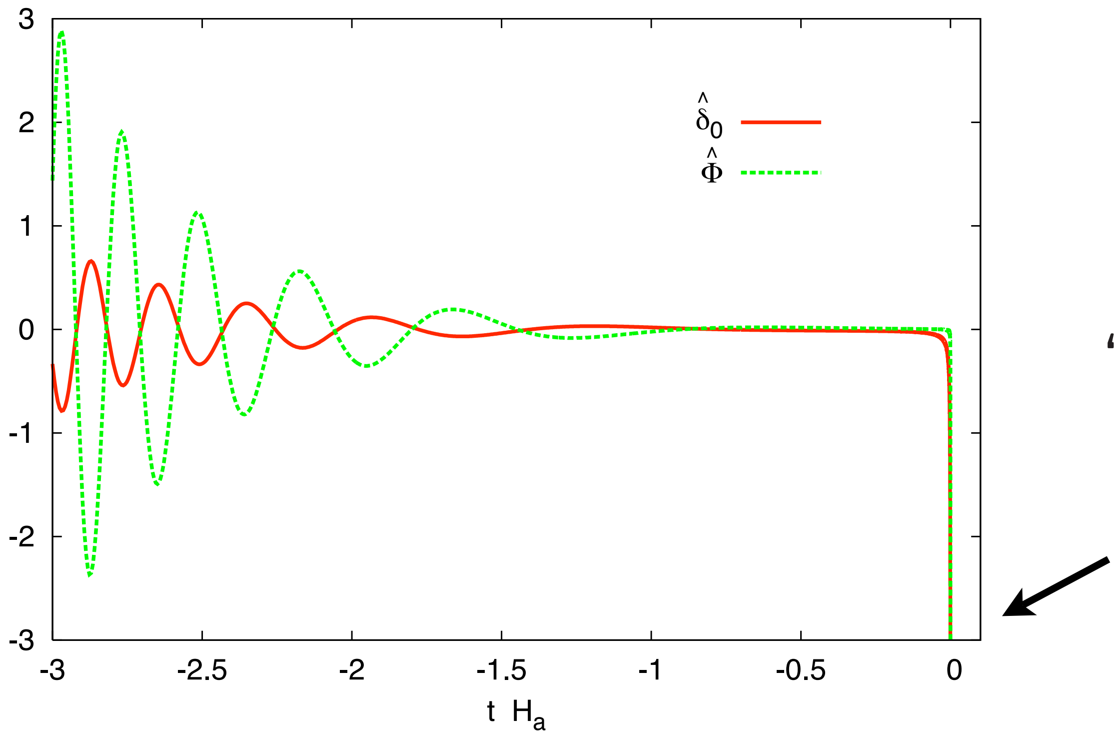
through modulated pertrbations

Dvali, Gruzinov, Zaldarriaga '03

Kofman '03



Complete study ($\delta g_{\mu\nu}$, δA_μ) shows that this model is unstable



Divergency at

"horizon crossing"

$$p_x^2 = \left(2 + \frac{m^2}{M_p^2}\right) H_a \, H_b$$

One mode becomes a ghost at that point

Kinetic term vanishes $\rightarrow$ perturbations diverge

Problems in theories with fixed A^2 also pointed out by Clayton '01

So what ?

Linearized computation blows up; maybe nonlinear evolution ok

Linearized computation $\rightarrow$ CMB

Assume singularity cured, any other problem ?

nonlinear interactions: $|0\rangle \rightarrow$ ghost-nonghost; UV ∞

For a gravitationally coupled ghost today, $\Lambda < 3\,\mathrm{MeV}$

Cline et al' 03

$U(1)$ hard breaking, A_L interactions p/m enhanced

Quantum theory out of control at $E \gtrsim m \sim H$

whole sub-horizon regime

Unclear UV completion $\quad \pm|DH|^2 \rightarrow \pm m^2 A^2$

Ghost condensation ?

Scalar-Vector Coupling

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1 - V_1(\phi_1) - \frac{1}{4} f^2(\phi_1) F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \partial_\mu \phi_2 \partial^\mu \phi_2 - V_2(\phi_2) \right]$$

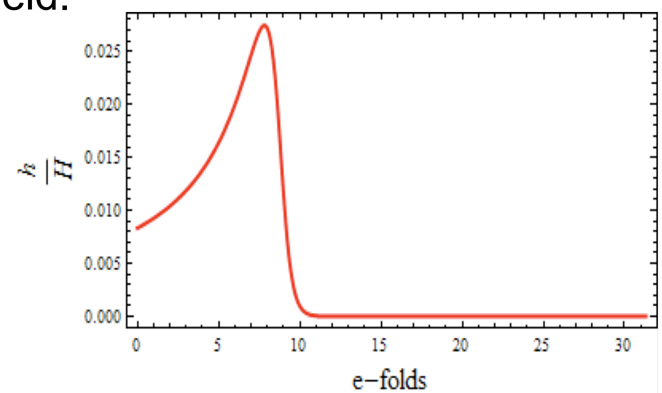
Isotropic expansion

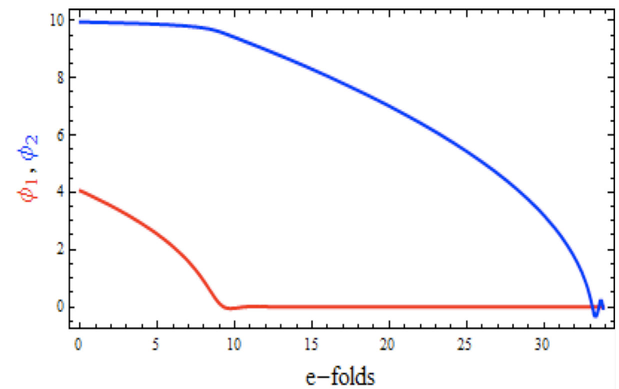
supports anisotropic expansion
due to vector field.

$$V_1(\phi_1) = m_1^2 \phi_1^2 / 2$$

$$V_2(\phi_2) = m_2^2 \phi_2^2 / 2$$

$$m_2 / m_1 < 1$$

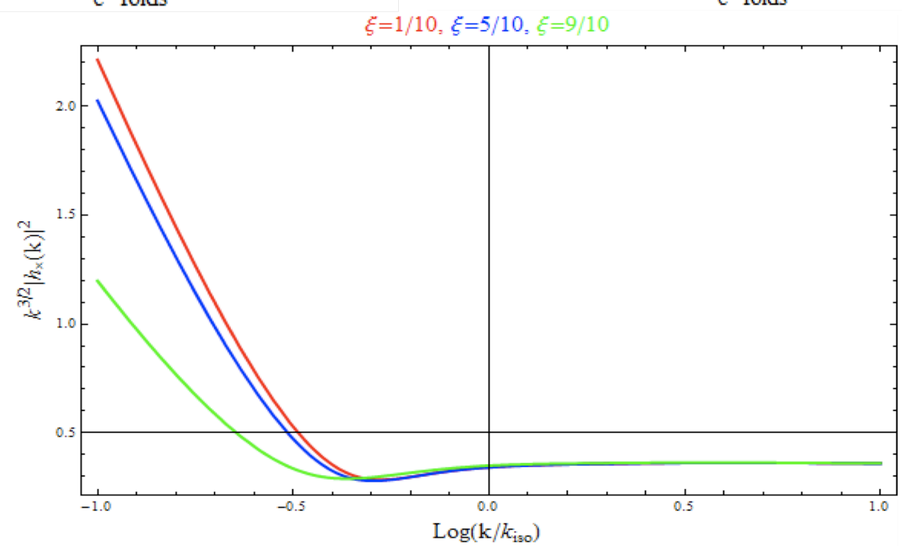




$$f(\phi_1) = \exp\left[\frac{\phi_1^2}{M_p^2}\right]$$

Power Spectrum for $h_\times$

$$k_x = |k|\, \xi$$



Conclusions

- Some evidence of broken statistical isotropy

- Full computations in simplest non FRW
  scalar-tensor coupling; $P_+ \neq P_\times$; Nondiagonal $C_{\ell\ell' mm'}$
  Easy to extend further

- Problems with specific realizations