Jet Definitions

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Does good theory matter?

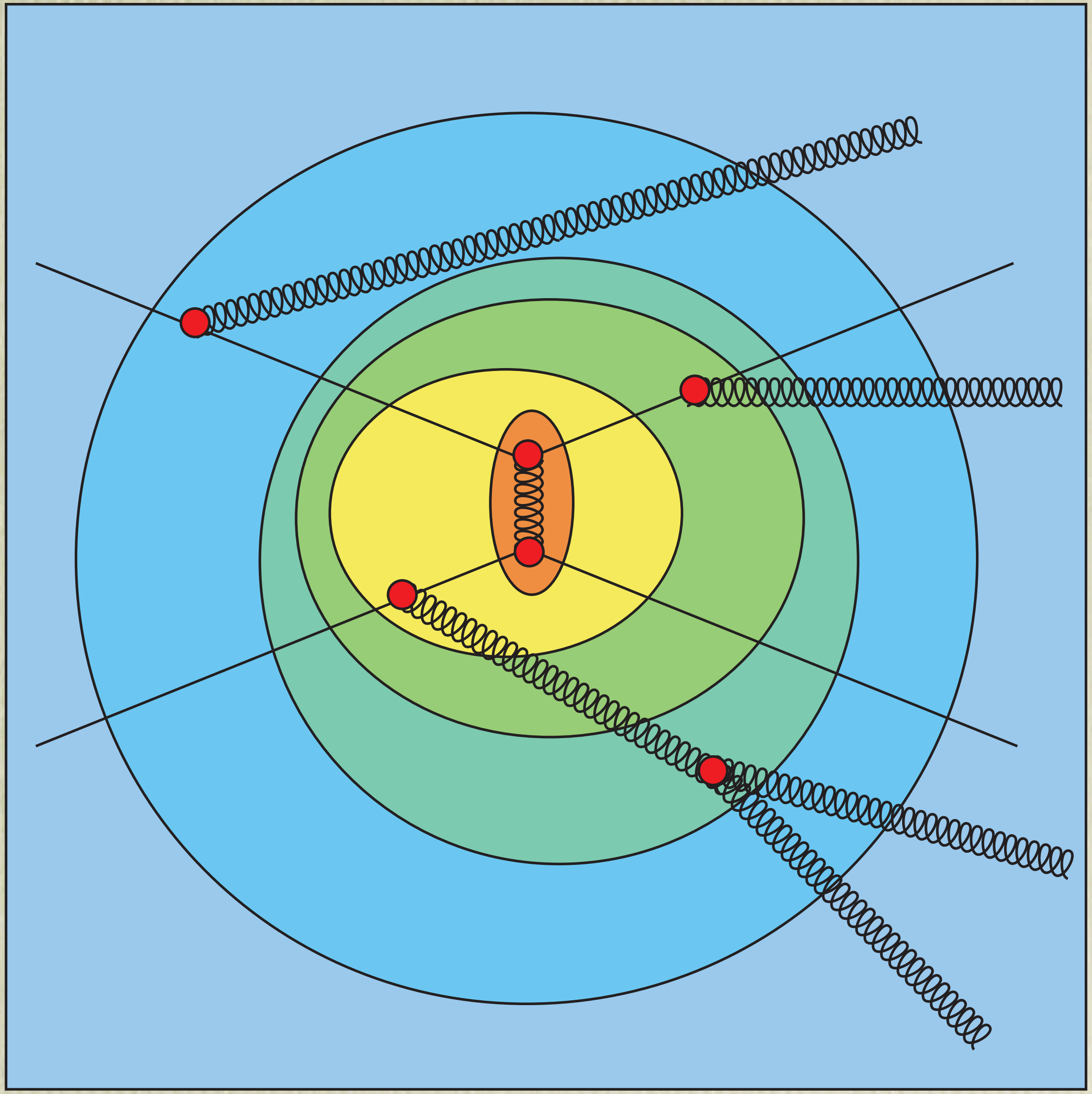
- I will discuss “infrared safe” observables.
- For these the theoretical error is
  - under control
  - not too big.
- Could anything go wrong with not-so-good theory?

- One could “see” a new physics effect that isn’t there.
- One could adjust parameters so as to get rid of a new physics effect that is there.

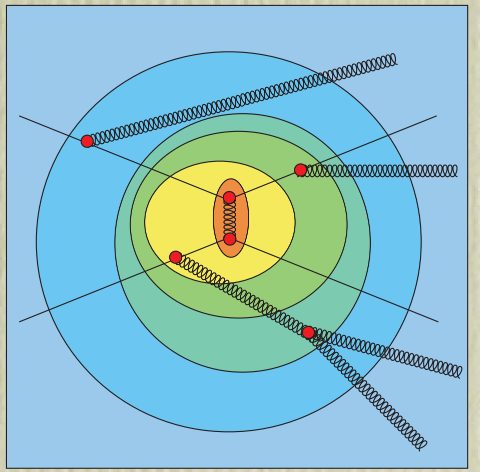
Odysseus between Scyla and Charybdis
Johann Heinrich Füssli, 1794-1796

Parton shower structure

- Standard Model or new physics hard interaction.
- Not-so-hard gluon emission.
- Softer gluon emission ...
- Finally, hadrons.



Infrared safety



- The softer stuff is less reliably calculated.
- So use measurements that are insensitive to soft physics.

Example of infrared danger

From G. Wolf at Multiparticle Dynamics 1983.

- Use e^+e^- annihilation event shapes and hadron energy spectrum to measure strong coupling.
  - $\alpha_s(M_Z) = 0.13 \pm 0.01$, independent jet model.
  - $\alpha_s(M_Z) = 0.17 \pm 0.01$, string model.
- Why?
  - Measured quantities not infrared safe.
  - Theory mixed short and long distance physics.

Perturbative definition

Consider electron-positron annihilation.

Define quantity to be measured by

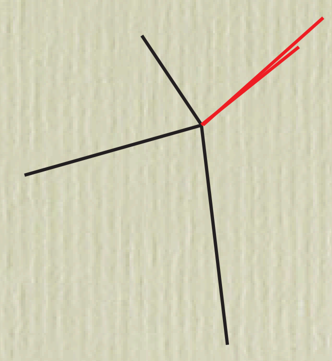
$$
\begin{aligned}
\mathcal{I} \;=\; & \frac{1}{2!}\int d\Omega_2\, \frac{d\sigma[2]}{d\Omega_2}\, \mathcal{S}_2(p_1^\mu, p_2^\mu) \\
& + \frac{1}{3!}\int d\Omega_2 dE_3 d\Omega_3\, \frac{d\sigma[3]}{d\Omega_2 dE_3 d\Omega_3}\, \mathcal{S}_3(p_1^\mu, p_2^\mu, p_3^\mu) \\
& + \frac{1}{4!}\int d\Omega_2 dE_3 d\Omega_3 dE_4 d\Omega_4 \\
& \quad \times \frac{d\sigma[4]}{d\Omega_2 dE_3 d\Omega_3 dE_4 d\Omega_4}\, \mathcal{S}_4(p_1^\mu, p_2^\mu, p_3^\mu, p_4^\mu) \\
& + \cdots .
\end{aligned}
$$

Then the measurement is specified by functions

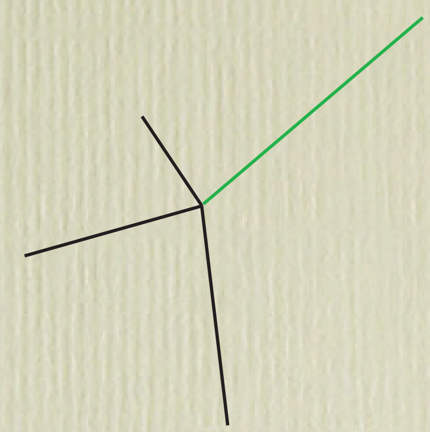
$$\mathcal{S}_n(p_1^\mu, p_2^\mu, \ldots, p_n^\mu)$$

IR safety for the measurement functions

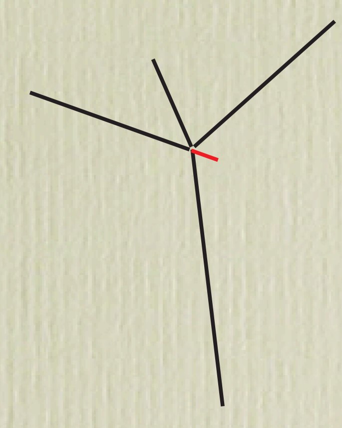
$$\mathcal{S}_{n+1}(p_1^\mu, \ldots, (1-\lambda)p_n^\mu, \lambda p_n^\mu) = \mathcal{S}_n(p_1^\mu, \ldots, p_n^\mu)$$



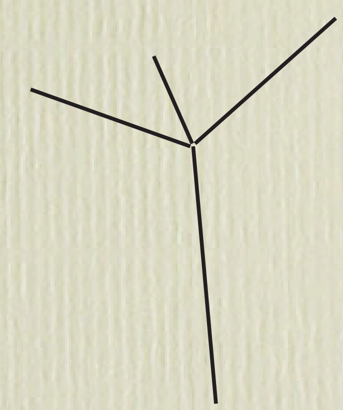
$\approx$



$$\mathcal{S}_{n+1}(p_1^\mu, \ldots, p_n^\mu, 0) = \mathcal{S}_n(p_1^\mu, \ldots, p_n^\mu)$$



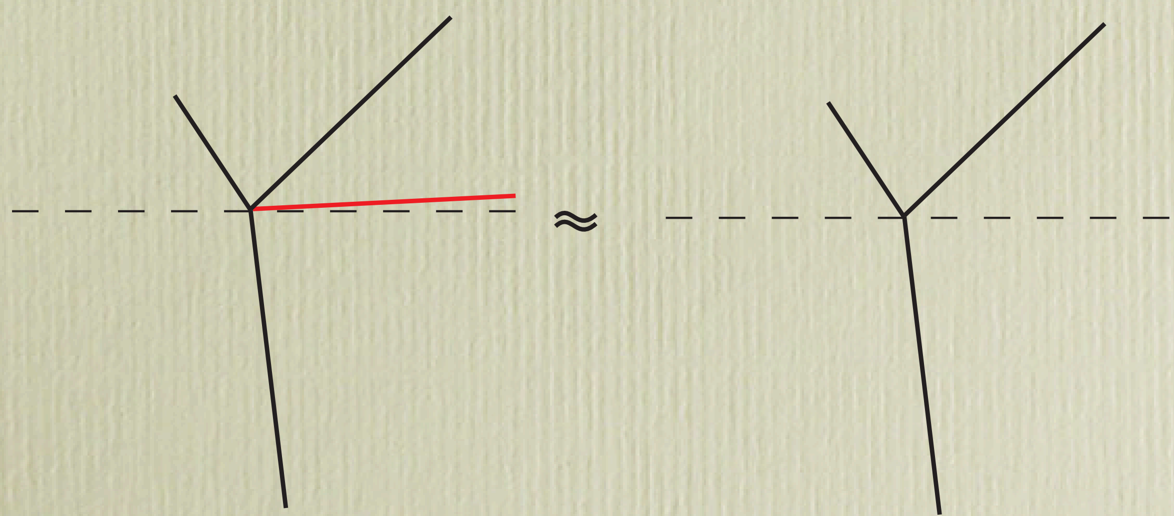
$\approx$



Additionally for hadron-hadron collisions

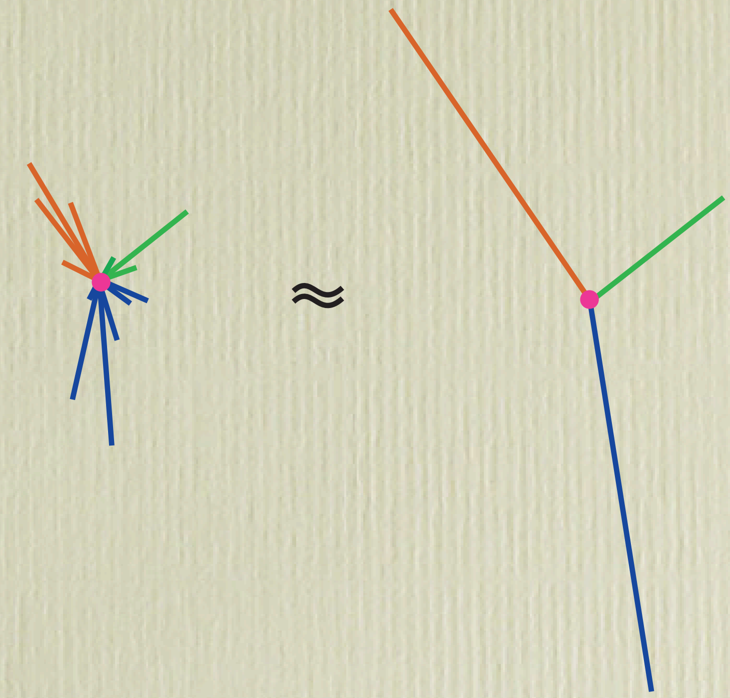
$$\mathcal{S}_{n+1}(p_1^\mu, \ldots, p_n, \lambda p_A^\mu) = \mathcal{S}_n(p_1^\mu, \ldots, p_n^\mu).$$

$$\mathcal{S}_{n+1}(p_1^\mu, \ldots, p_n, \lambda p_B^\mu) = \mathcal{S}_n(p_1^\mu, \ldots, p_n^\mu).$$



What does IR safety mean?

The physical meaning is that for an IR-safe quantity, the physical event with hadron jets should give approximately the same measurement as a parton event.



The calculational meaning is that infinities cancel.

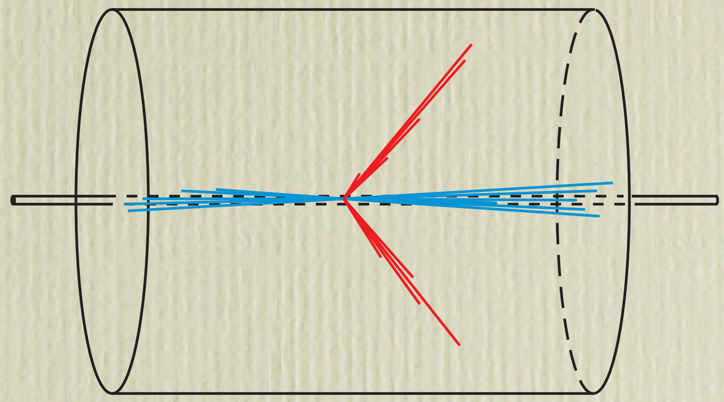
Jet Definitions

- Consider a jet cross section differential in transverse energy and rapidity,

$$\frac{d\sigma}{dE_T\, d\eta}$$

- Substantial E_T at large angles means care is needed in definition.

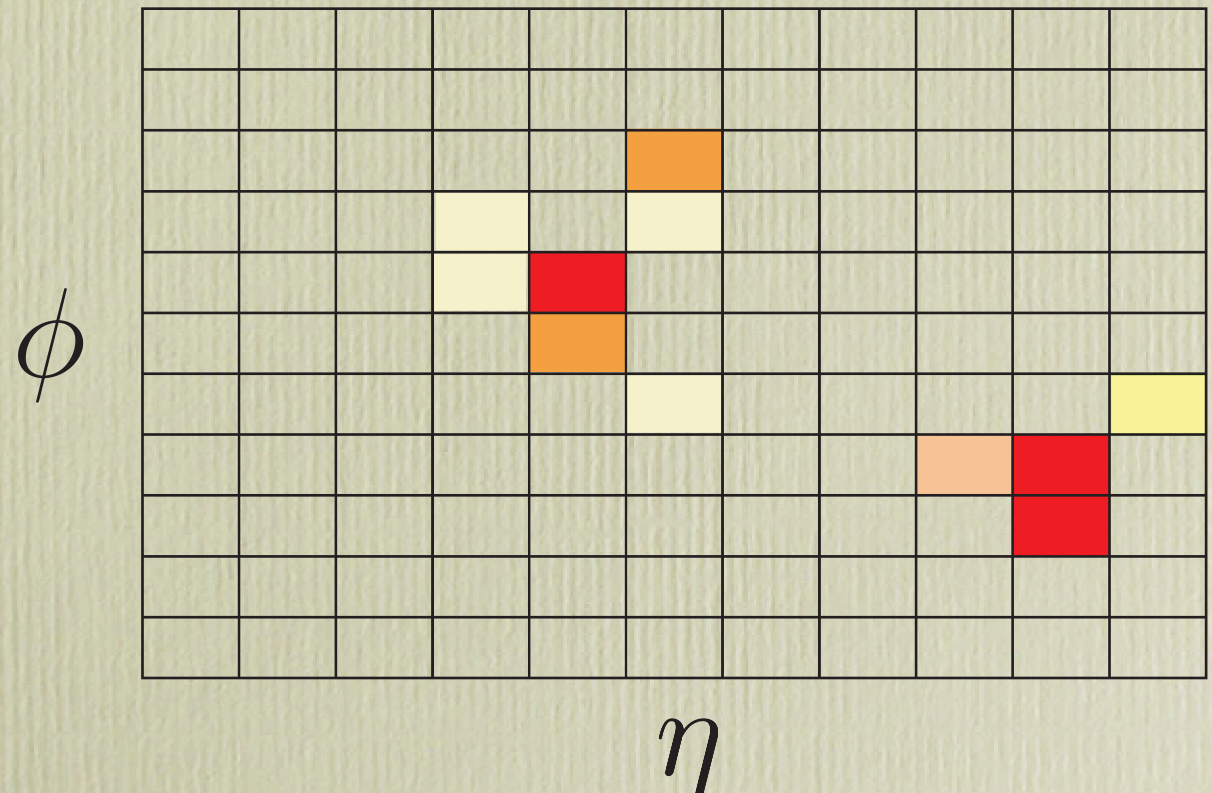
- Need IR-safe definition.



Cone definition basics

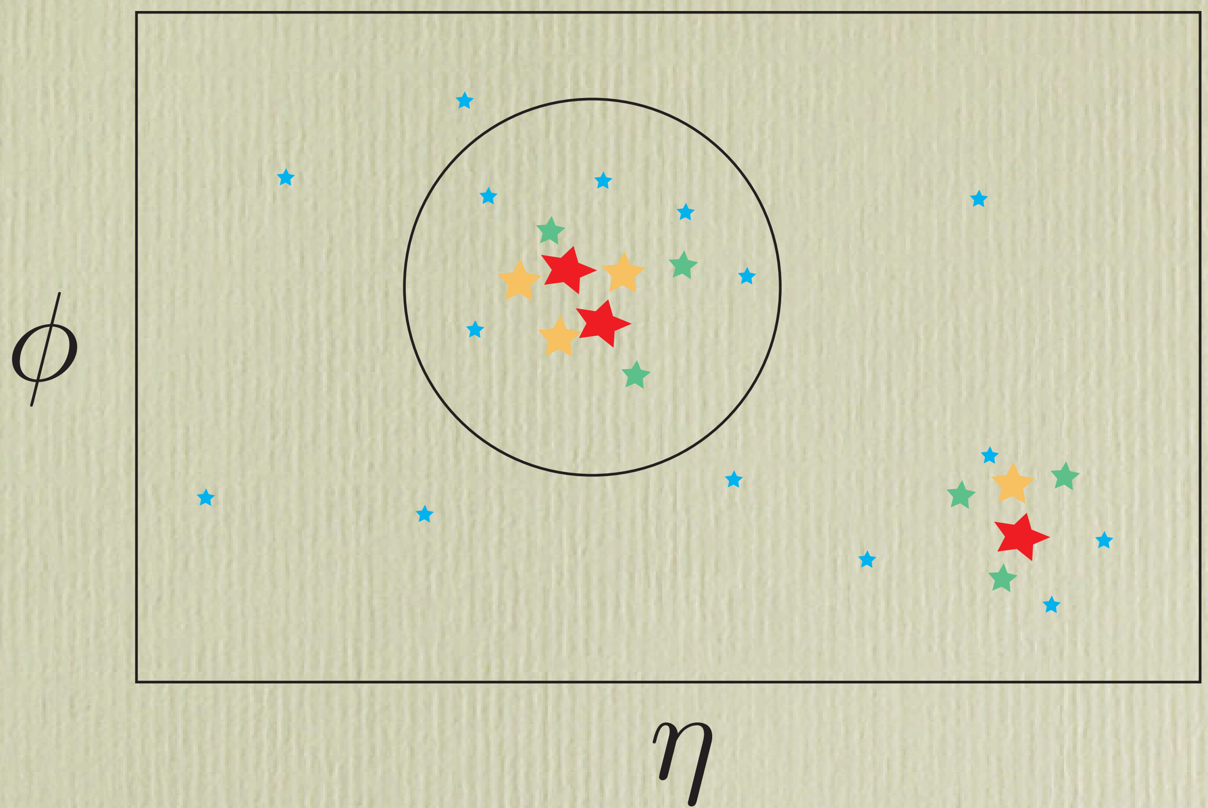
I recall the “Snowmass” definition.

Divide calorimeter into cells *i*.



Definition based on cell variables (E_{Ti}, η_i, ϕ_i)

There is a jet axis, (η_J, ϕ_J)



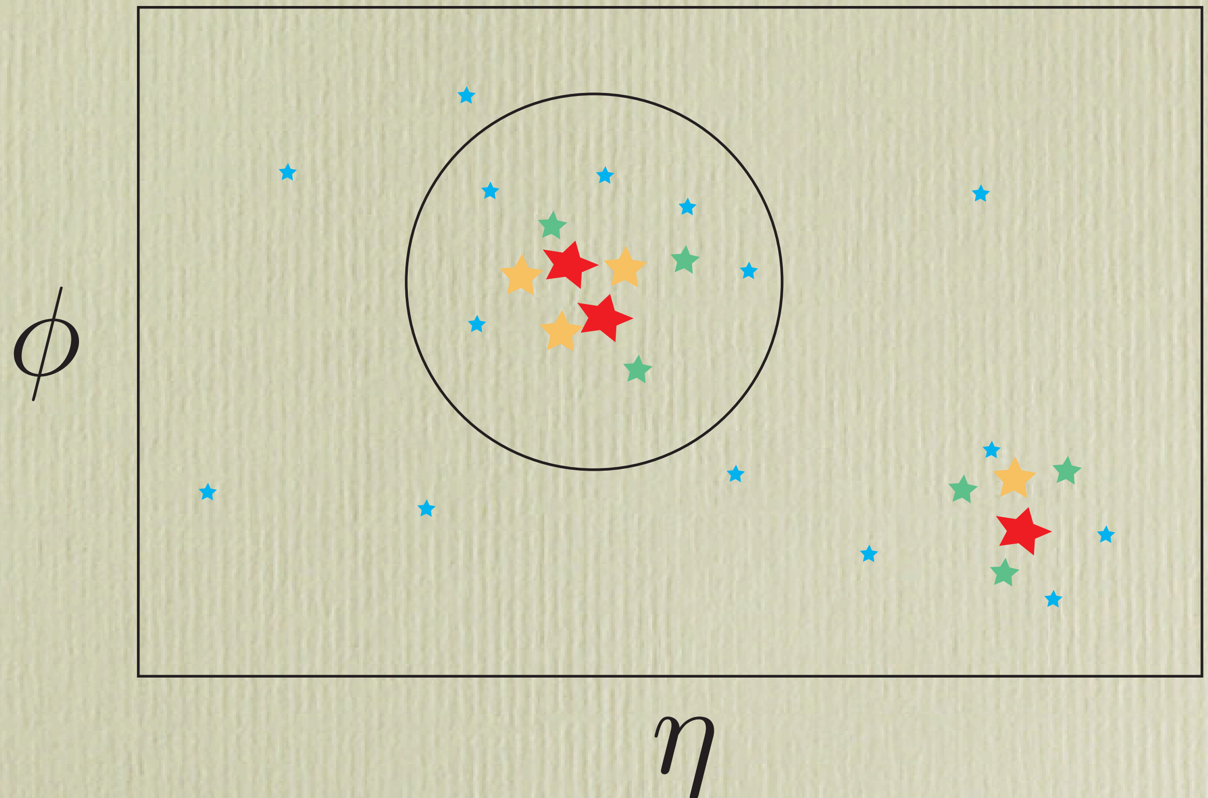
A jet consists of all of the cells with $(\eta_i - \eta_J)^2 + (\phi_i - \phi_J)^2 < R^2$

$$E_{TJ} = \sum_{i \in \text{cone}} E_{Ti}$$

A typical value for R is 0.7.

A jet consists of all of the cells with $(\eta_i - \eta_J)^2 + (\phi_i - \phi_J)^2 < R^2$

$$E_{TJ} = \sum_{i \in \text{cone}} E_{Ti}$$



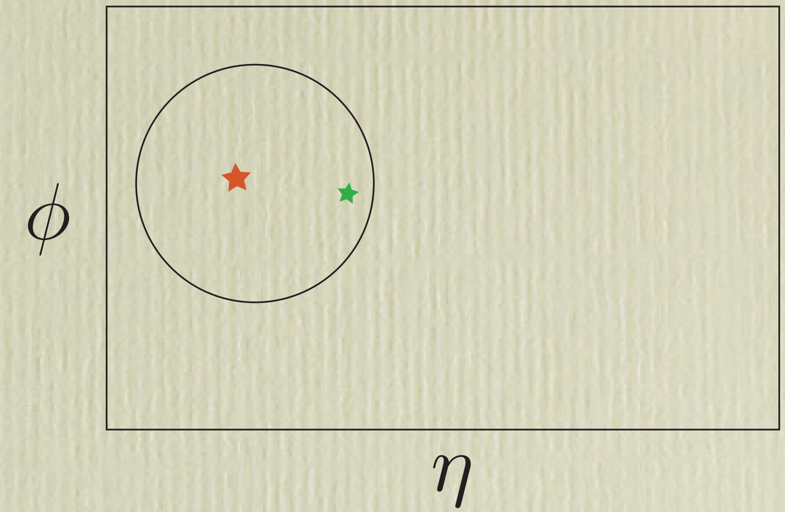
$$\eta_J = \frac{1}{E_{T,J}} \sum_{i \in \text{cone}} E_{Ti} \eta_i \qquad \phi_J = \frac{1}{E_{T,J}} \sum_{i \in \text{cone}} E_{Ti} \phi_i$$

There is some "fine print," but we will come to that later.

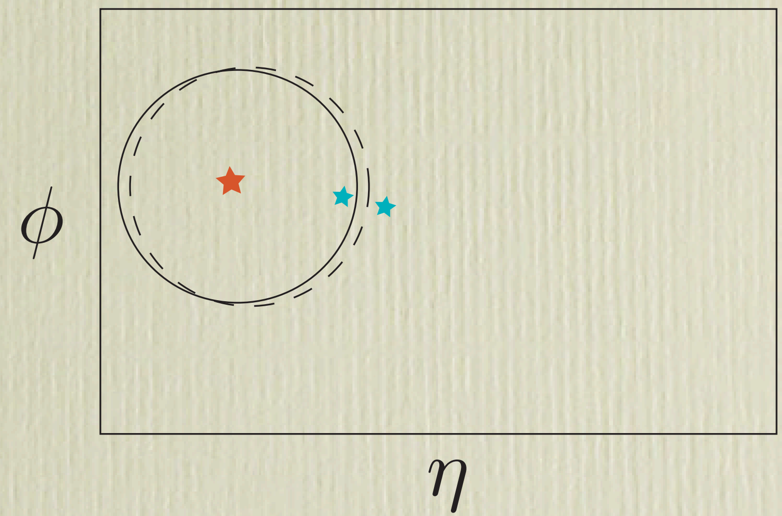
Is this infrared safe?

Consider an example.

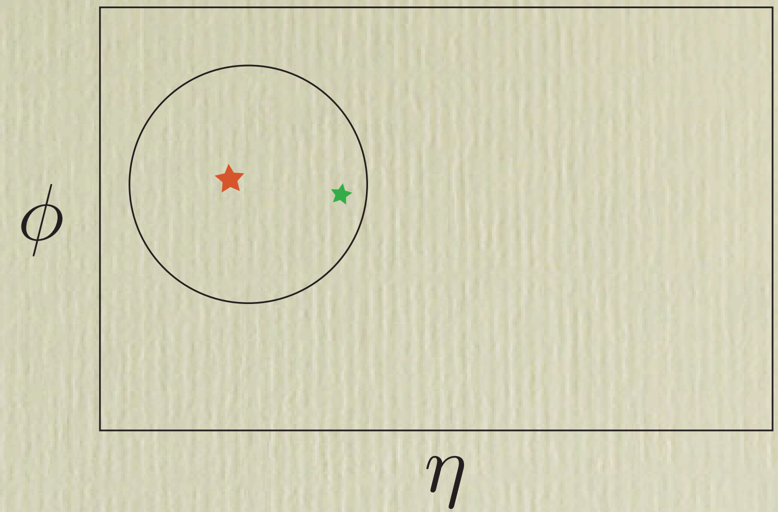
- Here is a two parton jet.



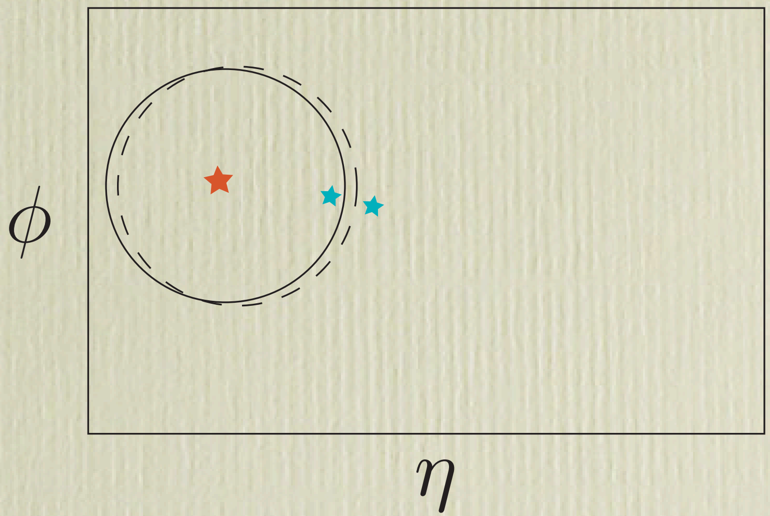
- Here one parton has split into two almost collinear partons.
- The new jet has less E_T.



- But if the splitting angle becomes smaller than the offset of the parton pair from the cone edge, both fit inside the jet.



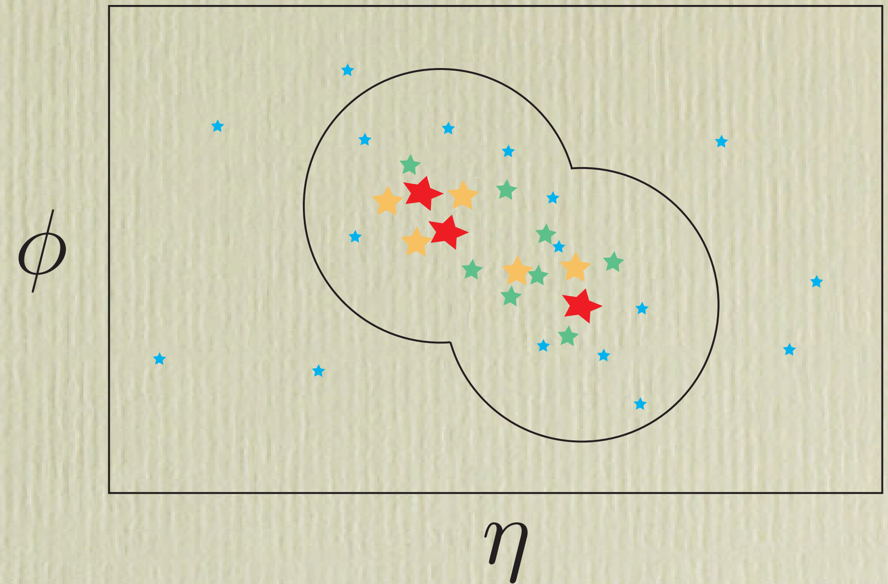
- In a perturbative calculation, there is still a singularity.
- But the singularity is integrable.

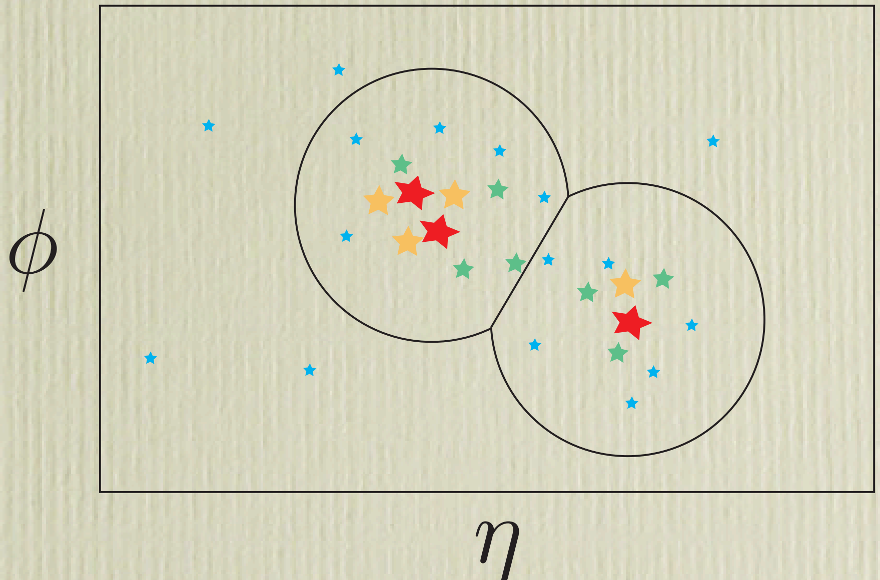


So IR safety is OK, despite cone edges.

Some fine print

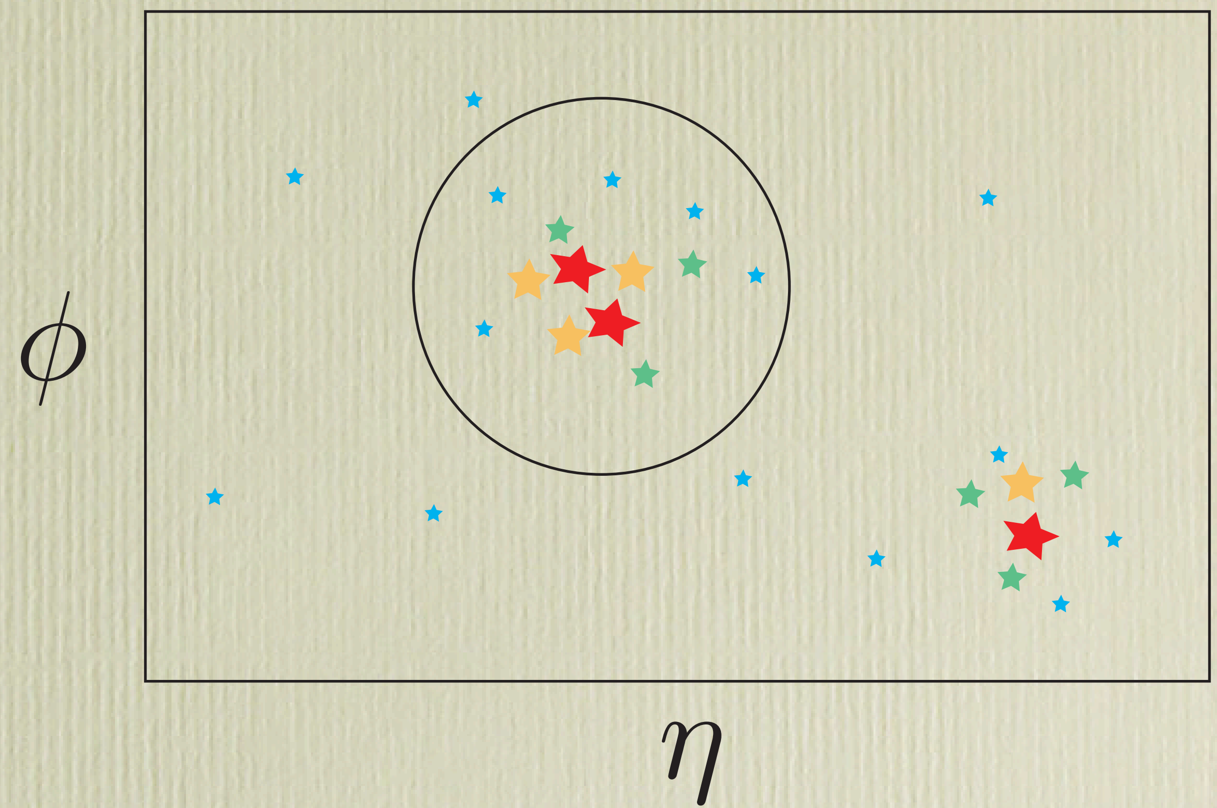
- Cones can overlap.
- Merge them if a substantial fraction of the E_T is in the overlap region.
- Otherwise split them.
- This is ugly, but IR safe.





Seeds

- To actually do this, start with a trial jet direction and iterate.
- Starting anywhere near the center of the cone shown will give the jet with that cone.
- Method has been to start with seeds, cells with E_T above a (low) threshold.

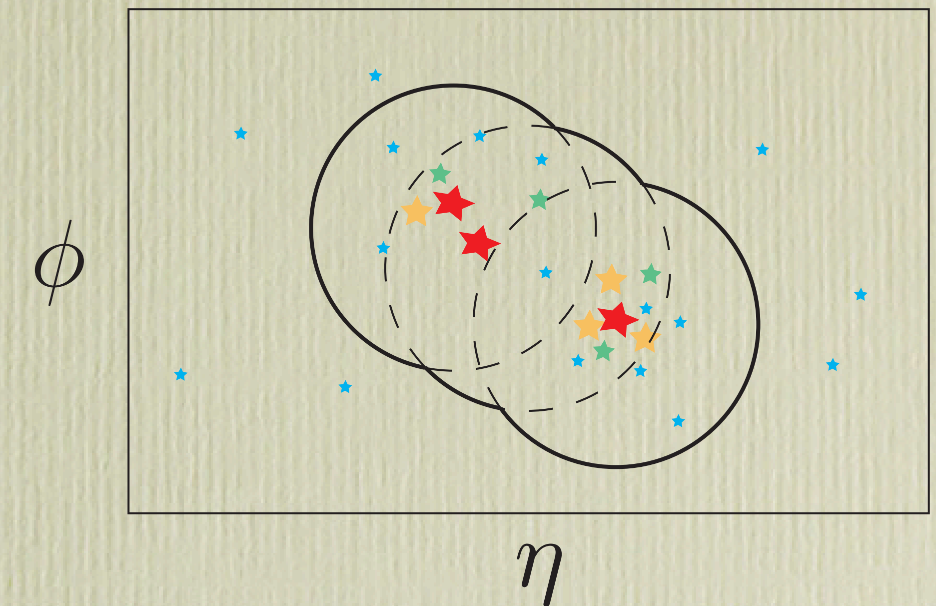


From a tiny seed ...

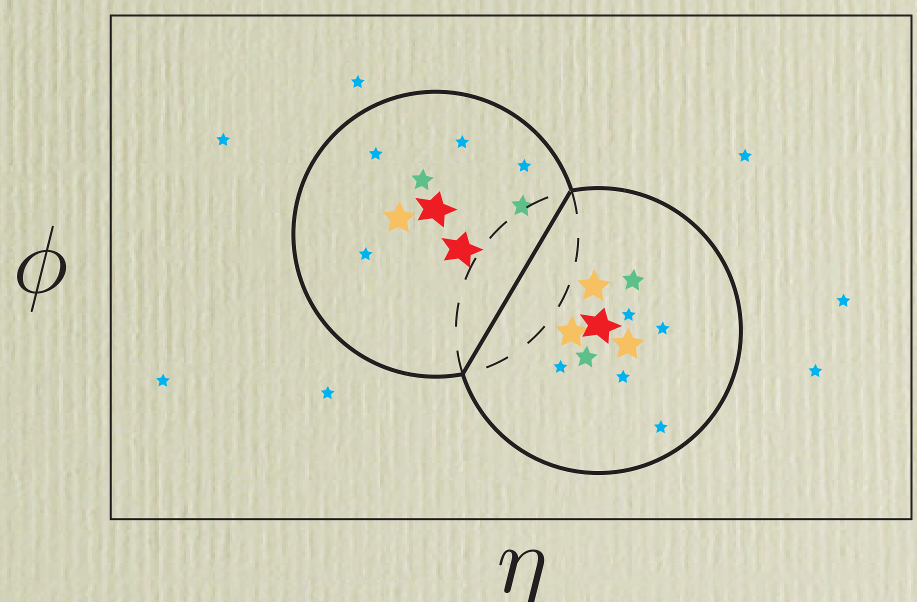
a giant jet can grow.

Seedy example

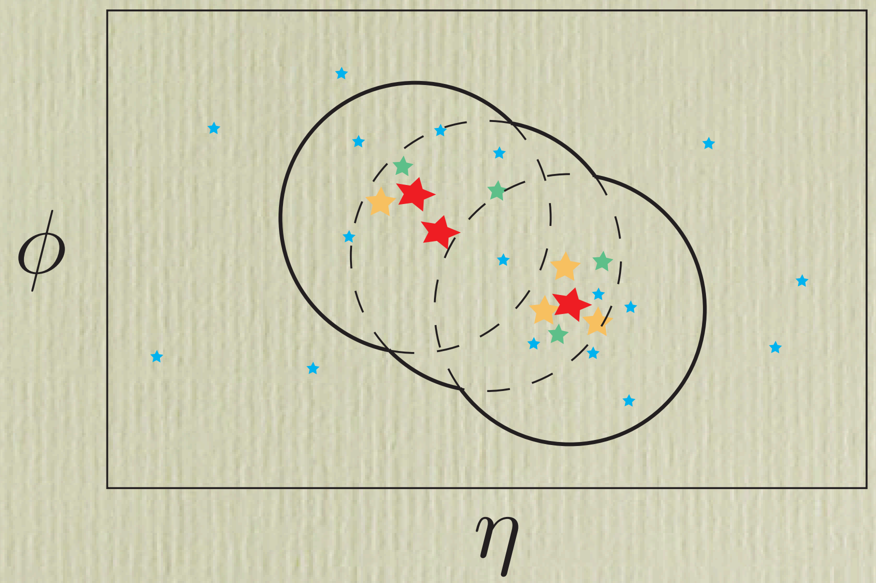
- With seed: three jets, merged.



- No seed: two jets, split.



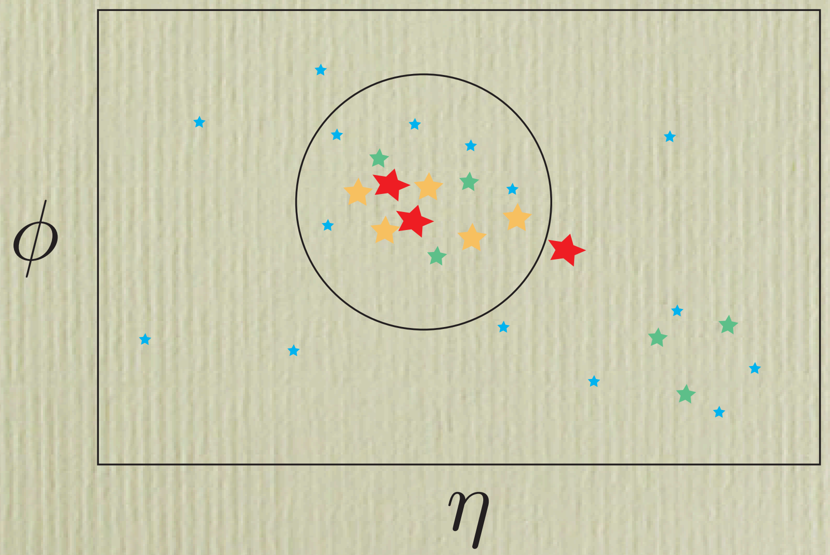
What to do?



- Use "seedless" algorithm. (*I.e.* seeds everywhere.)
- Fix algorithm (*e.g.* with extra seeds between jets) and demonstrate numerically that result is close to seedless result.

On the walls of cone jets

- Sometimes a tower with substantial E_T is just walled out.
- I think that is just the nature of cone algorithms.



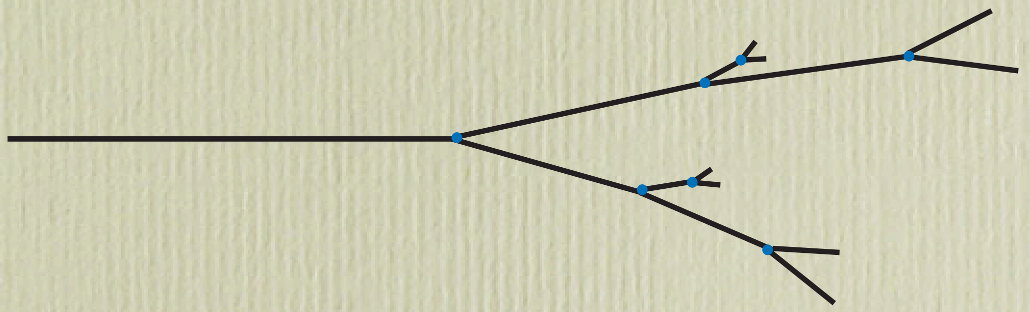
Something there is that doesn't love a wall,

...

Before I built a wall I'd ask to know
What I was walling in or walling out,
And to whom I was like to give offence.

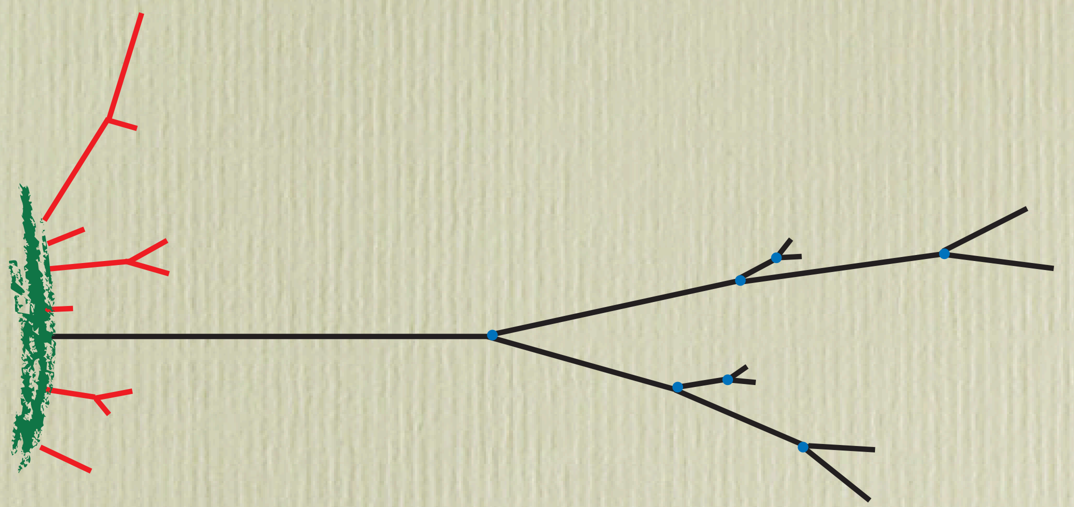
What people really want

- In some approximation, the final states are created by parton showers.



- You might want to deconstruct the final state to produce "early stage" partons as jets.
- A resolution parameter would tell how far back to go.

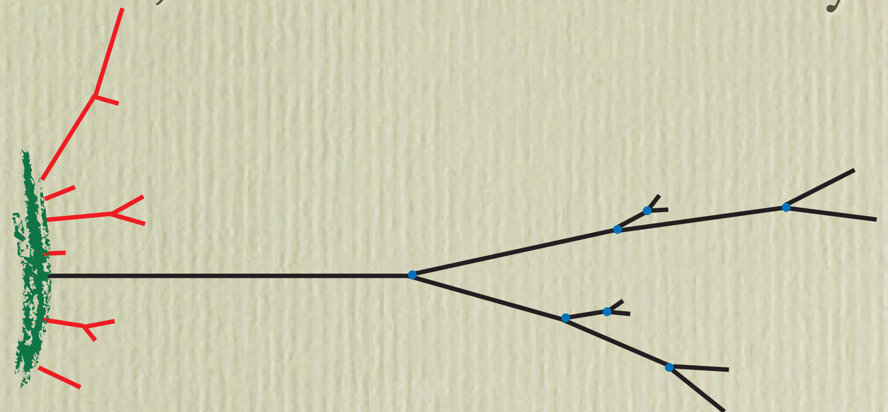
Beware of reality



- Soft gluons are really not associated with any particular jet.
- There can be lots of other (typically low E_T) particles that have nothing to do with your jet.

The k_T jet algorithm

Use E_T, η, ϕ variables; account for the many low E_T particles.



- Choose a resolution parameter *R*.
- Start with a list of protojets, specified by their p_j^μ .
- Start with an empty list of finished jets.
- Result is a list of finished jets with their momenta.
- Many are low E_T debris; just ignore these.

1. For each pair of protojets define

$$d_{ij} = \min(E_{T,i}^2, E_{T,j}^2)\,[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$

For each protojet define

$$d_i = E_{T,i}^2$$

2. Find the smallest of all the d_{ij} and the d_i. Call it $d_{\min}$
3. If $d_{\min}$ is a d_{ij}, merge protojets i and j into a new protojet k with

$$\begin{aligned} E_{T,k} &= E_{T,i} + E_{T,j} \\ \eta_k &= [E_{T,i}\,\eta_i + E_{T,j}\,\eta_j]/E_{T,k} \\ \phi_k &= [E_{T,i}\,\phi_i + E_{T,j}\,\phi_j]/E_{T,k} \end{aligned}$$

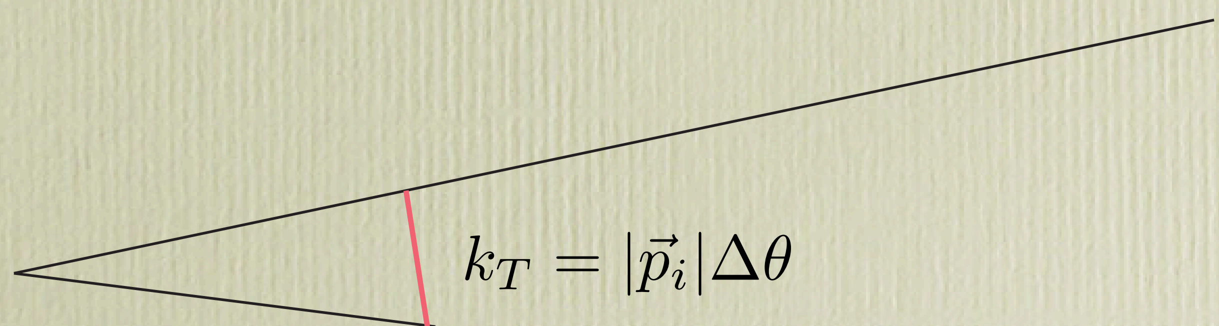
4. If $d_{\min}$ is a d_i, then protojet i is "not mergable." Remove it from the list of protojets and add it to the list of jets.
5. If protojets remain, go to 1.

Why the name?

$$d_{ij} = \min(E_{T,i}^2, E_{T,j}^2)\,[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$

is essentially

$$d_{ij} = k_T^2/R^2$$



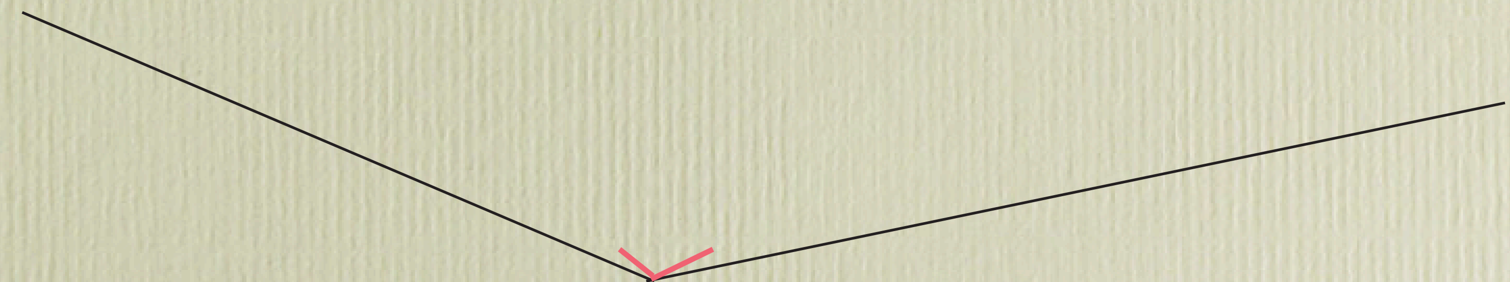
Why use k_T ?

The algorithm uses

$$d_{ij} = \min(E_{T,i}^2, E_{T,j}^2)\,[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$

Why not use

$$d_{ij} = E_{T,i}E_{T,j}[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$



- We want to avoid grouping the two soft partons with each other as the first step.

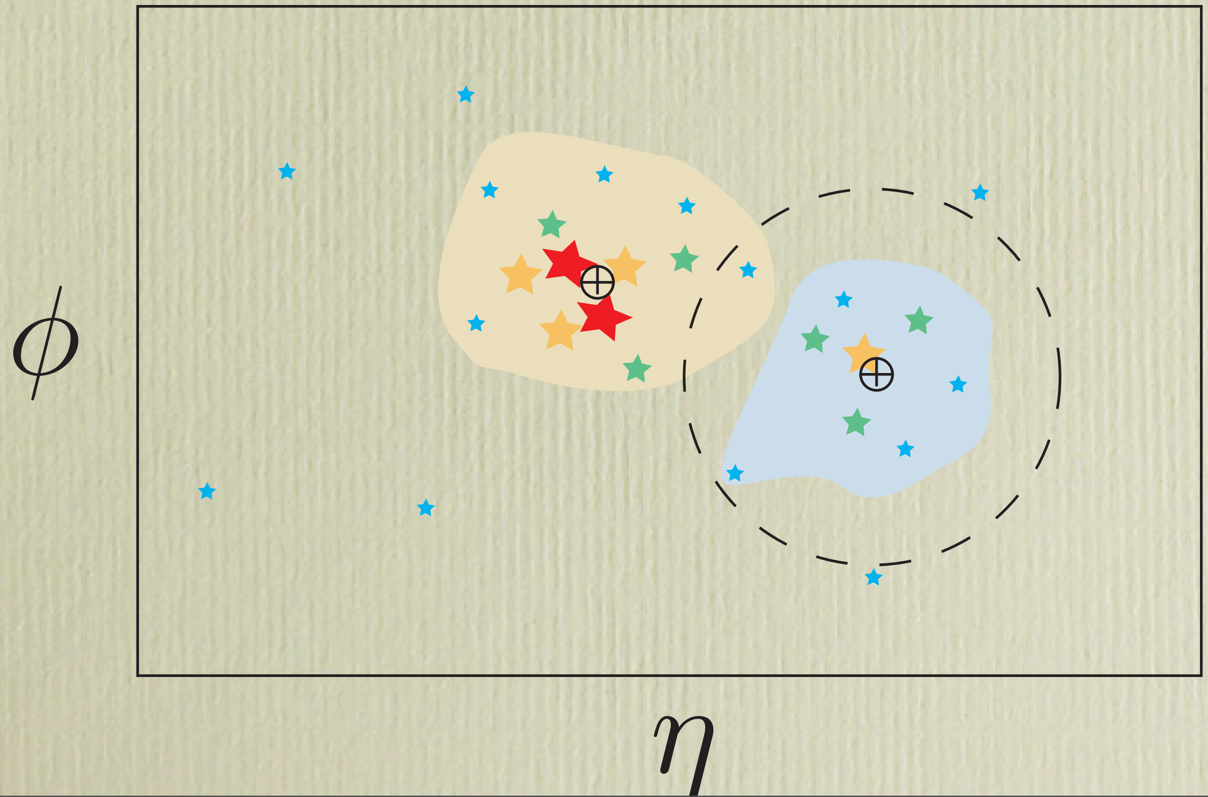
The "no merge" condition

$$d_{ij} = \min(E_{T,i}^2, E_{T,j}^2)\,[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$

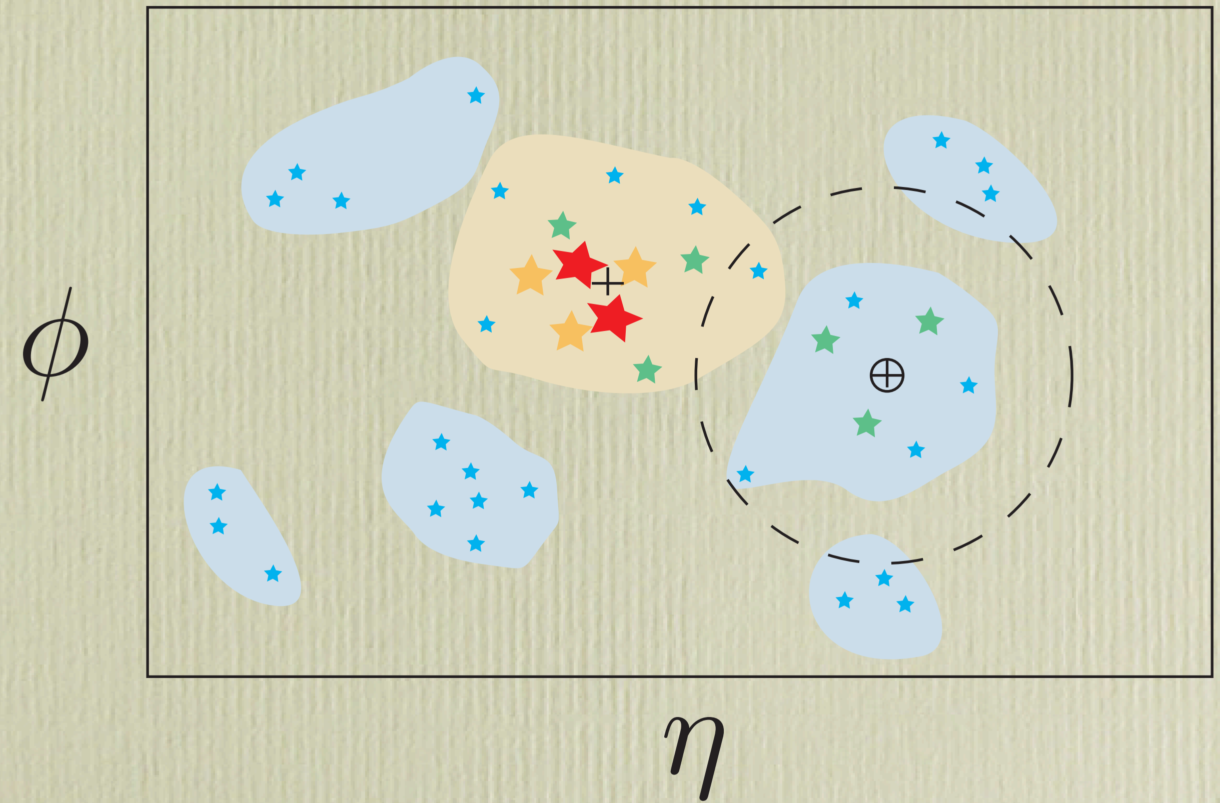
$$d_i = E_{T,i}^2$$

Protojet *i* is not mergable if $d_i < d_{ij}$. That is, if

$$[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2] > R^2$$



Why the "no merge" condition?



- There will be many soft jets.
- They should not merge into a few giant gets.

Is this IR safe?

- Two collinear partons get merged in the first step.
- Then they are equivalent to one “parent” parton.
- A very low E_T parton will get into a jet, but contributes almost nothing to the total momentum of the jet.
- So the algorithm is infrared safe.

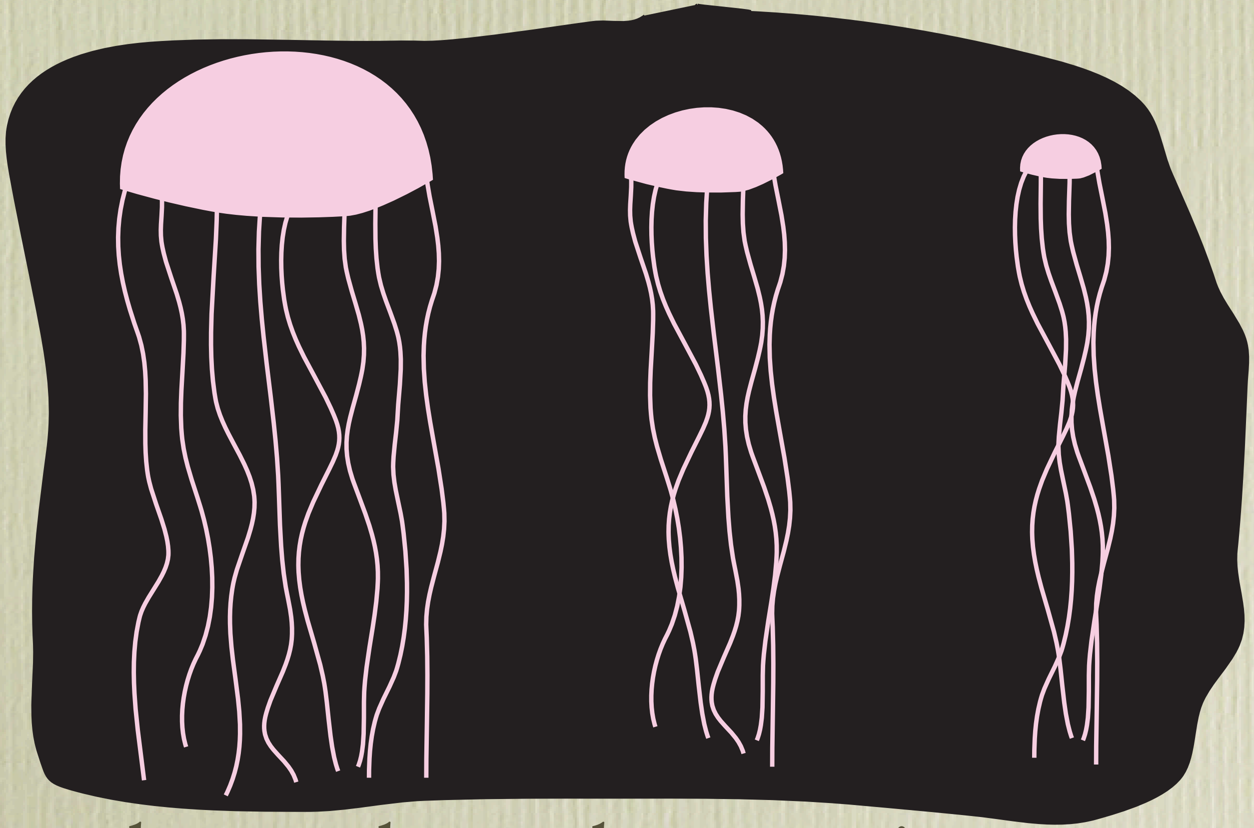
Conclusions

- I have outlined what “infrared safety” means.
- We looked at a simple cone jet definition and the k_T jet definition.
- Both are infrared safe (if we don’t use seeds).
- The cone definition gets added pieces to try to make it look more like it puts partons together in a reverse shower.
- The k_T definition does this without modification.

If there is time, we can look at jellyfish...

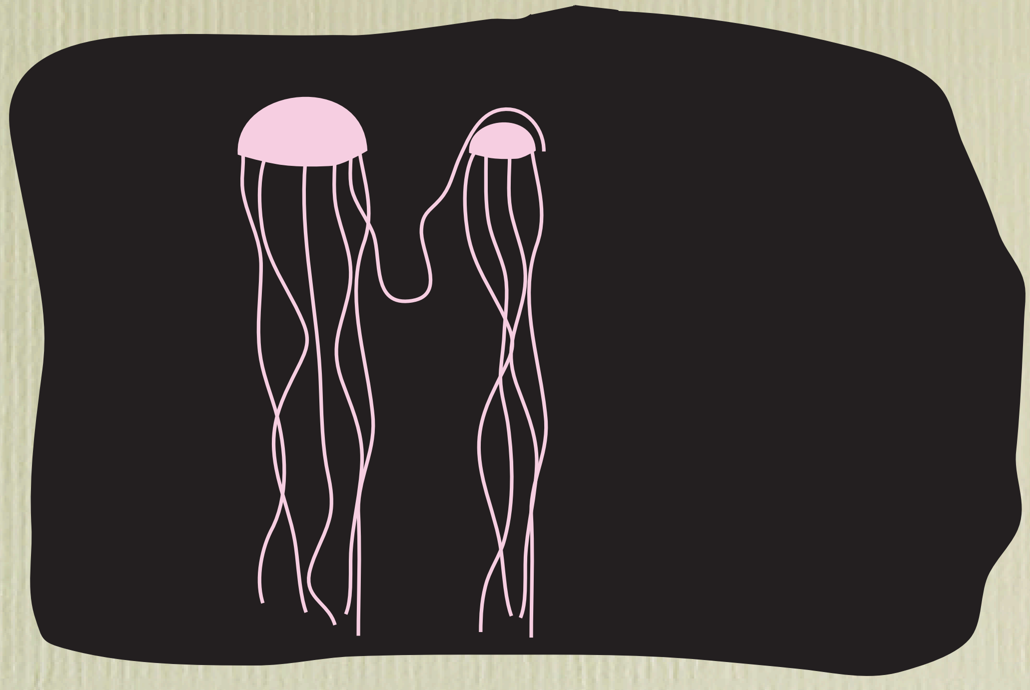
Cannibalistic jellyfish

Jellyfish of the species *protojetius cannibalis* come in a range of sizes.

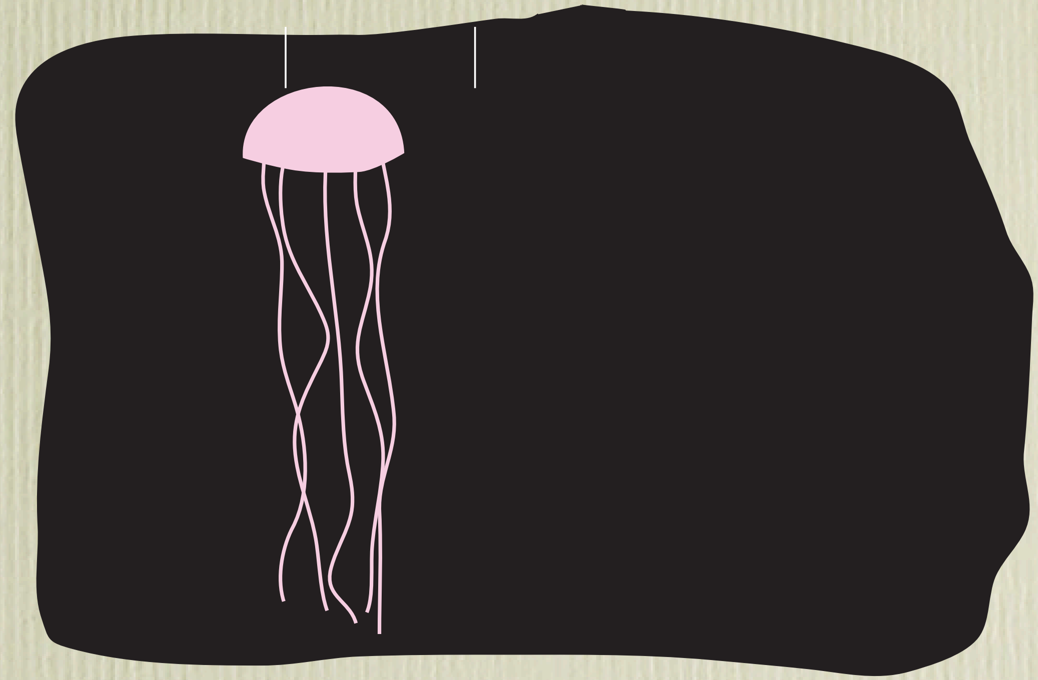


The tentacles are always the same size.

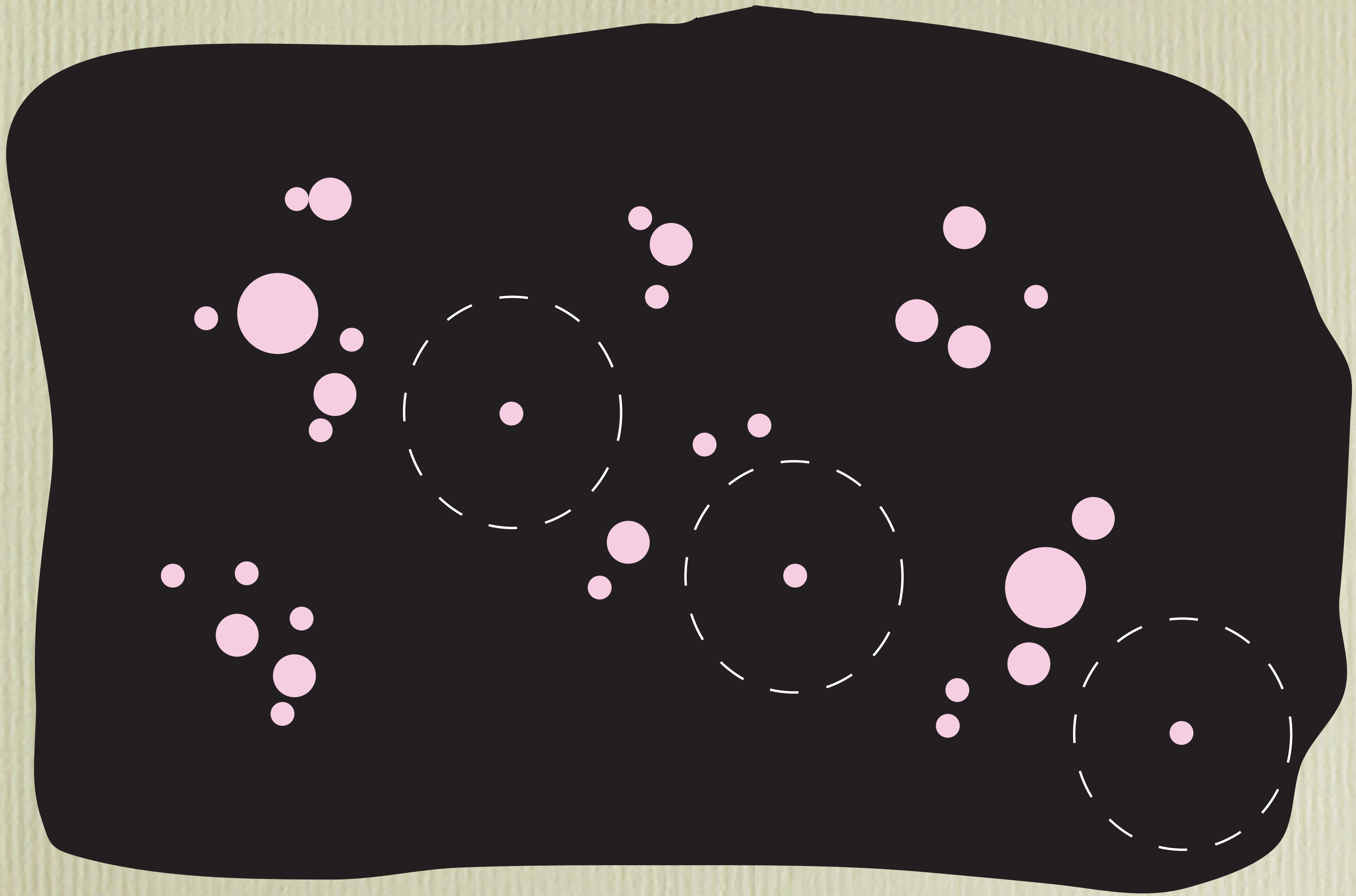
- Sometimes a big one has a little one for lunch.



- Then the big one is bigger.
- But the center-of-mass stays where it was.



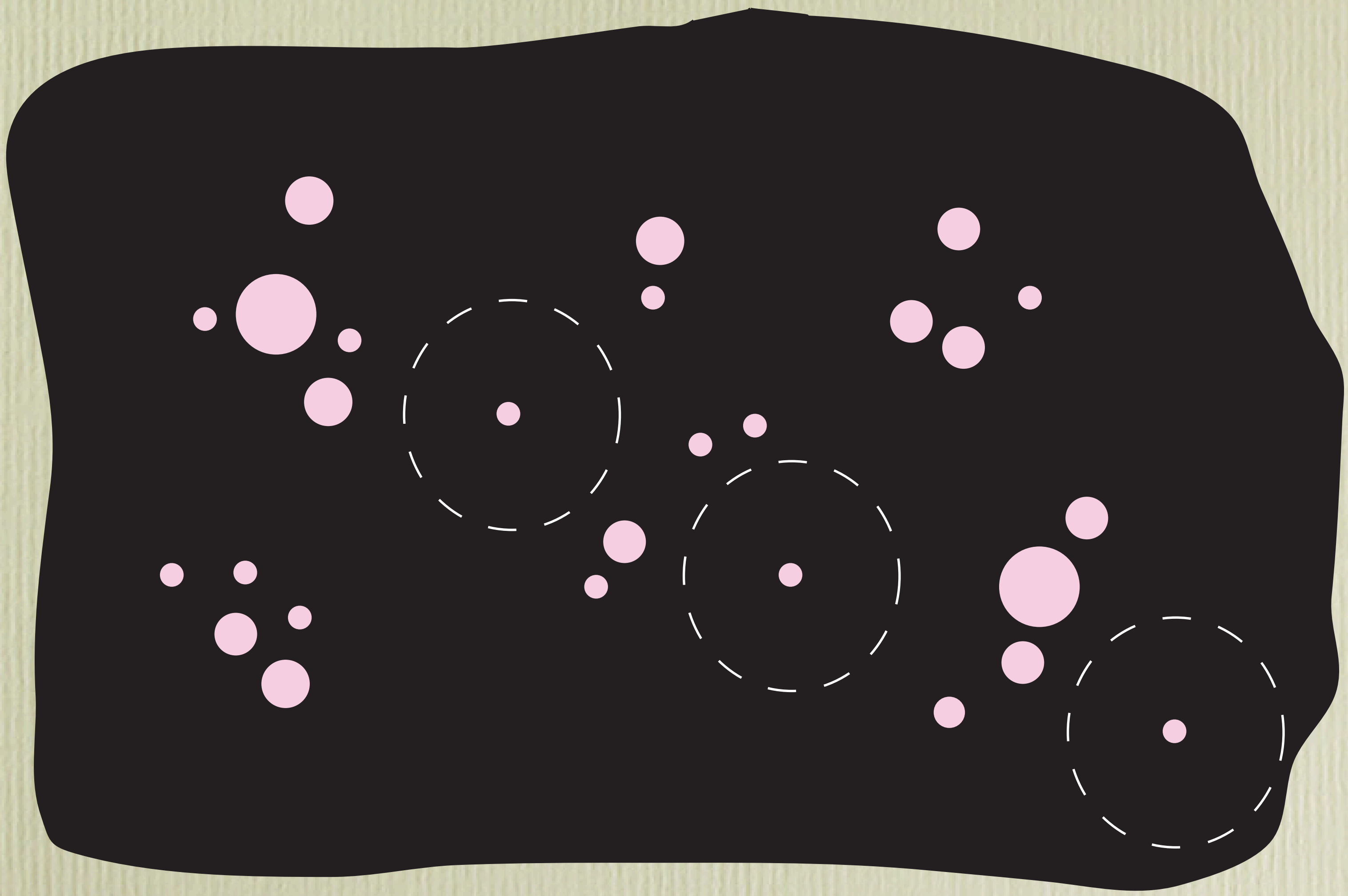
At sunrise, the jellyfish would like a snack.



They look for a jellyfish that is small and near.

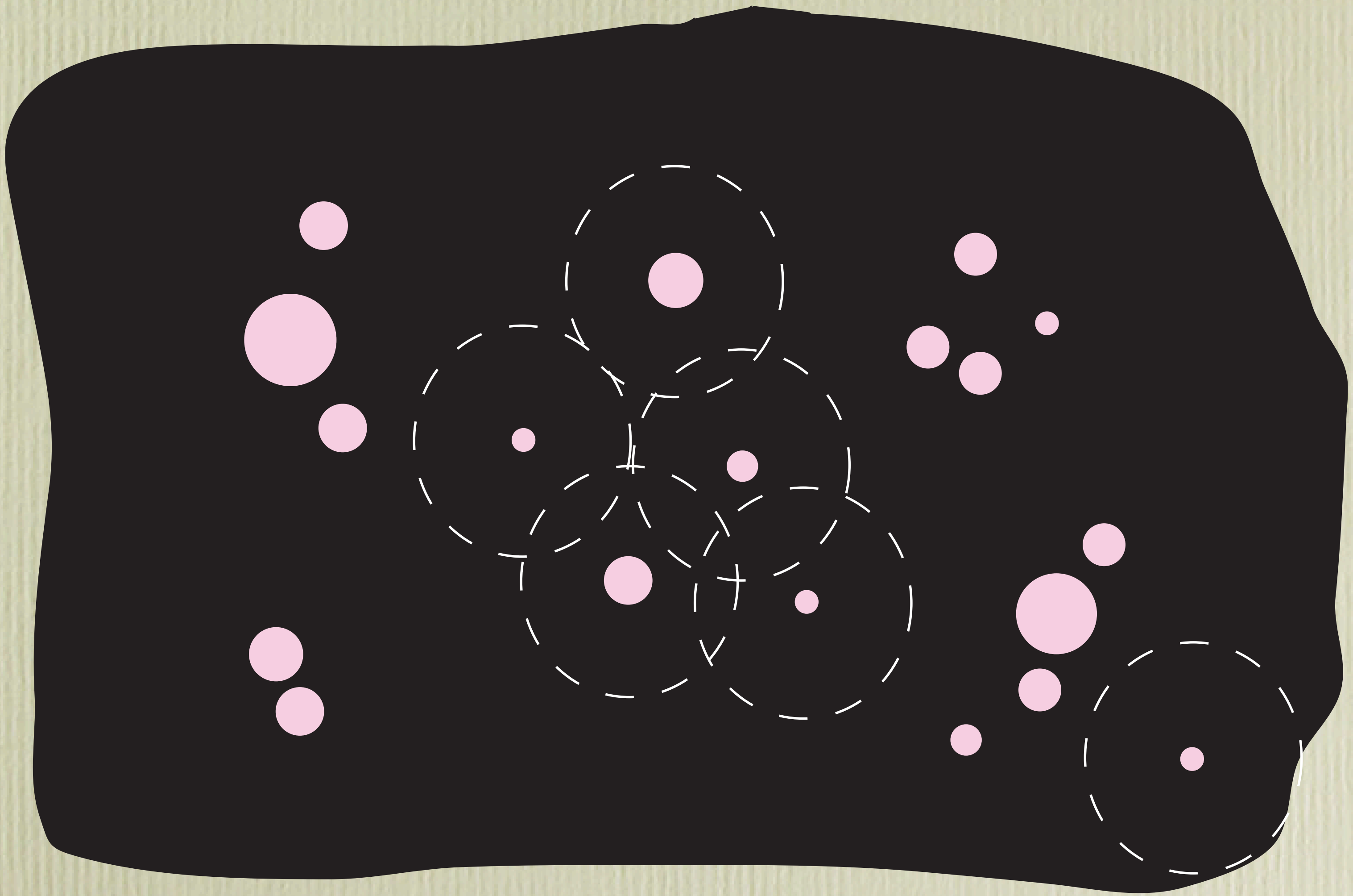
A few are out of range of predation.

After the snack, there are fewer jellyfish.



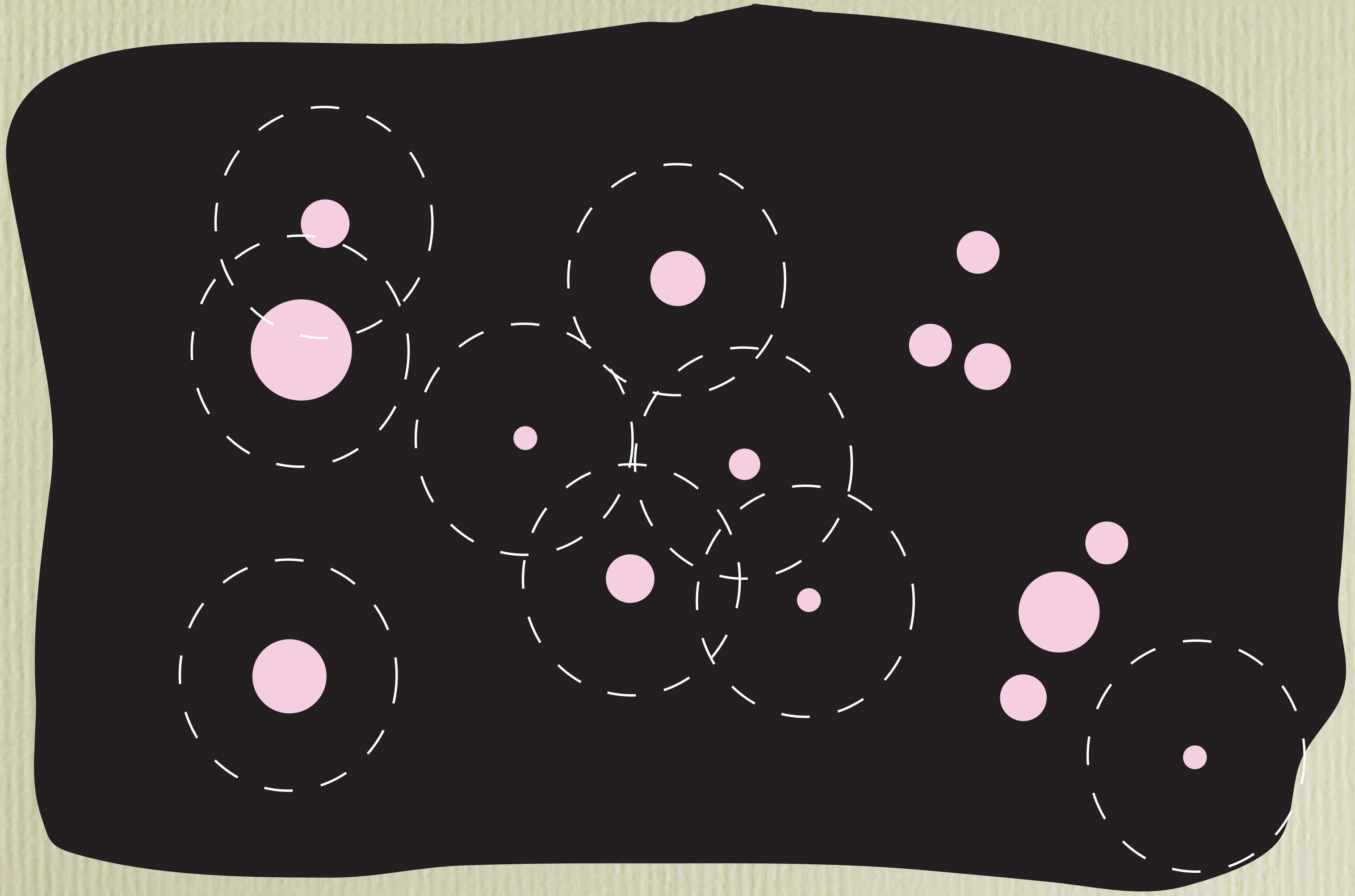
Then they get hungry for a real breakfast, and reach further and for bigger jellyfish.

By now, several jellyfish are out of reach from further predation.



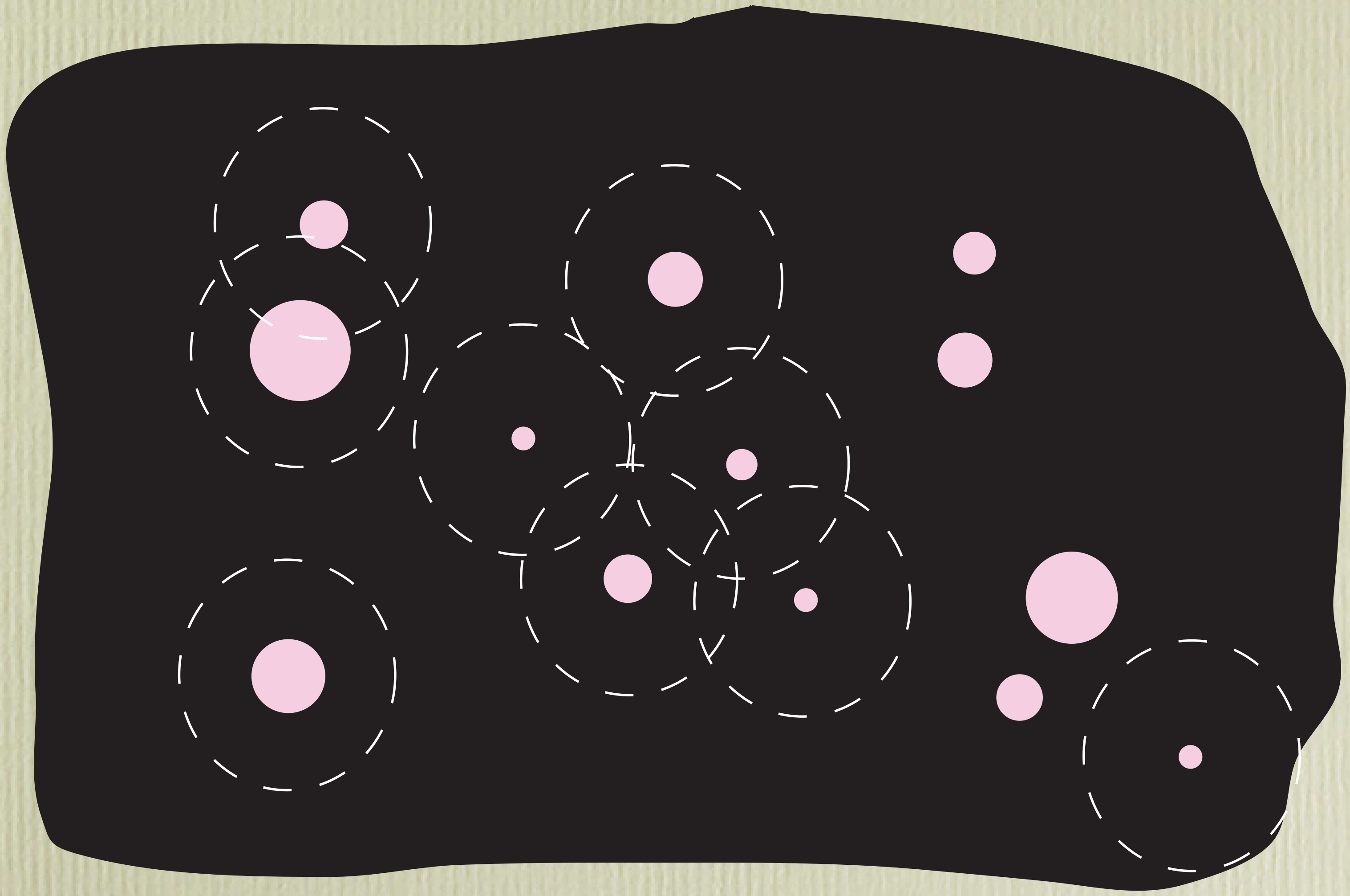
For lunch, they start reaching further and for bigger jellyfish.

Still, they are hungry...



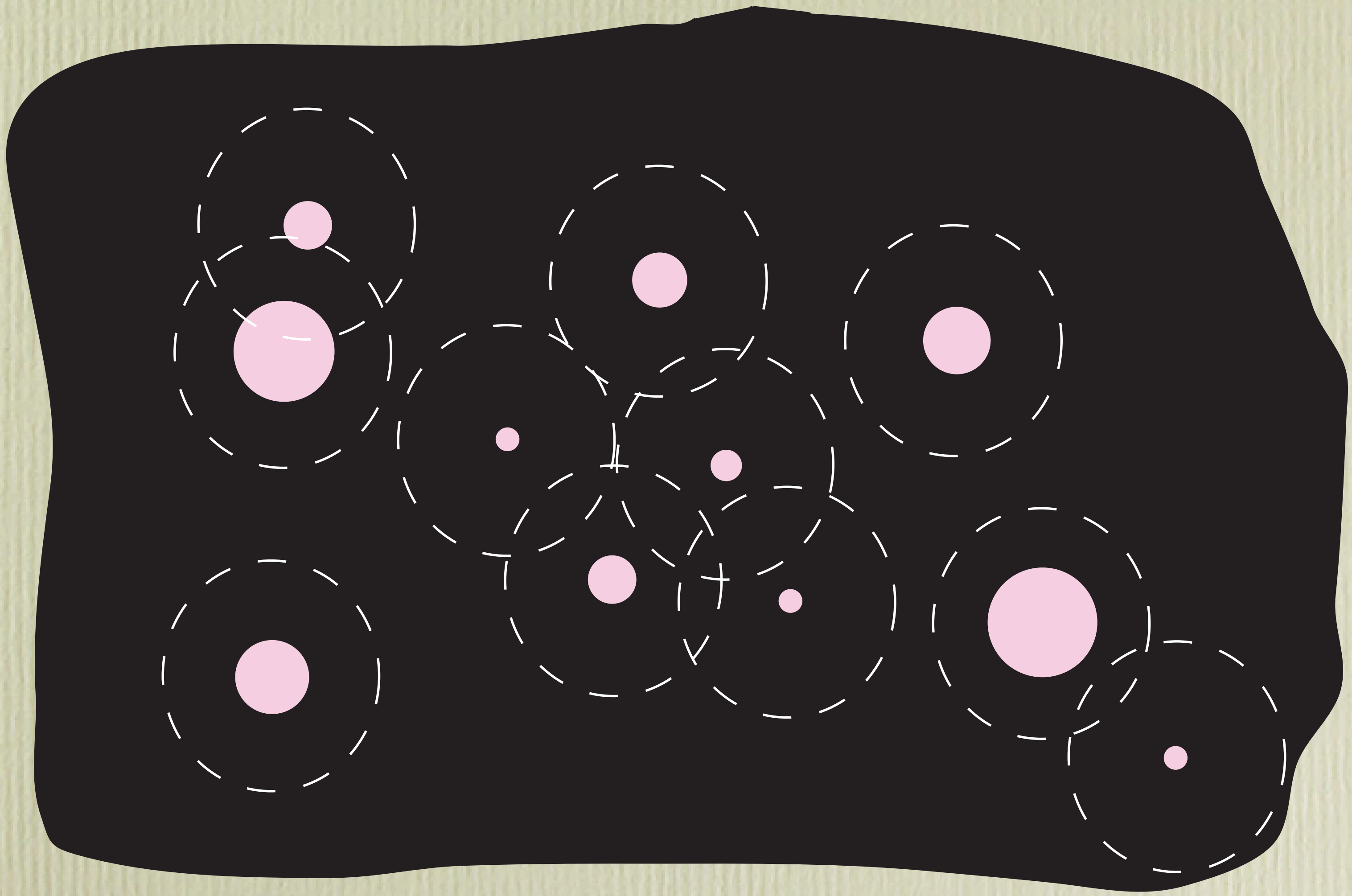
but only a few jellyfish are within reach.

Still, they are hungry...

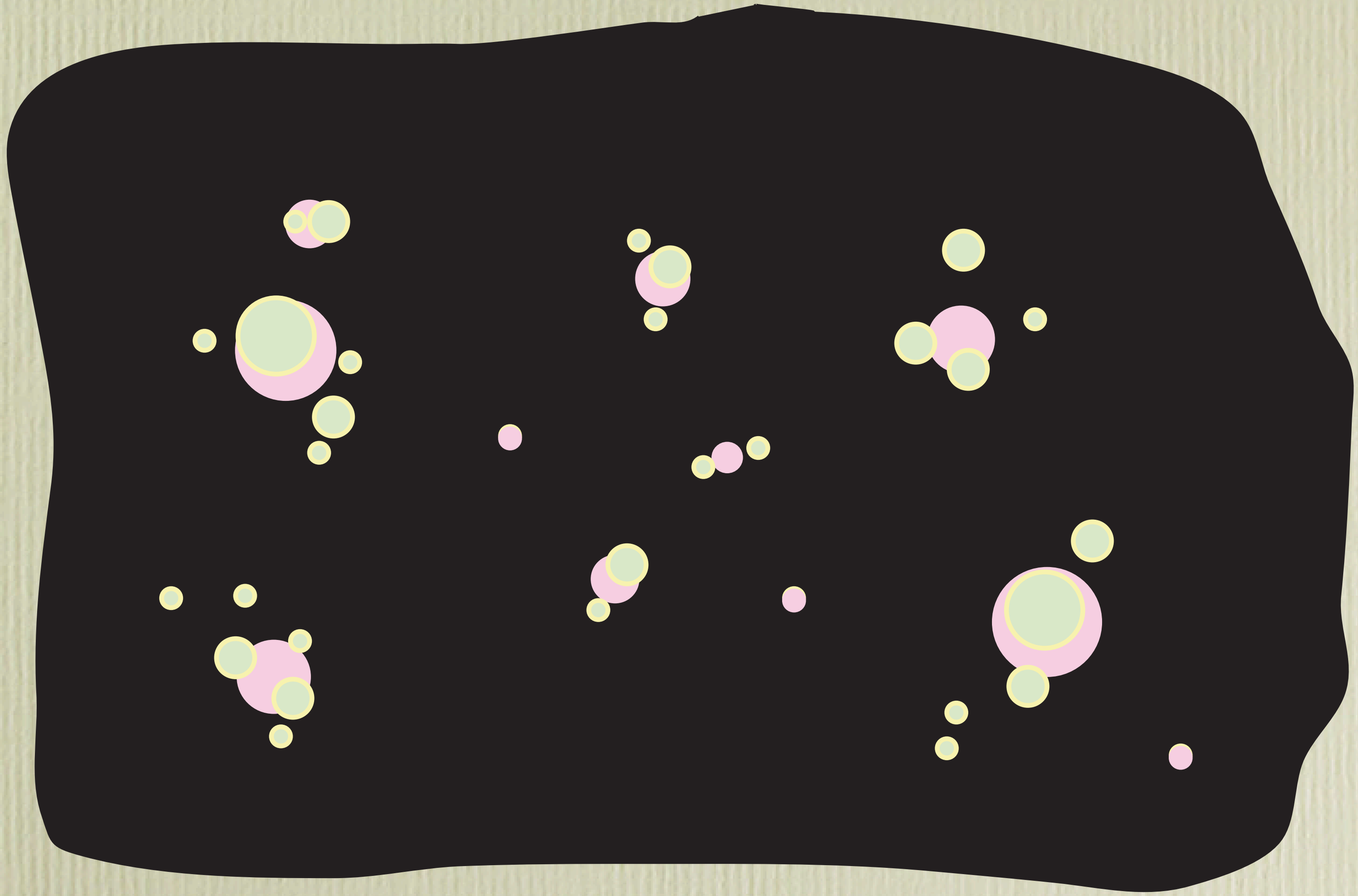


and one more feeding stage is possible.

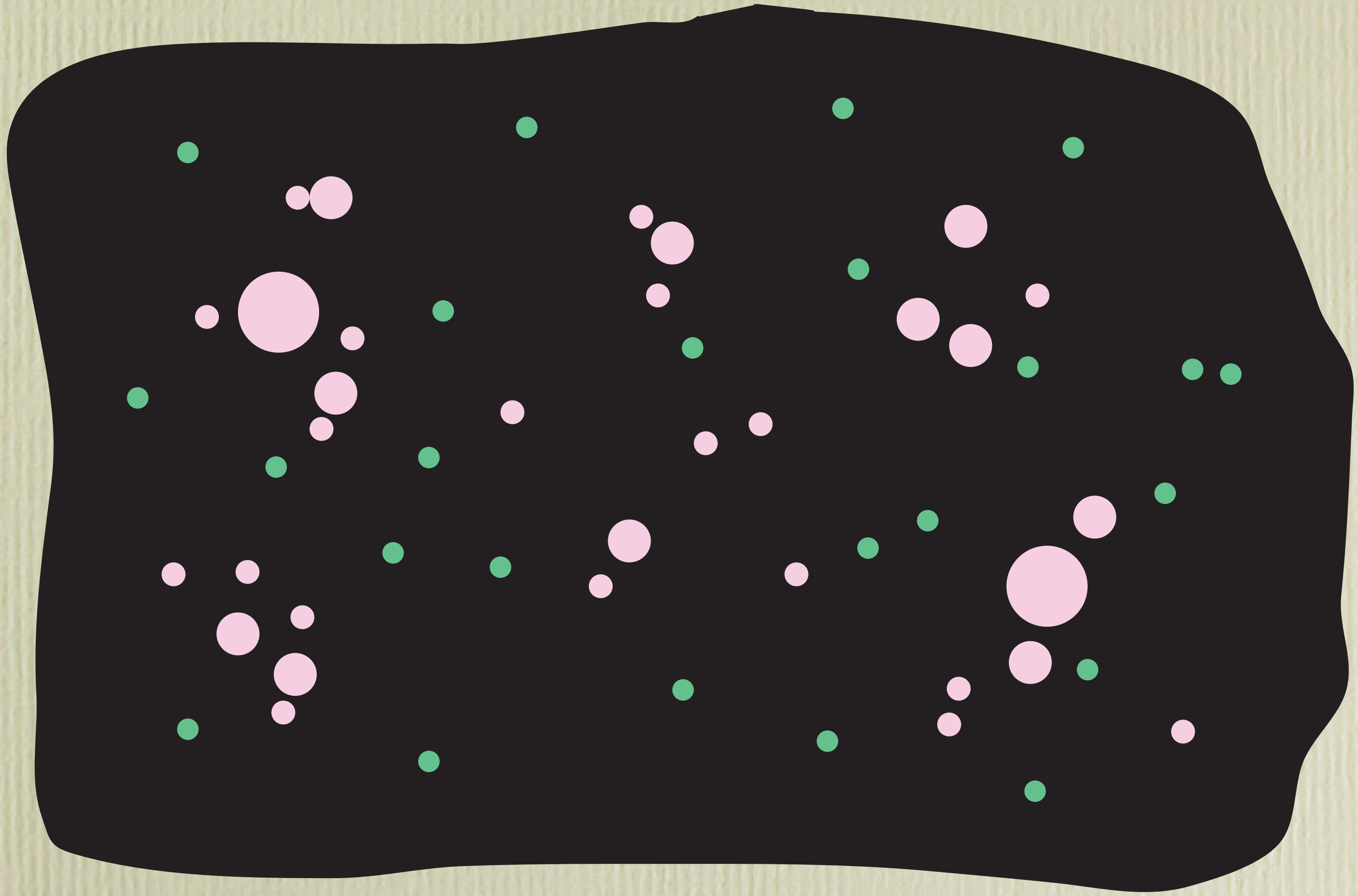
Feeding is over.



The shape of the feeding area for each final jellyfish is irregular.



What would happen if there were lots of little green jellyfish also?



Is the final pattern much affected?