The Affleck–Dine–Seiberg superpotential

SUSY QCD Symmetry

$SU(N)$ with F flavors where $F < N$

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
Φ, Q	$\square$	$\square$	$\mathbf{1}$	1	$\frac{F-N}{F}$
$\overline{\Phi}$, $\overline{Q}$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	-1	$\frac{F-N}{F}$

Recall that the auxiliary D^a fields:

$$D^a = g(\Phi^{*jn}(T^a)_n^m \Phi_{mj} - \overline{\Phi}^{jn}(T^a)_n^m \overline{\Phi}^*_{mj})$$

where $j = 1 \ldots F$; $m, n = 1 \ldots N$, $a = 1 \ldots N^2 - 1$,
D-term potential:

$$V = \frac{1}{2} D^a D^a$$

Classical Moduli Space

D-flat moduli space

$$\langle \overline{\Phi}^* \rangle = \langle \Phi \rangle = \begin{pmatrix} v_1 & & \\ & \ddots & \\ & & v_F \\ 0 & \dots & 0 \\ \vdots & & \vdots \\ 0 & \dots & 0 \end{pmatrix}$$

where $\langle \Phi \rangle$ is a $N \times F$ matrix
generic point in the moduli space $SU(N) \to SU(N-F)$

$$N^2 - 1 - ((N-F)^2 - 1) = 2NF - F^2$$

of $2NF$ chiral s.multiplets only F^2 singlets are massless
super Higgs mechanism: vector s.multiplet "eats" a chiral s.multiplet

Light “Mesons”

describe F^2 light degrees of freedom in a gauge invariant way by $F \times F$ matrix

$$M_i^j = \overline{\Phi}^{jn} \Phi_{ni}$$

where we sum over the color index n

M is a chiral superfield which is a product of chiral superfields, the only renormalization of M is the product of wavefunction renormalizations for Φ and $\overline{\Phi}$

Chiral Symmetries

axial $U(1)_A$ symmetry is explicitly broken by instantons
$U(1)_R$ symmetry remains unbroken

mixed anomalies between the global current and two gluons

$U(1)_R$: multiply the R-charge by the index

gaugino contributes $1 \cdot N$

each of the $2F$ quarks contributes $((F-N)/F - 1) \cdot \frac{1}{2}$

$$A_{Rgg} = N + \left(\tfrac{F-N}{F} - 1\right) \tfrac{1}{2} 2F = 0$$

$U(1)_A$: gauginos do not contribute

$$A_{Agg} = 1 \cdot 2F \cdot \tfrac{1}{2}$$

Spurious Symmetry

keep track of selection rules from the broken $U(1)_A$
define a spurious symmetry

$$
\begin{aligned}
Q &\to e^{i\alpha} Q \\
\overline{Q} &\to e^{i\alpha} \overline{Q} \\
\theta_{\rm YM} &\to \theta_{\rm YM} + 2F\alpha
\end{aligned}
$$

holomorphic intrinsic scale transforms as

$$
\Lambda^b \to e^{i2F\alpha} \Lambda^b
$$

construct the effective superpotential from: W^a, Λ, and M

	$U(1)_A$	$U(1)_R$
W^aW^a	0	2
Λ^b	$2F$	0
$\det M$	$2F$	$2(F-N)$

$\det M$ is only $SU(F) \times SU(F)$ invariant made out of M

Effective Wilsonian Superpotential

terms have the form

$$\Lambda^{bn}(W^a W^a)^m (\det M)^p$$

periodicity of $\theta_{YM} \Rightarrow$ only have powers of Λ^b
(for $m = 1$ perturbative term $b \ln(\Lambda)\, W^a W^a$ because of anomaly)
superpotential is neutral under $U(1)_A$ and has charge 2 under $U(1)_R$

$$0 = n\, 2F + p\, 2F$$
$$2 = 2m + p\, 2(F - N)$$

solution is

$$n = -p = \frac{1-m}{N-F}$$

$b = 3N - F > 0$, sensible $\Lambda \to 0$ limit if $n \geq 0$, implies $m \leq 1$(because $N > F$)
$W^a W^a$ contains derivatives, locality requires $m \geq 0$ and integer

Effective Wilsonian Superpotential

only two possible terms: $m = 0$ and $m = 1$

$m = 1$ term is field strength term

periodicity of $\theta_{\mathrm{YM}} \Rightarrow$ coefficient proportional to $b \ln \Lambda$.

$m = 0$ term is the Affleck–Dine–Seiberg (ADS) superpotential:

$$W_{\mathrm{ADS}}(N, F) = C_{N,F} \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{1/(N-F)}$$

where $C_{N,F}$ is renormalization scheme-dependent

Consistency of $W_{\rm ADS}$: moduli space

Consider giving a large VEV, v, to one flavor
$SU(N) \to SU(N-1)$ and one flavor is "eaten" by the Higgs mechanism
$2N-1$ broken generators

effective theory has $F-1$ flavors and $2F-1$ gauge singlets since

$$2NF - (2N-1) - (2F-1) = 2(N-1)(F-1)$$

low-energy effective theory for the $SU(N-1)$ gauge theory with $F-1$ flavors (gauge singlets interact only through irrelevant operators)
running holomorphic gauge coupling, g_L

$$\frac{8\pi^2}{g_L^2(\mu)} = b_L \ln\left(\frac{\mu}{\Lambda_L}\right)$$

$$b_L = 3(N-1) - (F-1) = 3N - F - 2$$

Λ_L is the holomorphic intrinsic scale of the low-energy effective theory

$$\Lambda_L \quad \equiv \quad |\Lambda_L| e^{i\theta_{\rm YM}/b_L} = \mu e^{2\pi i \tau_L/b_L}$$

Consistency of W_{ADS}: moduli space

low-energy coupling should match onto high-energy coupling

$$\frac{8\pi^2}{g^2(\mu)} = b \ln\left(\frac{\mu}{\Lambda}\right)$$

at the scale v in $\overline{DR}$:

$$\frac{8\pi^2}{g^2(v)} = \frac{8\pi^2}{g_L^2(v)}$$

$$\left(\frac{\Lambda}{v}\right)^b = \left(\frac{\Lambda_L}{v}\right)^{b_L}$$

$$\frac{\Lambda^{3N-F}}{v^2} = \Lambda_{N-1,F-1}^{3N-F-2}$$

subscript shows the number of colors and flavors: $\Lambda_{N-1,F-1} \equiv \Lambda_L$

Consistency of $W_{\rm ADS}$: moduli space

represent the light $(F-1)^2$ degrees of freedom as an $(F-1)\times(F-1)$ matrix $\widetilde{M}$

$$\det M = v^2 \det\widetilde{M} + \dots \ ,$$

where $\dots$ represents terms involving the decoupled gauge singlet fields
Plugging into $W_{\rm ADS}(N,F)$ and using

$$\left(\frac{\Lambda^{3N-F}}{v^2}\right)^{1/(N-F)} = \left(\Lambda^{3N-F-2}_{N-1,F-1}\right)^{1/((N-1)-(F-1))}$$

reproduce $W_{\rm ADS}(N-1,F-1)$
provided that $C_{N,F}$ is only a function of $N-F$

Consistency of W_{ADS}: moduli space

equal VEVs for all flavors
$SU(N) \rightarrow SU(N-F)$ and all flavors are "eaten"
from matching running couplings:

$$\left(\frac{\Lambda}{v}\right)^{3N-F} = \left(\frac{\Lambda_{N-F,0}}{v}\right)^{3(N-F)}$$

we then have

$$\frac{\Lambda^{3N-F}}{v^{2F}} = \Lambda_{N-F,0}^{3(N-F)}$$

So the effective superpotential is given by

$$W_{\text{eff}} = C_{N,F}\Lambda_{N-F,0}^{3}$$

reproduces holomorphy arguments for gaugino condensation in pure SUSY Yang-Mills

Consistency of W_{ADS}: mass terms

mass, m, for one flavor
low-energy effective theory is $SU(N)$ with $F-1$ flavors
Matching gauge couplings at m:

$$\left(\frac{\Lambda}{m}\right)^{b} = \left(\frac{\Lambda_L}{m}\right)^{b_L}$$
$$m\Lambda^{3N-F} = \Lambda_{N,F-1}^{3N-F+1}$$

holomorphy $\Rightarrow$ superpotential must have the form

$$W_{\mathrm{exact}} = \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)} f(t) \ ,$$

where

$$t = m M_F^F \left(\frac{\Lambda^{3N-F}}{\det M}\right)^{-1/(N-F)} \ ,$$

since mM_F^F is mass term in superpotential, it has $U(1)_A$ charge 0, and R-charge 2, so t has R-charge 0

Consistency of $W_{\rm ADS}$: mass terms

Taking the limit $\Lambda \to 0$, $m \to 0$, must recover our previous results with the addition of a small mass term

$$f(t) = C_{N,F} + t$$

in double limit t is arbitrary so this is the exact form

$$W_{\rm exact} = C_{N,F} \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{1/(N-F)} + m M_F^F$$

Consistency of W_{ADS}: mass terms

equations of motion for M_F^F and M_F^j

$$
\begin{aligned}
\frac{\partial W_{\mathrm{exact}}}{\partial M_F^F} &= C_{N,F}\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)}\left(\frac{-1}{N-F}\right)\frac{\mathrm{cof}(M_F^F)}{\det M}+m=0 \\
\frac{\partial W_{\mathrm{exact}}}{\partial M_F^j} &= C_{N,F}\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)}\left(\frac{-1}{N-F}\right)\frac{\mathrm{cof}(M_F^j)}{\det M}=0
\end{aligned}
$$

(where $\mathrm{cof}(M_i^F)$ is the cofactor of the matrix element M_i^F) imply that

$$
\frac{C_{N,F}}{N-F}\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)} = mM_F^F \qquad (*)
$$

and that $\mathrm{cof}(M_i^F)=0$. Thus, M has the block diagonal form

$$
M=\begin{pmatrix} \widetilde{M} & 0 \\ 0 & M_F^F \end{pmatrix}
$$

Consistency of W_{ADS}: mass terms

Plugging (*) into the exact superpotential we find

$$W_{\mathrm{exact}}(N, F-1) = (N-F+1)\left(\frac{C_{N,F}}{N-F}\right)^{(N-F)/(N-F+1)} \times \left(\frac{\Lambda_{N,F-1}^{3N-F+1}}{\det \widetilde{M}}\right)^{1/(N-F+1)}$$

$\propto W_{\mathrm{ADS}}(N, F-1)$. For consistency, we have a recursion relation:

$$C_{N,F-1} = (N-F+1)\left(\frac{C_{N,F}}{N-F}\right)^{(N-F)/(N-F+1)}$$

instanton calculation reliable for $F = N-1$ (gauge group is completely broken), determines $C_{N,N-1} = 1$ in the $\overline{\mathrm{DR}}$ scheme

$$C_{N,F} = N - F$$

Consistency of $W_{\rm ADS}$: mass terms

masses for all flavors

Holomorphy $\Rightarrow$

$$W_{\rm exact} = C_{N,F} \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{1/(N-F)} + m^i_j M^j_i$$

where m^i_j is the quark mass matrix. Equation of motion for M

$$M^j_i \quad = \quad (m^{-1})^j_i \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{1/(N-F)} \qquad (**)$$

taking the determinant and plugging the result back in to (**) gives

$$\bar{\Phi}^j \Phi_i = M^j_i \quad = \quad (m^{-1})^j_i \left(\det m \, \Lambda^{3N-F} \right)^{1/N}$$

result involves Nth root $\Rightarrow$ N distinct vacua, differ by the phase of M

Consistency of W_{ADS}: mass terms

Matching the holomorphic gauge coupling at mass thresholds

$$\Lambda^{3N-F} \det m = \Lambda_{N,0}^{3N}$$

So

$$W_{eff} = N\Lambda_{N,0}^{3}$$

reproduces holomorphy result for gaugino condensation and determines coefficient (up to phase)

$$\langle \lambda^a \lambda^a \rangle = -32\pi^2 e^{2\pi i k/N} \Lambda_{N,0}^{3}$$

where $k = 1...N$. Starting with $F = N - 1$ flavors can derive the correct ADS effective superpotential for $0 \leq F < N - 1$, and gaugino condensation for $F = 0$

justifies the assumption that there was a mass gap in SUSY YM

Generating W_{ADS} from instantons

Recall ADS superpotential

$$W_{\text{ADS}} \propto \Lambda^{b/(N-F)}$$

instanton effects are suppressed by

$$e^{-S_{\text{inst}}} \propto \Lambda^b$$

So for $F = N - 1$ it is possible that instantons can generate W_{ADS}

$SU(N)$ can be completely broken

allows for reliable instanton calculation

Generating W_{ADS} from instantons

With equal VEVs W_{ADS} predicts quark masses of order

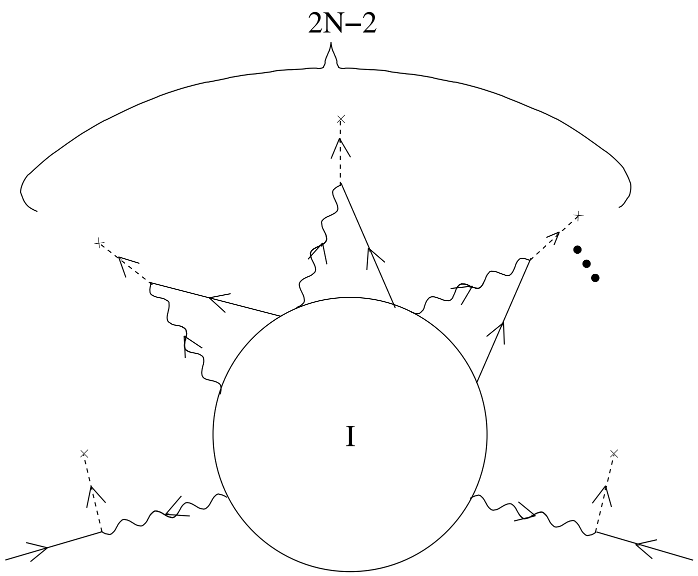
$$\frac{\partial^2 W_{\mathrm{ADS}}}{\partial \Phi_i \partial \overline{\Phi}^j} \sim \frac{\Lambda^{2N+1}}{v^{2N}}$$

and a vacuum energy density of order

$$\left| \frac{\partial W_{\mathrm{ADS}}}{\partial \Phi_i} \right|^2 \sim \left| \frac{\Lambda^{2N+1}}{v^{2N-1}} \right|^2$$

Generating $W_{\rm ADS}$ from instantons

single instanton vertex has $2N$ gaugino legs and $2F = 2N - 2$ quark legs



quark legs connected to gaugino legs by a scalar VEV, two gaugino legs converted to quark legs by the insertion of VEVs

fermion mass is generated

Generating $W_{\rm ADS}$ from instantons

instanton calculation $\longrightarrow$ quark mass

$$m \quad \sim \quad e^{-8\pi^2/g^2(1/\rho)} v^{2N} \rho^{2N-1} \sim (\Lambda\rho)^b \, v^{2N} \rho^{2N-1} \sim \Lambda^{2N+1} v^{2N} \rho^{4N}$$

dimensional analysis works because integration over ρ dominated by

$$\rho^2 = \frac{b}{16\pi^2 v^2}$$

quark legs ending at the same spacetime point gives $\mathcal{F}$ component of M, and vacuum energy of the right size

can derive the ADS superpotential for smaller values of F from $F = N-1$, so we can derive gaugino condensation for zero flavors from the instanton calculation with $N-1$ flavors

Generating W_{ADS} from $\langle\lambda\lambda\rangle$

For $F < N - 1$ can't use instantons since at generic point in moduli space $SU(N) \to SU(N-F) \supset SU(2)$

IR effective theory splits into and $SU(N-F)$ gauge theory and F^2 gauge singlets described by M

two sectors coupled by irrelevant operators

$SU(N-F)$ gauginos have an anomalous R-symmetry

R-symmetry spontaneously broken by squark VEVs not anomalous

QCD Analogy:

$SU(2)_L \times SU(2)_R$ spontaneously broken

axial anomaly of the quarks is reproduced in the low-energy theory by an irrelevant operator (the Wess–Zumino term) which gives $\pi^0 \to \gamma\gamma$

Generating W_{ADS} from $\langle\lambda\lambda\rangle$

In SUSY QCD the correct term is present since

$$\tau = \frac{3(N-F)}{2\pi i} \ln\left(\frac{\Lambda_{N-F,0}}{\mu}\right)$$

depends on $\ln \det M$ through matching condition

$$\Lambda^{3N-F} = \Lambda^{3(N-F)}_{N-F,0} \det M$$

relevant term in effective theory $\propto$

$$\begin{aligned}&\int d^2\theta \ln\det(M)\, W^a W^a + h.c.\\ &= \left[Tr(\mathcal{F}_M M^{-1})\lambda^a\lambda^a + \mathrm{Arg}(\det M) F^{a\mu\nu}\widetilde{F}^a_{\mu\nu} + \ldots\right] + h.c.\end{aligned}$$

where $\mathcal{F}_M$ is the auxiliary field for M

second term can be seen to arise through triangle diagrams involving the fermions in the massive gauge supermultiplets

Generating W_{ADS} from $\langle\lambda\lambda\rangle$

Arg(det$M$) transforms under chiral rotation as Nambu–Goldstone boson of the spontaneously broken R-symmetry:

$$\mathrm{Arg}(\det M) \rightarrow \mathrm{Arg}(\det M) + 2F\alpha$$

equation of motion for $\mathcal{F}_M$ gives

$$\mathcal{F}_M \quad = \quad \frac{\partial W}{\partial M} = M^{-1}\langle\lambda^a\lambda^a\rangle \propto M^{-1}\Lambda^3_{N-F,0} \propto M^{-1}\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)}$$

gives vacuum energy density that agrees with the ADS calculation

potential energy implies that a nontrivial superpotential generated, only superpotential consistent with holomorphy and symmetry is W_{ADS}

for $F < N-1$ flavors, gaugino condensation generates W_{ADS}

Vacuum structure

$$\begin{aligned} V_{\text{ADS}} &= \sum_i \left|\frac{\partial W_{\text{ADS}}}{\partial Q_i}\right|^2 + \left|\frac{\partial W_{\text{ADS}}}{\partial \overline{Q}_i}\right|^2 \\ &= \sum_i |\mathcal{F}_i|^2 + |\overline{\mathcal{F}}_i|^2 \ , \end{aligned}$$

is minimized as $\det M \to \infty$, so there is a "run-away vacuum"

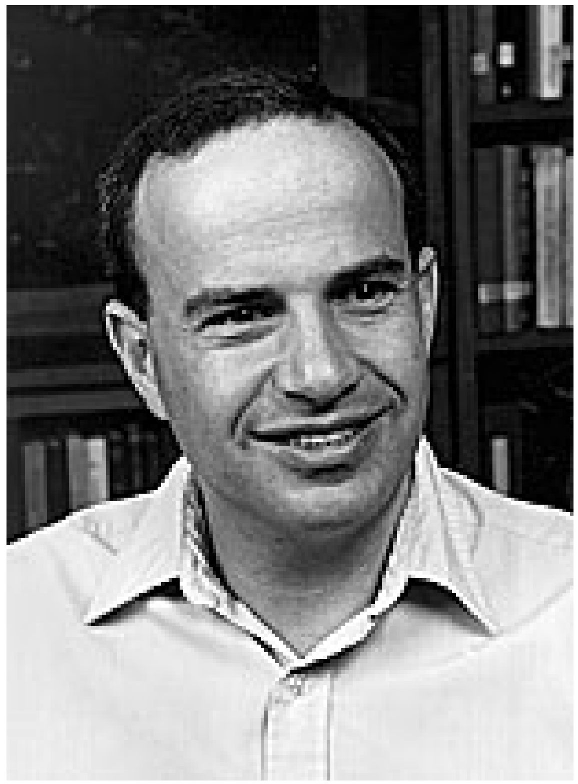
potential loop-hole: wavefunction renormalization effects not included, could produce local minima, could not produce new vacua unless renormalization factors were singular

cannot happen unless particles are massless at point in the moduli space, also produces singularity in the superpotential

At $\det M = 0$, massive gauge supermultiplets become massless

recent progress on theory without VEVs

Intriligator—Seiberg—Shih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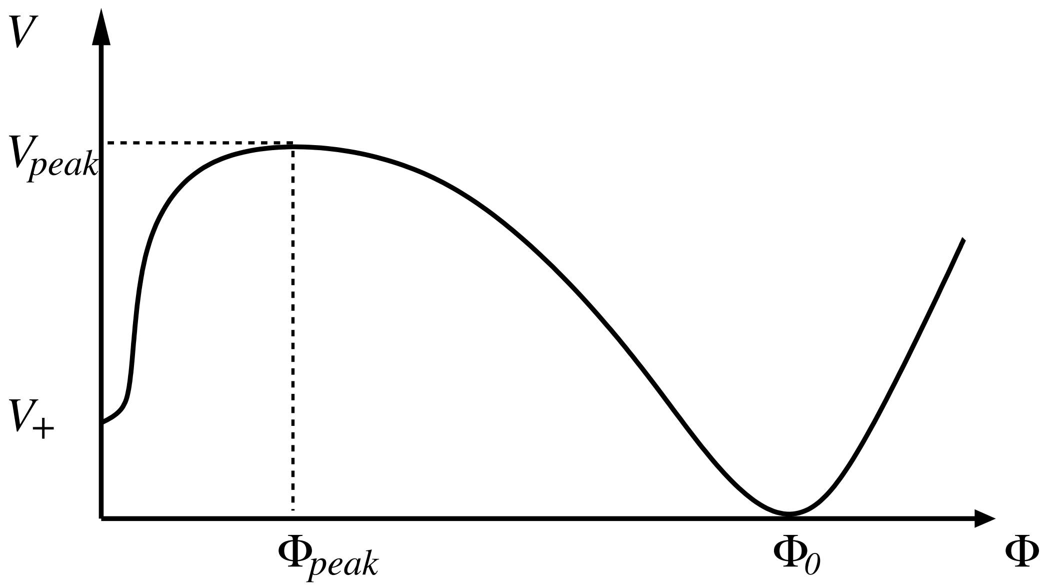
Intriligator—Seiberg—Shih



tunnelling $\propto e^{-S}$

$S \gg 1$ if $F < 3N/2$