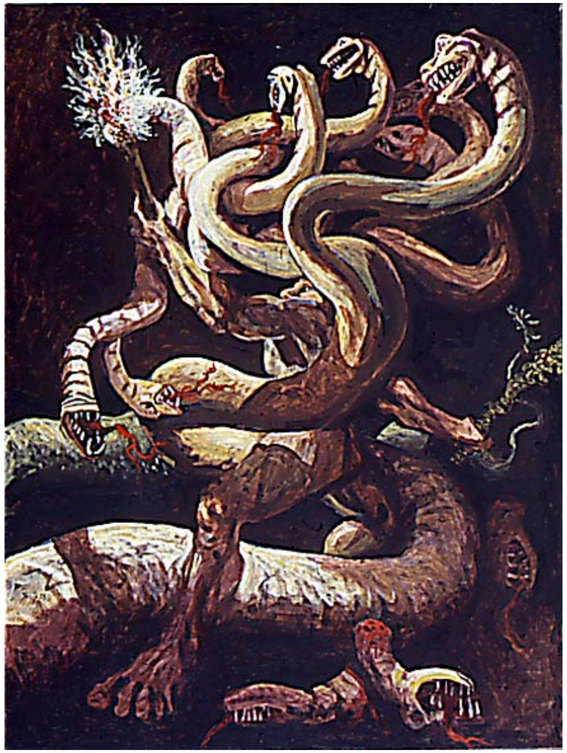
A Parton Shower built for Matching with QCD Matrix Elements



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MC4BSM 09
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The problem of matching of parton showers to QCD matrix elements is a very important one for LHC physics.

It is not possible to obtain reliable predictions for BSM background processes such as W + jets and tt + jets from parton shower Monte Carlos alone.

Peter Skands has introduced the idea of matching in some detail. In this talk, I will describe some progress in my approach to this subject. So far, my method gives a working code only for the simplest case, the final-state shower in $h^0 \to ng$.

There are two basic approaches to matching:

Additive:

Use matrix elements in a particular (hard scattering) region of phase space; use parton showers in the rest of phase space.

e.g. method of CKKW (Catani-Krauss-Kuhn-Webber):
use matrix elements when partons momentum transfers are greater than Q_0^2; use parton showers when the momentum transfers are less.

Multiplicative:

Use partons showers in all of phase space, but reweight events to take matrix elements into account.

All of the codes actually used now by experimenters to analyze data are of the additive type:

W, Z, t + jets, with matrix elements up to 4-jet emission

ALPGEN, MADEVENT, SHERPA, HELAC

(see the talk of F. Maltoni)

correction of parton showers to incorporate exact 1-loop calculations

MC@NLO   Frixione, Webber, Nason
POWHEG   Frixione, Nason, Oleari

Any codes matched to PYTHIA or HERWIG are necessarily additive, because these showers do not cover all of phase space.

For concreteness, concentrate on the simplest case:

the shower in $h^0 \to ng$

Begin by writing -- using $h^0 \to ng$ QCD tree amplitudes only:

$$\mathrm{Prob}(n) = \int d\Pi_n |\mathcal{M}(h \to ng)|^2$$

After the 2 gluon term, all contributions to the sum are infinite.

This is corrected by inclusion of loop amplitudes. These combine with tree amplitudes to cancel infrared divergences. The finite terms left over give $\mathcal{O}(\alpha_s)$ corrections (e.g. 'K-factors').

However, QCD loop amplitudes are difficult to compute and, in a Monte Carlo, expensive to evaluate.

A parton shower deals with this in the following way:

Let t be an ordering variable among the parton emissions, e.g. $t = \log(m_h^2/s_{ij}),\ \ s_{ij} = (k_i + k_j)^2$ ('virtuality ordering'). Each emission is assigned a definite value of t.

Then let $S(t_n, t_{n+1}) = \int_{t_n}^{t_{n+1}} d\Pi\ |m(\to g)|^2$

for one emission. This is the Sudakov integral. The probability that there is no emission between t_n and t_{n+1} is

$$\exp[-S(t_n, t_{n+1})]$$

Including these probabilities or Sudakov factors, the total probability of a Higgs decay becomes

$$\mathrm{Prob}(n) = \int d\Pi_n |\mathcal{M}(h \to ng)|^2 \prod_i e^{-S(t_i, t_{i+1})}$$

With appropriate choice of $S(t_i, t_{i+1})$, $\sum_n \mathrm{Prob}(n) = 1$

This method incorporates the most important effects of loop diagrams, though it does not capture the K-factors and other finite radiative corrections.

In a parton shower, the full emission amplitude is taken to factorize by stages. At each stage, one takes the emission amplitude to be the Altarelli-Parisi splitting function. This is correct in the collinear limit (only).

My goal here is to apply a formula

$$\mathrm{Prob}(n) = \int d\Pi_n |\mathcal{M}(h \to ng)|^2 \prod_i e^{-S(t_i, t_{i+1})}$$

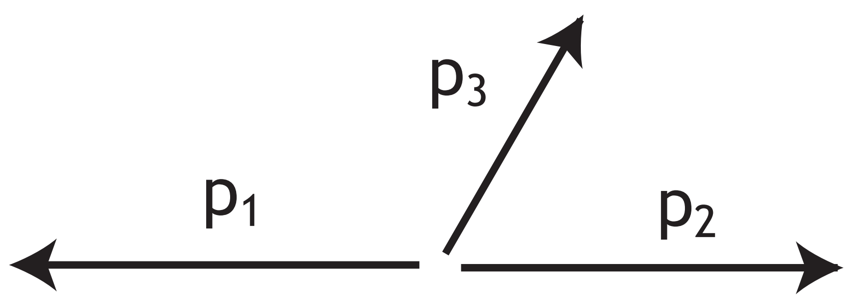
where $\mathcal{M}(h \to ng)$ is the exact QCD tree amplitude to as high a level as my computer has the strength to compute it.

[Caution: Here, 'exact' = leading order in N_c only.]

Bauer, Tackmann, and Thaler have emphasized that, to use the multiplicative method, it is necessary for the parton shower to exactly cover phase space. Here is my solution to this problem. To be most effective, I should preferentially generate points in phase space in the soft and collinear regions.

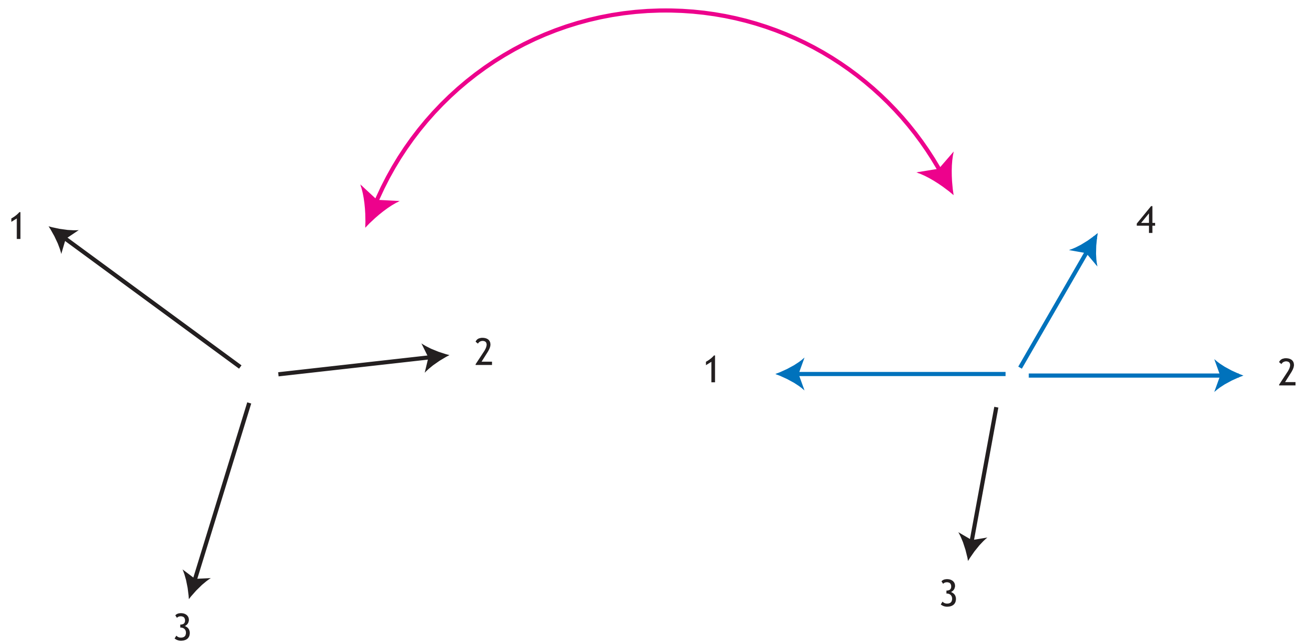
An effective trick has been introduced by Draggiotis, van Hameren, and Kleiss as the basis of their SARGE algorithm

Start with two back-to-back lightlike vectors.   Add a third lightlike vector $p_3 = \xi_1 p_1 + \xi_2 p_2 + p_\perp$



Then boost and rescale to the original CM frame and energy.

To add the fourth vector, pick two neighbors, boost these back-to-back, add a vector as before, and then boost the entire system back to the CM frame.



Effectively, the entire event recoils when a new vector is added.

The logarthmic integral over the parameters reproduces massless phase space

$$\int \frac{d^3p_3}{(2\pi)^2 2p_3}\,\frac{2p_1\cdot p_2}{2p_1\cdot p_3\,2p_3\cdot p_2} = \frac{1}{(4\pi)^2}\int\frac{d\xi_1}{\xi_1}\int\frac{d\xi_2}{\xi_2}\int\frac{d\phi}{2\pi}$$

Applying this operation repeatedly, we build up phase space with all of the QCD denominators for emission of final-state radiation that are found in the exact, leading- N_c amplitudes.

$$\int d\Pi_n \frac{1}{2p_1\cdot p_2\,2p_2\cdot p_3\,\cdots 2p_n\cdot p_1}$$
$$= \frac{1}{8\pi Q^4}\prod_i\left[\frac{1}{(4\pi)^2}\int\frac{d\xi_{1i}}{\xi_{1i}}\int\frac{d\xi_{2i}}{\xi_{2i}}\int\frac{d\phi}{2\pi}\right]$$

This is an exact formula for massless phase space with QCD denominators, but only if we integrate over every point in phase space exactly once.

Draggiotis, van Hameren, and Kleiss suggested adding the vectors 1, 2, 3 in fixed (color) order. This requires very large values for the ξ_i to reproduce some phase space configurations.

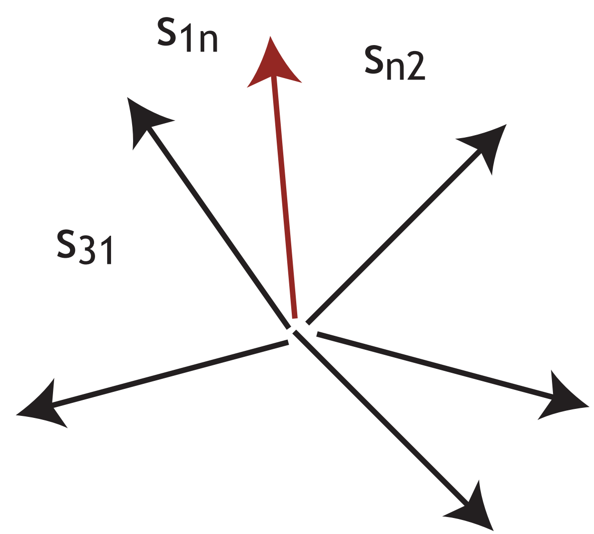
An alternative approach is to choose arbitrarily at each step one interval in which to insert a new vector. We call the set of such choices a chamber. It is then necessary to define the limits of each chamber so that the full set of chambers tiles phase space.

Here is a useful definition of a chamber:

Let the nth vector be inserted between 1 and 2. Then allow all values of ξ_1, ξ_2, ϕ such that

s_{1n} is the smallest invariant mass of two neighbors,
and $s_{n2} < s_{13}$

Reversing the inequality defines a second chamber in which n is radiated on the left side of 1.

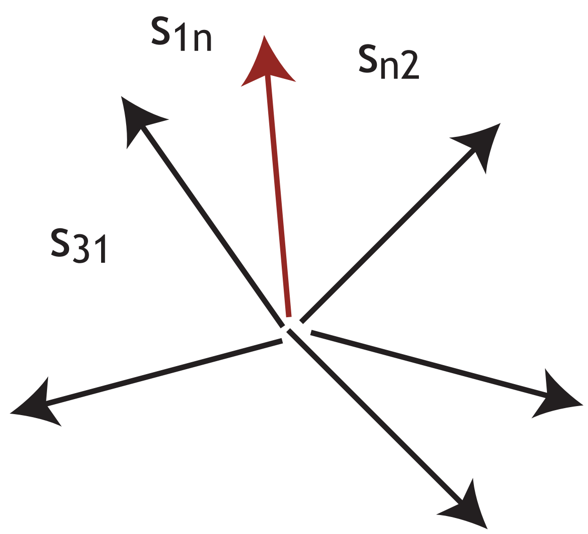


These prescriptions put reasonable upper limits on the ξ_{1j} integrals.

The ordering of virtualities s_{ij} is similar to the ordering in a parton shower. In fact, we can identify s_{ij} with the evolution variable of a parton shower.

Here is the proof that this method tiles phase space:

Just go backward. For an n-gluon configuration,

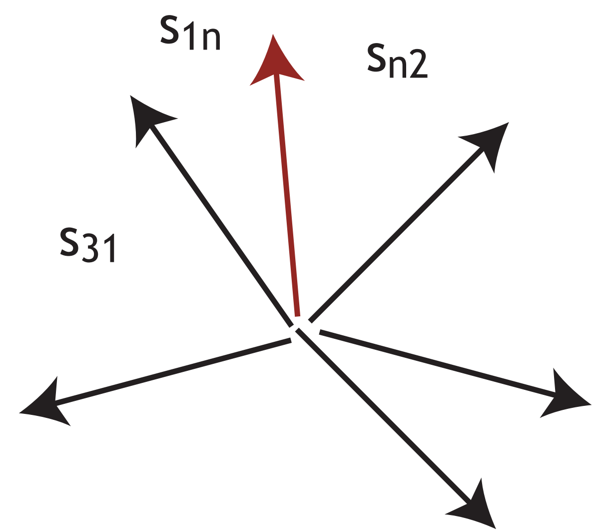


pick the smallest $s_{i,i+1}$ to be the emission chamber, and choose the smaller of the $s_{i,i+1}$ on the two sides to complete the dipole. Proceding in this way, each point in phase space gives a unique path back to the 2-gluon state.

We can look at the emission in the chamber

between 1 and 2, on the side of 1

as an emission from the gluon 1
in the antenna (in the sense of Skands, Weinzierl, et al.) of gluons 1 and 2.



At each stage in the shower, I choose an antenna and an emission side at random.

The correspondence to Altarelli-Parisi is

$$(1-z) = \frac{1}{(1+\xi_1+\xi_2)}$$

and

$$\int \frac{d\xi_2}{\xi_2} \int \frac{d\xi_1}{\xi_1} \approx \int \frac{dQ^2}{Q^2} \int \frac{dz}{z(1-z)}$$

So, using the SARGE meaure and choosing all weights = 1 corresponds to the formula

$$\mathrm{Prob}(n) = \int d\Pi_n |\mathcal{M}(h \to ng)|^2 \prod_i e^{-S(t_i, t_{i+1})}$$

with

$$|\mathcal{M}(h \to ng)|^2 = \frac{m_h^8}{s_{12} s_{23} \cdots s_{n1}}$$

This is very convenient, because it is an exact result in QCD that

$$\mathcal{M}(h \to g_1^+ g_2^+ \cdots g_n^+) = \frac{m_h^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$$

Dixon, Glover, Khoze

In addition, each antenna automatically has color-coherence it its emission

$$\int d\Pi \frac{1}{s_{12} s_{23}}$$

This is the more correct expression of the physics implemented in PYTHIA and HERWIG by angular ordering.

For a simple parton shower, I choose for the numerators of the splitting functions:

$$1\ ,\ \frac{1}{(1+\xi_1+\xi_2)^4}\ ,\ \frac{\xi_1^4}{(1+\xi_1+\xi_2)^4}\ ,\ \frac{\xi_2^4}{(1+\xi_1+\xi_2)^4}$$

for $+ \to (++,-+,+-,--)$ respectively. Note that, in the last of these functions, the numerator cancels the collinear singularity.

We could also use more complicated weights. In particular, the prescription

$$w = \frac{|\mathcal{M}(h\to ng)|^2/|\mathcal{M}(h\to (n-1)g)|^2}{(2p_1\cdot p_2)/(2p_1\cdot p_n)(2p_n\cdot p_2)}$$

reweights the emissions to the probabilities given by exact tree-level matrix elements. We can use this prescription as long as our computer has the strength to compute the matrix elements.

To generate QCD tree amplitudes, I use the Britto-Cachazo-Feng recursion formula for on-shell, color-ordered amplitudes:

$$
\begin{aligned}
i\mathcal{M}(1\cdots n) \quad &= \quad \sum_{splits} i\mathcal{M}(b+1\,\cdots\,\hat{i}\,\cdots\,a-1\ -\hat{Q}) \\
&\qquad\qquad \cdot\frac{1}{s_{a\cdots b}}\cdot i\mathcal{M}(a\,\cdots\,\hat{j}\,\cdots b\,\hat{Q})
\end{aligned}
$$

The BCF formula recursively breaks amplitudes down (numerically, on the fly) to the simpler exact results for $h^0\to 2g, 3g$, and $h^0\to$ all + or all - gluons.

Now look at some results from the simulation:

All results refer to a Higgs of mass 1000 GeV, showered to an infrared cutoff scale of 2 GeV. Since we are doing shower physics, not Higgs physics, I use the effective interaction

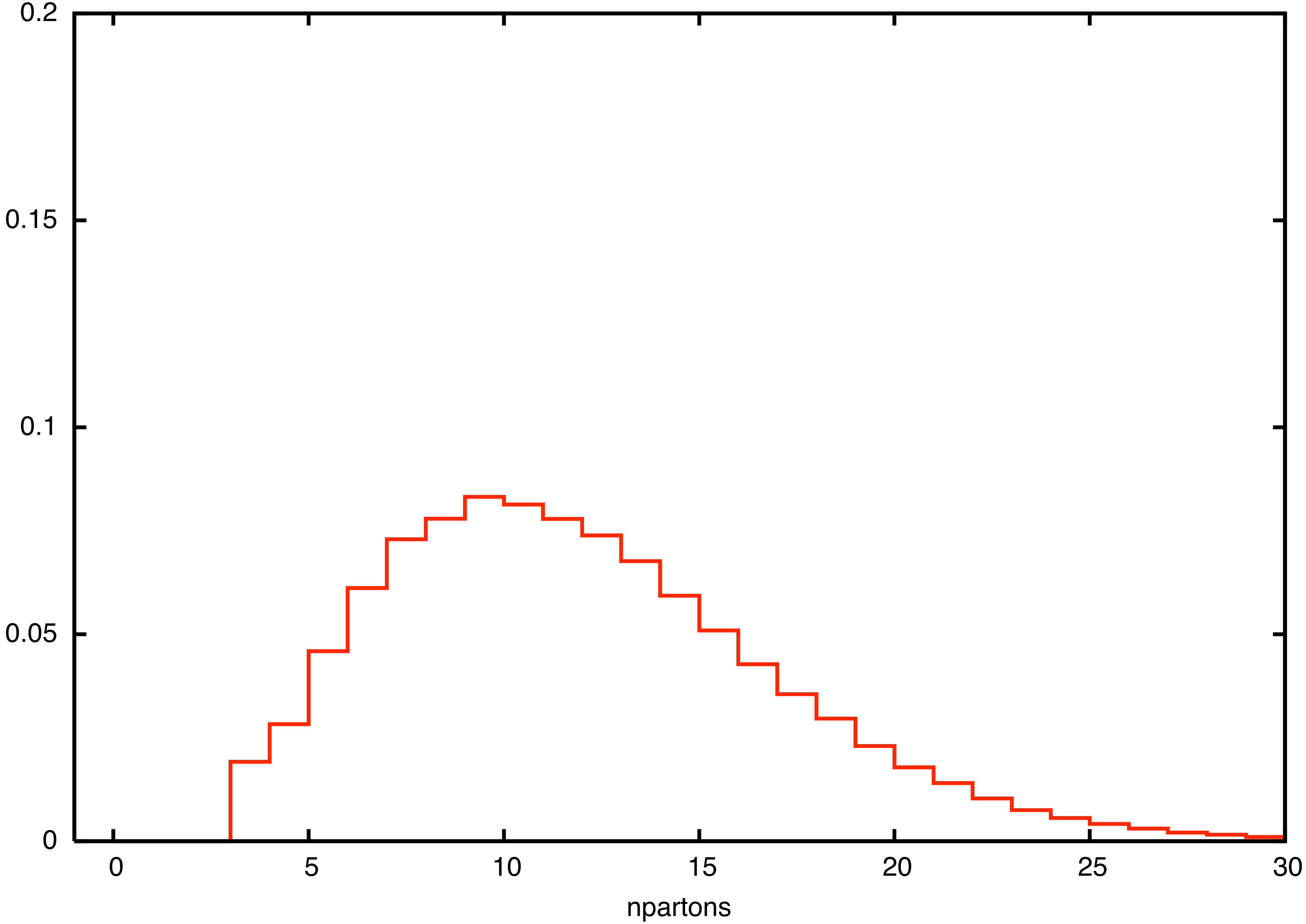
$$\delta\mathcal{L} = \frac{\alpha_s}{12\pi v} h\, F_{\mu\nu}F^{\mu\nu}$$

without apology.

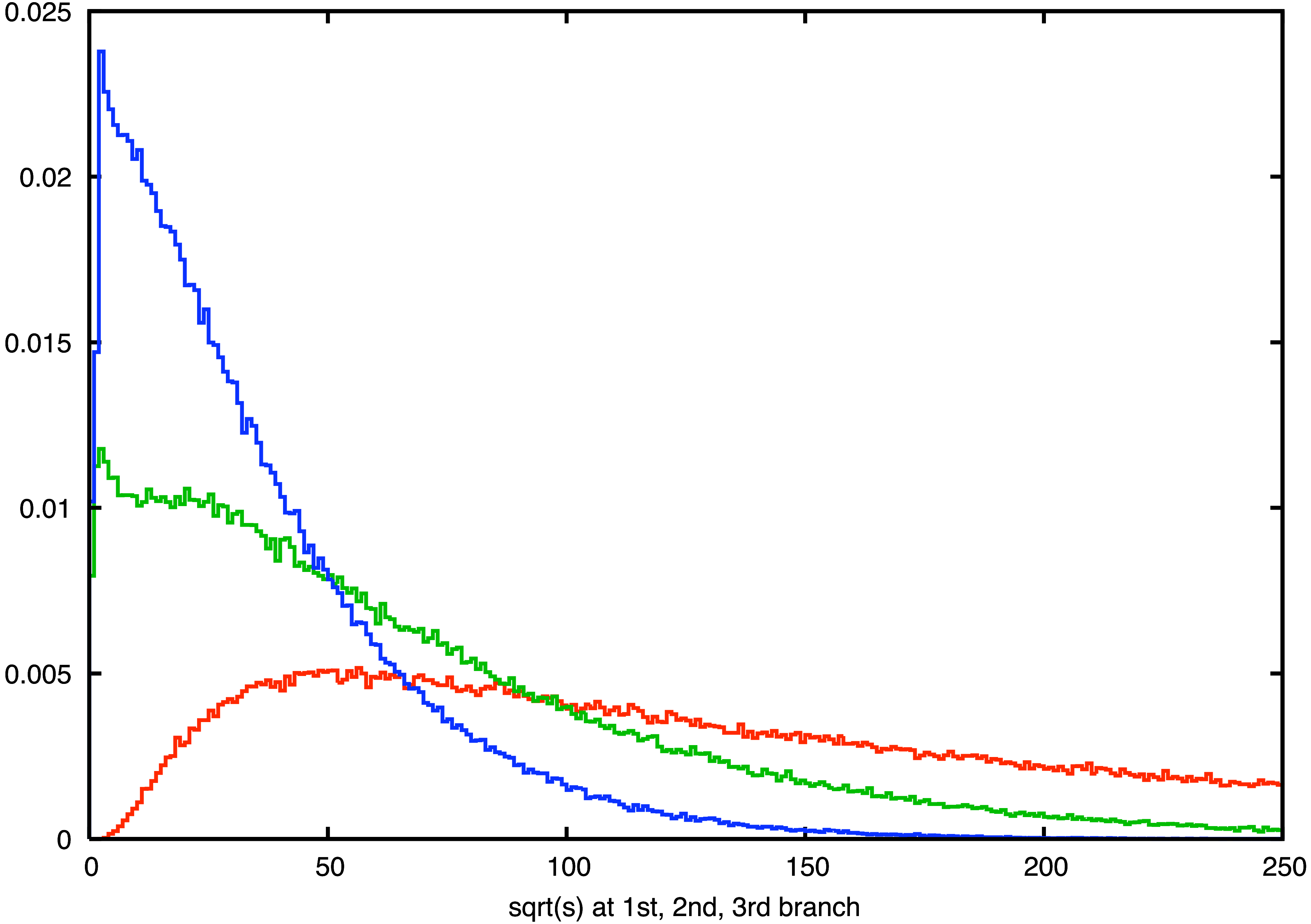
First, the simple shower without matching. This runs at

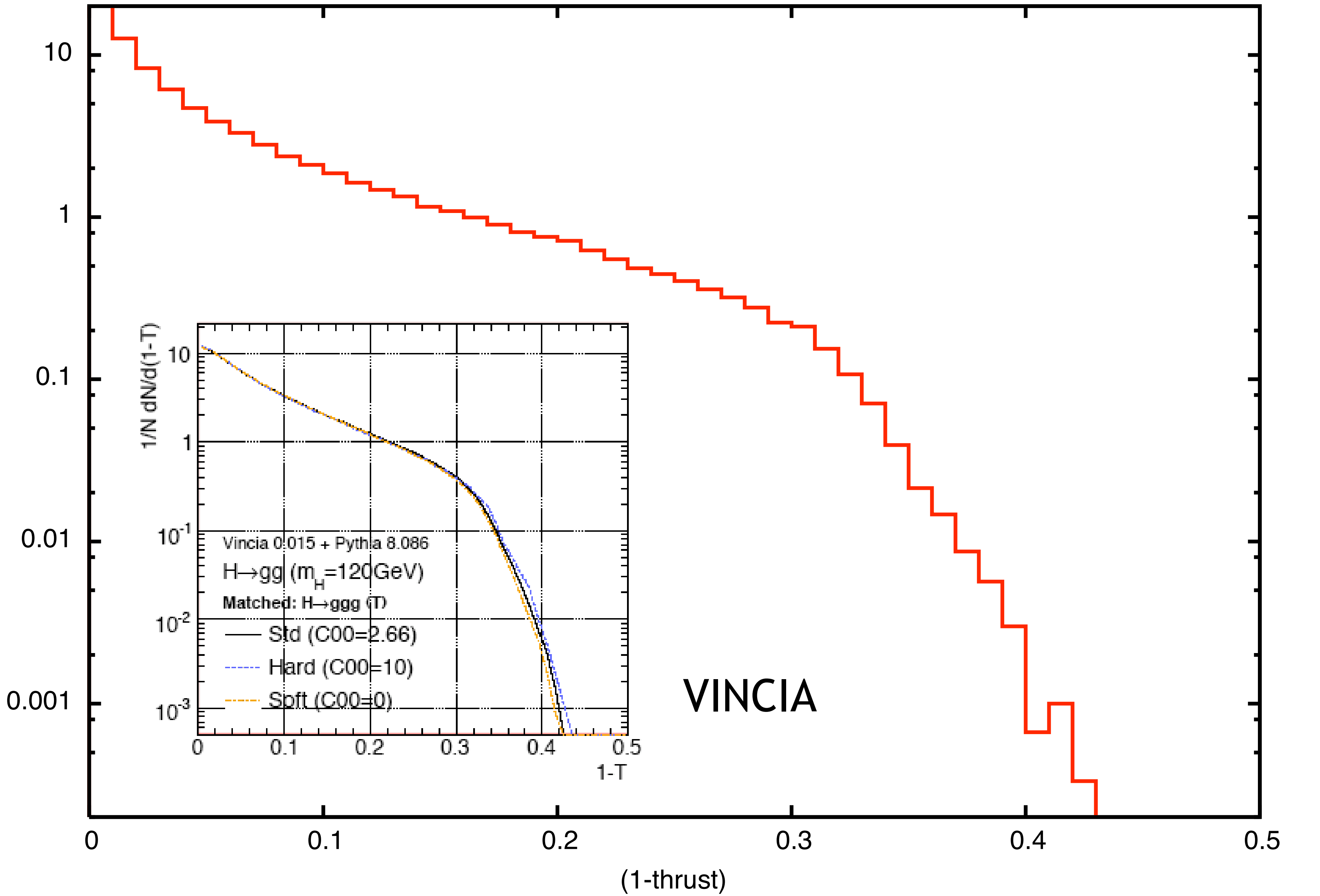
4 events / msec

on my MacBook.

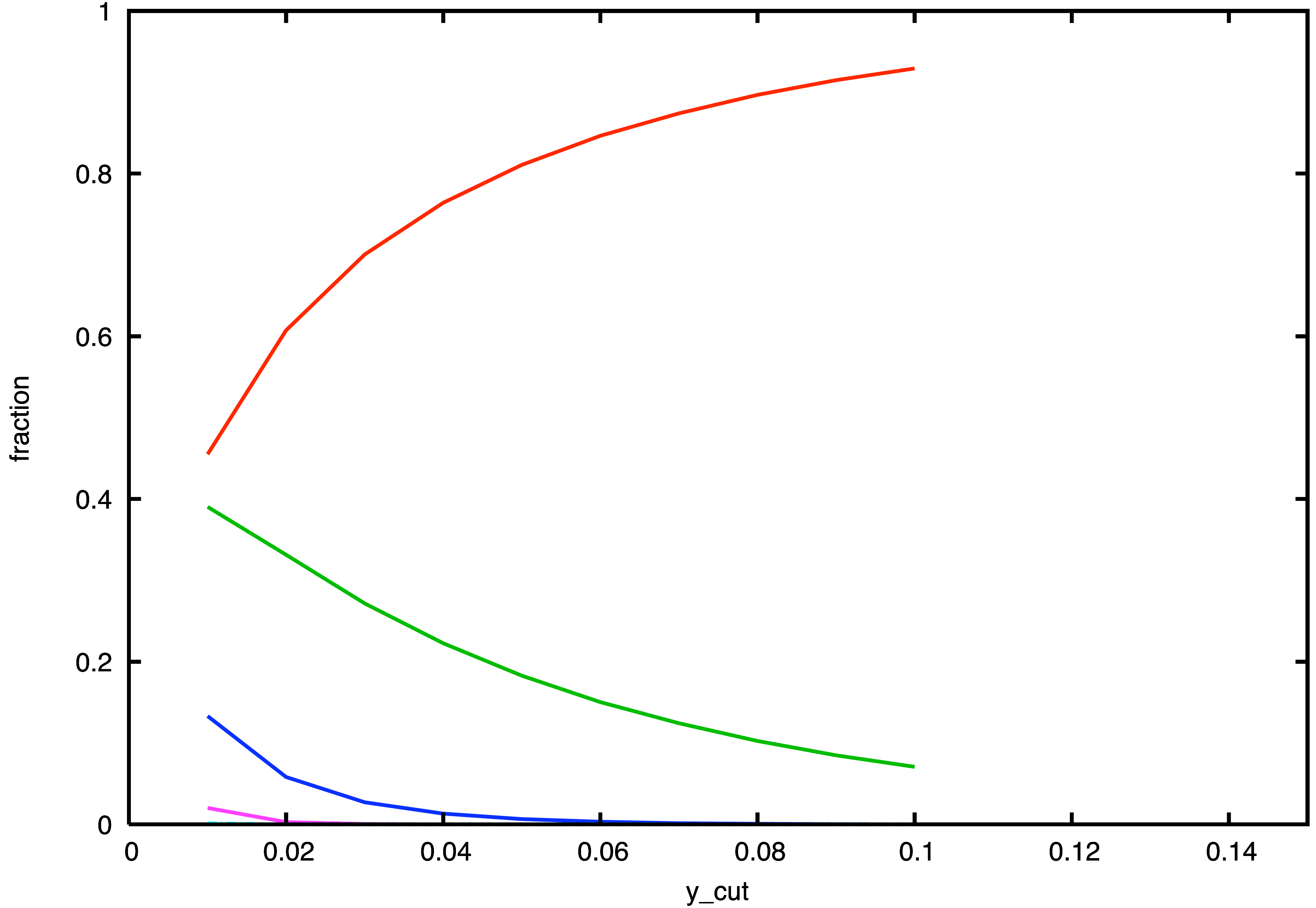


0.2
0.15
0.1
0.05
0
0
5
10
15
20
25
30
npartons

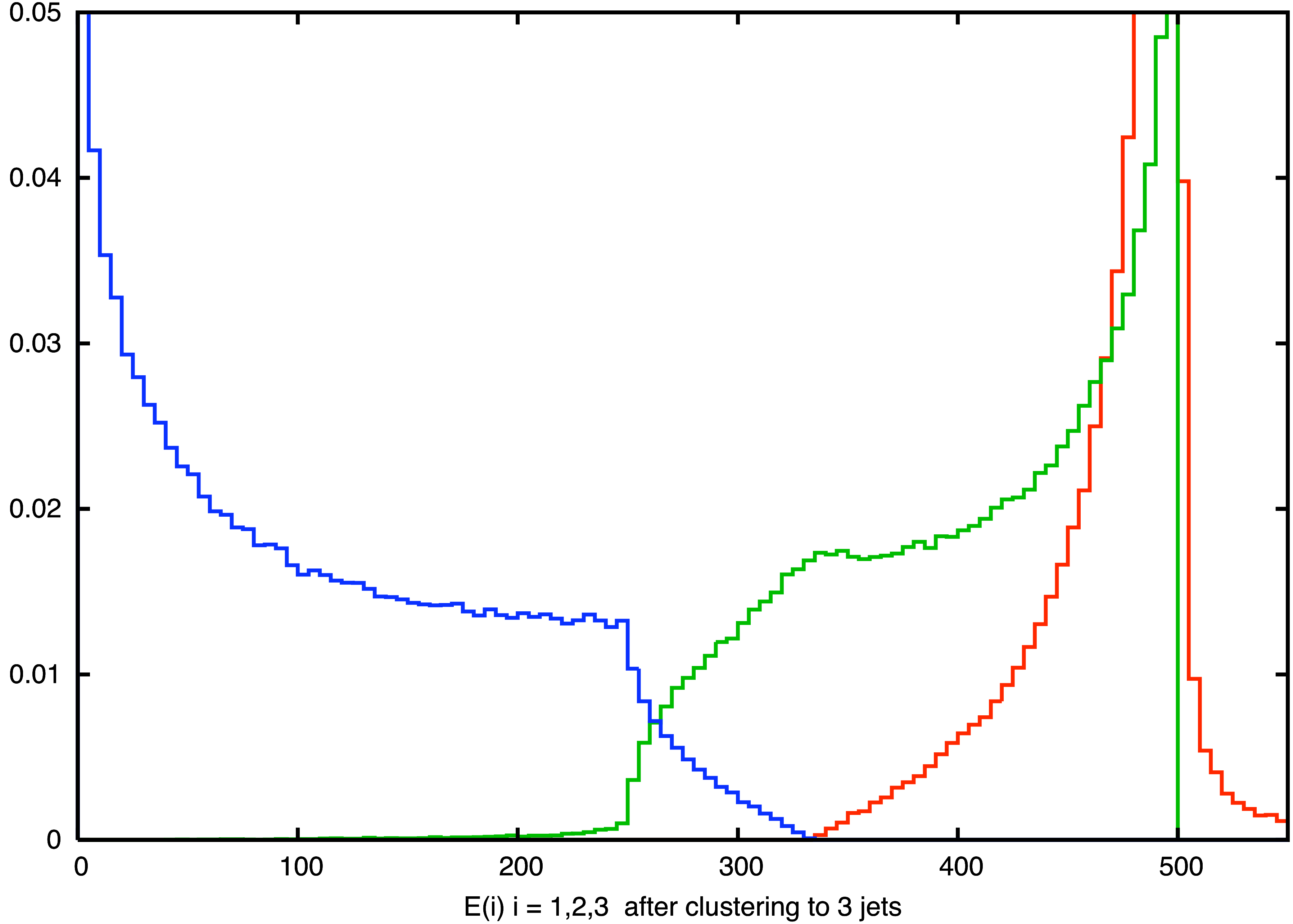

0.025
0.02
0.015
0.01
0.005
0
0
50
100
150
200
250
sqrt(s) at 1st, 2nd, 3rd branch

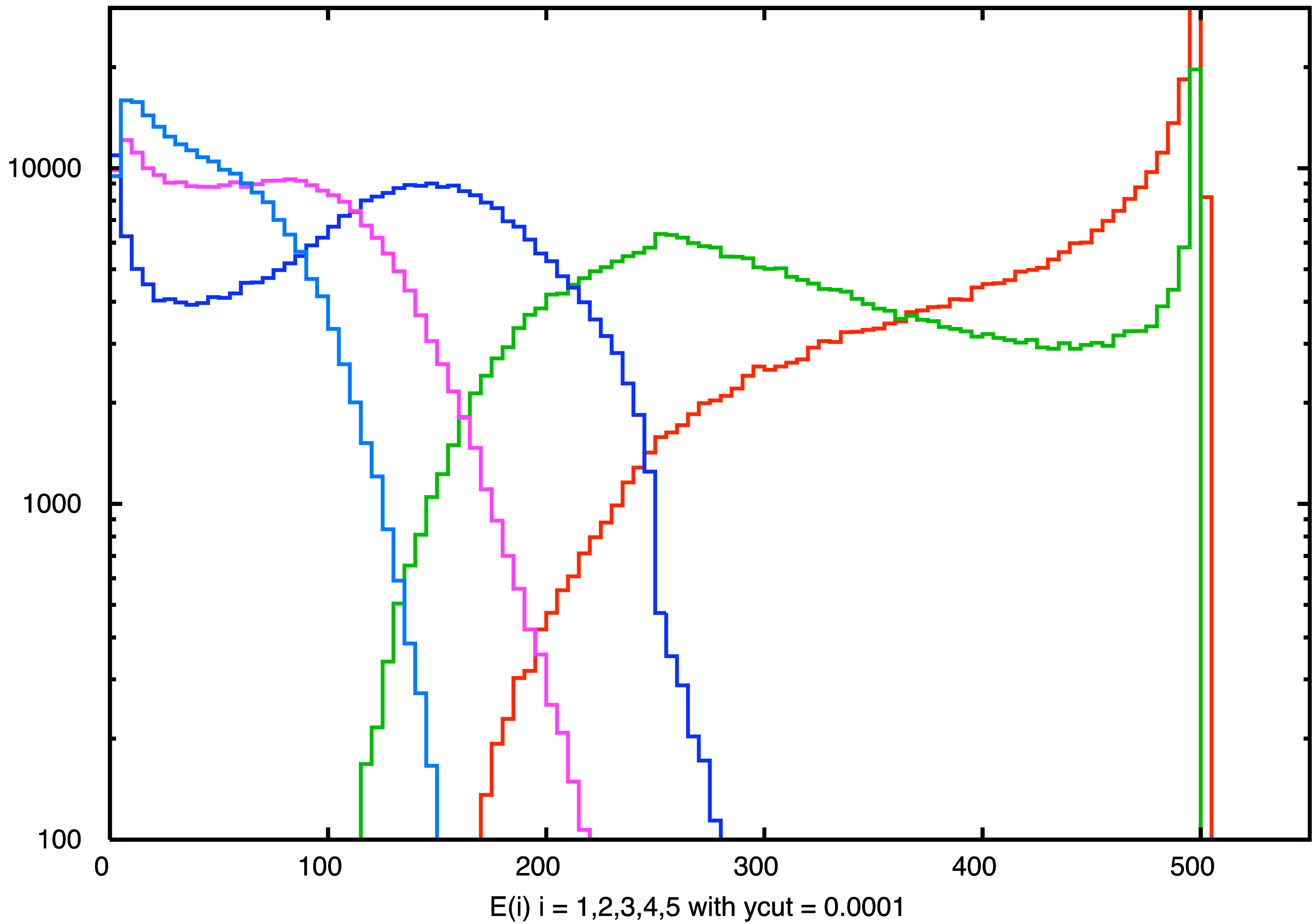


10
1
0.1
0.01
0.001
0
0.1
0.2
0.3
0.4
0.5
(1-thrust)
VINCIA
1/N dN/d(1-T)
Vincia 0.015 + Pythia 8.086
H→gg (m_H=120GeV)
Matched: H→ggg (T)
Std (C00=2.66)
Hard (C00=10)
Soft (C00=0)
1-T



1
0.8
0.6
0.4
0.2
0
fraction
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
y_cut

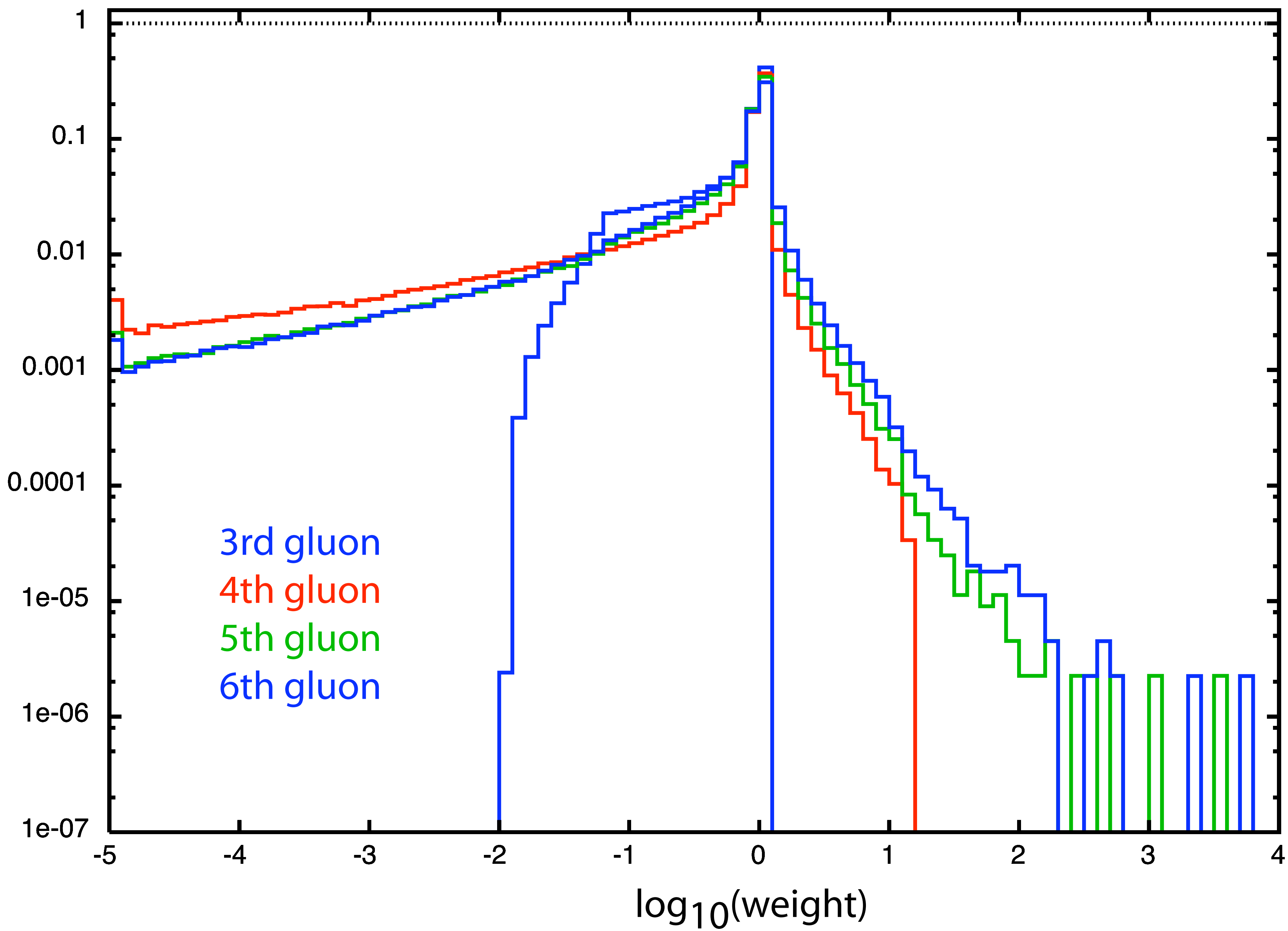

0.05
0.04
0.03
0.02
0.01
0
0
100
200
300
400
500
E(i) i = 1,2,3 after clustering to 3 jets


10000
1000
100
0
100
200
300
400
500
E(i) i = 1,2,3,4,5 with ycut = 0.0001

Now add matching to matrix elements.

There is a small problem here. For the PYTHIA rejection algorithm to work properly, we must choose $g(t)$ to bound all possible weights. But, large weights can appear !

One large weight W means that typical branches are selected with probability $1/W$. This dramatically slows the process.



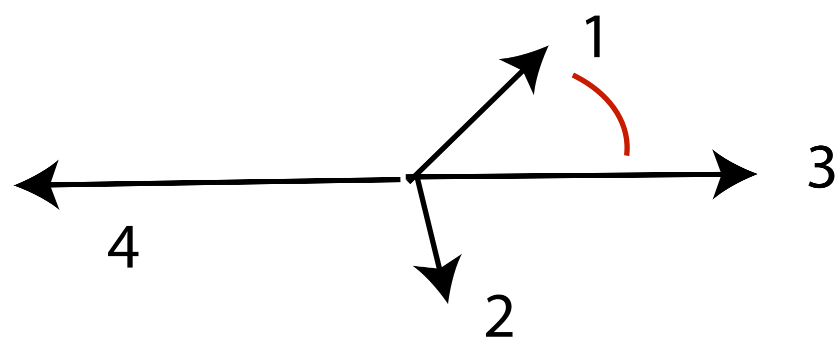
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
-5
-4
-3
-2
-1
0
1
2
3
4
$\log_{10}$(weight)
3rd gluon
4th gluon
5th gluon
6th gluon

In the formula

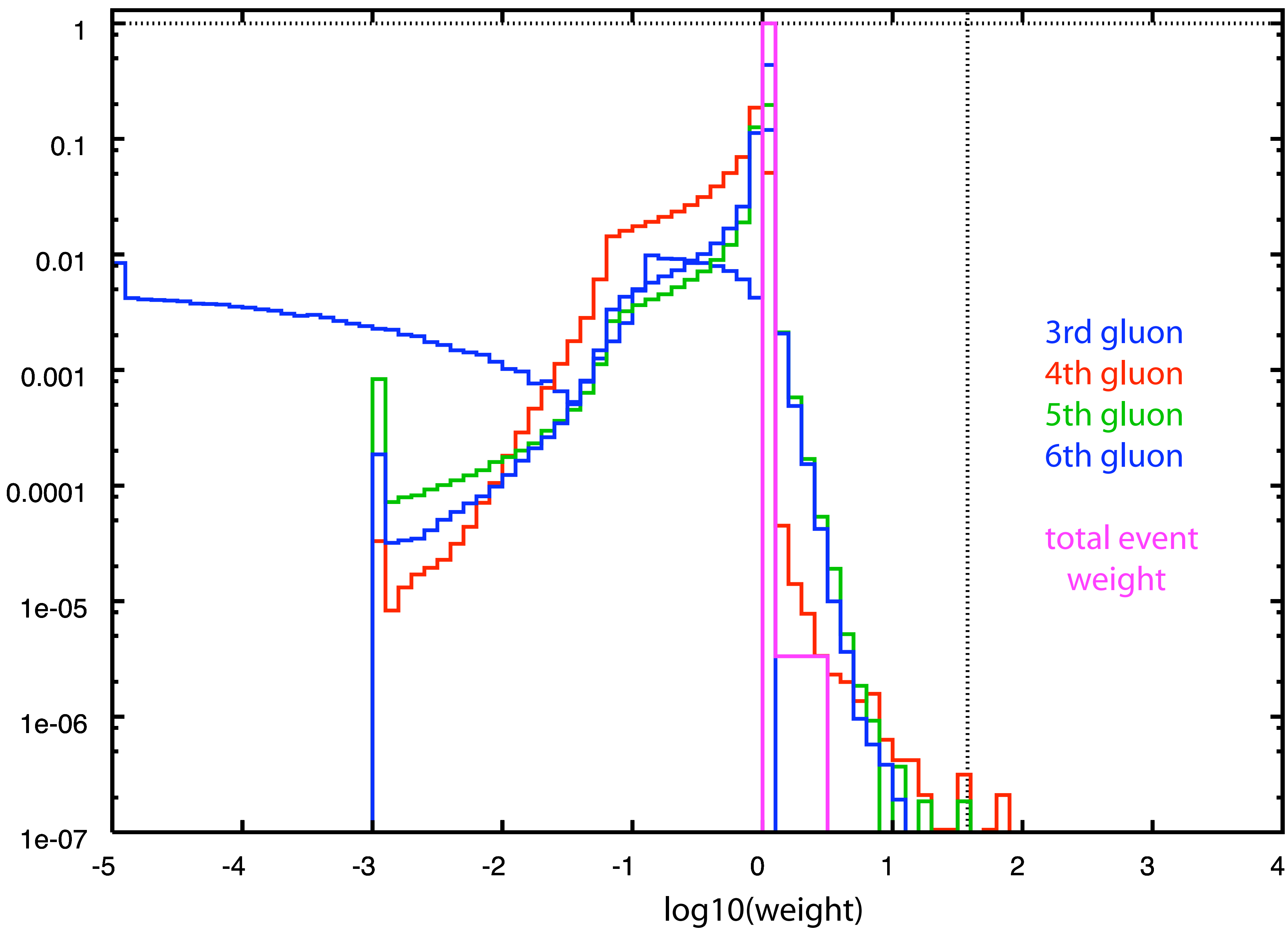
$$\mathcal{M} = \frac{A m_h^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

$|A|^2$ never gets bigger than about 1.5. However, some A's are very small, and, in a series of emissions, a large A can follow a small A. This leads to large weights $|A_n/A_{n-1}|^2$.

The problem occurs because the chamber prescription above sometimes emits a high-energy gluon between two lower-energy gluons.



A better prescription is: $z_1 < z_3$ or $(s_{12} + s_{14}) < (s_{32} + s_{34})$
This also tiles phase space precisely.



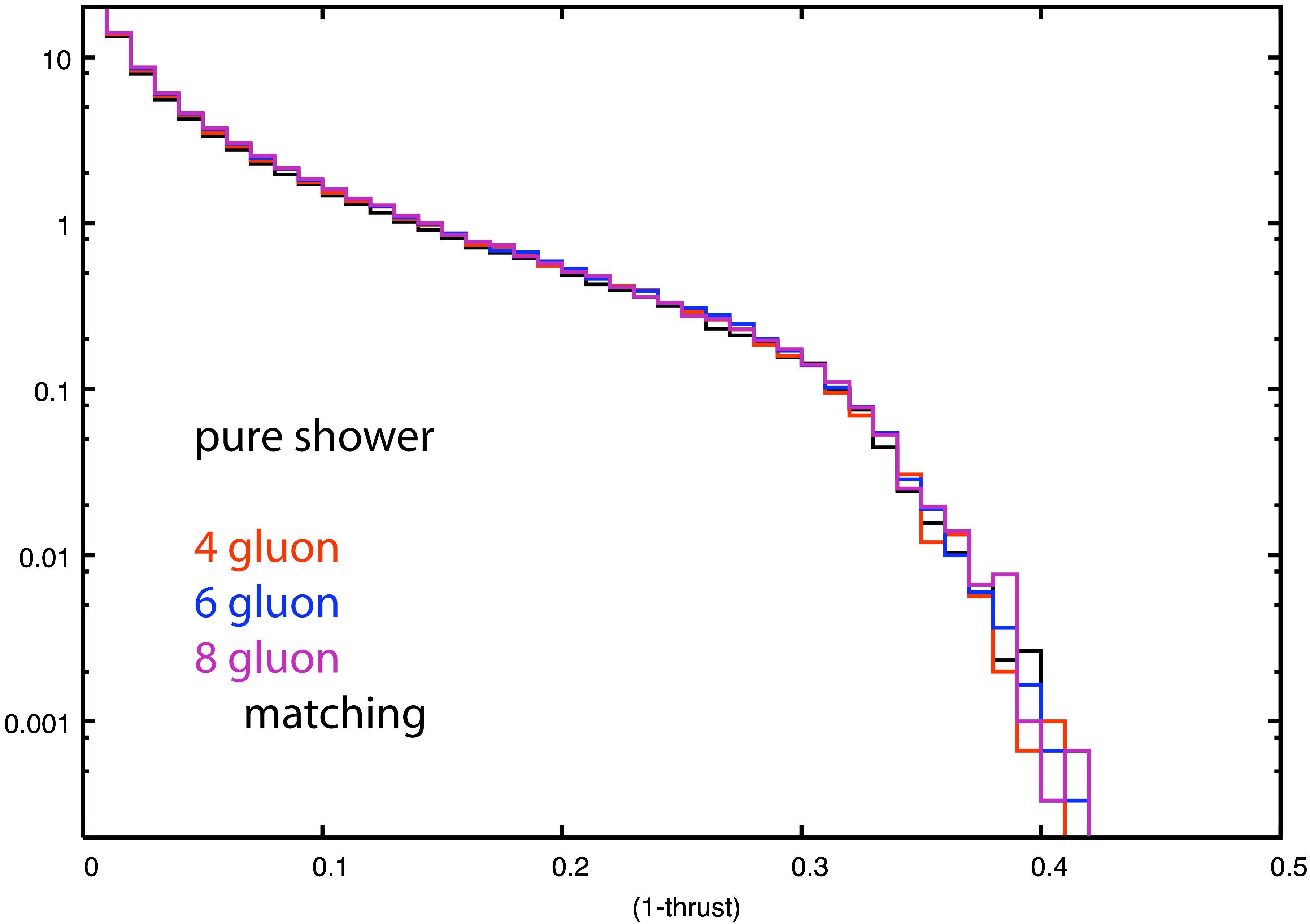
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
-5
-4
-3
-2
-1
0
1
2
3
4
log10(weight)
3rd gluon
4th gluon
5th gluon
6th gluon
total event weight

With this change, here is the speed of event generation (msec/event)

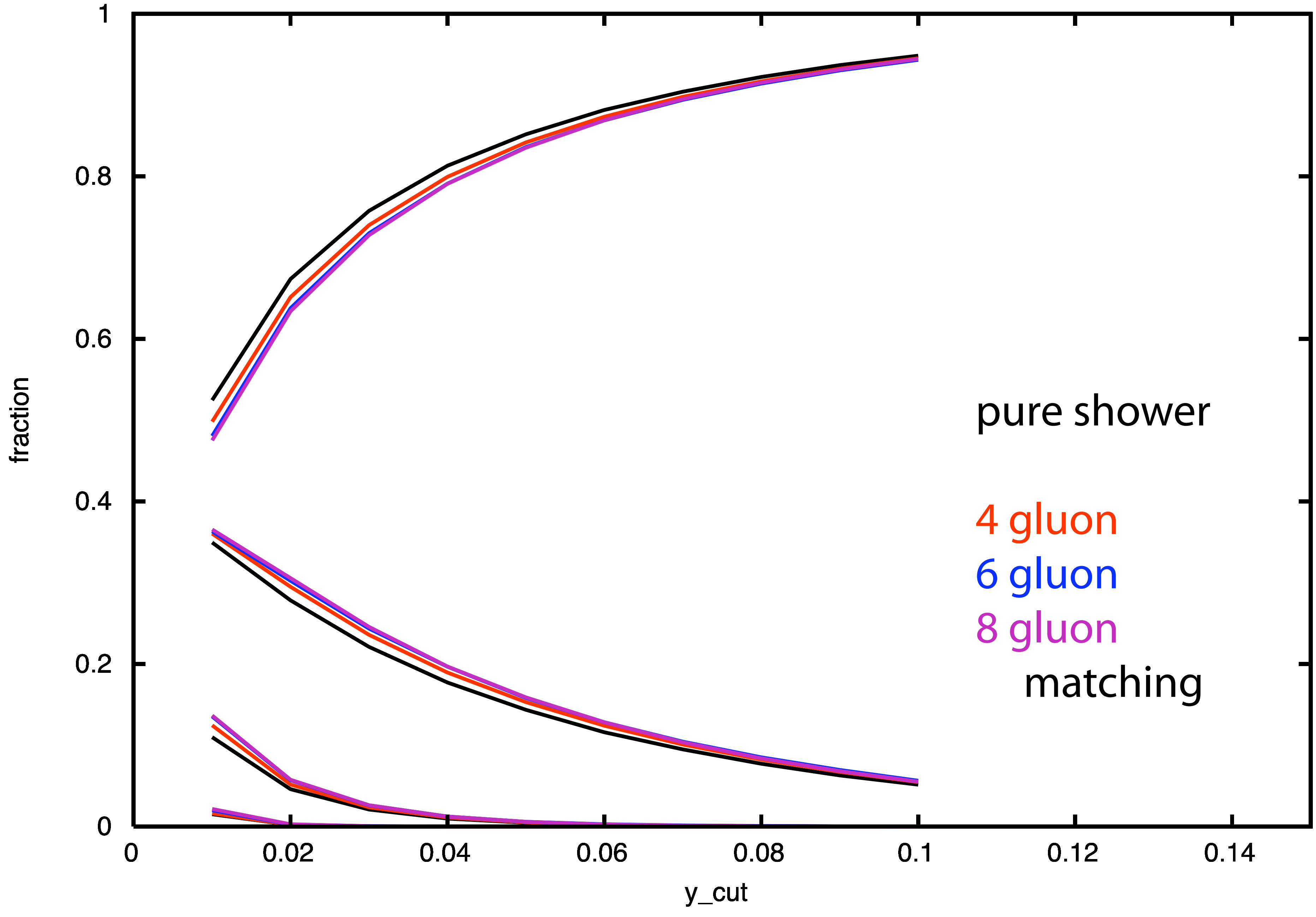
pure shower: 0.42

matching to	4 gluons	6 gluons	8 gluons
	2.1	6.4	78.

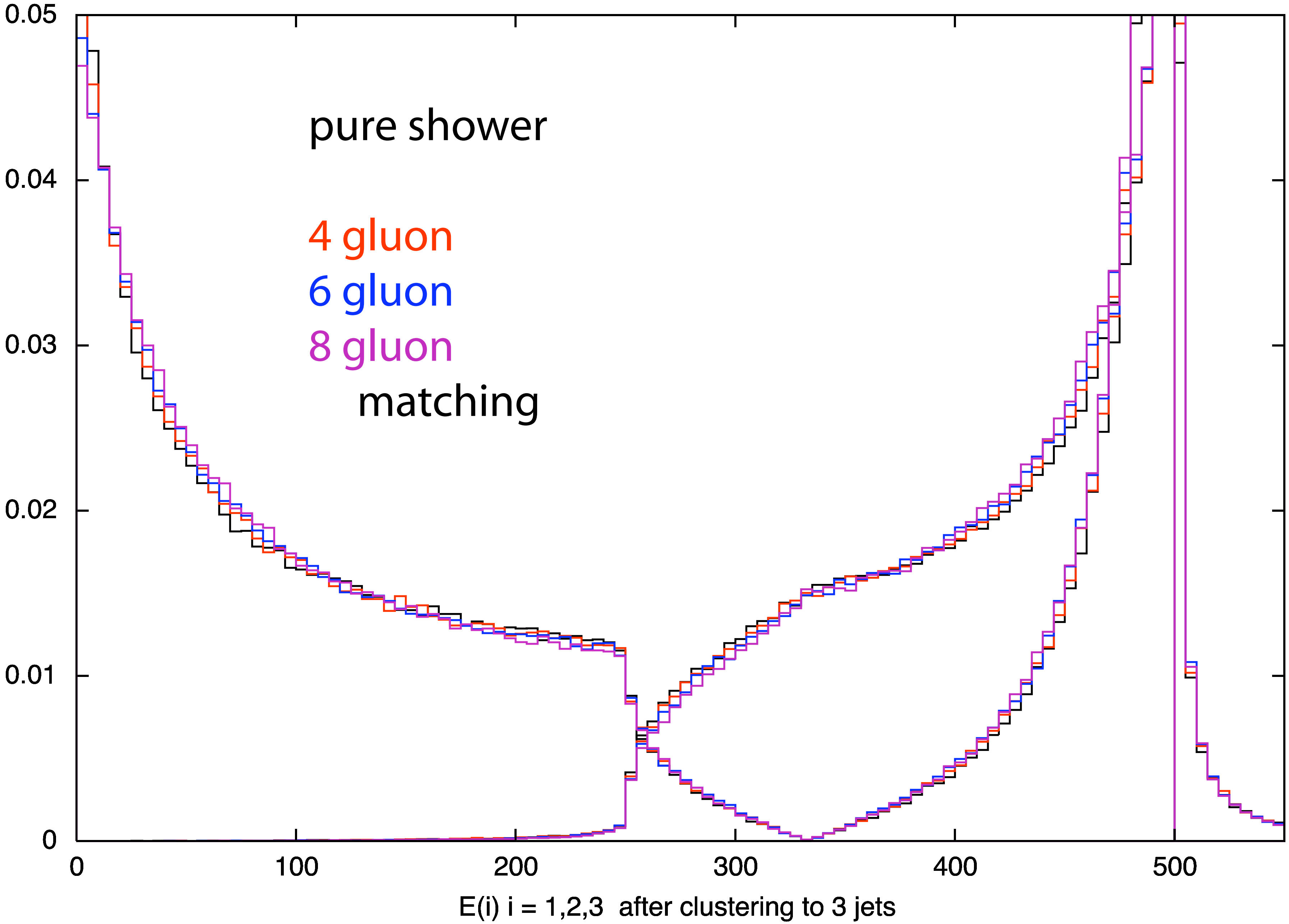
Here are some results of these simulations.



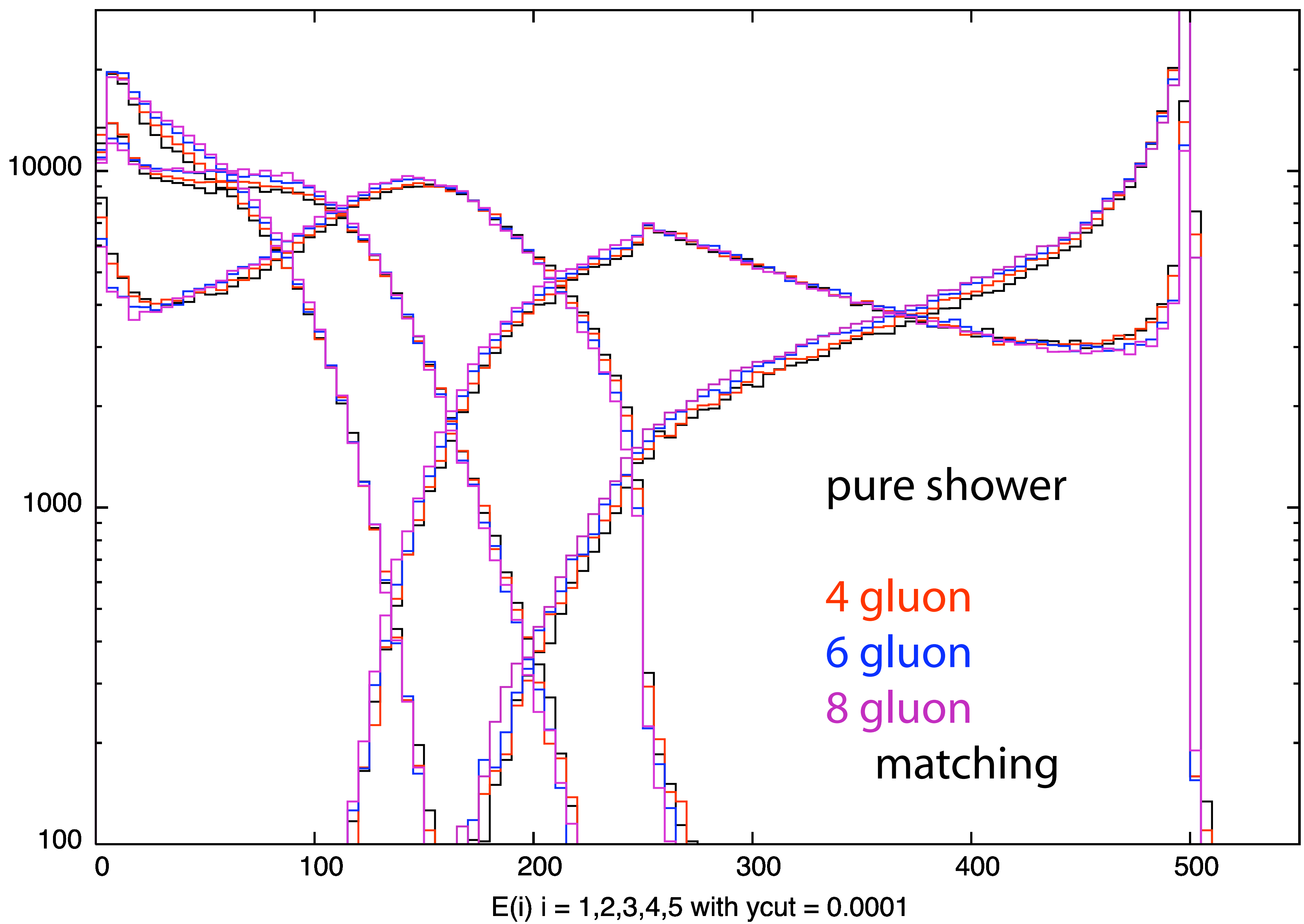
10
1
0.1
0.01
0.001
0
0.1
0.2
0.3
0.4
0.5
(1-thrust)
pure shower
4 gluon
6 gluon
8 gluon
matching

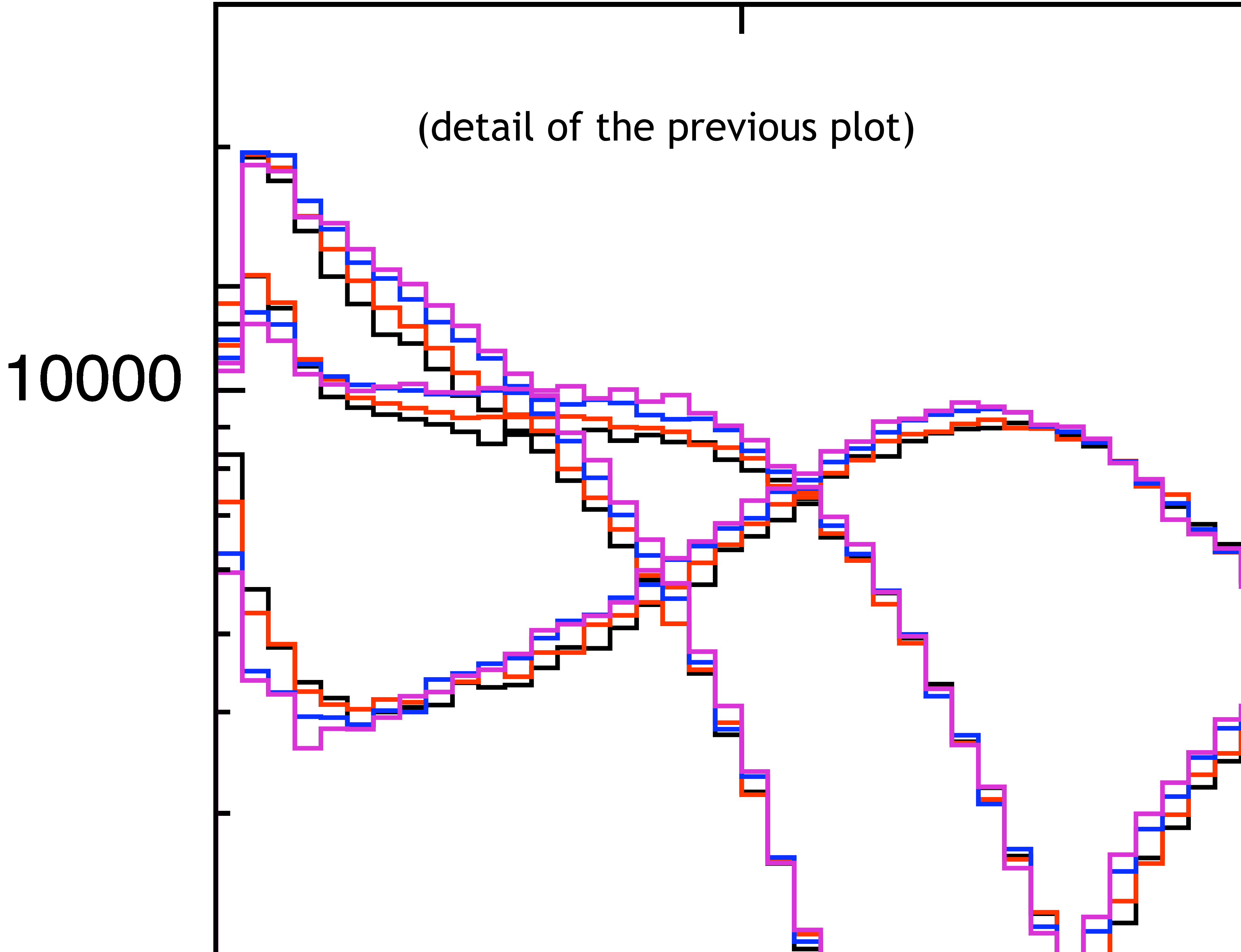


fraction
1
0.8
0.6
0.4
0.2
0
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
y_cut
pure shower
4 gluon
6 gluon
8 gluon
matching



0.05
0.04
0.03
0.02
0.01
0
0
100
200
300
400
500
pure shower
4 gluon
6 gluon
8 gluon
matching
E(i) i = 1,2,3 after clustering to 3 jets


10000
1000
100
0
100
200
300
400
500
pure shower
4 gluon
6 gluon
8 gluon
matching
E(i) i = 1,2,3,4,5 with ycut = 0.0001



(detail of the previous plot)
10000

Conclusions:

This is a proof of principle for a new way to incorporate exact matrix elements into a parton shower. Only the simplest situation has been implemented so far.

The method generalizes to processes with massive particles in the final state and to processes with initial state radiation. However, these generators are not yet running (so you should still be skeptical).

Still, there is promise that this method might develop into an interesting tool for modeling multijet QCD processes.