Mass and spin measurements in missing energy events at hadron colliders

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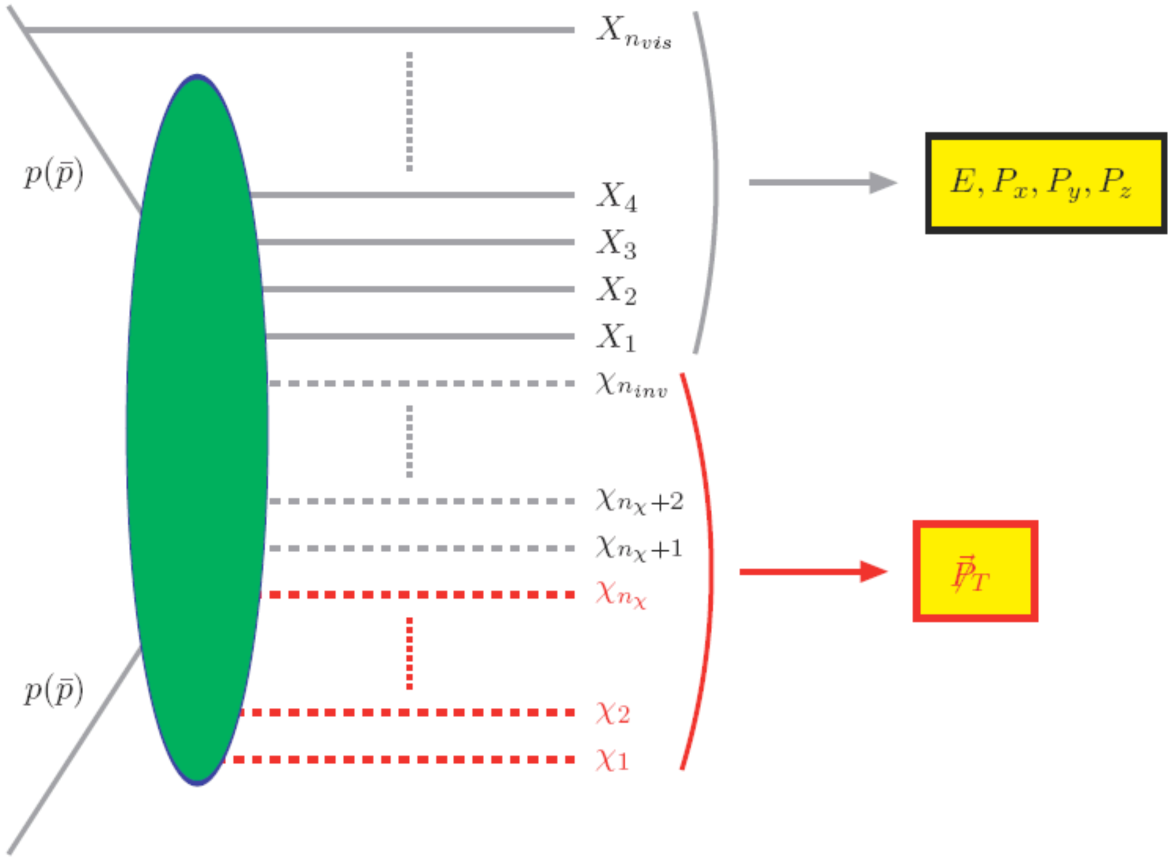
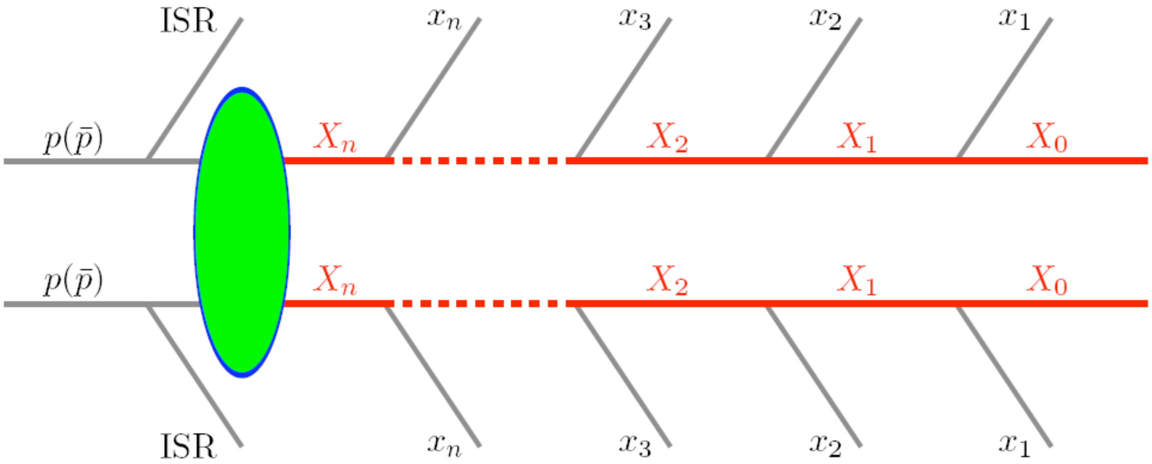
MET events: experimentalist's view



- What is going on here?

MET events: theorist's view



- Pair production of new particles (conserved R, KK, T parity)
- Motivated by dark matter + SUSY, UED, LHT
  - How do you tell the difference? (Cheng, KM, Schmaltz 2002)
- SM particles x_i seen in the detector, originate from two chains
  - How well can I identify the two chains? Should I even try?
    - What about ISR jets versus jets from particle decays?
- "WIMPs" X_0 are invisible, momenta unknown, except p_T sum
  - How well can I reconstruct the WIMP momenta? Should I even try?
    - What about SM neutrinos among the x_i's?

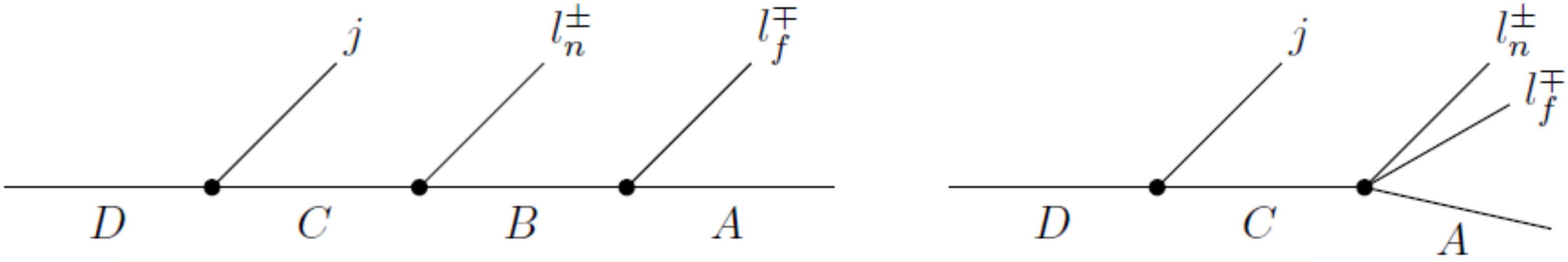
In place of an outline

Missing momenta reconstruction?	Mass measurements		Spin measurements
	Inclusive	2 symmetric chains	
None	Inv. mass endpoints and boundary lines		Inv. mass shapes
	$M_{eff,} M_{est}, H_T$	Wedgebox	
Approximate	$S_{min,}$ M_{Tgen}	M_{T2}, M_{2C}, $M_{3C,}$ $M_{CT,}$ $M_{T2}(n,p,c)$	As usual
Exact	?	Polynomial method	As usual

pessimism (vertical axis, top) ↓ **optimism** (vertical axis, bottom)

pessimism → **optimism**

The classic endpoint method



$$\underbrace{(m_D)^2}_{\text{overall scale}} \quad \boldsymbol{R_{CD} = \left(\frac{M_C}{M_D}\right)^2} \quad \boldsymbol{R_{BC} = \left(\frac{M_B}{M_C}\right)^2} \quad \boldsymbol{R_{AB} = \left(\frac{M_A}{M_B}\right)^2}$$

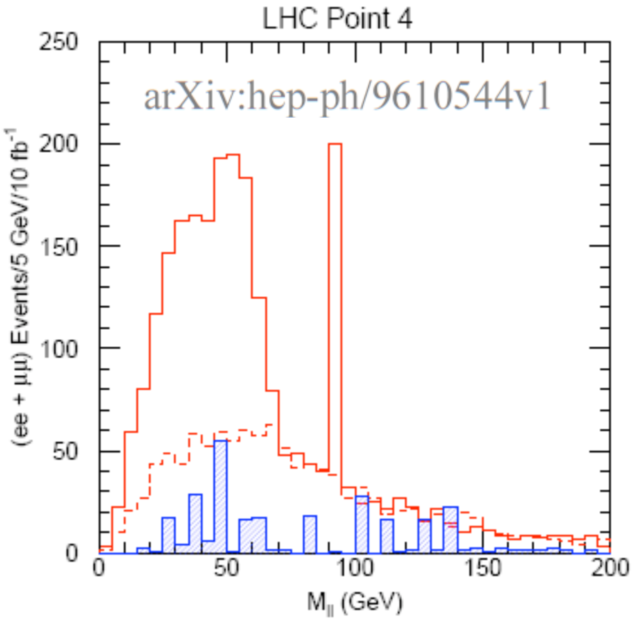
- Identify a sub-chain as shown
- Form all possible invariant mass distributions
  - M_{ll}, M_{jll}, $M_{jl(lo)}$, $M_{jl(hi)}$
- Remove combinatorial background (OF and ME subtraction)
- Measure the endpoints and solve for the masses of A,B,C,D
- 4 measurements, 4 unknowns. Should be sufficient. Not so fast:
  - The measurements may not be independent
  - Piecewise defined functions -> multiple solutions?

Combinatorics problems

- Lepton combinatorics
- Solution: OF subtraction

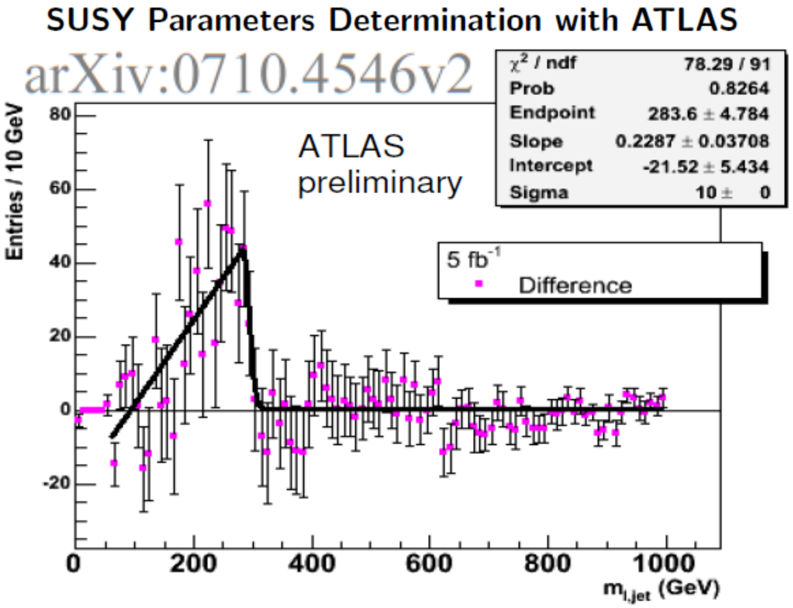
$$\left.\frac{d\sigma}{dM}\right|_{\rm sub} = \left.\frac{d\sigma}{dM}\right|_{e^+e^-} + \left.\frac{d\sigma}{dM}\right|_{\mu^+\mu^-} - \left.\frac{d\sigma}{dM}\right|_{e^+\mu^-} - \left.\frac{d\sigma}{dM}\right|_{e^-\mu^+}$$

I. Hinchliffe, F.E. Paige, M.D. Shapiro, J. Soderqvist, W. Yao

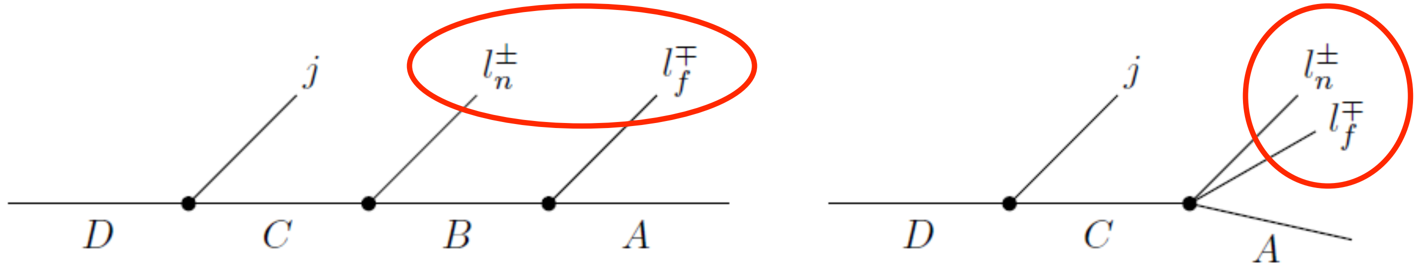


- Jet combinatorics
- Solution: Mixed Event subtraction

N. Ozturk (Texas U., Arlington), for the ATLAS Collaboration

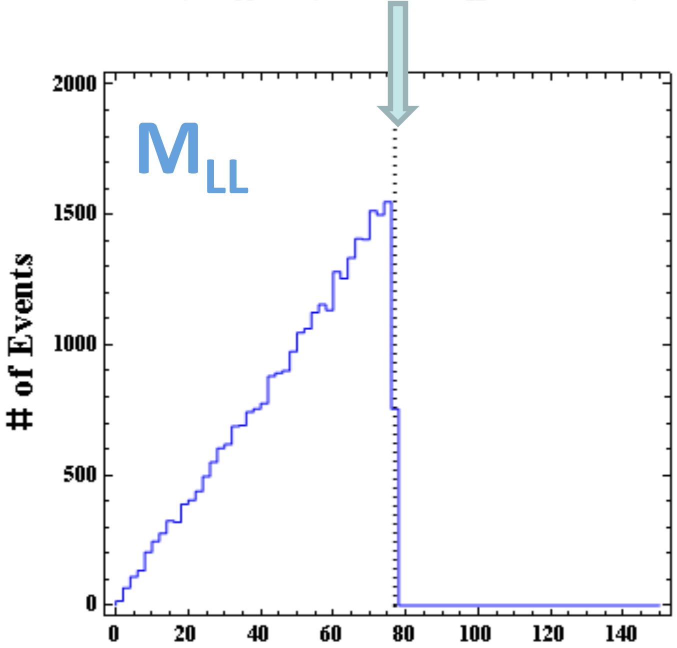


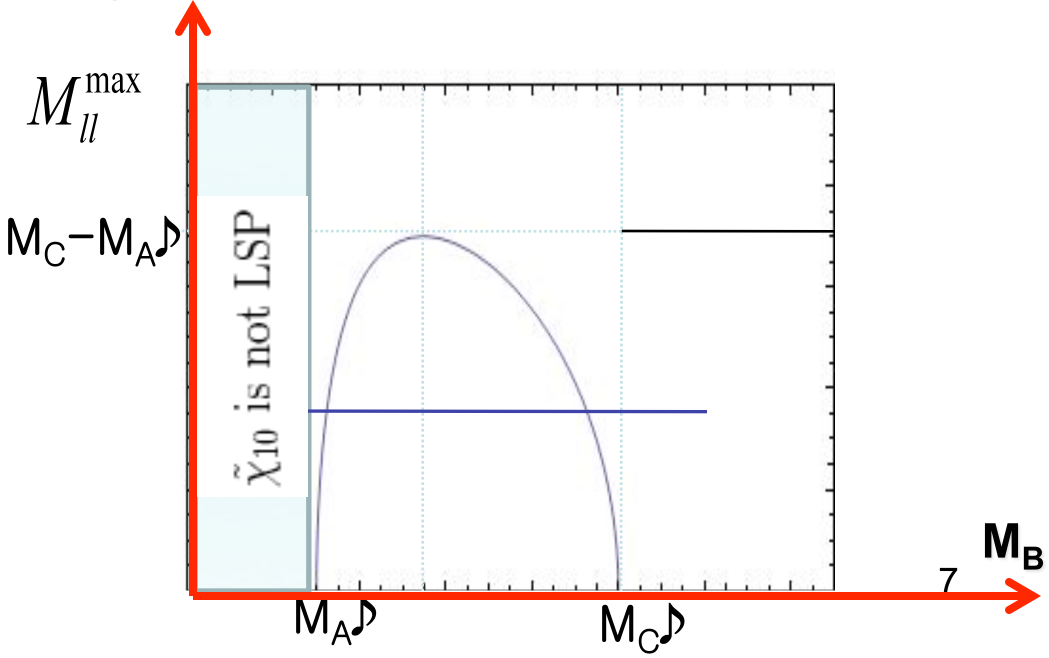
Example: dilepton invariant mass



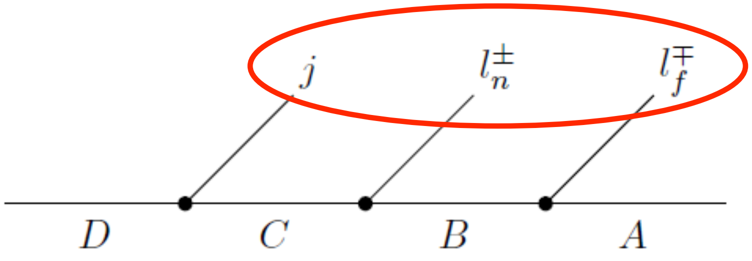
$$a \equiv (m_{ll}^{max})^2 = m_D^2\, R_{CD}\,(1-R_{BC})\,(1-R_{AB})$$ **B on-shell**

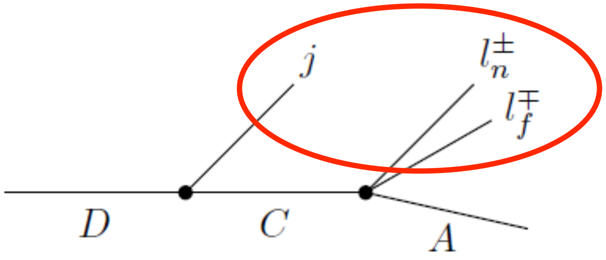
$$a \equiv (m_{ll}^{max})^2 = m_D^2\, R_{CD}\,(1-\sqrt{R_{AC}})^2$$ **B off-shell**

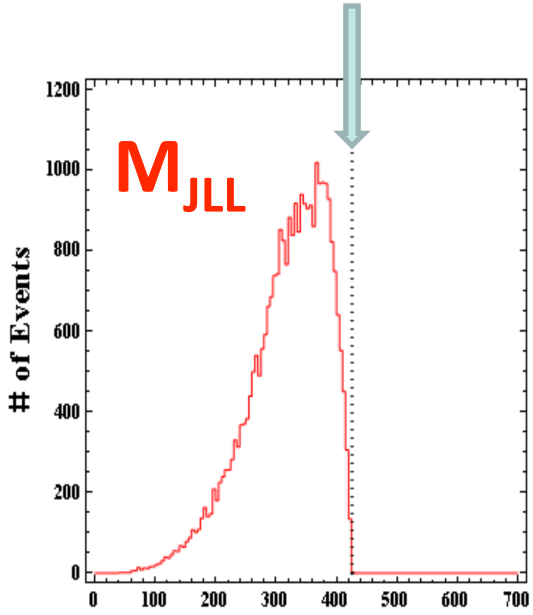




Jet-lepton-lepton invariant mass







$$b \equiv \left(m_{jll}^{max}\right)^2 = \begin{cases} m_D^2(1-R_{CD})(1-R_{AC}), & \text{for } R_{CD} < R_{AC}, \quad \text{case } (1,-) \\ m_D^2(1-R_{BC})(1-R_{CD}R_{AB}), & \text{for } R_{BC} < R_{CD}R_{AB}, \quad \text{case } (2,-) \\ m_D^2(1-R_{AB})(1-R_{BD}), & \text{for } R_{AB} < R_{BD}, \quad \text{case } (3,-) \\ m_D^2\left(1-\sqrt{R_{AD}}\right)^2, & \text{otherwise}, \quad \text{case } (4,-) \end{cases}$$

$$b \equiv \left(m_{jll}^{max}\right)^2 = \begin{cases} m_D^2(1-R_{CD})(1-R_{AC}), & \text{for } R_{CD} < R_{AC}, \quad \text{case } (5,-) \\ m_D^2(1-\sqrt{R_{AD}})^2, & \text{otherwise}, \quad \text{case } (6,-) \end{cases}$$

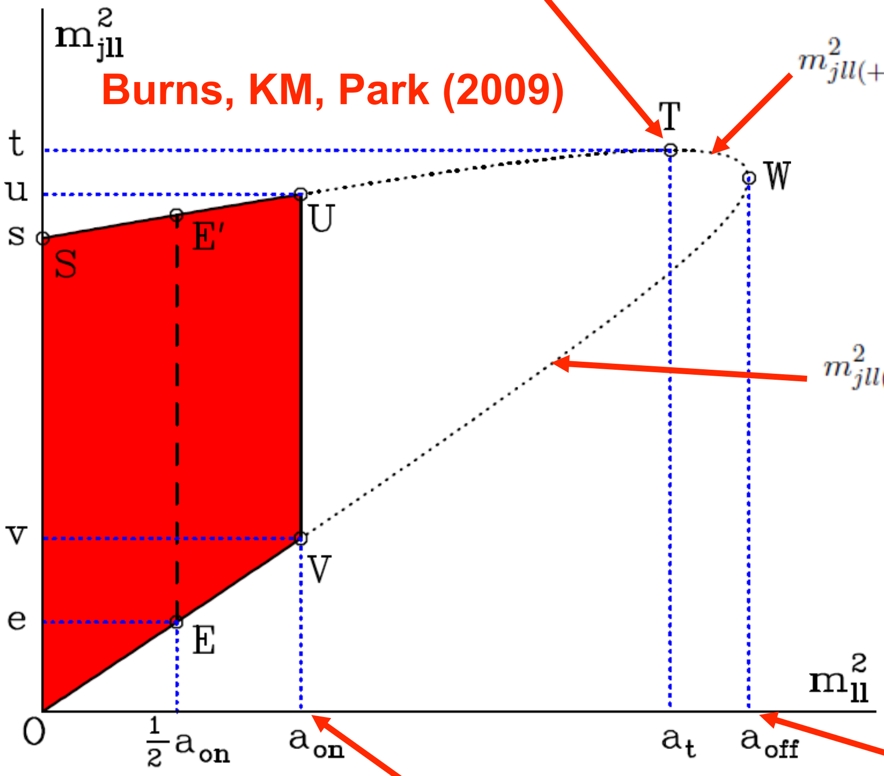
- There are 6 different cases to consider: $(N_{jll},-)$

M_{JLL} versus M_{LL} scatter plot

Bounded by a hyperbola OWS and a line UV

Lester,Parker,White 06

$$\left(\frac{dm^2_{jll(+)}}{dm^2_{ll}}\right)_{m^2_{ll}=a_t} = 0$$



Burns, KM, Park (2009)

$$m^2_{jll(+)}(m^2_{ll}) = \frac{1+R_{CD}}{2}\frac{m^2_{ll}}{R_{CD}} + \frac{1}{2}m^2_D(1-R_{CD})(1-R_{AC}) + \frac{1-R_{CD}}{2}\left\{\left[\left(\frac{m^2_{ll}}{R_{CD}}\right) - m^2_D(1+R_{AC})\right]^2 - 4m^4_D R_{AC}\right\}^{\frac{1}{2}}$$

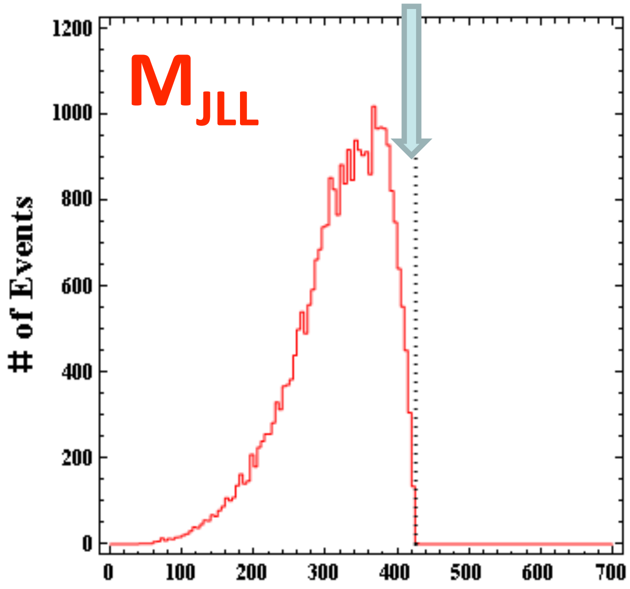
$$m^2_{jll(-)}(m^2_{ll}) = \frac{1+R_{CD}}{2}\frac{m^2_{ll}}{R_{CD}} + \frac{1}{2}m^2_D(1-R_{CD})(1-R_{AC}) - \frac{1-R_{CD}}{2}\left\{\left[\left(\frac{m^2_{ll}}{R_{CD}}\right) - m^2_D(1+R_{AC})\right]^2 - 4m^4_D R_{AC}\right\}^{\frac{1}{2}}$$

$$a_{\text{off}} \equiv m^2_D\, R_{CD}\, (1-\sqrt{R_{AC}})^2$$

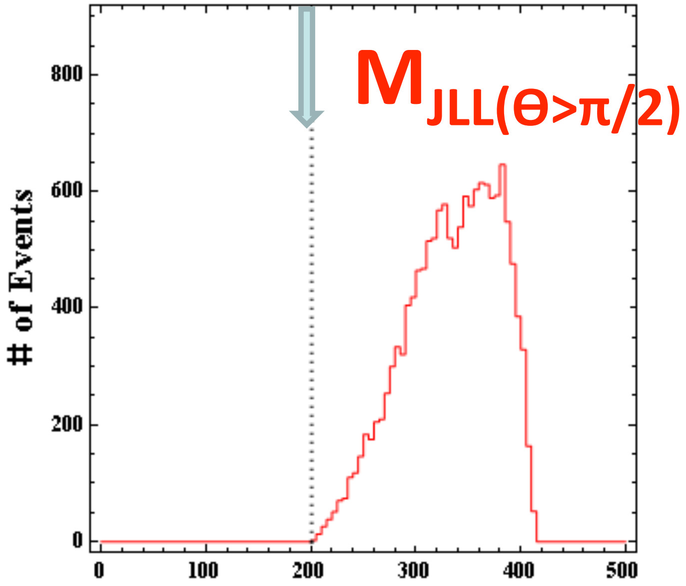
$$a_{\text{on}} \equiv m^2_D\, R_{CD}\,(1-R_{BC})\,(1-R_{AB})$$

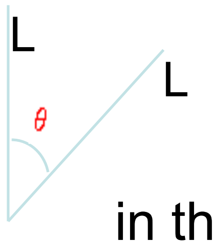
The $M_{JLL(\Theta>\pi/2)}$ invariant mass "threshold"

- Needed whenever $\left(m_{jll}^{max}\right)^2 = \left(m_{jl(hi)}^{max}\right)^2 + \left(m_{ll}^{max}\right)^2$



$$\frac{m_{ll}^{max}}{\sqrt{2}} < m_{ll}$$





in the rest frame of C

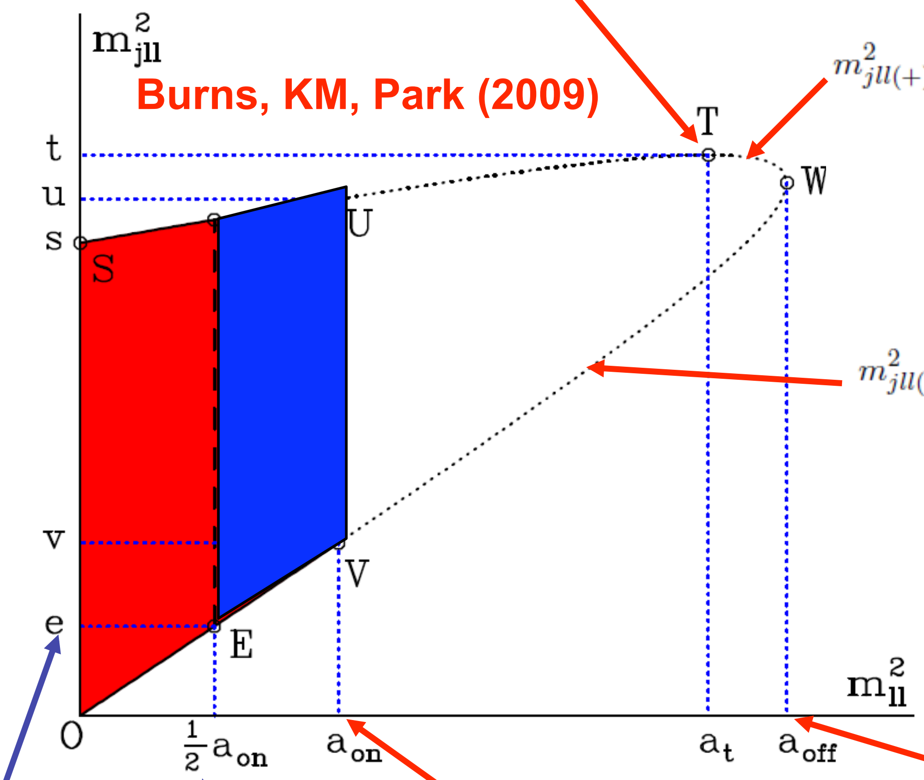
$$e \equiv \left(m_{jll(\theta>\frac{\pi}{2})}^{min}\right)^2 = \frac{1}{4}m_D^2\left\{(1-R_{AB})(1-R_{BC})(1+R_{CD})\right.$$
$$+ (1-R_{AB})(1+R_{BC})(1-R_{CD})$$
$$+ (1+R_{AB})(1-R_{BC})(1-R_{CD})$$
$$\left.- (1-R_{CD})\sqrt{(1+R_{AB})^2(1+R_{BC})^2-16R_{AC}}\right\}$$

M_{JLL} versus M_{LL} scatter plot

Bounded by a hyperbola OWS and a line UV

Lester,Parker,White 06

$$\left(\frac{dm^2_{jll(+)}}{dm^2_{ll}}\right)_{m^2_{ll}=a_t} = 0$$



Burns, KM, Park (2009)

$$m^2_{jll(+)}(m^2_{ll}) = \frac{1+R_{CD}}{2}\frac{m^2_{ll}}{R_{CD}} + \frac{1}{2}m^2_D(1-R_{CD})(1-R_{AC}) + \frac{1-R_{CD}}{2}\left\{\left[\left(\frac{m^2_{ll}}{R_{CD}}\right) - m^2_D(1+R_{AC})\right]^2 - 4m^4_D R_{AC}\right\}^{\frac{1}{2}}$$

$$m^2_{jll(-)}(m^2_{ll}) = \frac{1+R_{CD}}{2}\frac{m^2_{ll}}{R_{CD}} + \frac{1}{2}m^2_D(1-R_{CD})(1-R_{AC}) - \frac{1-R_{CD}}{2}\left\{\left[\left(\frac{m^2_{ll}}{R_{CD}}\right) - m^2_D(1+R_{AC})\right]^2 - 4m^4_D R_{AC}\right\}^{\frac{1}{2}}$$

$$a_{\text{off}} \equiv m^2_D\, R_{CD}\,(1-\sqrt{R_{AC}})^2$$

$$a_{\text{on}} \equiv m^2_D\, R_{CD}\,(1-R_{BC})\,(1-R_{AB})$$

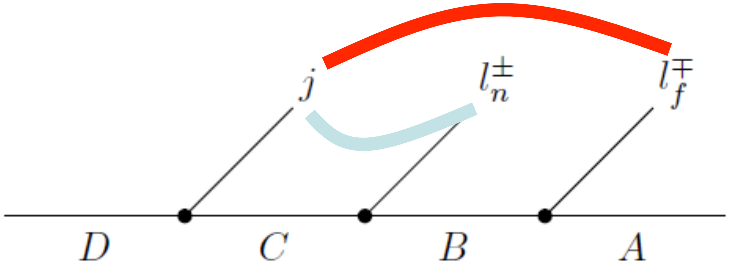
The $M_{JLL(\Theta>\pi/2)}$ invariant mass "threshold"

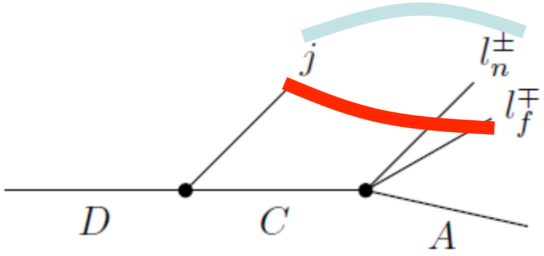
“Low” jet-lepton pair invariant mass

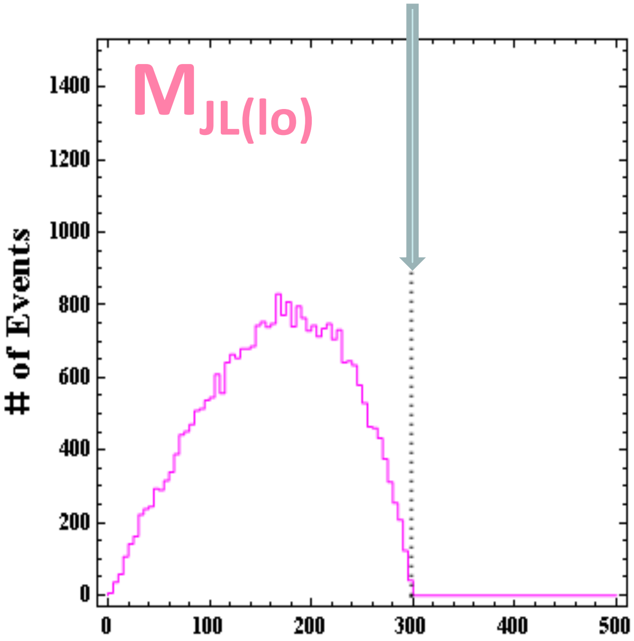
- Four additional cases: (-,N_{jl})

$$m_{jl(lo)} \equiv \min\left\{m_{jl_n}, m_{jl_f}\right\}$$

$$m_{jl(hi)} \equiv \max\left\{m_{jl_n}, m_{jl_f}\right\}$$







$$c \equiv \left(m^{max}_{ql(lo)}\right)^2 = \begin{cases} \left(m^{max}_{jl_n}\right)^2, & \text{for } (2-R_{AB})^{-1} < R_{BC} < 1, \quad \text{case } (-,1), \\ \left(m^{max}_{jl(eq)}\right)^2, & \text{for } R_{AB} < R_{BC} < (2-R_{AB})^{-1}, \text{ case } (-,2), \\ \left(m^{max}_{jl(eq)}\right)^2, & \text{for } 0 < R_{BC} < R_{AB}, \quad \text{case } (-,3); \end{cases}$$

$$c \equiv \left(m^{max}_{ql(lo)}\right)^2 = \frac{1}{2} m_D^2 (1 - R_{CD})(1 - R_{AC}), \quad \text{case } (-,4);$$

$$\left(m^{max}_{jl_n}\right)^2 = m_D^2 \left(1 - R_{CD}\right) \left(1 - R_{BC}\right),$$

$$\left(m^{max}_{jl_f}\right)^2 = m_D^2 \left(1 - R_{CD}\right) \left(1 - R_{AB}\right),$$

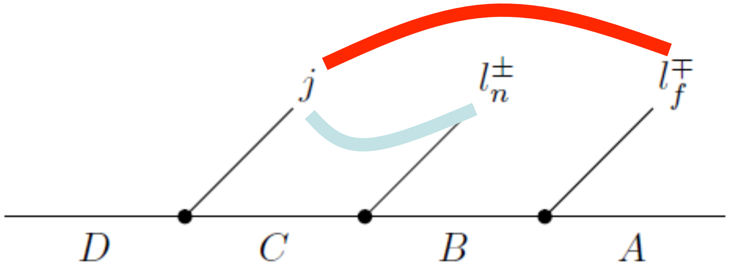
$$\left(m^{max}_{jl(eq)}\right)^2 = m_D^2 \left(1 - R_{CD}\right) \left(1 - R_{AB}\right) \left(2 - R_{AB}\right)^{-1}$$

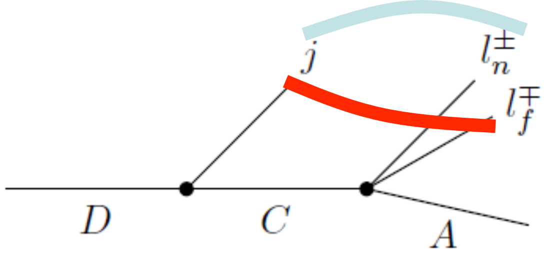
“High” jet-lepton pair invariant mass

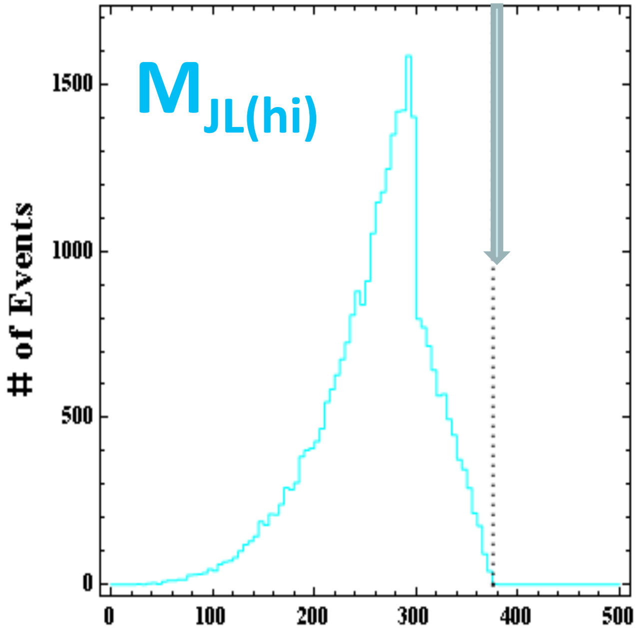
- The same 4 cases: (-,N_{jl})

$$m_{jl(lo)} \equiv \min\left\{m_{jl_n}, m_{jl_f}\right\}$$

$$m_{jl(hi)} \equiv \max\left\{m_{jl_n}, m_{jl_f}\right\}$$







$$d \equiv \left(m_{jl(hi)}^{max}\right)^2 = \begin{cases} \left(m_{jl_f}^{max}\right)^2, & \text{for } (2-R_{AB})^{-1} < R_{BC} < 1, \quad \text{case } (-,1) \\ \left(m_{jl_f}^{max}\right)^2, & \text{for } R_{AB} < R_{BC} < (2-R_{AB})^{-1}, \text{case } (-,2) \\ \left(m_{jl_n}^{max}\right)^2, & \text{for } 0 < R_{BC} < R_{AB}, \quad \text{case } (-,3) \end{cases}$$

$$d \equiv \left(m_{jl(hi)}^{max}\right)^2 = m_D^2(1-R_{CD})(1-R_{AC}), \qquad \text{case } (-,4)$$

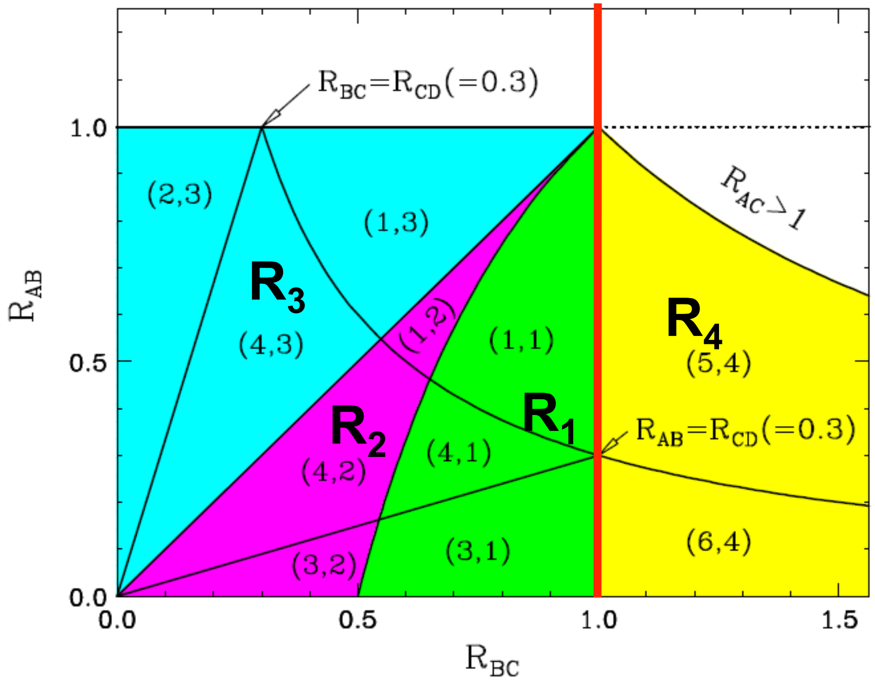
$$\left(m_{jl_n}^{max}\right)^2 = m_D^2\,(1-R_{CD})\,(1-R_{BC})\,,$$

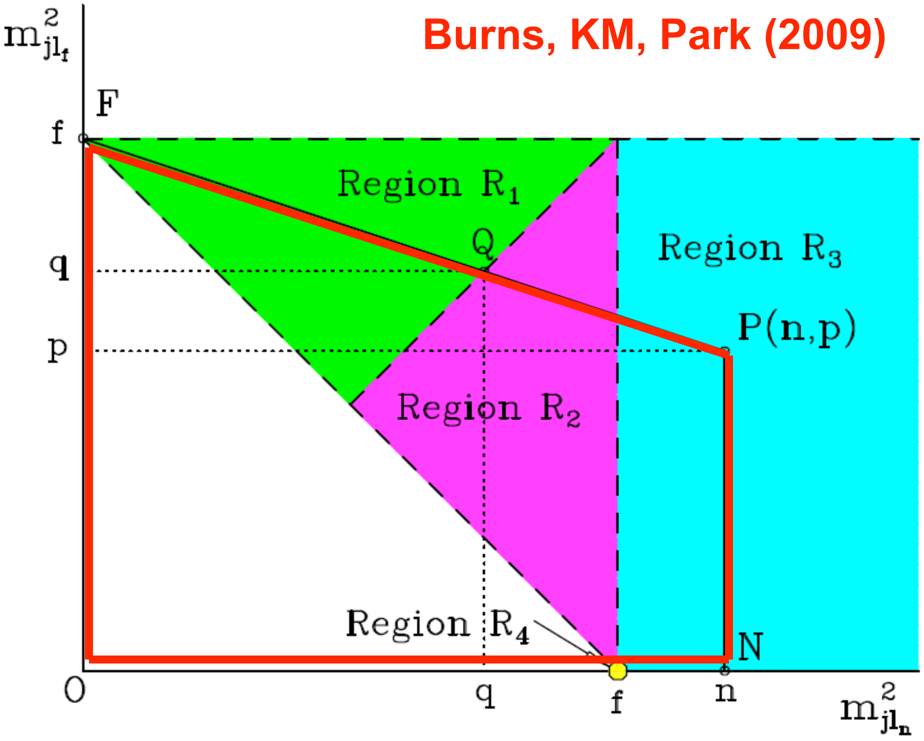
$$\left(m_{jl_f}^{max}\right)^2 = m_D^2\,(1-R_{CD})\,(1-R_{AB})\,,$$

$$\left(m_{jl(eq)}^{max}\right)^2 = m_D^2\,(1-R_{CD})\,(1-R_{AB})\,(2-R_{AB})^{-1}$$

Understanding JL shapes

- Start with “near” versus “far” JL pairs (unobservable)
- The shape is a right-angle trapezoid ONPF
- Notice the correspondence between regions and point P




Burns, KM, Park (2009)

- Notice available measurements: n, f, p, perhaps also q

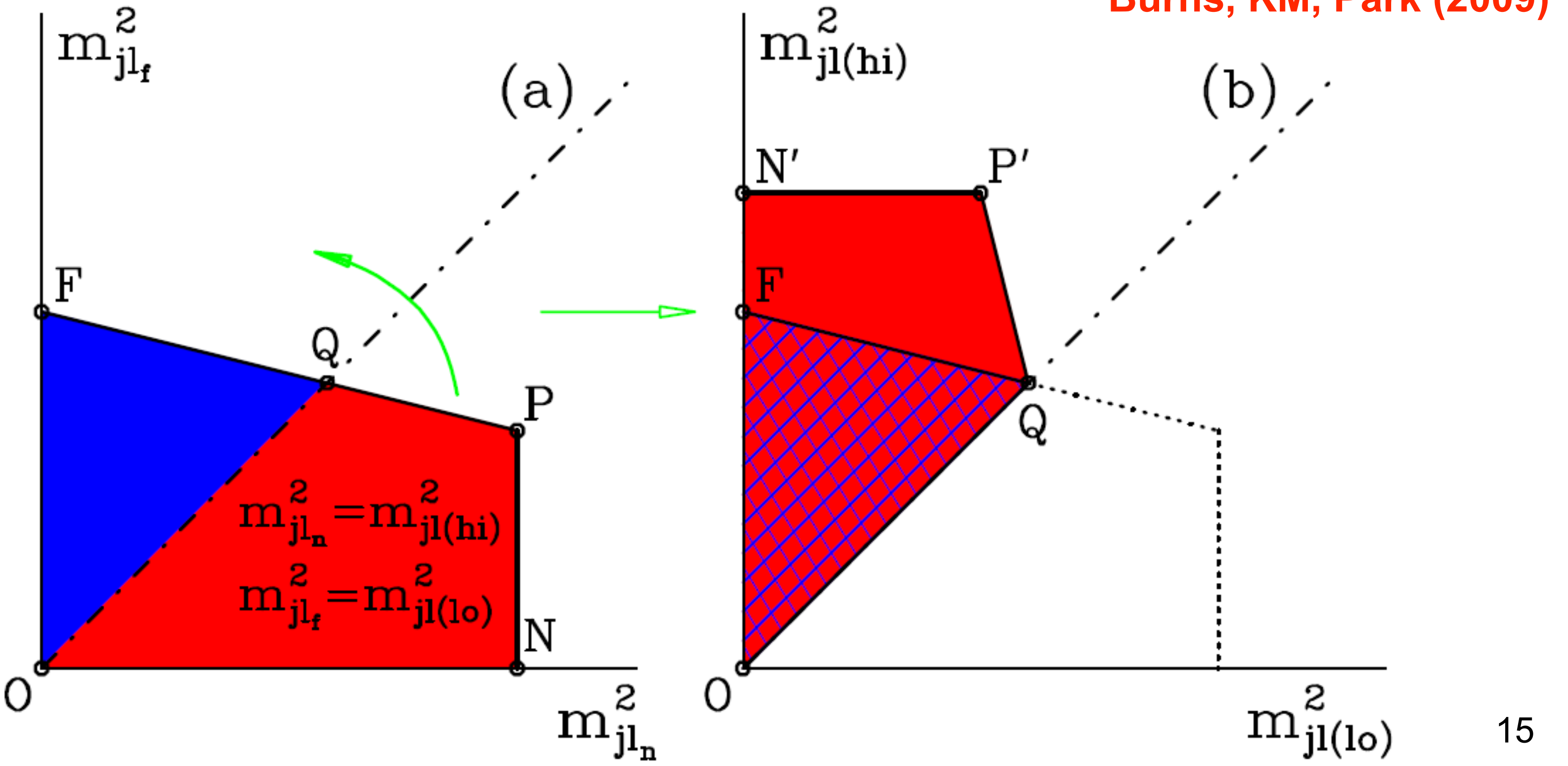
From “near-far” to “low-high”

- This reordering is simply origami: a 45 degree fold

$$m_{jl(lo)} \equiv \min\left\{m_{jl_n}, m_{jl_f}\right\}$$

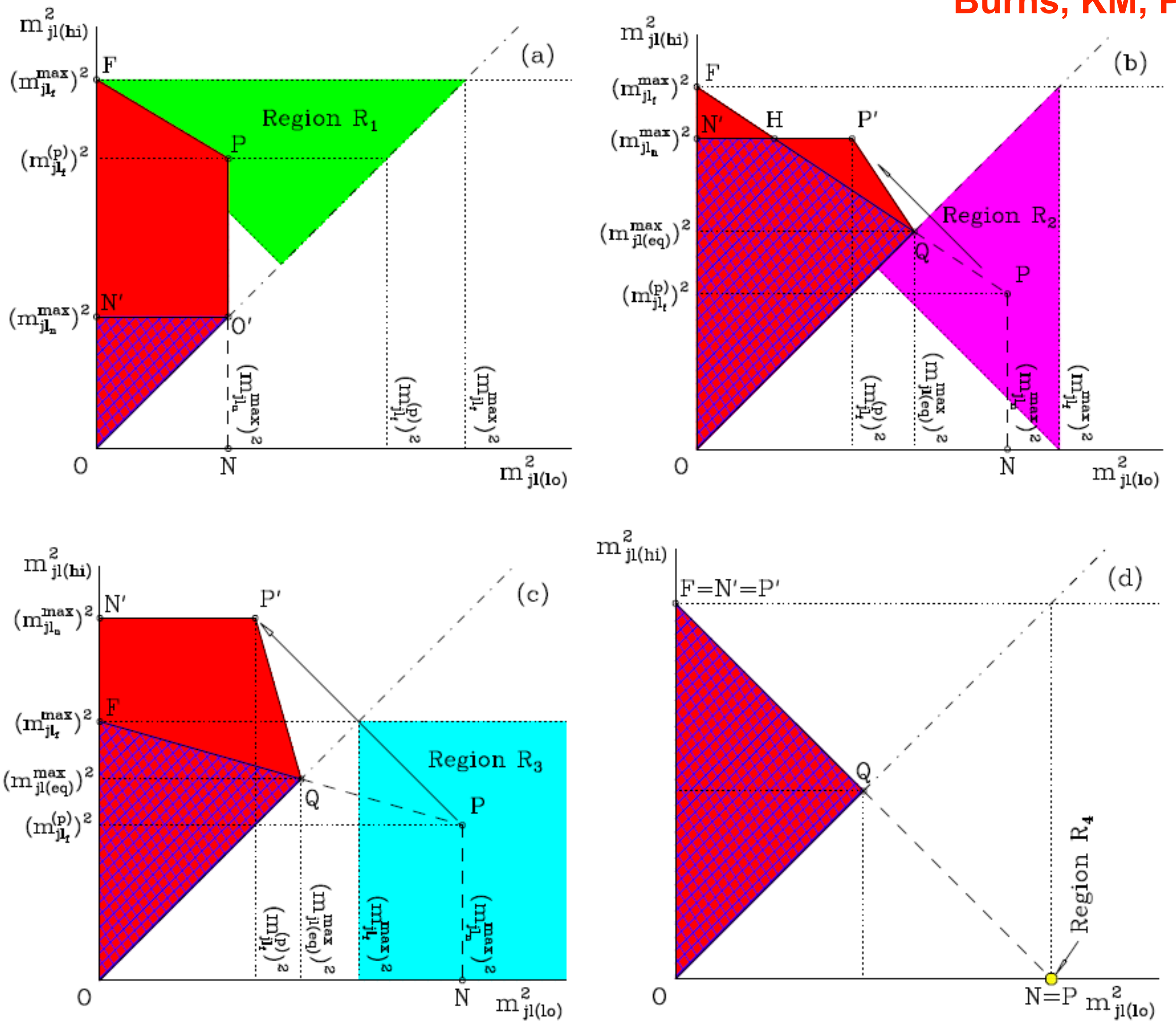
$$m_{jl(hi)} \equiv \max\left\{m_{jl_n}, m_{jl_f}\right\}$$

Burns, KM, Park (2009)



The four basic JL shapes

Burns, KM, Park (2009)



The LHC inverse problem

- Find the four masses of A, B, C, D, given the 5 endpoints

$$a = (m_{ll}^{max})^2, \quad b = (m_{jll}^{max})^2, \quad c = \left(m_{jl(lo)}^{max}\right)^2, \quad d = \left(m_{jl(hi)}^{max}\right)^2, \quad e = \left(m_{jll(\theta>\frac{\pi}{2})}^{min}\right)^2$$

- Solution:

Burns, KM, Park (2009)

$$(a,\ b,\ c,\ d \text{ and } e)$$

$$g \equiv 2e - a$$

$$G_1 \equiv \frac{g\,(2d-g) - 2c\,(d-g)}{g}, \qquad \alpha_1 \equiv \frac{a+G_1}{G_1}, \qquad \beta_1 \equiv \frac{d}{G_1}, \qquad \gamma_1 \equiv \frac{c}{G_1};$$

$$G_2 \equiv \frac{g\,(2d-g)\,(d-c)}{g\,(d-c) + 2c\,(d-g)}, \qquad \alpha_2 \equiv \frac{a+G_2}{G_2}, \qquad \beta_2 \equiv \frac{d}{G_2}, \qquad \gamma_2 \equiv \frac{c}{d-c};$$

$$G_3 \equiv \frac{(g\,(2d-g) - 2c\,(d-g))\,d}{gd + 2c\,(d-g)}, \qquad \alpha_3 \equiv \frac{a+G_3}{G_3}, \qquad \beta_3 \equiv \frac{c\,(d+G_3)}{dG_3}, \qquad \gamma_3 \equiv \frac{d}{G_3};$$

$$G_4 \equiv \frac{d^2\left(-1+\sqrt{2\frac{d}{g}-1}\right)}{3d - 2g - d\sqrt{2\frac{d}{g}-1}}, \qquad \alpha_4 \equiv 1 + \frac{aG_4}{d^2}, \qquad \beta_4 \equiv \gamma_4 \equiv \frac{d+G_4}{2G_4}.$$

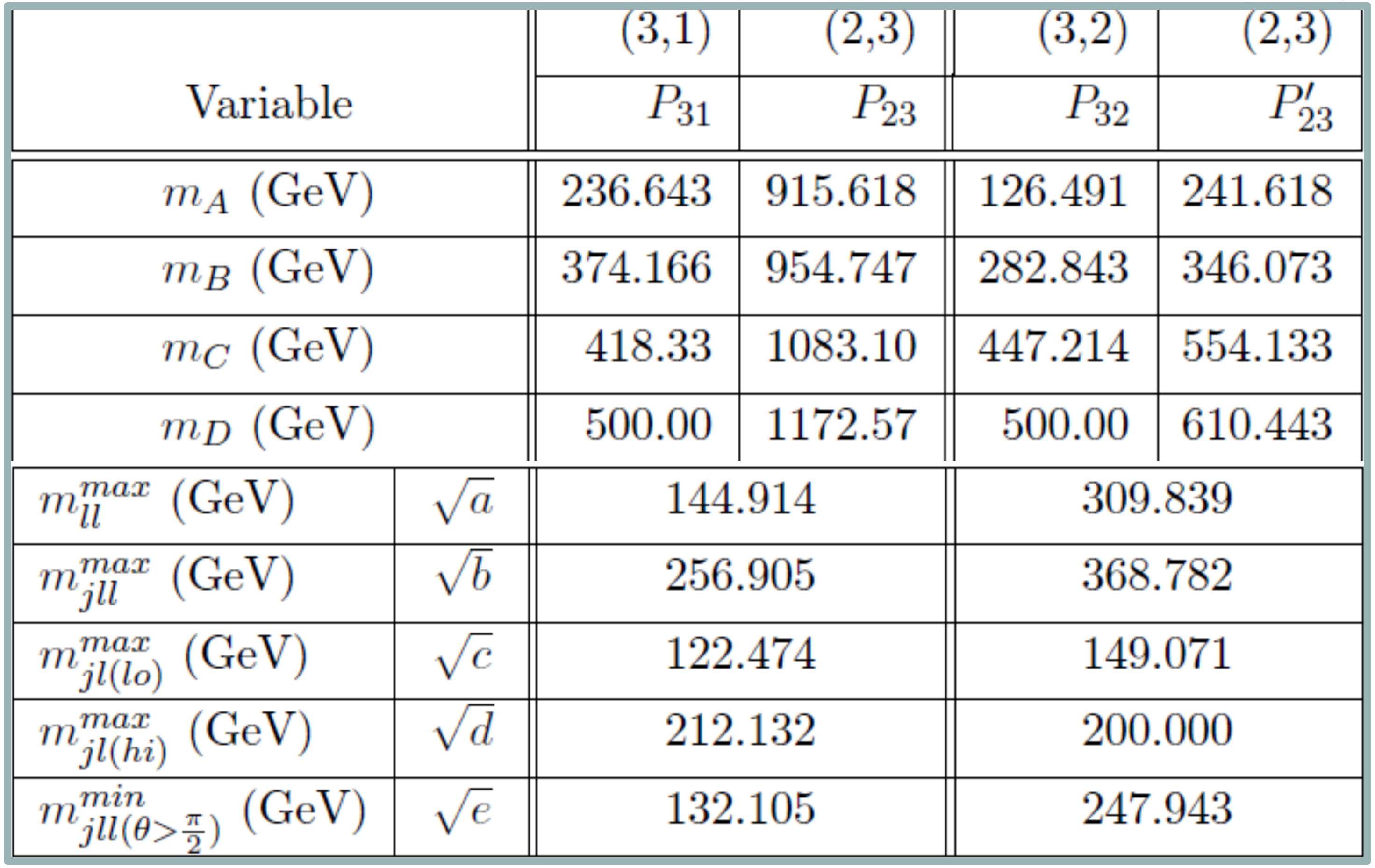
$$m_A^2 = G_i\,(\alpha_i - 1)\,(\beta_i - 1)\,(\gamma_i - 1) \qquad m_C^2 = G_i\,(\alpha_i - 1)\,\beta_i\,\gamma_i,$$

$$m_B^2 = G_i\,(\alpha_i - 1)\,(\beta_i - 1)\,\gamma_i, \qquad m_D^2 = G_i\,\alpha_i\,\beta_i\,\gamma_i.$$

Mass ambiguities

- Exact spectrum duplication in (3,1), (3,2) and (2,3)

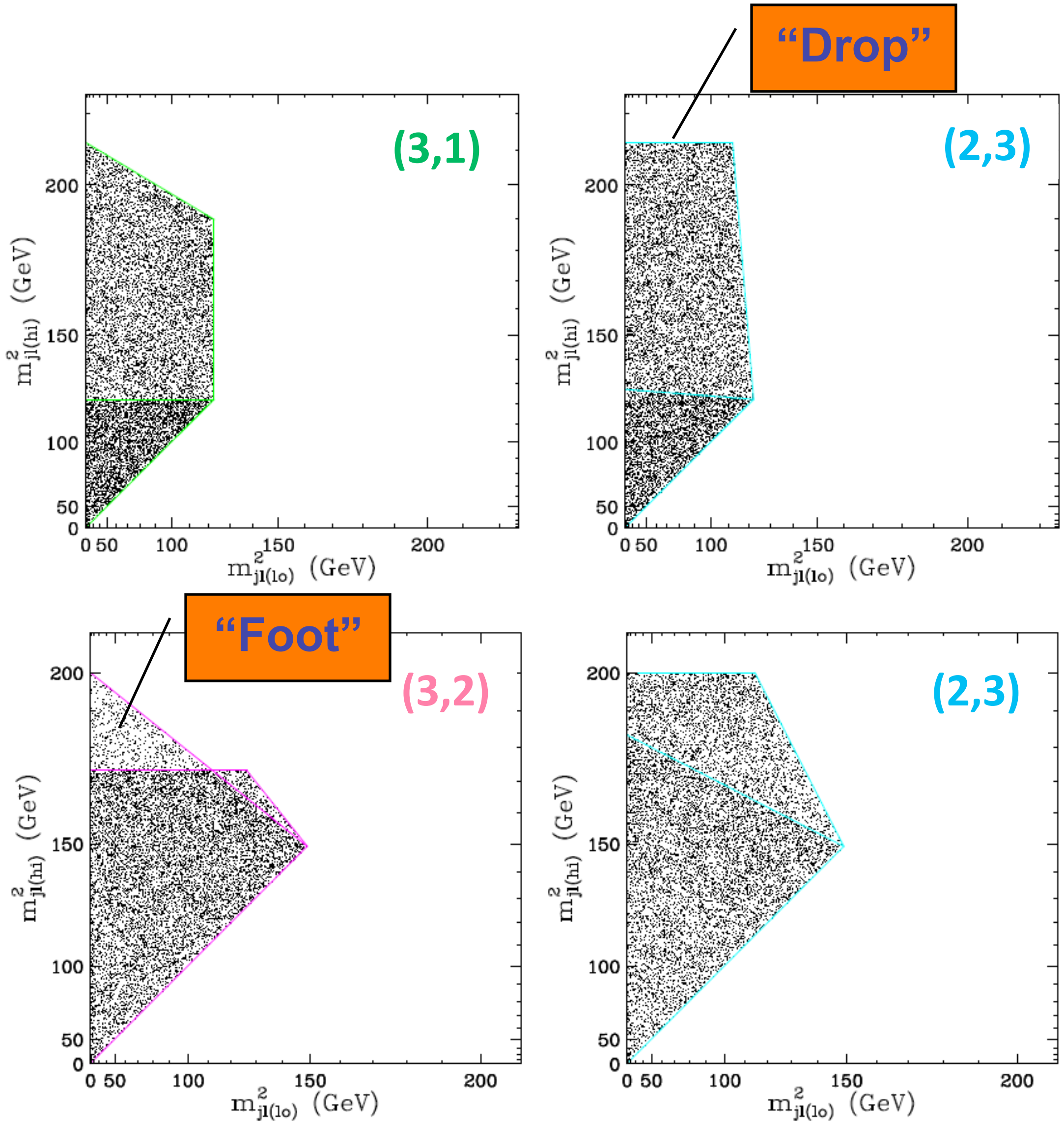
Burns, KM, Park (2009)



Variable		(3,1)	(2,3)	(3,2)	(2,3)
		P_{31}	P_{23}	P_{32}	P'_{23}
m_A (GeV)		236.643	915.618	126.491	241.618
m_B (GeV)		374.166	954.747	282.843	346.073
m_C (GeV)		418.33	1083.10	447.214	554.133
m_D (GeV)		500.00	1172.57	500.00	610.443
m_{ll}^{max} (GeV)	$\sqrt{a}$	144.914		309.839	
m_{jll}^{max} (GeV)	$\sqrt{b}$	256.905		368.782	
$m_{jl(lo)}^{max}$ (GeV)	$\sqrt{c}$	122.474		149.071	
$m_{jl(hi)}^{max}$ (GeV)	$\sqrt{d}$	212.132		200.000	
$m_{jll(\theta>\frac{\pi}{2})}^{min}$ (GeV)	$\sqrt{e}$	132.105		247.943	

JL scatter plots resolve the ambiguity

- R_1 versus R_3
- R_2 versus R_3

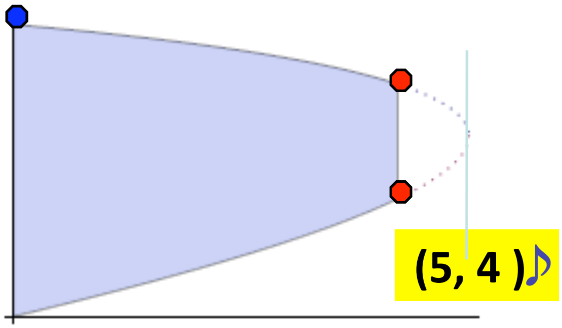


“Drop”

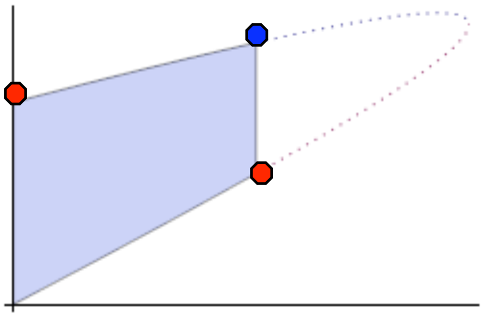
“Foot”

Invariant mass endpoints: summary

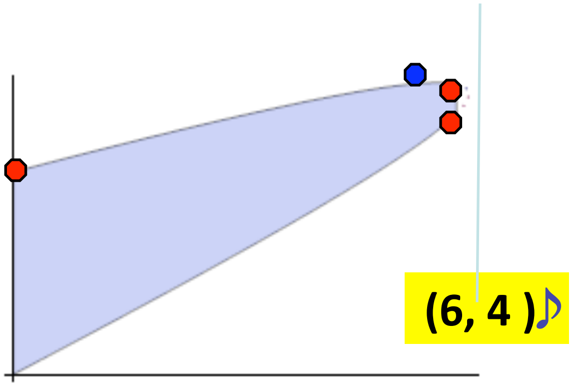
- 2D plots contain more information than their 1D projections
  - The shapes reveal the relevant region – which set of formulas applies
  - Easy to understand the usual 1D endpoints
  - Special points on the boundaries offer additional measurements
  - Altogether there could be as many as 11



Region (1, -)♪



Region (2, -) , (3, -) ♪



Region (4, -)♪

In place of an outline

Missing momenta reconstruction?	Mass measurements		Spin measurements
	Inclusive	2 symmetric chains	
None	Inv. mass endpoints and boundary lines		Inv. mass shapes
	$M_{eff,}M_{est},H_T$	Wedgebox	
Approximate	$S_{min,}$ M_{Tgen}	M_{T2}, M_{2C}, $M_{3C,}$ $M_{CT,}$ M_{T2}(n,p,c)	As usual
Exact	?	Polynomial method	As usual

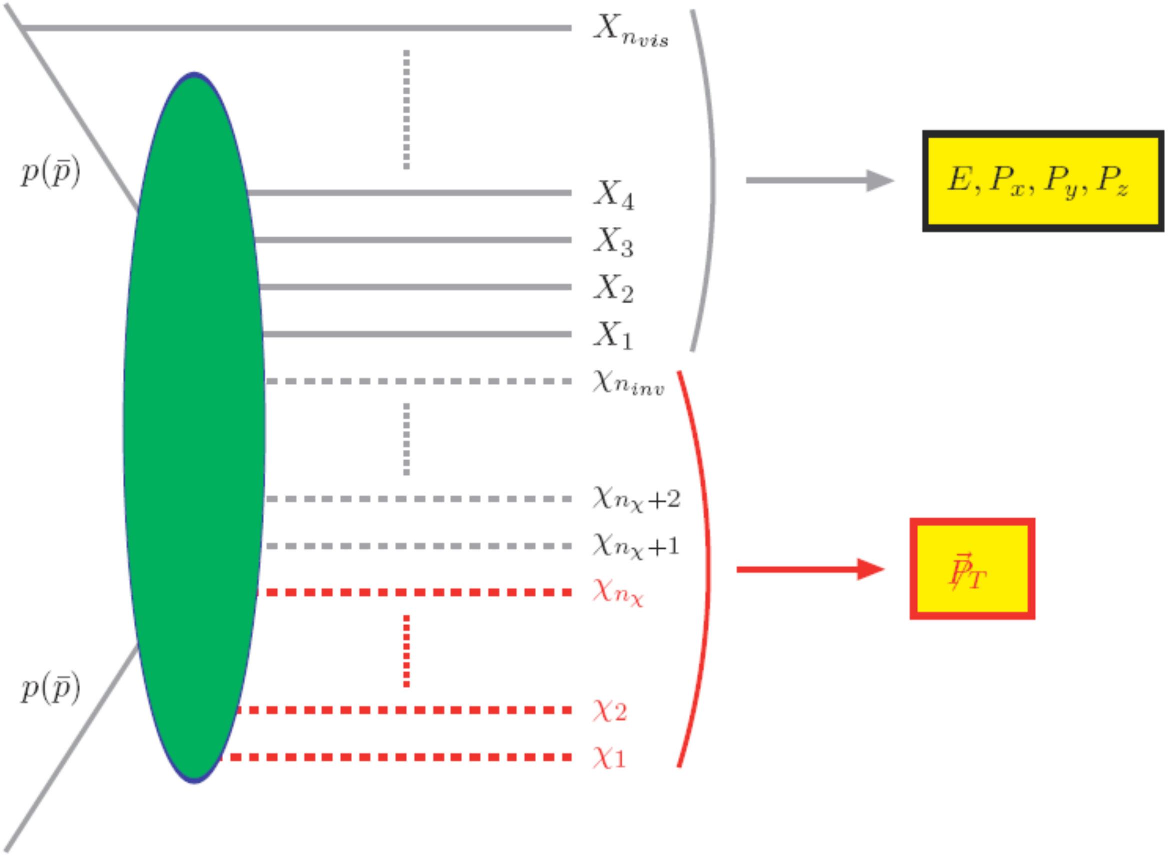
pessimism

optimism

pessimism

optimism

MET events: experimentalist’s view



- What is going on here?

M_{eff}

- Used by Frank E. Paige at 1996 Snowmass (hep-ph/9609373).
- What is it? It’s neither a mass, nor very effective. ☺

II. EFFECTIVE MASS ANALYSIS

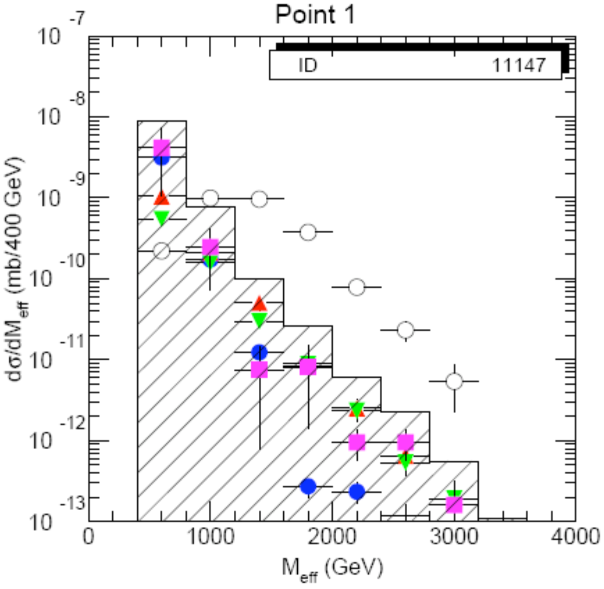
The first step after discovering a deviation from the SM is to estimate the mass scale. SUSY production at the LHC is dominated by gluinos and squarks, which decay into jets plus missing energy. The mass scale can be estimated using the effective mass, defined as the scalar sum of the p_T's of the four hardest jets and the missing transverse energy $\not{E}_T$,

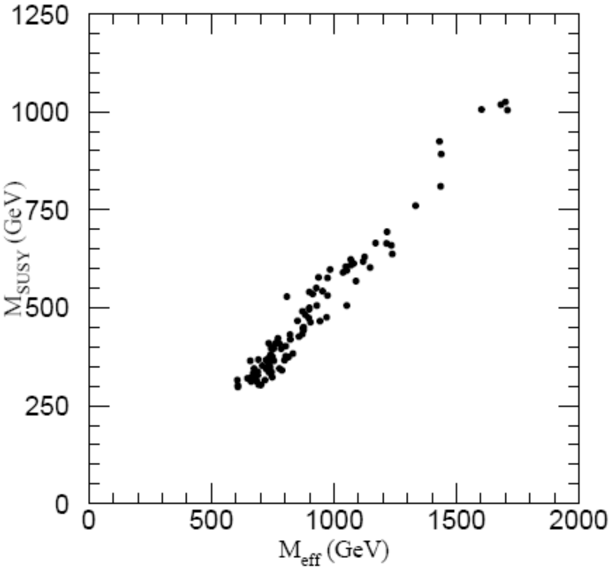
$$M_{\rm eff} = p_{T,1} + p_{T,2} + p_{T,3} + p_{T,4} + \not{E}_T \,.$$

The peak of the $M_{\rm eff}$ mass distribution, or alternatively the point at which the signal and background are equal, provides a good first estimate of the SUSY mass scale, which is defined to be

$$M_{\rm SUSY} = \min(M_{\tilde{g}}, M_{\tilde{u}_R})$$

(The choice of $M_{\tilde{u}_R}$ as the typical squark mass is arbitrary.) The





M_{est}

- Proposed by Dan Tovey in hep-ph/0006276

2 Measurement Technique

(1) $M_{\mathrm{est}} = |p_{T(1)}| + |p_{T(2)}| + |p_{T(3)}| + |p_{T(4)}| + E_T^{\mathrm{miss}}$,

(2) $M_{\mathrm{est}} = |p_{T(1)}| + |p_{T(2)}| + |p_{T(3)}| + |p_{T(4)}|$,

(3) $M_{\mathrm{est}} = \sum_i |p_{T(i)}| + E_T^{\mathrm{miss}}$,

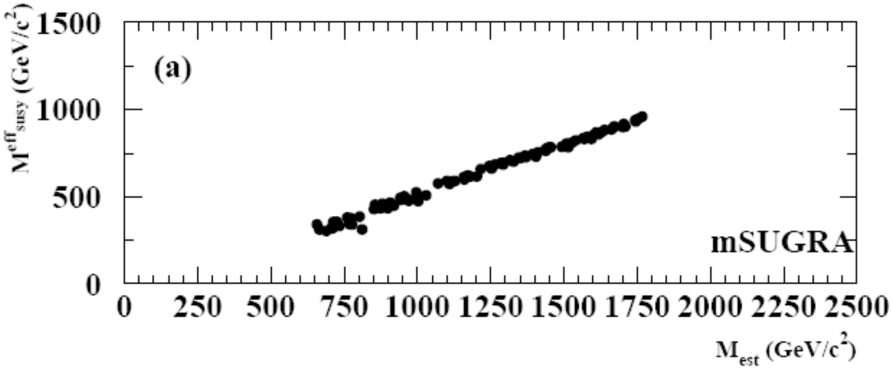
(4) $M_{\mathrm{est}} = \sum_i |p_{T(i)}|$.

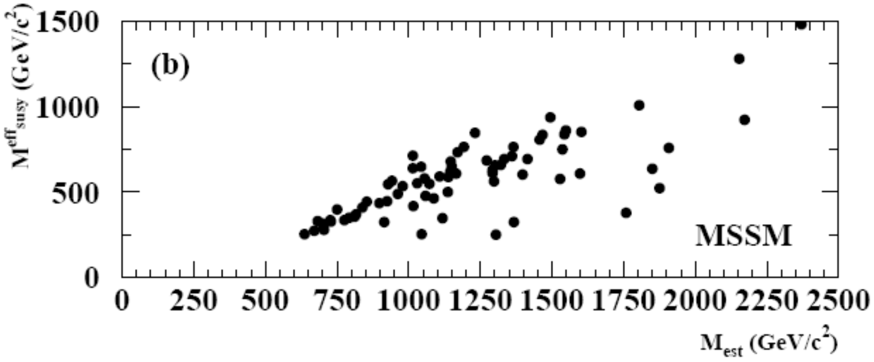
3 Definition of Mass Scale

$$M_{\mathrm{susy}} = \frac{\sum_i \sigma_i m_i}{\sum_i \sigma_i}.$$

$$M_{\mathrm{susy}}^{\mathrm{eff}} = \left(M_{\mathrm{susy}} - \frac{M_\chi^2}{M_{\mathrm{susy}}} \right)$$

- You measure M_{est} and interpret it as $M^{\mathrm{eff}}(M_{\mathrm{susy}}, M_{\mathrm{chi}})$
- The relation is very model-dependent:





2 different philosophies

- The problem with measuring the mass scale: Anything you try to measure will depend on both the mass of the parent particles (M_{susy}) as well as the LSP mass ($M_{chi)}$).
- How should one deal with it? 2 approaches:
  - **Option I.** Define an experimental observable which does not depend on the unknown LSP mass M_{chi} and then interpret it in terms of some function of both M_{susy} and M_{chi}.
    - Example: $M_{est}=M^{eff}(M_{susy},M_{chi})$
  - **Option II.** Define an experimental observable which does depend on the unknown LSP mass M_{chi} and then interpret it in terms of M_{susy}.
    - Cambridge variable: $M_{T2}{}^{max}(M_{chi})=M_{susy}$
    - Gator variable: $S_{min}(M_{chi})=(2M_{susy})^2$
- IMHO the second option is better.

Gator variable: S_{min}

Konar, Kong, KM 2008

- The minimum value of the Mandelstam variable consistent with the measured values of the total energy E and total visible momentum (P_x,P_y,P_z)

$$\hat{s}^{1/2}_{min}(M_{inv}) = \sqrt{E^2 - P_z^2} + \sqrt{\not{E}_T^2 + M_{inv}^2}$$

$$\hat{s}^{1/2}_{min}(M_{inv}) = \sqrt{E_T^2 + M^2} + \sqrt{\not{E}_T^2 + M_{inv}^2}$$

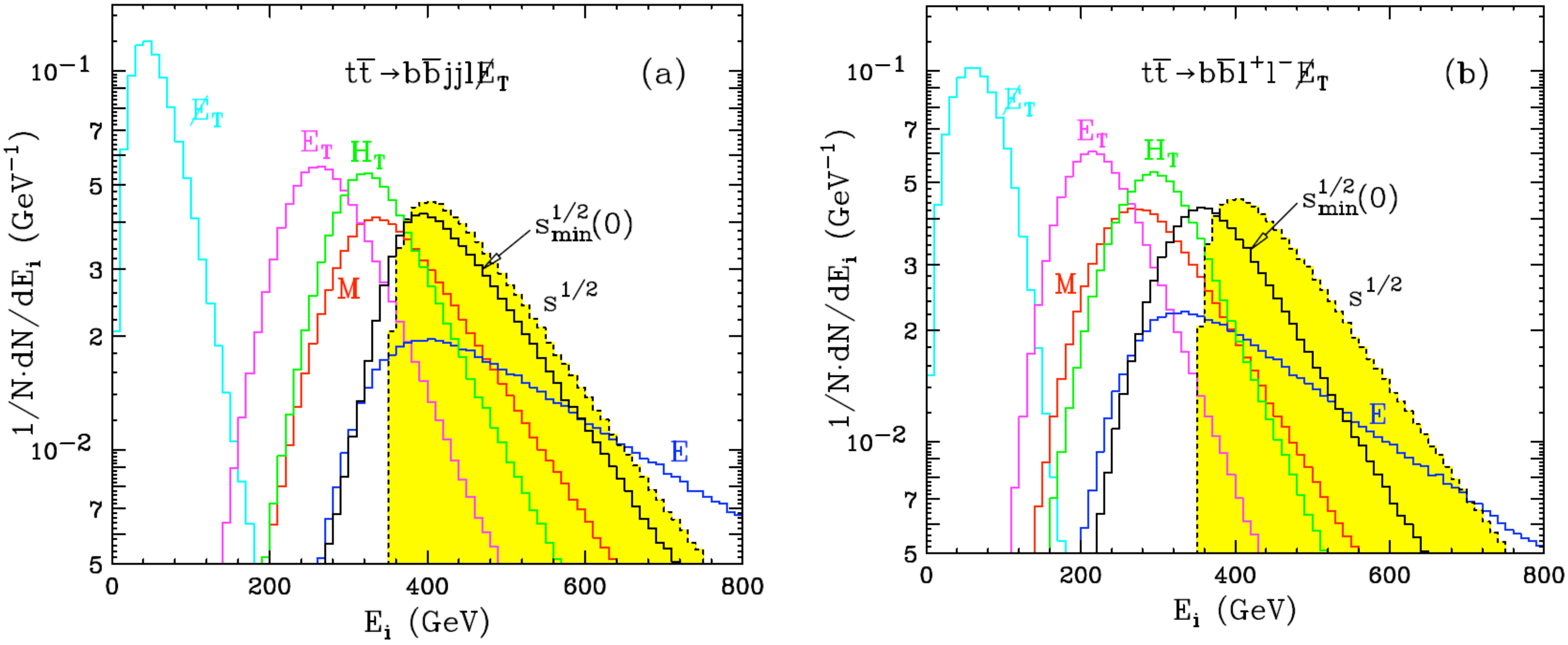
$$M_{inv} \equiv \sum_{i=1}^{n_{inv}} m_i = \sum_{i=1}^{n_\chi} m_i$$

- Advantages:
  - Uses all available information (not just transverse quantities)
  - Model-independent: no need for any event reconstruction
  - Very general: arbitrary number and type of missing particles
  - Inclusive
  - Global
  - Clear physical meaning

What is S_{min} good for?

- As a conservative approximation to the true value of S:

Konar, Kong, KM 2008



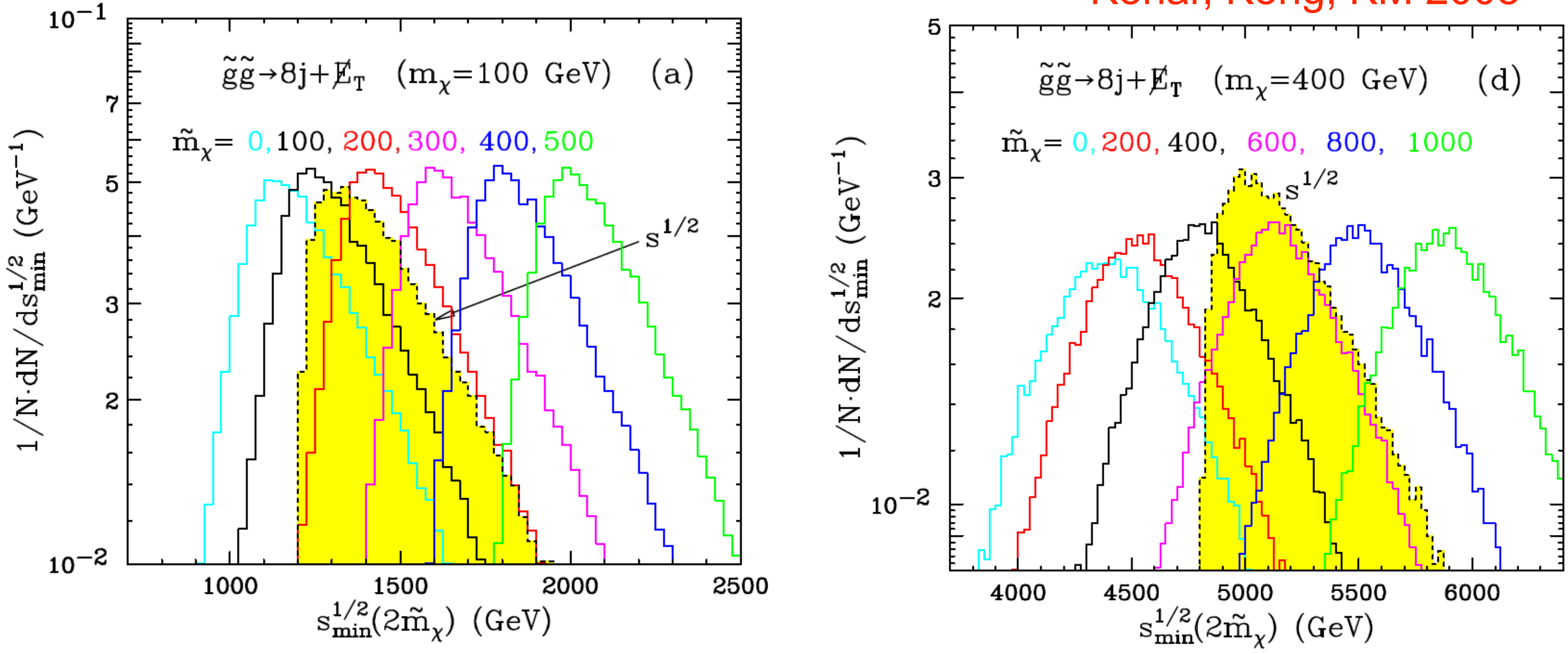
- Disclaimer: no ISR or multiple parton interactions

What is S_{min} good for?

- One can measure SUSY masses in terms of the LSP mass:

$$\left(\hat{s}^{1/2}\right)_{thr} \approx \left(\hat{s}^{1/2}_{min}(2m_\chi)\right)_{peak}$$

Konar, Kong, KM 2008



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In place of an outline

Missing momenta reconstruction?	Mass measurements		Spin measurements
	Inclusive	2 symmetric chains	
None	Inv. mass endpoints and boundary lines		Inv. mass shapes
	$M_{eff,}M_{est},H_T$	Wedgebox	
Approximate	$S_{min,}$ M_{Tgen}	M_{T2}, M_{2C}, $M_{3C,}$ $M_{CT,}$ M_{T2}(n,p,c)	As usual
Exact	?	Polynomial method	As usual

pessimism

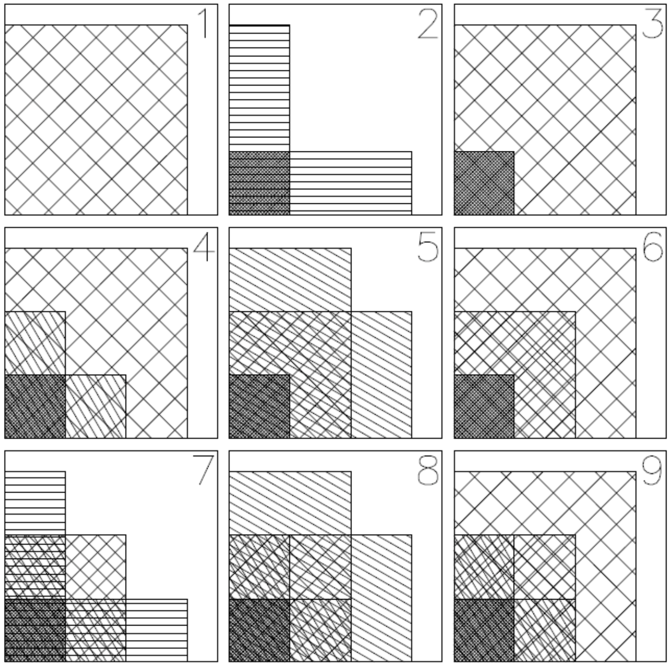
optimism

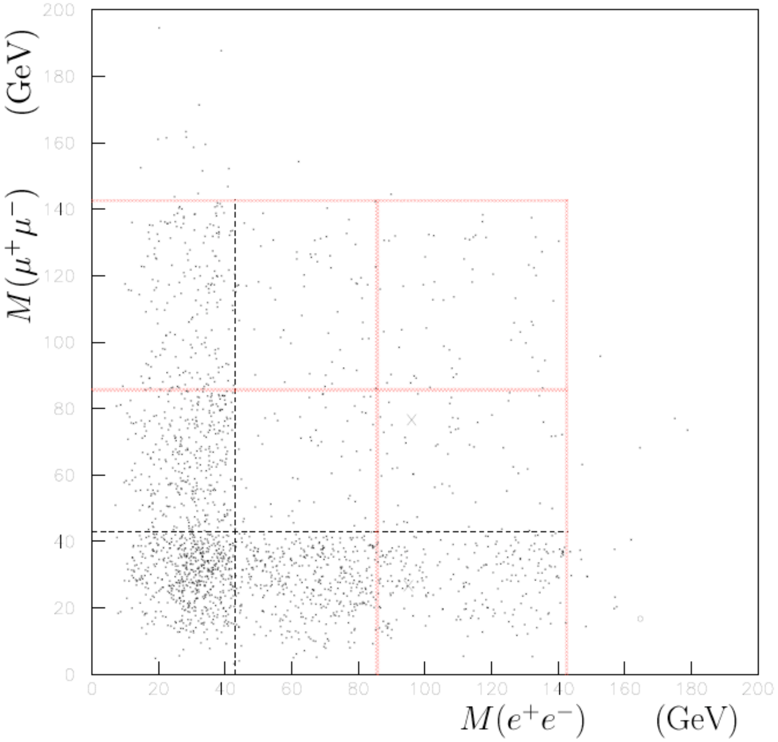
pessimism **optimism**

Wedgebox technique

- Scatter plot of the invariant masses of the visible decay products on both sides

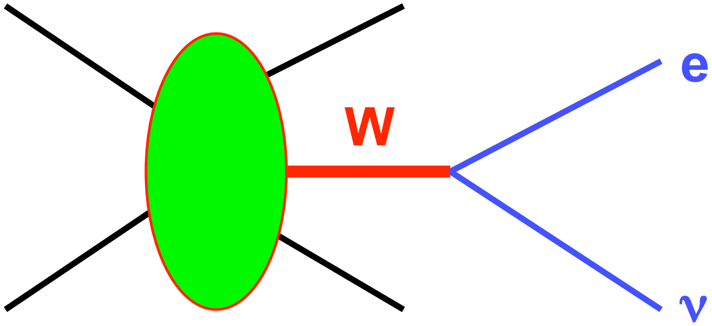
Bisset,Kersting,Li,Moortgat,Moretti,Xie 2005

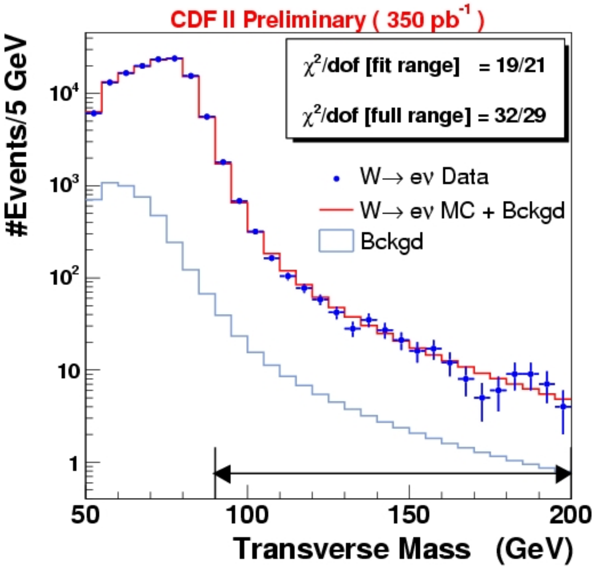




The Cambridge variable M_{T1}

- Single semi-invisibly decaying particle





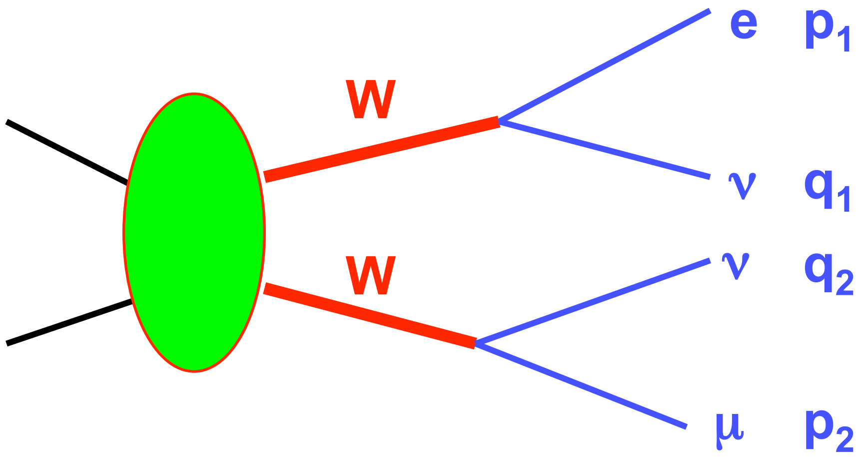
- Use the transverse mass distribution

$$M_W^2 \geq m_T^2(e,\nu) \equiv \left(\left|\vec{p}_{eT}\right| + \left|\vec{p}_{\nu T}\right|\right)^2 - \left(\vec{p}_{eT} + \vec{p}_{\nu T}\right)^2$$

The Cambridge variable M_{T2}

Lester,Summers 99
Barr,Lester,Stephens 03

- A pair of semi-invisibly decaying particles



$$\not{E}_T = \vec{q}_1 + \vec{q}_2 = -(\vec{p}_1 + \vec{p}_2)$$

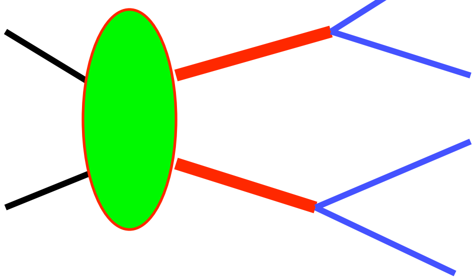
- If q_1 and q_2 were known, use the larger m_T
- Since q_1 and q_2 unknown, minimize the larger m_T

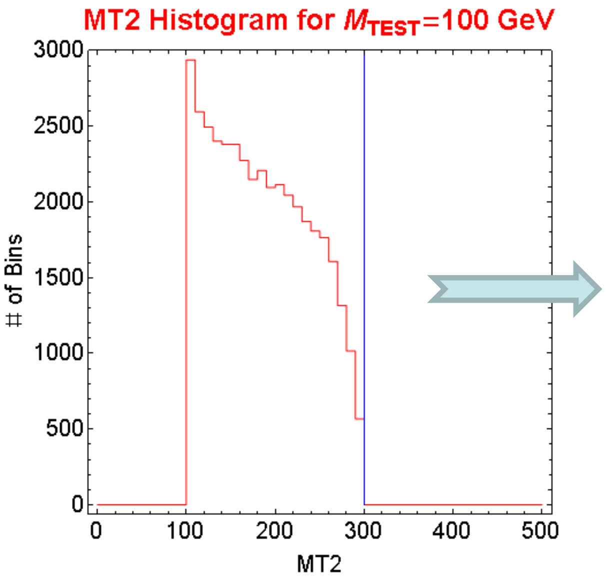
$$M_W^2 \geq m_{T2}^2 \equiv \min_{\vec{q}_1+\vec{q}_2=-(\vec{p}_1+\vec{p}_2)} \{\max\{m_T^2(\vec{p}_1,\vec{q}_1), m_T^2(\vec{p}_2,\vec{q}_2)\}\}$$

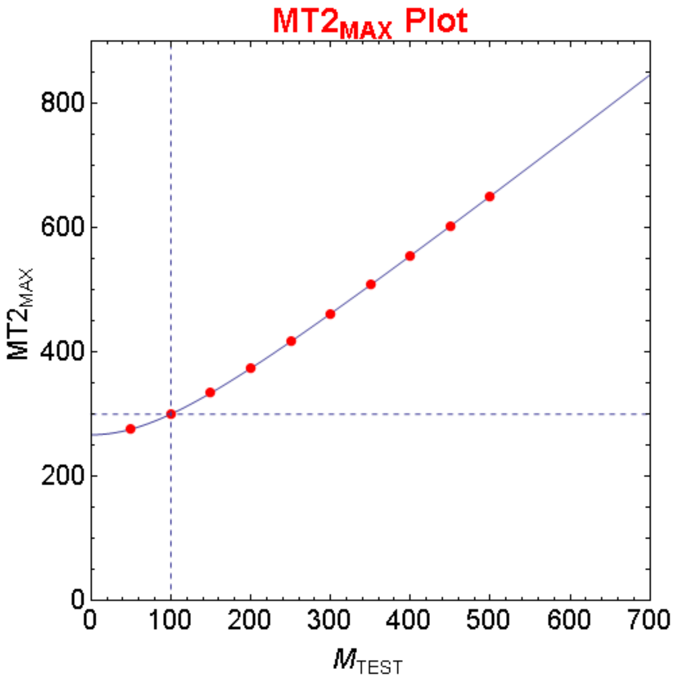
What is m_{T2} good for?

- Also applies when the missing particle is massive
- Provides one parent-child mass relation
  - Vary the child (LSP) mass, read the endpoint of m_{T2}, identify with the parent (slepton) mass

Lester,Summers 99





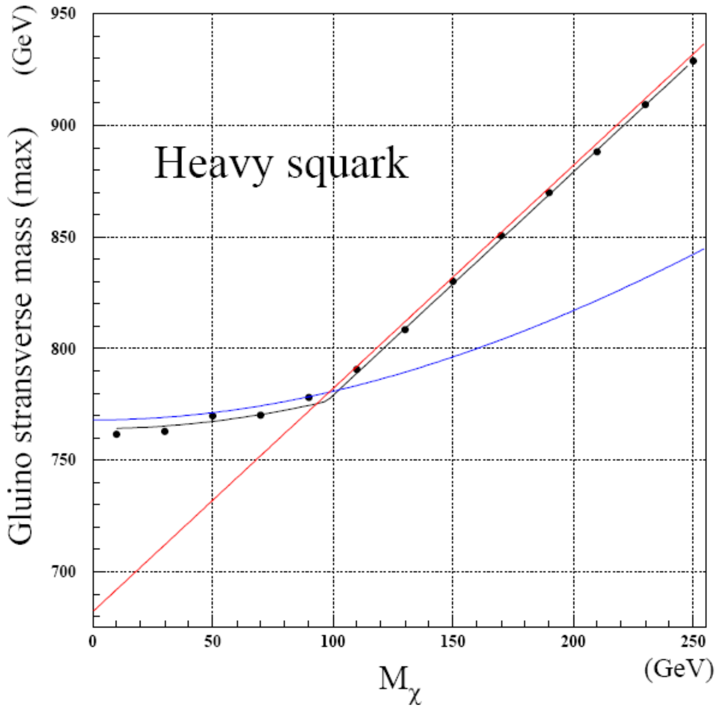


- So what? We still don't know exactly the LSP mass

LSP mass measurement from kinks

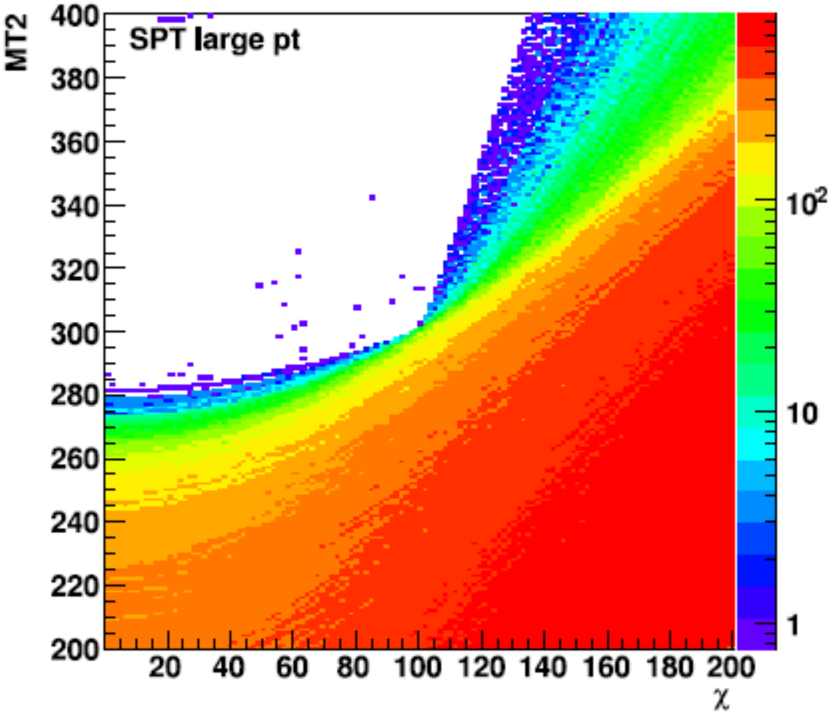
- N>1 particles on each side

Cho,Choi,Kim,Park 2007



- Large p_T recoil due to ISR

Barr,Gripaios,Lester 2007

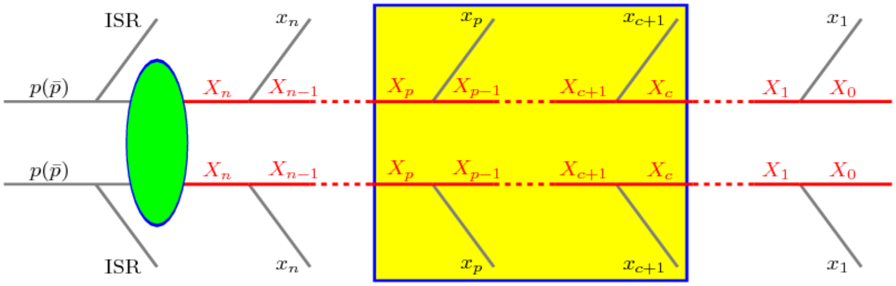


- The kink is at the true masses of the parent and the child

Subsystem M_{T2}(n,p,c)

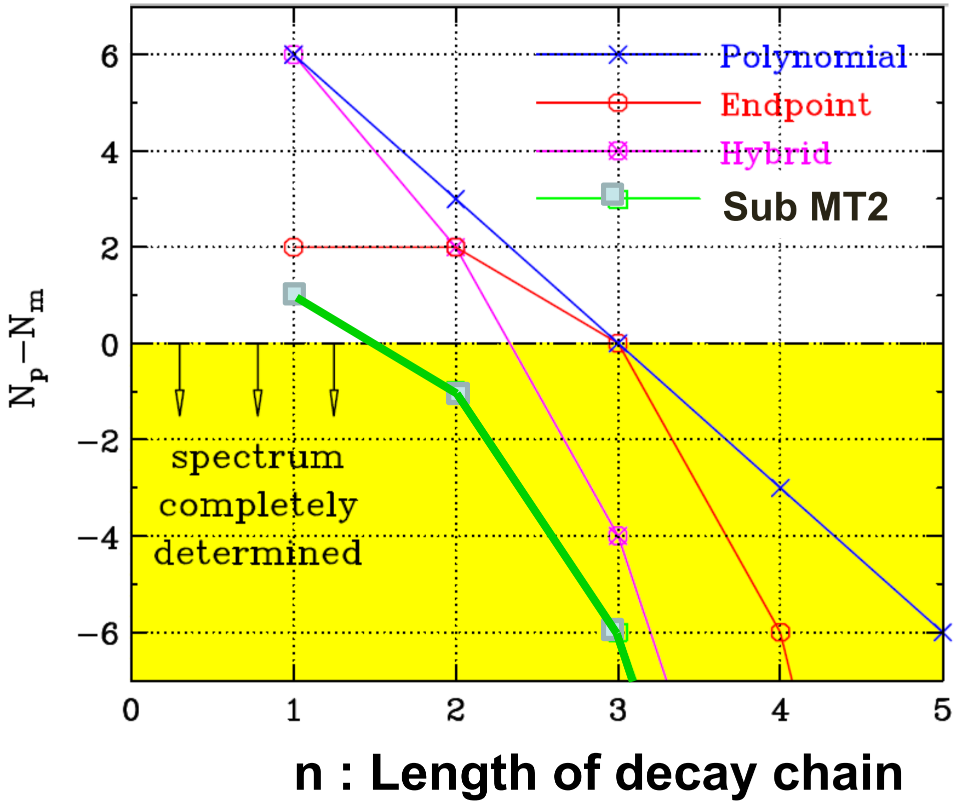
Serna 2008

Burns, Kong, KM, Park 2008



N_P : Number of unknowns

N_m : Number of measurements



N_P= number of BSM particles

= n+1

$$N_m = \sum_{p=1}^{n} p\,(n-p+1) = \frac{1}{6}\,n\,(n+1)\,(n+2)$$

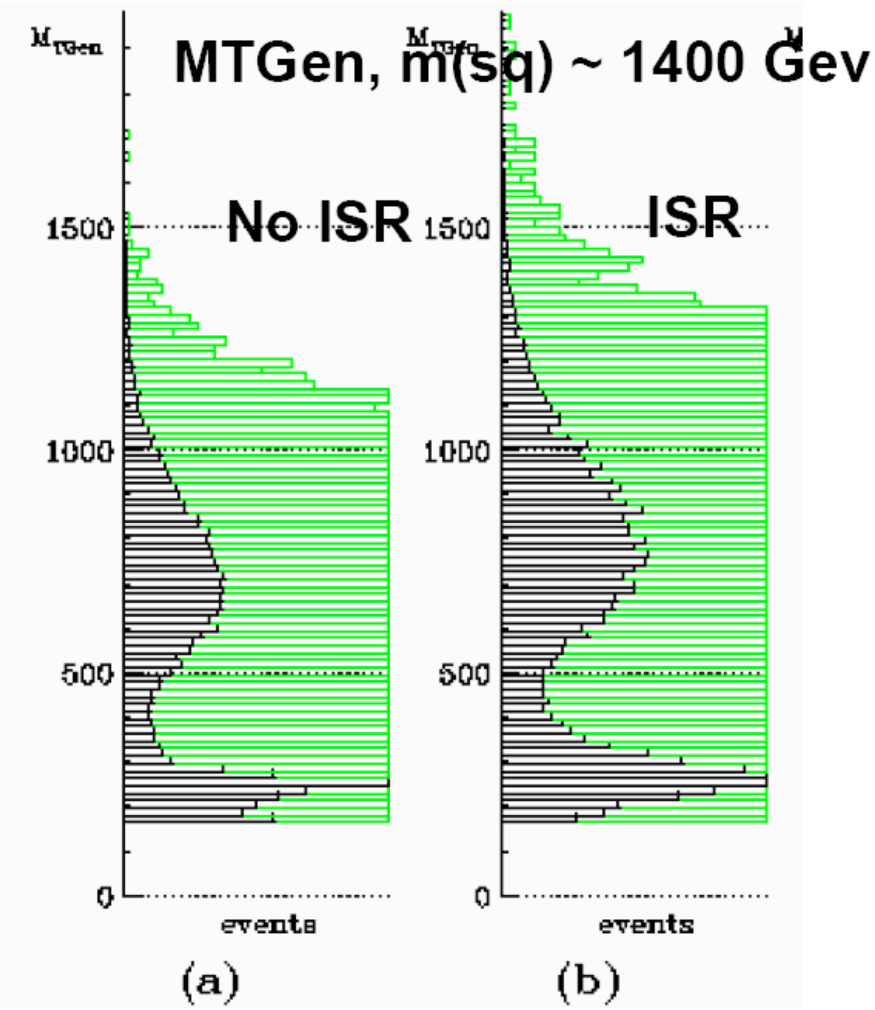
How many undetermined parameters (masses) are left?

$$N_p - N_m = \frac{1}{6}(n+1)(6-2n-n^2)$$

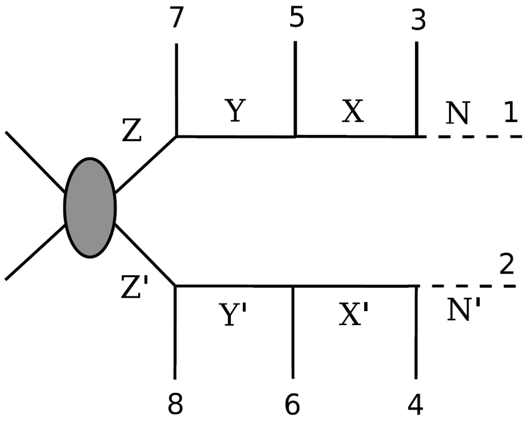
M_{Tgen}

Lester,Barr 2008

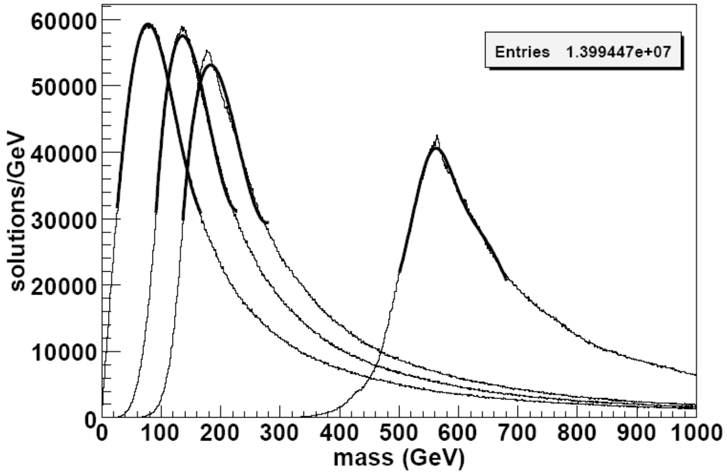
- Inclusive application of M_{T2}: minimize M_{T2} over all possible partitions of the visible decay products between two chains
  - Brute force way to deal with combinatorial issue
  - Preserves the endpoint, provides a measure of the scale
  - Endpoint smeared in the presence of ISR
  - Does not measure the LSP mass
  - Difficult to interpret when many processes contribute



Polynomial method



Cheng,Gunion,Han,Marandella,McElrath 2007
Cheng,Engelhardt,Gunion,Han,McElrath 2007



$$
\begin{array}{lrcl}
(M_Z^2 \;=) & (p_1+p_3+p_5+p_7)^2 & = & (p_2+p_4+p_6+p_8)^2, \\
(M_Y^2 \;=) & (p_1+p_3+p_5)^2 & = & (p_2+p_4+p_6)^2, \\
(M_X^2 \;=) & (p_1+p_3)^2 & = & (p_2+p_4)^2, \\
(M_N^2 \;=) & p_1^2 & = & p_2^2.
\end{array}
$$

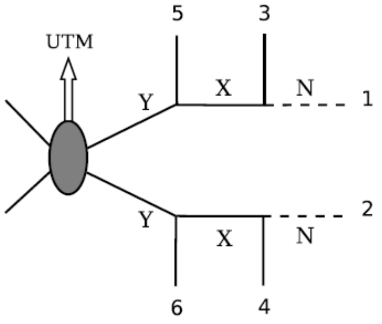
$$p_1^x + p_2^x = p_{miss}^x, \quad p_1^y + p_2^y = p_{miss}^y$$

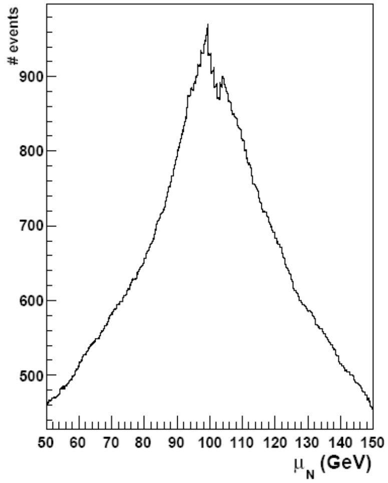
$$
\begin{array}{rcccl}
q_1^2 & = & q_2^2 & = & p_2^2, \\
(q_1+q_3)^2 & = & (q_2+q_4)^2 & = & (p_2+p_4)^2, \\
(q_1+q_3+q_5)^2 & = & (q_2+q_4+q_6)^2 & = & (p_2+p_4+p_6)^2, \\
\end{array}
$$

$$(q_1+q_3+q_5+q_7)^2 = (q_2+q_4+q_6+q_8)^2$$

$$q_1^x + q_2^x = q_{miss}^x, \quad q_1^y + q_2^y = q_{miss}^y$$

Cheng,Han 2008





In place of an outline

Missing momenta reconstruction?	Mass measurements		Spin measurements
	Inclusive	2 symmetric chains	
None	Inv. mass endpoints and boundary lines		Inv. mass shapes
	$M_{eff,}M_{est},H_T$	Wedgebox	
Approximate	$S_{min,}$ M_{Tgen}	M_{T2}, M_{2C}, $M_{3C,}$ $M_{CT,}$ M_{T2}(n,p,c)	As usual
Exact	?	Polynomial method	As usual

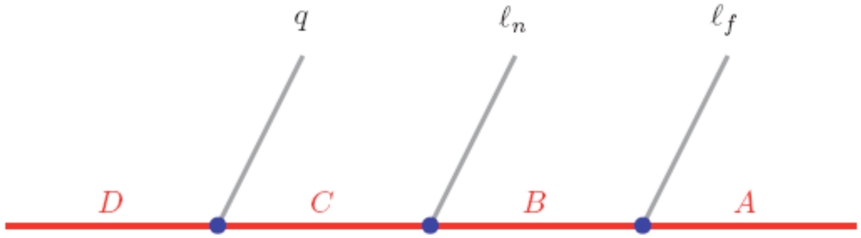
pessimism

optimism

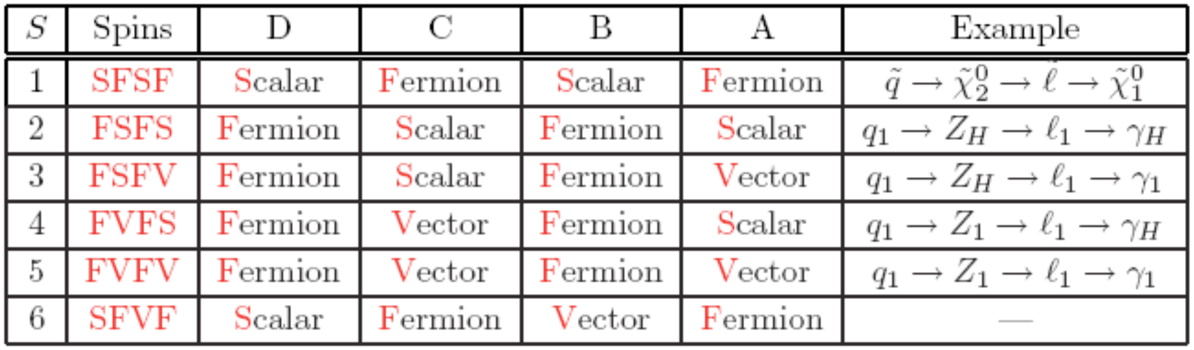
pessimism **optimism**

Why is it difficult to measure the spin?

- Missing energy signatures arise from something like:



- Several alternative explanations:

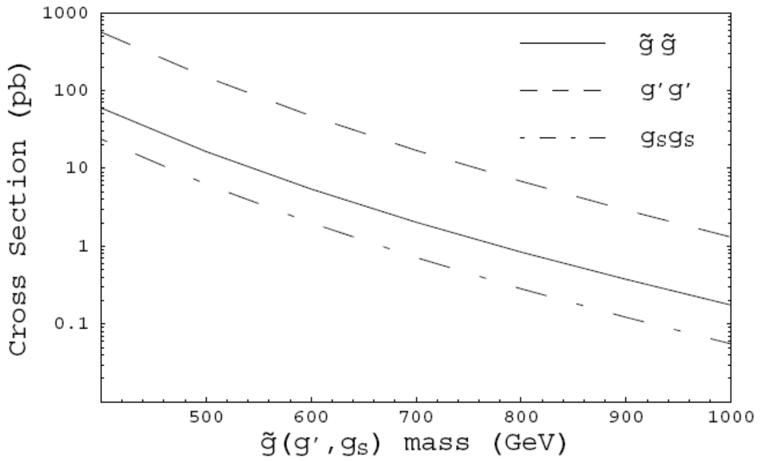


S	Spins	D	C	B	A	Example
1	SFSF	Scalar	Fermion	Scalar	Fermion	$\tilde{q} \to \tilde{\chi}_2^0 \to \tilde{\ell} \to \tilde{\chi}_1^0$
2	FSFS	Fermion	Scalar	Fermion	Scalar	$q_1 \to Z_H \to \ell_1 \to \gamma_H$
3	FSFV	Fermion	Scalar	Fermion	Vector	$q_1 \to Z_H \to \ell_1 \to \gamma_1$
4	FVFS	Fermion	Vector	Fermion	Scalar	$q_1 \to Z_1 \to \ell_1 \to \gamma_H$
5	FVFV	Fermion	Vector	Fermion	Vector	$q_1 \to Z_1 \to \ell_1 \to \gamma_1$
6	SFVF	Scalar	Fermion	Vector	Fermion	—

Spin measurements from production cross-section

- The cross-section knows about the spin: measure the cross-section and you will know the spin.
- Are we really measuring the production cross-section?

Kane,Petrov,Shao,Wang 2008



$$Rate = L\left[\sigma(XX) + \sigma(XY)B(Y \rightarrow X) + \sigma(YY)B^2(Y \rightarrow X)\right]B^2(X \rightarrow SM)$$

- How can we be sure that
  - There is no contribution from indirect production of particle Y?
    - Think of W pair production from top quarks
  - The branching fraction B(X->SM) is 100 %?
- The spin cannot be determined by measuring 1 number
  - Must look at distributions

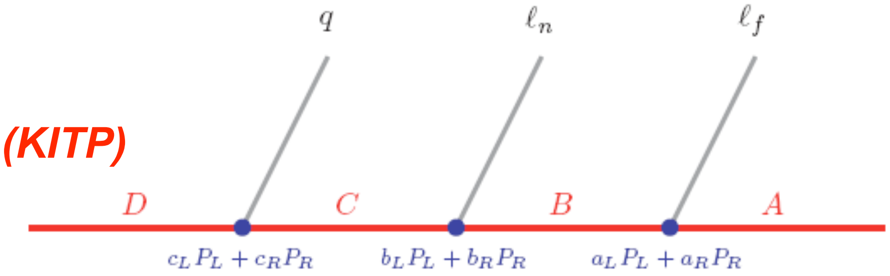
What is a good distribution to look at?

- Invariant mass distributions

  Athanasiou et al 06, Kilic,Wang,Yavin 07, Csaki,Heinonen,Perelstein 07, S. Thomas (KITP)

- Advantages:
  - well studied
  - know about spin
- Disadvantage: know about many other things, not all of which are measured!
  - Masses M_A, M_B, M_C, M_D (x,y,z)
  - Couplings and mixing angles (g_L and g_R)
  - Particle-antiparticle (D/D*) fraction (f/f*) (f+f*=1)
- Ask the right question:

Given the data, can any of these spin configurations give a good fit for any values of the other relevant parameters?



$$x = \frac{m_C^2}{m_D^2} \qquad y = \frac{m_B^2}{m_C^2} \qquad z = \frac{m_A^2}{m_B^2}$$

$$\tan\varphi_a = \frac{|a_R|}{|a_L|}\,, \qquad \tan\varphi_b = \frac{|b_R|}{|b_L|}\,, \qquad \tan\varphi_c = \frac{|c_R|}{|c_L|}$$

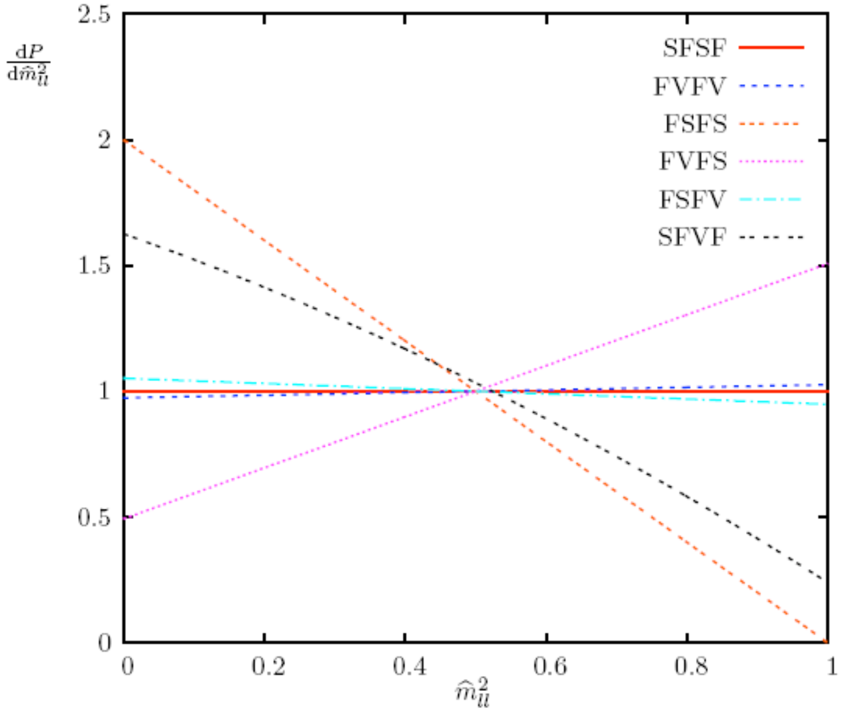
$$\boxed{g_{L,R} \equiv \mathbf{U_F}^{\dagger}\; g^{(0)}_{L,R}\; \mathbf{U_B}}$$

S	Spins	D	C	B	A
1	SFSF	Scalar	Fermion	Scalar	Fermion
2	FSFS	Fermion	Scalar	Fermion	Scalar
3	FSFV	Fermion	Scalar	Fermion	Vector
4	FVFS	Fermion	Vector	Fermion	Scalar
5	FVFV	Fermion	Vector	Fermion	Vector
6	SFVF	Scalar	Fermion	Vector	Fermion

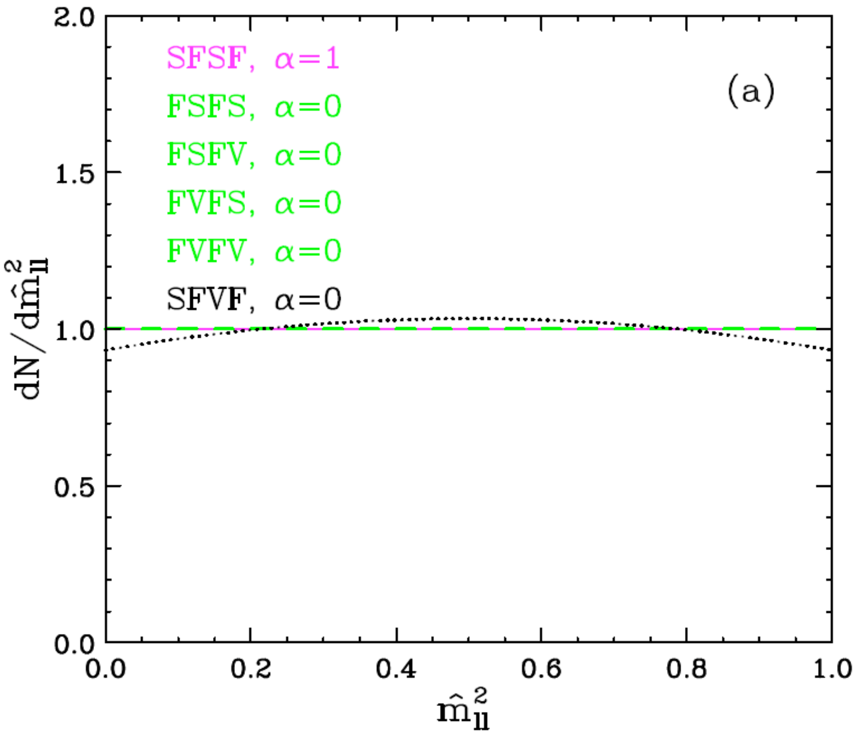
Does this really make any difference?

- Yes! Dilepton invariant mass distribution. Data from SPS1a.

Athanasiou, Lester, Smillie, Webber 06



Burns, Kong, KM, Park 08



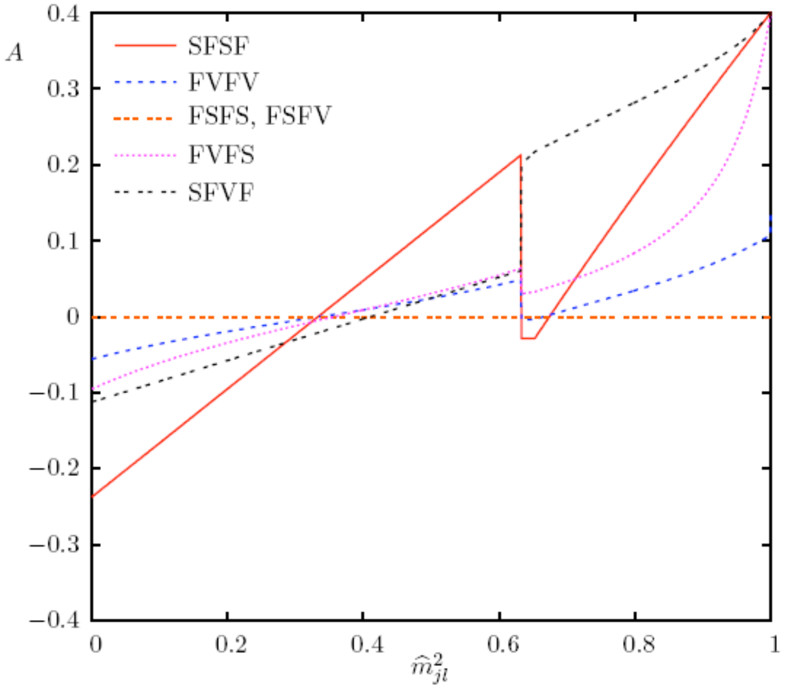
- Spins vary
- Everything else fixed to SPS1a values
- Easy to distinguish!

- Mass spectrum fixed to SPS1a values
- Everything else varies
- Difficult to distinguish!

Does this really make any difference?

- Yes! Lepton charge (Barr) asymmetry. Data: “UED” with SPS1a mass spectrum.

Athanasiou, Lester, Smillie, Webber 06



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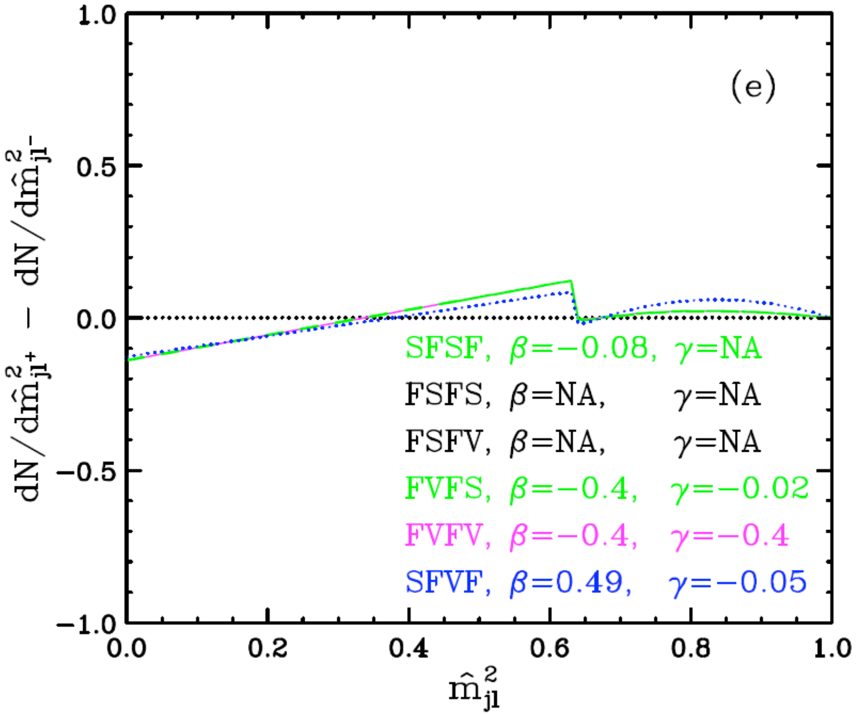
Burns, Kong, KM, Park 08



- Mass spectrum fixed to SPS1a values
- Everything else varies
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How do we do it?

Burns, Kong, KM, Park 08

- Separate the spin dependence from all the rest
  - Parameterize conveniently the effect from “all the rest”

$$\left(\frac{dN}{dm^2}\right)_S = F_{S;\delta}(m^2) + \alpha F_{S;\alpha}(m^2) + \beta F_{S;\beta}(m^2) + \gamma F_{S;\gamma}(m^2)$$

- Measure both the spin (S) as well as all the rest: α, β, γ

Data from	Can this data be fitted by model					
	SFSF	FSFS	FSFV	FVFS	FVFV	SFVF
SFSF	yes	no	no	no	no	no
FSFS	no	yes	maybe	no	no	no
FSFV	no	yes	yes	no	no	no
FVFS	no	no	no	yes	maybe	no
FVFV	no	no	no	yes	yes	no
SFVF	no	no	no	no	no	yes

In place of a summary

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pessimism → **optimism**

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