SUSY breaking and the MSSM

Spontaneous SUSY breaking at tree-level

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Spontaneous SUSY Breaking

$$\langle 0|H|0\rangle > 0$$

implies that SUSY is broken.

$$V = \mathcal{F}^{i*}\mathcal{F}_i + \frac{g^2}{2} D^a D^a \ ,$$

find models where $\mathcal{F}_i = 0$ or $D^a = 0$ cannot be simultaneously solved

then use this SUSY breaking sector to generate the soft SUSY breaking

O'Raifeartaigh model

have nonzero $\mathcal{F}$-terms

$$W_{O'R} = -k^2\Phi_1 + m\Phi_2\Phi_3 + \frac{y}{2}\Phi_1\Phi_3^2.$$

scalar potential is

$$\begin{aligned} V &= |\mathcal{F}_1|^2 + |\mathcal{F}_2|^2 + |\mathcal{F}_3|^2 \\ &= |k^2 - \frac{y}{2}\phi_3^{*2}|^2 + |m\phi_3^*|^2 + |m\phi_2^* + y\phi_1^*\phi_3^*|^2. \end{aligned}$$

no solution where both $\mathcal{F}_1 = 0$ and $\mathcal{F}_2 = 0$
For large m, minimum is at $\phi_2 = \phi_3 = 0$ with ϕ_1 undetermined

vacuum energy density is

$$V = |\mathcal{F}_1|^2 = k^4\ .$$

O'Raifeartaigh model

Around $\phi_1 = 0$, the mass spectrum of scalars is

$$0,\ \ 0,\ \ m^2,\ \ m^2,\ \ m^2 - yk^2,\ \ m^2 + yk^2.$$

There are also three fermions with masses

$$0,\ \ m,\ \ m.$$

Note that these masses satisfy a sum rule for tree-level breaking

$$\mathrm{Tr}[M^2_{\mathrm{scalars}}] = 2\mathrm{Tr}[M^2_{\mathrm{fermions}}]$$

O'Raifeartaigh model: One Loop

For $k^2 \neq 0$, loop corrections will give a mass to ϕ_1

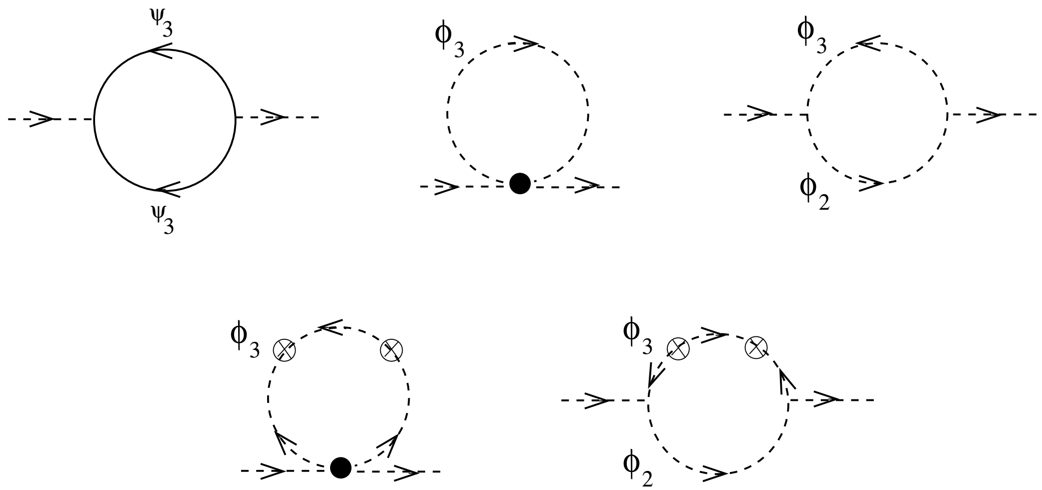


Figure 1: Crosses mark an insertion of yk^2.

yk^2 insertions must appear with an even number in order to preserve the orientation of the arrows flowing into the vertices
correction to the ϕ_1 mass from the top three graphs vanishes by SUSY

O'Raifeartaigh model: One Loop

bottom two graphs give

$$-im_1^2 = \int \frac{d^4p}{2\pi^4}(-iy^2)\frac{iy^2k^4}{(p^2-m^2)^3} + (iym)^2\frac{i}{p^2-m^2}\frac{iy^2k^4}{(p^2-m^2)^3} \ ,$$

yields a finite, positive, result

$$m_1^2 = \frac{y^4k^4}{48\pi^2m^2} = \frac{y^4}{48\pi^2}\frac{|\mathcal{F}_1|^2}{m^2} \ .$$

the classical flat direction is lifted by quantum corrections, the potential is stable around $\phi_1 = 0$

the massless fermion ψ_1 stays massless since it is the Nambu–Goldstone particle for the broken SUSY generator, a *goldstino*

ψ_1 is the fermion in the multiplet with the nonzero $\mathcal{F}$ component.

Fayet–Iliopoulos mechanism

uses a nonzero D-term for a $U(1)$ gauge group
add a term linear in the auxiliary field to the theory:

$$\mathcal{L}_{\rm FI} = \kappa^2 D \ ,$$

where κ is a constant parameter with dimensions of mass
scalar potential is

$$V = \tfrac{1}{2} D^2 - \kappa^2 D + g D \textstyle\sum_i q_i \phi^{i*} \phi_i \ ,$$

and the D equation of motion gives

$$D = \kappa^2 - g \textstyle\sum_i q_i \phi^{i*} \phi_i \ .$$

If the ϕ_is have large positive mass squared terms, $\langle \phi \rangle = 0$ and $D = \kappa^2$

in the MSSM, however, squarks and sleptons cannot have superpotential mass terms

Problems

Fayet–Iliopoulos and O'Raifeartaigh models set the scale of SUSY breaking by hand. To get a SUSY breaking scale that is naturally small compared to the Planck scale, M_{Pl}, we need an asymptotically free gauge theory that gets strong through RG evolution at some much smaller scale

$$\Lambda \sim e^{-8\pi^2/(bg_0^2)} M_{Pl} \ ,$$

and breaks SUSY nonperturbatively
can't use renormalizable tree-level couplings to transmit SUSY breaking, since SUSY does not allow scalar–gaugino–gaugino couplings

we expect that SUSY breaking occurs dynamically in a "hidden sector" and is communicated by non-renormalizable interactions or through loop effects. If the interactions that communicate SUSY breaking to the MSSM ("visible") sector are flavor-blind it is possible to suppress FCNCs

Gauge-Mediated Scenario

add "messenger" chiral supermultiplets where the fermions and bosons are split and which couple to the SM gauge groups
MSSM superpartners get masses through loops:

$$m_{\rm soft} \sim \frac{\alpha_i}{4\pi} \frac{\langle \mathcal{F} \rangle}{M_{\rm mess}}$$

If $M_{\rm mess} \sim \sqrt{\langle \mathcal{F} \rangle}$, then the SUSY breaking scale can be as low as $\sqrt{\langle \mathcal{F} \rangle} \sim 10^4$–$10^5$ GeV.

Gravity-Mediated Scenario

interactions with the SUSY breaking sector are suppressed by powers of M_{Pl} hidden sector field X with a nonzero $\langle \mathcal{F}_X \rangle$, then MSSM soft terms of the order

$$m_{\text{soft}} \sim \frac{\langle \mathcal{F}_X \rangle}{M_{Pl}} \ .$$

To get m_{soft} to come out around the weak scale we need $\sqrt{\langle \mathcal{F}_X \rangle} \sim 10^{10}$–$10^{11}$ GeV. Alternatively, if SUSY is broken by a gaugino condensate $\langle 0|\lambda^a \lambda^b|0\rangle = \delta^{ab}\Lambda^3 \neq 0$, then

$$m_{\text{soft}} \sim \frac{\Lambda^3}{M_{Pl}^2},$$

which requires $\Lambda \sim 10^{13}$ GeV. This can, of course, be rewritten as: $\langle \mathcal{F}_X \rangle = \Lambda^3/M_{Pl}$.

Effective Lagrangian

Below the M_{Pl} is:

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} \;=\; & -\int d^4\theta\, \frac{X^*}{M_{Pl}} \hat{b}'^{ij}\psi_i\psi_j + \frac{XX^*}{M_{Pl}^2}\left(\hat{m}^i_j\psi_i\psi^{j*} + \hat{b}^{ij}\psi_i\psi_j\right) + h.c. \\
& -\int d^2\theta\, \frac{X}{2M_{Pl}}\left(\hat{M}_3 G^\alpha G_\alpha + \hat{M}_2 W^\alpha W_\alpha + \hat{M}_1 B^\alpha B_\alpha\right) + h.c. \\
& -\int d^2\theta\, \frac{X}{M_{Pl}} \hat{a}^{ijk}\psi_i\psi_j\psi_k + h.c.
\end{aligned}
$$

where G_α, W_α, B_α, and ψ_i are the chiral superfields of the MSSM, and the hatted symbols are dimensionless

If $\langle X \rangle = \langle \mathcal{F} \rangle$ then

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} \;=\; & -\frac{\langle \mathcal{F}_X \rangle}{2M_{Pl}}\left(\hat{M}_3\widetilde{G}\widetilde{G} + \hat{M}_2\widetilde{W}\widetilde{W} + \hat{M}_1\widetilde{B}\widetilde{B}\right) + h.c. \\
& -\frac{\langle \mathcal{F}_X \rangle\langle \mathcal{F}^*_X \rangle}{M_{Pl}^2}\left(\hat{m}^i_j\widetilde{\psi}_i\widetilde{\psi}^{j*} + \hat{b}^{ij}\widetilde{\psi}_i\widetilde{\psi}_j\right) + h.c. \\
& -\frac{\langle \mathcal{F}_X \rangle}{M_{Pl}}\hat{a}^{ijk}\widetilde{\psi}_i\widetilde{\psi}_j\widetilde{\psi}_k - \frac{\langle \mathcal{F}^*_X \rangle}{M_{Pl}}\int d^2\theta\, \hat{b}'^{ij}\psi_i\psi_j + h.c.
\end{aligned}
$$

Assumptions

assume $\hat{M}_i = \hat{M}$, $\hat{m}^i_j = \hat{m}\delta^i_j$

we have generated a μ-term with $\mu^{ij} = \hat{b}'\delta^i_{H_u}\delta^j_{H_d}\langle\mathcal{F}^*_X\rangle/M_{Pl}$ assuming $\hat{a}^{ijk} = \hat{a}Y^{ijk}$ and $\hat{b}^{ij} = \hat{b}\delta^i_{H_u}\delta^j_{H_d}$, then soft parameters have a universal form (when renormalized at M_{Pl})

gaugino masses are equal

$$M_i = m_{1/2} = \hat{M}\frac{\langle\mathcal{F}_X\rangle}{M_{Pl}} \ ,$$

the scalar masses are universal

$$\mathbf{m^2_f} = m^2_{H_u} = m^2_{H_d} = m^2_0 = \hat{m}\frac{|\langle\mathcal{F}_X\rangle|^2}{M^2_{Pl}} \ ,$$

A and b terms are given by

$$\mathbf{A_f} \quad = \quad A\,\mathbf{Y_f} = \hat{a}\frac{\langle\mathcal{F}_X\rangle}{M_{Pl}}\,\mathbf{Y_f} \ , \ \ b = B\mu = \frac{\hat{b}}{\hat{b}'}\frac{\langle\mathcal{F}_X\rangle}{M_{Pl}}\mu \ .$$

μ^2 and b are naturally of the same order of magnitude if $\hat{b}$ and $\hat{b}'$ are of the same order of magnitude

Justified Assumptions?

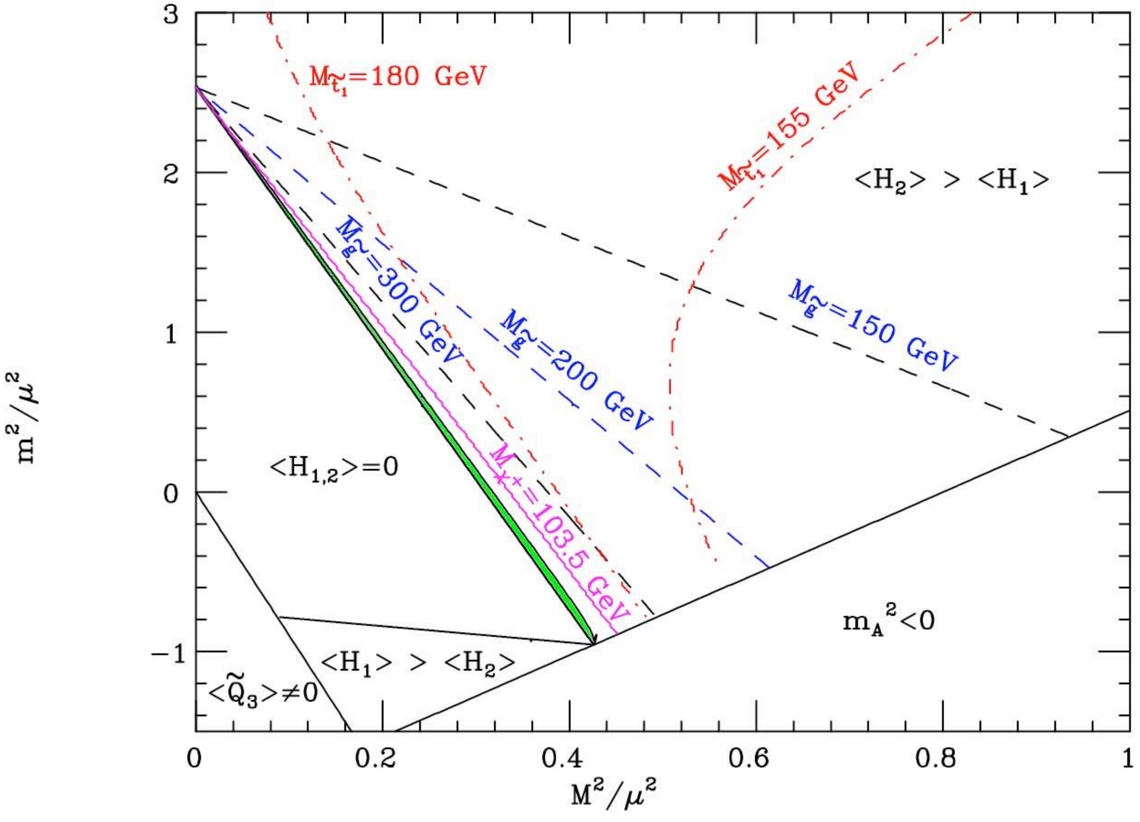
the assumptions avoid problems with FCNCs. Since gravity is flavor-blind, it might seem that this a natural result of gravity mediation. However, the equivalence principle does not guarantee these universal terms, since nothing forbids a Kähler function of the form

$$K_{\text{bad}} = f(X^\dagger, X)^i_j \, \psi^{\dagger j} \psi_i \ ,$$

which leads directly to off-diagonal terms in the matrix $\hat{m}^i_j$

Taking μ and the four SUSY breaking parameters and running them down from the unification scale (rather than the Planck scale as one would expect) is referred to as the *minimal supergravity* scenario

MSugra



scalar mass m^2, gaugino mass M, $A = 0$

Giudice, Rattazzi, hep-ph/0606105

The goldstino

Consider the fermions in a general SUSY gauge theory. Take a basis $\Psi = (\lambda^a, \psi_i)$. The mass matrix is

$$\mathbf{M}_{\mathrm{fermion}} = \begin{pmatrix} 0 & \sqrt{2}g_a(\langle\phi^*\rangle T^a)^i \\ \sqrt{2}g_a(\langle\phi^*\rangle T^a)^j & \langle W^{ij}\rangle \end{pmatrix}$$

eigenvector with eigenvalue zero:

$$\begin{pmatrix} \langle D^a\rangle/\sqrt{2} \\ \langle \mathcal{F}_i\rangle \end{pmatrix}$$

eigenvector is only nontrivial if SUSY is broken. The corresponding canonically normalized massless fermion field is the goldstino:

$$\Pi = \frac{1}{F_\Pi}\left(\frac{\langle D^a\rangle}{\sqrt{2}}\lambda^a + \langle\mathcal{F}_i\rangle\psi_i\right)$$

where

$$F_\Pi^2 = \sum_a \frac{\langle D^a\rangle^2}{2} + \sum_i \langle\mathcal{F}_i\rangle^2$$

The Goldstino

masslessness of the goldstino follows from two facts. First the superpotential is gauge invariant,

$$(\phi^* T^a)^i W_i^* = -(\phi^* T^a)^i \mathcal{F}_i = 0$$

second, the first derivative of the scalar potential

$$\frac{\partial V}{\partial \phi_i} = -W_i^* \frac{\partial W^i}{\partial \phi_i} - g_a(\phi^* T^a)^j D^a$$

vanishes at its minimum

$$\langle \frac{\partial V}{\partial \phi_i} \rangle = \langle \mathcal{F}_i \rangle \langle W^{ij} \rangle - g_a(\langle \phi^* \rangle T^a)^j \langle D^a \rangle = 0$$

The Supercurrent

$$
\begin{aligned}
J^\mu_\alpha &= iF_\Pi(\sigma^\mu\bar{\Pi})_\alpha + (\sigma^\nu\overline{\sigma}^\mu\psi_i)_\alpha\, D_\nu\phi^{*i} - \tfrac{1}{2\sqrt{2}}(\sigma^\nu\overline{\sigma}^\rho\sigma^\mu\bar{\lambda}^a)_\alpha\, F^a_{\nu\rho}\ , \\
&\equiv iF_\Pi(\sigma^\mu\bar{\Pi})_\alpha + j^\mu_\alpha\ .
\end{aligned}
$$

terms included in j^μ_α contain two or more fields.
supercurrent conservation:

$$
\partial_\mu J^\mu_\alpha = iF_\Pi(\sigma^\mu\partial_\mu\bar{\Pi})_\alpha + \partial_\mu j^\mu_\alpha = 0 \qquad (*)
$$

effective Lagrangian for the goldstino

$$
\mathcal{L}_{\text{goldstino}} = i\bar{\Pi}\overline{\sigma}^\mu\partial_\mu\Pi + \tfrac{1}{F_\Pi}(\Pi\,\partial_\mu j^\mu + h.c.).
$$

The EQOM for Π is just eqn (*)

goldstino–scalar–fermion and goldstino–gaugino–gauge boson interactions allow the heavier superpartner to decay interaction terms have two derivatives, coupling is proportional to the difference of mass squared

Eat the Goldstino

Nambu–Goldstone boson can be eaten by a gauge boson

for gravity, Poincaré symmetry, and hence SUSY, must be a local SUSY spinor $\epsilon^\alpha \to \epsilon^\alpha(x)$: *supergravity*

spin-2 graviton has spin-3/2 fermionic superpartner, *gravitino*, $\widetilde{\Psi}_\mu^\alpha$, which transforms inhomogeneously under local SUSY transformations:

$$\delta\widetilde{\Psi}_\mu^\alpha = -\partial_\mu \epsilon^\alpha + \ldots\ .$$

gravitino is the "gauge" particle of local SUSY transformations

when SUSY is spontaneously broken, the gravitino acquires a mass by "eating" the goldstino: the other *super Higgs* mechanism

gravitino mass:

$$m_{3/2} \sim \frac{\langle \mathcal{F}_X \rangle}{M_{Pl}}$$

Gravitino Mass

In gravity-mediated SUSY breaking, the gravitino mass $\sim m_{\rm soft}$
In gauge-mediated SUSY breaking the gravitino is much lighter than the MSSM sparticles if $M_{\rm mess} \ll M_{Pl}$, so the gravitino is the LSP.

For a superpartner of mass $m_{\widetilde{\psi}} \approx 100$ GeV, and $\sqrt{\langle \mathcal{F}_X \rangle} < 10^6$ GeV

$$m_{3/2} < 1 \text{ keV}$$

the decay $\widetilde{\psi} \to \psi\,\Pi$ can be observed inside a collider detector

The goldstino theorem

no matter how SUSY is spontaneously broken, even if it is dynamical, there is a goldstino. Using the SUSY algebra it follows

$$\langle 0|\{Q_\alpha, J^{\mu\dagger}_{\dot\alpha}(y)\}|0\rangle = \sqrt{2}\sigma^\nu_{\alpha\dot\alpha}\langle 0|T^\mu_\nu(y)|0\rangle = \sqrt{2}\sigma^\nu_{\alpha\dot\alpha}\, E\,\eta^\mu_\nu \ ,$$

where E is the vacuum energy density. When $E \neq 0$, SUSY is spontaneously broken. Taking the location of the current to be at the origin, and writing out Q_α as an integral over a dummy spatial variable

$$\begin{aligned}\sqrt{2}\sigma^\mu_{\alpha\dot\alpha}\, E &= \langle 0|\{\textstyle\int d^3x J^0_\alpha(x), J^{\mu\dagger}_{\dot\alpha}(0)\}|0\rangle \\ &= \textstyle\sum_n \int d^3x \left(\langle 0|J^0_\alpha(x)|n\rangle\langle n|J^{\mu\dagger}_{\dot\alpha}(0)|0\rangle + \langle 0|J^{\mu\dagger}_{\dot\alpha}(0)|n\rangle\langle n|J^0_\alpha(x)|0\rangle\right)\end{aligned}$$

where we have inserted a sum over a complete set of states. Choosing $x^0 = 0$, use the generator of translations (P^μ) to show that

$$\begin{aligned}\langle 0|J^0_\alpha(x)|n\rangle &= \langle 0|e^{iP.x}J^0_\alpha(0)e^{-iP.x}|n\rangle \\ &= \langle 0|J^0_\alpha(0)e^{-i\vec{p}_n.\vec{x}}|n\rangle\end{aligned}$$

The goldstino theorem

So we have

$$\sqrt{2}\sigma^{\mu}_{\alpha\dot{\alpha}}\, E \quad = \quad \sum_n (2\pi)^3 \delta(\vec{p}_n) \left(\begin{array}{l} \langle 0|J^0_\alpha(0)|n\rangle\langle n|J^{\mu\dagger}_{\dot{\alpha}}(0)|0\rangle \\ +\langle 0|J^{\mu\dagger}_{\dot{\alpha}}(0)|n\rangle\langle n|J^0_\alpha(0)|0\rangle \end{array} \right)$$

write the term in parenthesis as $f_n(E_n, \vec{p}_n,)$ We can also write our anti-commutator as

$$\begin{aligned} \sqrt{2}\sigma^{\mu}_{\alpha\dot{\alpha}}\, E &= \int d^4x \left(\langle 0|J^0_\alpha(x) J^{\mu\dagger}_{\dot{\alpha}}(0)|0\rangle + \langle 0|J^{\mu\dagger}_{\dot{\alpha}}(0) J^0_\alpha(x)|0\rangle \right) \delta(x^0) \\ &= \int d^4x\, \partial_\rho \left(\langle 0|J^\rho_\alpha(x) J^{\mu\dagger}_{\dot{\alpha}}(0)|0\rangle \Theta(x^0) - \langle 0|J^{\mu\dagger}_{\dot{\alpha}}(0) J^\rho_\alpha(x)|0\rangle \Theta(-x^0) \right) \end{aligned}$$

where $\Theta(x^0)$ is the step function

E is related to the integral of a total divergence. Nonvanishing if there is a massless particle contributing to the two-point function.

The goldstino theorem

Inserting a sum over a complete set of states we have

$$\begin{aligned}
\sqrt{2}\sigma^{\mu}_{\alpha\dot{\alpha}}\, E &= \sum_n \int d^4x \partial_\rho \begin{pmatrix} \langle 0|J^\rho_\alpha(0) e^{-i\vec{p}_n.\vec{x}}|n\rangle\langle n|J^{\mu\dagger}_{\dot{\alpha}}(0)|0\rangle\Theta(x^0) \\ -\langle 0|J^{\mu\dagger}_{\dot{\alpha}}(0)|n\rangle\langle n|e^{i\vec{p}_n.\vec{x}} J^\rho_\alpha(0)|0\rangle\Theta(-x^0) \end{pmatrix} \\
&= \sum_n \int d^4x \left[\begin{array}{l} -ip_{n\rho} \begin{pmatrix} e^{-i\vec{p}_n.\vec{x}}\langle 0|J^\rho_\alpha(0)|n\rangle\langle n|J^{\mu\dagger}_{\dot{\alpha}}(0)|0\rangle\Theta(x^0) \\ +e^{i\vec{p}_n.\vec{x}}\langle 0|J^{\mu\dagger}_{\dot{\alpha}}(0)|n\rangle\langle n|J^\rho_\alpha(0)|0\rangle\Theta(-x^0) \end{pmatrix} \\ +\delta(x^0) \begin{pmatrix} e^{-i\vec{p}_n.\vec{x}}\langle 0|J^\rho_\alpha(0)|n\rangle\langle n|J^{\mu\dagger}_{\dot{\alpha}}(0)|0\rangle \\ +e^{i\vec{p}_n.\vec{x}}\langle 0|J^{\mu\dagger}_{\dot{\alpha}}(0)|n\rangle\langle n|J^\rho_\alpha(0)|0\rangle \end{pmatrix} \end{array} \right. \\
&= \sum_n (2\pi)^3\delta(\vec{p}_n) \left(f_n(E_n,\vec{p}_n,) - i\int_0^\infty dx^0\, e^{i\vec{E}_n.\vec{x}} E_n f_n(E_n,\vec{p}_n,) \right)
\end{aligned}$$

The goldstino theorem

Comparing the two eqns we see that

$$\int_0^\infty dx^0\, e^{iE_n x^0} E_n f_n(E_n, \vec{0},) = 0$$

and if SUSY is spontaneously broken

$$f_n(E_n, \vec{0},) \neq 0$$

The only possibility is that

$$f_n(E_n, \vec{0},) \propto \delta(E_n)$$

so a state contributes to our two-point function with the quantum numbers of J^0_α (i.e. a fermion) with $\vec{p} = 0$ and $E = 0$. In other words there must be a goldstino!