The MSSM

interactions of particles and sparticles

The field content of the MSSM

	bosons	fermions	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Q_i	$(\widetilde{u}_L, \widetilde{d}_L)_i$	$(u_L, d_L)_i$	$\square$	$\square$	$\frac{1}{6}$
$\overline{u}_i$	$\widetilde{u}^*_{Ri}$	$\overline{u}_i = u^\dagger_{Ri}$	$\overline{\square}$	$\mathbf{1}$	$-\frac{2}{3}$
$\overline{d}_i$	$\widetilde{d}^*_{Ri}$	$\overline{d}_i = d^\dagger_{Ri}$	$\overline{\square}$	$\mathbf{1}$	$\frac{1}{3}$
L_i	$(\widetilde{\nu}, \widetilde{e}_L)_i$	$(\nu, e_L)_i$	$\mathbf{1}$	$\square$	$-\frac{1}{2}$
$\overline{e}_i$	$\widetilde{e}^*_{Ri}$	$\overline{e}_i = e^\dagger_{Ri}$	$\mathbf{1}$	$\mathbf{1}$	1
H_u	(H_u^+, H_u^0)	$(\widetilde{H}_u^+, \widetilde{H}_u^0)$	$\mathbf{1}$	$\square$	$\frac{1}{2}$
H_d	(H_d^0, H_d^-)	$(\widetilde{H}_d^0, \widetilde{H}_d^-)$	$\mathbf{1}$	$\square$	$-\frac{1}{2}$
G	G^a_μ	$\widetilde{G}^a$	$\mathbf{Ad}$	$\mathbf{1}$	0
W	$W^3_\mu,\ W^\pm_\mu$	$\widetilde{W}^3,\ \widetilde{W}^\pm$	$\mathbf{1}$	$\mathbf{Ad}$	0
B	B_μ	$\widetilde{B}$	$\mathbf{1}$	$\mathbf{1}$	0

interactions of particles and sparticles

SM has three generations, i is a generation label

$$\begin{aligned} u_i &= (u, c, t), \qquad d_i = (d, s, b), \\ \nu_i &= (\nu_e, \nu_\mu, \nu_\tau), \quad e_i = (e, \mu, \tau). \end{aligned}$$

Higgs VEV breaks $SU(2)_L \times U(1)_Y \to U(1)$

$$Q = T_L^3 + Y$$

$$\tfrac{1}{e^2} = \tfrac{1}{g^2} + \tfrac{1}{g'^2} \ .$$

Two Higgs Doublets

Two Higgs doublets with opposite hypercharges are needed to cancel the $U(1)_Y^3$ and $U(1)_Y SU(2)_L^2$ anomalies from higgsinos
even number of fermion doublets to avoid the Witten anomaly for $SU(2)_L$.

The superpotential for the Higgs :

$$W_{\text{Higgs}} = \overline{u}\mathbf{Y_u}QH_u - \overline{d}\mathbf{Y_d}QH_d - \overline{e}\,\mathbf{Y_e}LH_d + \mu H_u H_d \ .$$

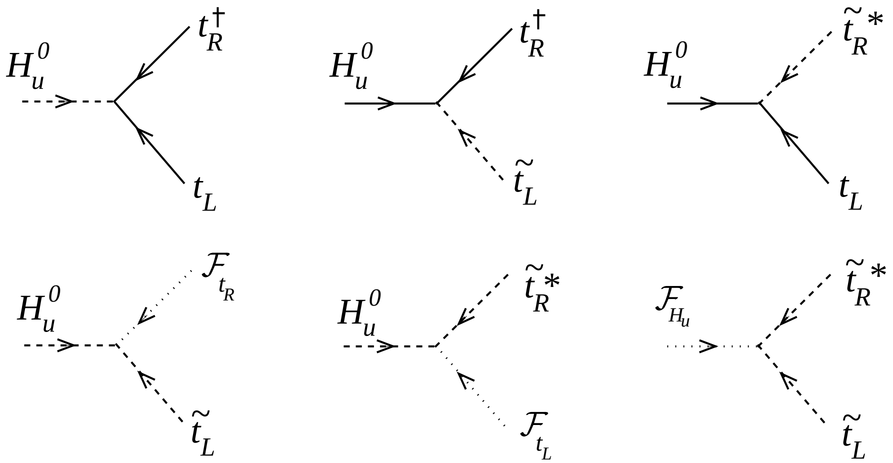
In the SM we can have Yukawa couplings with H or H^* but holomorphy requires both H_u and H_d in order to write Yukawa couplings for both u and d

Yukawa Couplings

$m_t \gg m_c, m_u;\ m_b \gg m_s, m_d;\ m_\tau \gg m_\mu, m_e,$

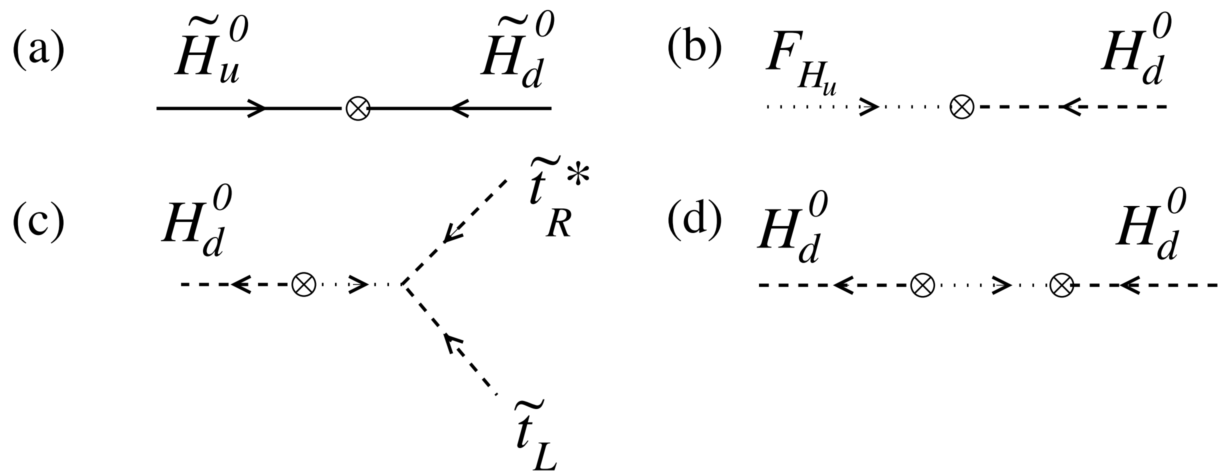
$$\mathbf{Y_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix},\ \mathbf{Y_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix},\ \mathbf{Y_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$\begin{aligned} W_{\rm Higgs} \quad = \quad & y_t(\bar{t}tH_u^0 - \bar{t}bH_u^+) - y_b(\bar{b}tH_d^- - \bar{b}bH_d^0) \\ & -y_\tau(\bar{\tau}\nu_\tau H_d^- - \bar{\tau}\tau H_d^0) + \mu(H_u^+H_d^- - H_u^0H_d^0)\ . \end{aligned}$$



μ-term

gives a mass to the higgsinos and a mixing term between a Higgs and the auxiliary $\mathcal{F}$ field of the other Higgs. Integrating out auxiliary fields yields the Higgs mass terms and the cubic scalar interactions



Higgs mass terms

$$\mathcal{L}_{\mu,\text{quadratic}} = -\mu(\widetilde{H}_u^+ \widetilde{H}_d^- - \widetilde{H}_u^0 \widetilde{H}_d^0) + h.c. \\ -|\mu|^2(|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2).$$

The D-term potential adds quartic terms with positive curvature, so there is a stable minimum at the origin with $\langle H_u \rangle = \langle H_d \rangle = 0$.

EWSB requires soft SUSY breaking terms.

without unnatural cancellations we will need $\mu \sim \mathcal{O}(m_{\text{soft}}) \sim \mathcal{O}(M_W)$ rather than $\mathcal{O}(M_{\text{Pl}})$. This is known as the μ-problem. perhaps μ is forbidden at tree-level so μ is then determined by the SUSY breaking mechanism which also determines m_{soft}.

cubic scalar

After integrating out auxiliary fields,

$$\mathcal{L}_{\mu,\text{cubic}} = \mu^* \Big(\widetilde{u}_R^* \mathbf{Y_u} \widetilde{u}_L H_d^{0*} + \widetilde{d}_R^* \mathbf{Y_d} \widetilde{d}_L H_u^{0*} + \widetilde{e}_R^* \mathbf{Y_e} \widetilde{e}_L H_u^{0*} + \widetilde{u}_R^* \mathbf{Y_u} \widetilde{d}_L H_d^{-*} + \widetilde{d}_R^* \mathbf{Y_d} \widetilde{u}_L H_u^{+*} + \widetilde{e}_R^* \mathbf{Y_e} \widetilde{\nu}_L H_u^{+*} \Big) + h.c.$$

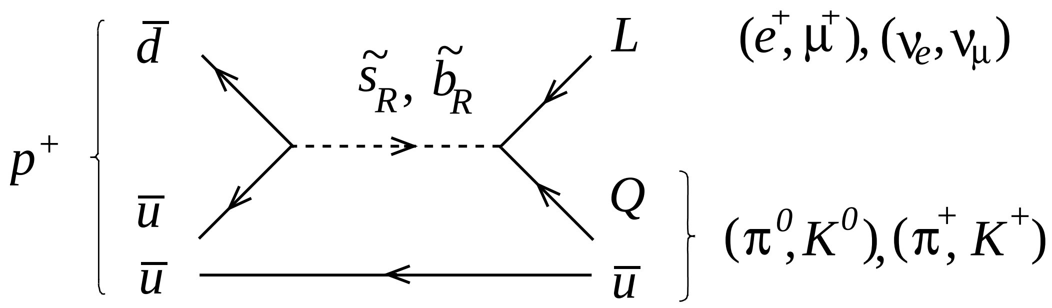
The quartic scalar interactions are obtained in a similar fashion.

other holomorphic renormalizable terms :

$$W_{\text{disaster}} = \alpha^{ijk} Q_i L_j \overline{d}_k + \beta^{ijk} L_i L_j \overline{e}_k + \gamma^i L^i H_u + \delta^{ijk} \overline{d}_i \overline{d}_j \overline{u}_k \ ,$$

W_{disaster} violates lepton and baryon number!

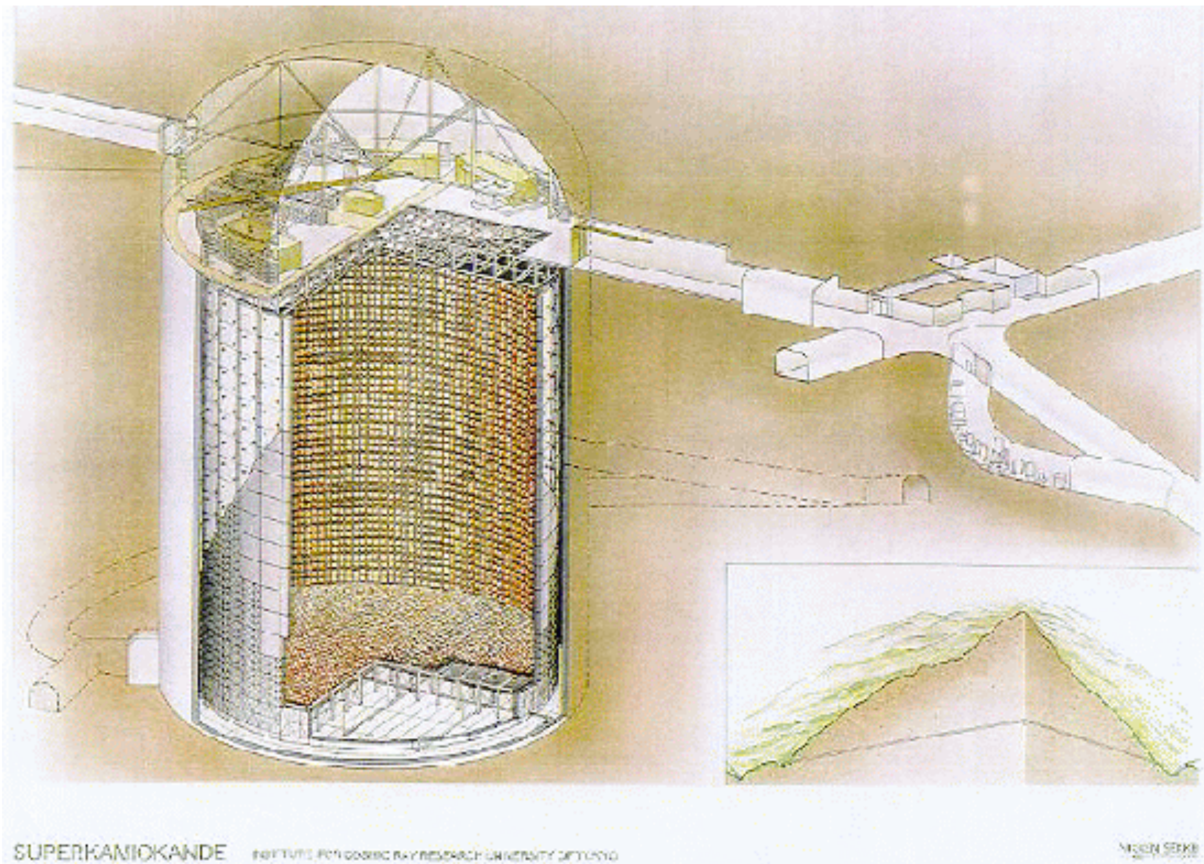
Rapid Proton Decay

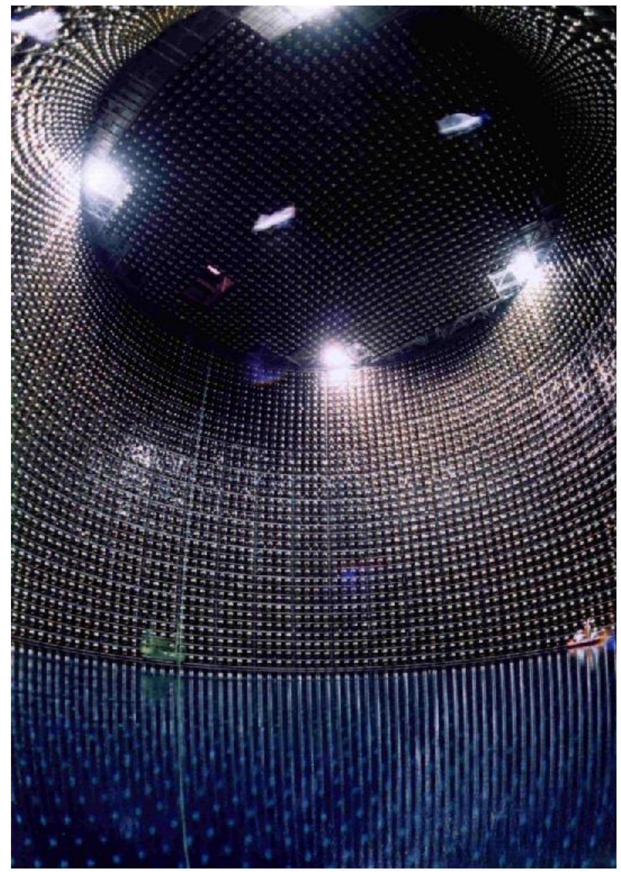


$$\Gamma_p \approx \frac{|\alpha\delta|^2}{m_{\tilde{q}}^4} \frac{m_p^5}{8\pi} \ ,$$

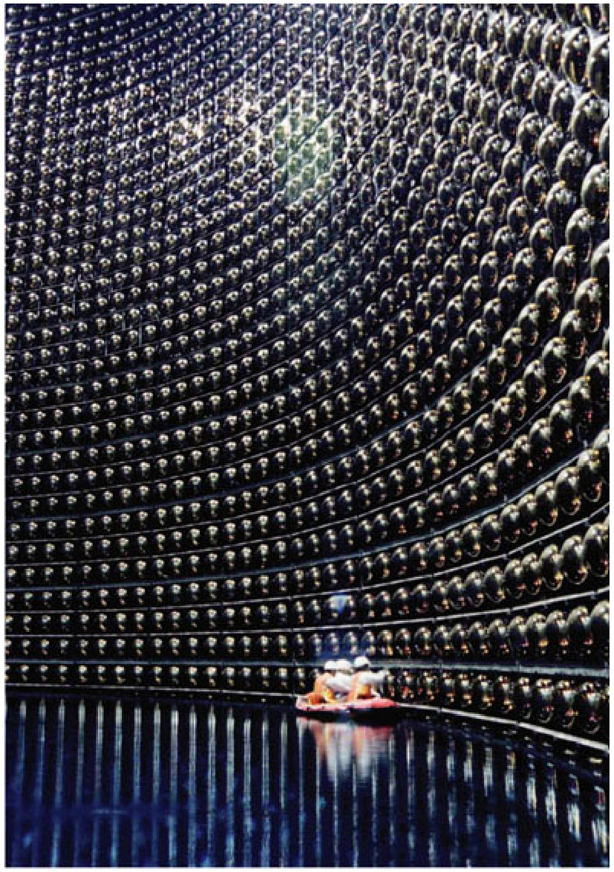
$$\tau_p = \frac{1}{\Gamma} \approx \frac{1}{|\alpha\delta|^2} \left(\frac{m_{\tilde{q}}}{1\ \mathrm{TeV}}\right)^4 2 \times 10^{-11}\,\mathrm{s}.$$

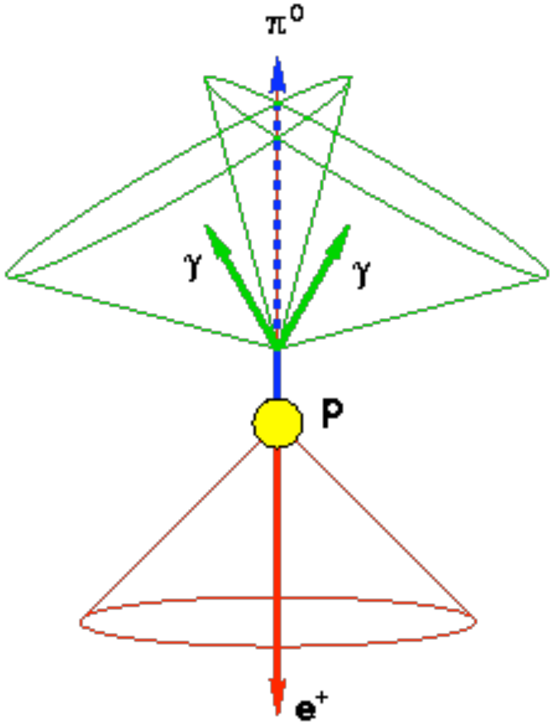
Super Kamiokande

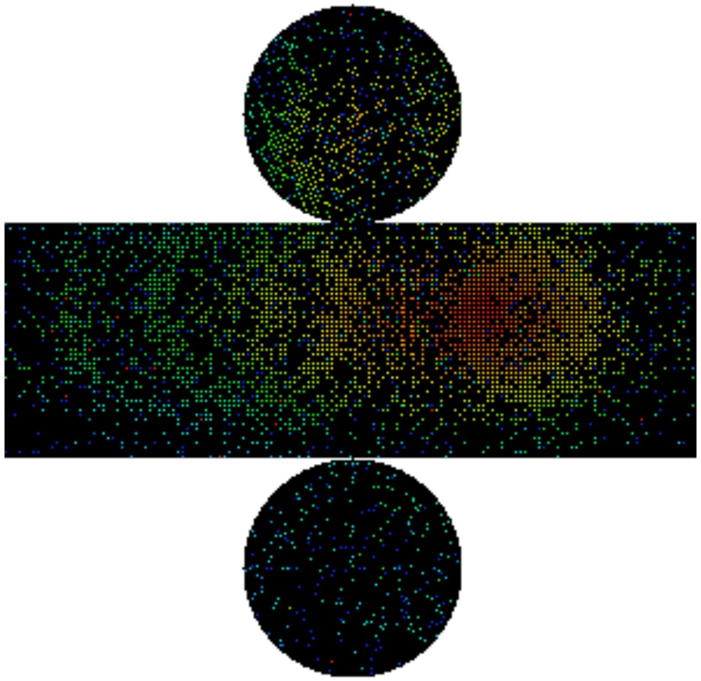




Super Kamiokande







Rapid Proton Decay

$$\tau_p \quad = \quad \frac{1}{\Gamma} \approx \frac{1}{|\alpha\delta|^2} \left(\frac{m_{\tilde{q}}}{1\,\mathrm{TeV}}\right)^4 2 \times 10^{-11}\,\mathrm{s}$$

Experimentally, $\tau_p > 10^{32}$ years $\approx 3 \times 10^{39}$ s

need $|\alpha\delta| < 10^{-25}$

R-Parity

invent a new discrete symmetry called R-parity:

$$\begin{aligned} (\text{observed particle}) &\rightarrow (\text{observed particle}) \, , \\ (\text{superpartner}) &\rightarrow -(\text{superpartner}) \, . \end{aligned}$$

Imposing this discrete R-parity forbids W_{disaster}
R-parity $\equiv$ to imposing a discrete subgroup of $B-L$
("matter parity") $P_M = (-1)^{3(B-L)}$ since

$$R = (-1)^{3(B-L)+F}$$

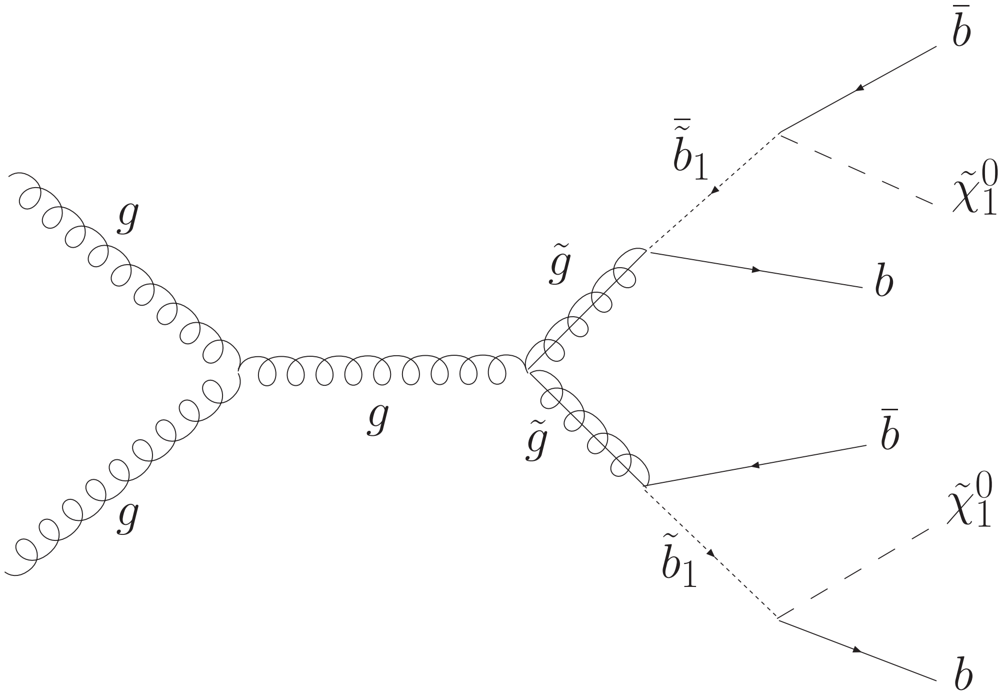
R-parity is part of the definition of the MSSM

R-Parity

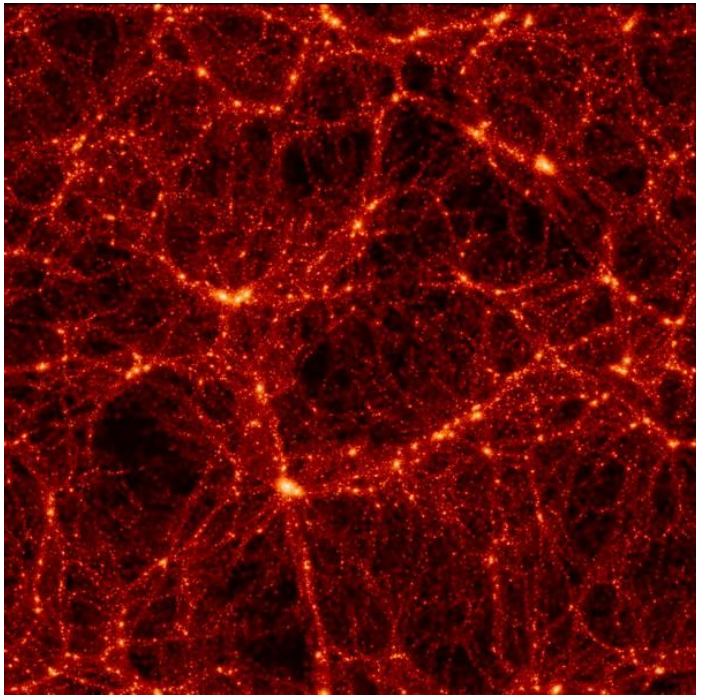
R-parity has important consequences:

- at colliders superpartners are produced in pairs;
- the lightest superpartner (LSP) is stable, and thus (if it is neutral) can be a dark matter candidate;
- each sparticle (besides the LSP) eventually decays into an odd number of LSPs.

R-Parity



g
g
g
$\tilde{g}$
$\tilde{g}$
$\bar{\tilde{b}}_1$
$\tilde{b}_1$
$\bar{b}$
$\tilde{\chi}_1^0$
b
$\bar{b}$
$\tilde{\chi}_1^0$
b



Soft SUSY Breaking

$$
\begin{aligned}
\mathcal{L}_{\rm soft}^{\rm MSSM} \quad = \quad & -\tfrac{1}{2}\left(M_3\widetilde{G}\widetilde{G} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B}\right) + h.c. \\
& - \left(\widetilde{\overline{u}}\,\mathbf{A_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{A_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{A_e}\,\widetilde{L}H_d\right) + h.c. \\
& -\widetilde{Q}^*\mathbf{m^2_Q}\,\widetilde{Q} - \widetilde{L}^*\mathbf{m^2_L}\,\widetilde{L} - \widetilde{\overline{u}}^*\mathbf{m^2_{\overline{u}}}\,\widetilde{\overline{u}} - \widetilde{\overline{d}}^*\mathbf{m^2_{\overline{d}}}\,\widetilde{\overline{d}} - \widetilde{\overline{e}}^*\mathbf{m^2_{\overline{e}}}\,\widetilde{\overline{e}} \\
& - m^2_{H_u}H_u^*H_u - m^2_{H_d}H_d^*H_d - (bH_uH_d + h.c.)\,.
\end{aligned}
$$

to $m_{\rm soft} \approx 1$ TeV in order to solve the hierarchy problem by canceling quadratic divergences:

$$M_i, \mathbf{A_f} \sim m_{\rm soft}\ , \quad \mathbf{m^2_f}, b \sim m^2_{\rm soft}\ .$$

105 more parameters than the SM!

Electroweak symmetry breaking

D-term potentials for the Higgs fields. The $SU(2)_L$ and $U(1)_Y$ D-terms are (with other scalars set to zero)

$$\begin{aligned} D^a|_{\text{Higgs}} &= -g\left(H_u^* \tau^a H_u + H_d^* \tau^a H_d\right), \\ D'|_{\text{Higgs}} &= -\frac{g'}{2}\left(|H_u^+|^2 + |H_u^0|^2 - |H_d^0|^2 - |H_d^-|^2\right) \end{aligned}$$

$$g = \frac{e}{\sin\theta_W} = \frac{e}{s_W}, \quad g' = \frac{e}{\cos\theta_W} = \frac{e}{c_W}$$

$$\begin{aligned} V(H_u, H_d) = {} & (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) \\ & + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) \\ & + b\left(H_u^+ H_d^- - H_u^0 H_d^0\right) + h.c. + \tfrac{1}{2} g^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 \\ & + \tfrac{1}{8}(g^2 + g'^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 \end{aligned}$$

Electroweak symmetry breaking

$SU(2)_L$ gauge transformation can set $\langle H_u^+ \rangle = 0$. If we look for a stable minimum along the charged directions we find

$$\frac{\partial V}{\partial H_u^+}|_{\langle H_u^+ \rangle = 0} = b H_d^- + \frac{g^2}{2} H_d^{0*} H_d^- H_u^{0*}$$

will not vanish for nonzero H_d^- for generic values of the parameters.

$$\begin{aligned} V(H_u^0, H_d^0) &= (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (b\, H_u^0 H_d^0 + h.c.) \\ &\quad + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2. \end{aligned}$$

origin is not a stable minimum requires:

$$b^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2).$$

stabilizing D-flat direction $H_u^0 = H_d^0$ where the b term is arbitrarily negative requires

$$2b < 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2.$$

Electroweak symmetry breaking

tight relation between b and μ

there is no solution if $m^2_{H_u} = m^2_{H_d}$. Typically, choose $m^2_{H_u}$ and $m^2_{H_d}$ to have opposite signs and different magnitudes

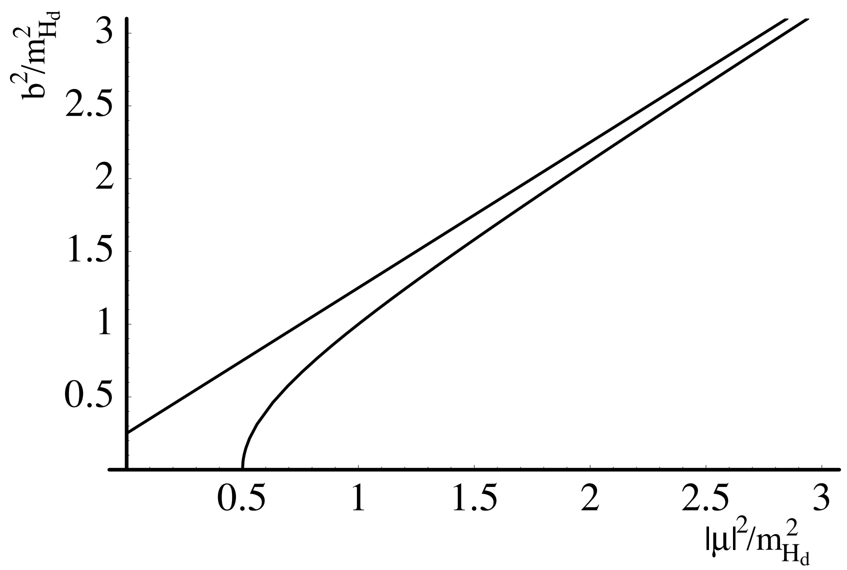


Figure 1: Above the top line the Higgs VEVs go to ∞, while below the bottom line the Higgs VEVs go to zero.

Electroweak symmetry breaking

$$\langle H_u^0 \rangle = \frac{v_u}{\sqrt{2}} \ ,$$
$$\langle H_d^0 \rangle = \frac{v_d}{\sqrt{2}} \ .$$

VEVs produce masses for the W and Z

$$M_W^2 = \frac{1}{4} g^2 v^2 \ ,$$
$$M_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2 \ ,$$

where we need to have

$$v^2 = v_u^2 + v_d^2 \approx (246 \,\mathrm{GeV})^2 \ ,$$

define an angle β:

$$s_\beta \equiv \sin\beta \equiv \frac{v_u}{v} \ , \ \ c_\beta \equiv \cos\beta \equiv \frac{v_d}{v} \ ,$$

with $0 < \beta < \pi/2$. From this definition it follows that

$$\tan\beta = v_u/v_d \ ,$$
$$\cos 2\beta = \frac{v_d^2 - v_u^2}{v^2} \ .$$

Electroweak symmetry breaking

imposing $\partial V/\partial H_u^0 = \partial V/\partial H_d^0 = 0$ gives

$$\begin{aligned} |\mu|^2 + m_{H_u}^2 &= b\cot\beta + (M_Z^2/2)\cos 2\beta \ . \\ |\mu|^2 + m_{H_d}^2 &= b\tan\beta - (M_Z^2/2)\cos 2\beta \ , \end{aligned}$$

this is another way of seeing the μ-problem.

Higgs scalar fields consist of eight real scalar degrees of freedom. three are eaten by the Z^0 and $W^\pm$. This leaves five degrees of freedom: $H^\pm$, the h_0 and H^0 which are CP even and the A^0 is CP odd.

shift the fields by their VEVs:

$$\begin{aligned} H_u^0 &\to \tfrac{v_u}{\sqrt{2}} + H_u^0 \ , \\ H_d^0 &\to \tfrac{v_d}{\sqrt{2}} + H_d^0 \ , \end{aligned}$$

Higgs spectrum

$$V \supset (\mathrm{Im} H_u^0, \mathrm{Im} H_d^0) \begin{pmatrix} b\cot\beta & b \\ b & b\tan\beta \end{pmatrix} \begin{pmatrix} \mathrm{Im} H_u^0 \\ \mathrm{Im} H_d^0 \end{pmatrix} .$$

Diagonalizing, we find the two mass eigenstates:

$$\begin{pmatrix} \pi^0 \\ A^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} s_\beta & -c_\beta \\ c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} \mathrm{Im} H_u^0 \\ \mathrm{Im} H_d^0 \end{pmatrix} .$$

would-be Nambu–Goldstone boson π^0 is massless

$$m_A^2 = \frac{b}{s_\beta c_\beta} \; .$$

Higgs spectrum

$$V \supset (H_u^{+*}, H_d^-) \begin{pmatrix} b\cot\beta + M_W^2 c_\beta^2 & b + M_W^2 c_\beta s_\beta \\ b + M_W^2 c_\beta s_\beta & b\tan\beta + M_W^2 s_\beta^2 \end{pmatrix} \begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix},$$

mass eigenstates

$$\begin{pmatrix} \pi^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} s_\beta & -c_\beta \\ c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix},$$

where $\pi^- = \pi^{+*}$ and $H^- = H^{+*}$.

$$m_{H^\pm}^2 = \quad m_A^2 + M_W^2 \ .$$

Higgs spectrum

$$V \supset (\mathrm{Re}H_u^0, \mathrm{Re}H_d^0) \left(\begin{array}{cc} b\cot\beta + M_Z^2 s_\beta^2 & -b - M_Z^2 c_\beta s_\beta) \\ -b - M_Z^2 c_\beta s_\beta) & b\tan\beta + M_Z^2 c_\beta^2 \end{array} \right) \left(\begin{array}{c} \mathrm{Re}H_u^0 \\ \mathrm{Re}H_d^0 \end{array} \right),$$

mass eigenstates

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \mathrm{Re}H_u^0 \\ \mathrm{Re}H_d^0 \end{pmatrix},$$

with masses

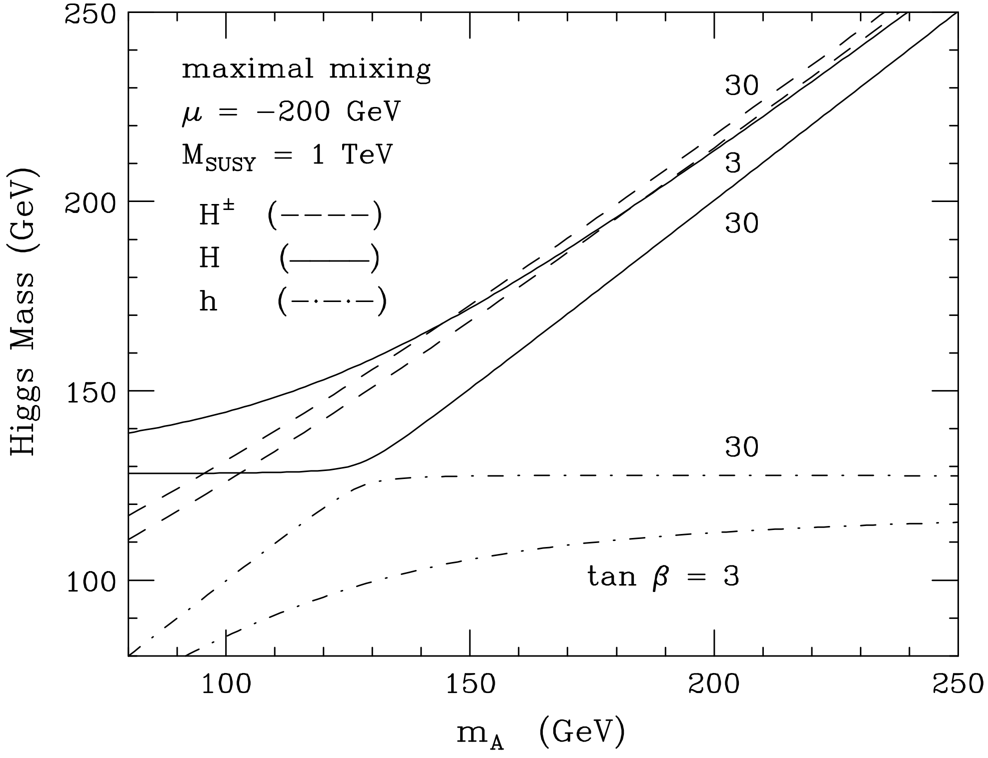
$$m_{h,H}^2 = \frac{1}{2}\left(m_A^2 + M_Z^2 \mp \sqrt{(m_A^2 + M_Z^2)^2 - 4M_Z^2 m_A^2 \cos^2 2\beta} \right),$$

and the mixing angle α is determined given by

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\frac{m_A^2 + m_Z^2}{m_H^2 - m_h^2}, \quad \frac{\cos 2\alpha}{\cos 2\beta} = -\frac{m_A^2 - m_Z^2}{m_H^2 - m_h^2} \; .$$

By convention, h^0 corresponds to the lighter mass eigenstate

Higgs spectrum



Carena, Haber, hep-ph/0208209

Higgs spectrum

Note that m_A, $m_H^{\pm}$, and $m_H \to \infty$ as $b \to \infty$ but m_h is maximized at $m_A = \infty$ so at tree-level there is an upper bound on the Higgs mass

$$m_h < |\cos 2\beta| M_Z \ ,$$

which is ruled out by experiment
There can be large one-loop corrections to the Higgs mass

The sparticle spectrum

gluino, $\widetilde{G}$, which is a color octet fermion with mass $|M_3|$

for squarks and sleptons masses have to diagonalize 6×6 matrices neglecting the intergenerational mixing stop mass terms are given by

$$\mathcal{L}_{\rm stop} = -\,(\,\widetilde{t}_L^* \quad \widetilde{t}_R^*\,)\,\mathbf{m}^{\mathbf{2}}_{\widetilde{\mathbf{t}}} \begin{pmatrix} \widetilde{t}_L \\ \widetilde{t}_R \end{pmatrix}$$

$$\mathbf{m}^{\mathbf{2}}_{\widetilde{\mathbf{t}}} = \begin{pmatrix} m^2_{Q33} + m_t^2 + \delta_u & v(A_{u33}\,s_\beta - \mu y_t c_\beta) \\ v(A_{u33}\,s_\beta - \mu y_t c_\beta) & m^2_{\overline{u}33} + m_t^2 + \delta_{\overline{u}} \end{pmatrix},$$

where

$$\delta_f = -gT_f^3\langle D^3\rangle - g'Y_f\langle D'\rangle = (T_f^3 - Q_f s_W^2)\cos 2\beta\, M_Z^2\ ,$$

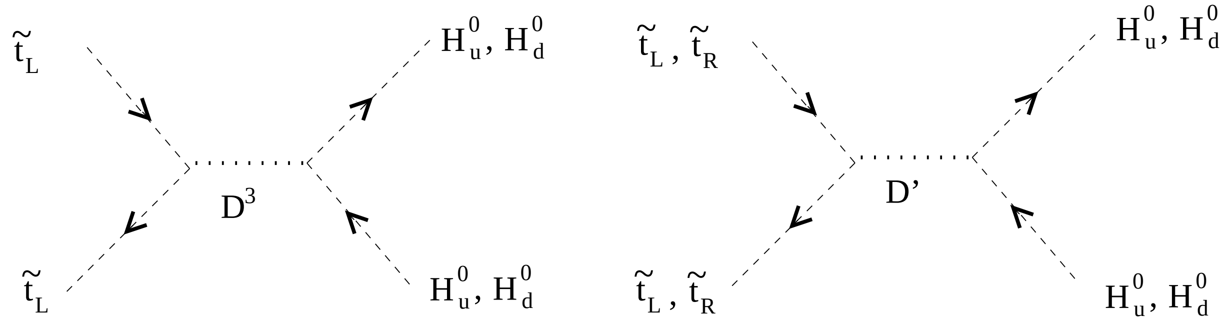
m^2_{Q33} and $m^2_{\overline{u}33}$ and A_{u33} are soft SUSY breaking terms

m_t^2 terms come from quartic with two Higgses

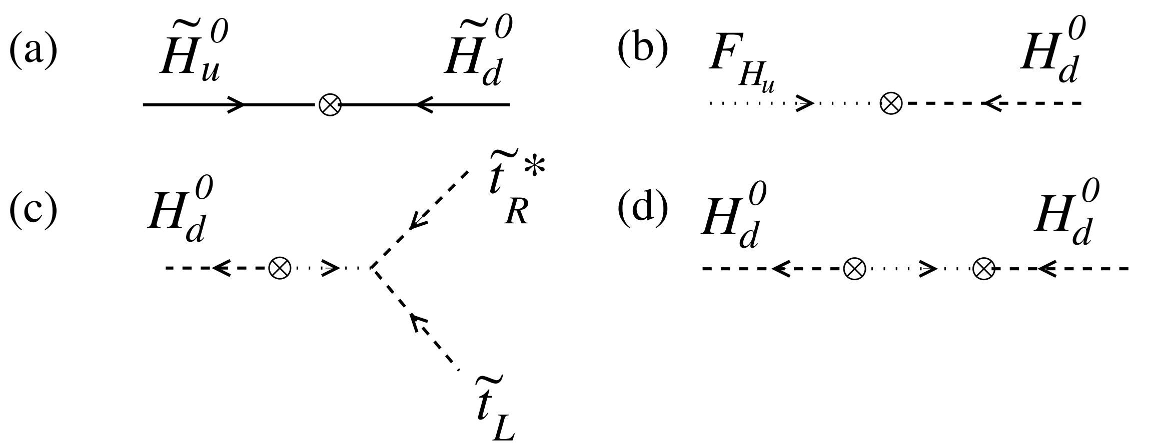
δ_f terms represent the contributions from quartic D-terms

terms $\propto \mu$ arise from integrating out the Higgs auxiliary fields

stop mixing



$\tilde{t}_L$
H_u^0, H_d^0
D^3
$\tilde{t}_L$
H_u^0, H_d^0
$\tilde{t}_L, \tilde{t}_R$
H_u^0, H_d^0
D'
$\tilde{t}_L, \tilde{t}_R$
H_u^0, H_d^0



(a)
$\tilde{H}_u^0$
$\tilde{H}_d^0$
(b)
F_{H_u}
H_d^0
(c)
H_d^0
$\tilde{t}_R^*$
$\tilde{t}_L$
(d)
H_d^0
H_d^0

The sparticle spectrum

for bottom squarks and tau sleptons

$$\mathbf{m}^2_{\tilde{\mathbf{b}}} = \begin{pmatrix} m^2_{Q33} + m^2_b + \delta_d & v(A_{d33}\, c_\beta - \mu y_b s_\beta) \\ v(A_{d33}\, c_\beta - \mu y_b s_\beta) & m^2_{\overline{d}33} + m^2_b + \delta_{\overline{d}} \end{pmatrix},$$

$$\mathbf{m}^2_{\tilde{\tau}} = \begin{pmatrix} m^2_{L33} + m^2_\tau + \delta_e & v(A_{e33}\, c_\beta - \mu y_\tau s_\beta) \\ v(A_{e33}\, c_\beta - \mu y_\tau s_\beta) & m^2_{\overline{e}33} + m^2_\tau + \delta_{\overline{e}} \end{pmatrix}$$

large Yukawa couplings or A-terms allow for large mixing and the possibility that the lower mass squared eigenvalue is driven negative This would break $U(1)_{\rm em}$ and/or $SU(3)_c$, and must be avoided

The sparticle spectrum

without soft SUSY breaking mass terms, 6×6 mixing matrices

$$\mathbf{m}^{\mathbf{2}}_{\widetilde{\mathbf{u}}} = \begin{pmatrix} \mathbf{m_u}^\dagger \mathbf{m_u} + \delta_u \, \mathbf{I} & \mathbf{\Delta}_u \\ \mathbf{\Delta_u}^\dagger & \mathbf{m_u} \mathbf{m_u}^\dagger + \delta_{\overline{u}} \, \mathbf{I} \end{pmatrix},$$
$$\mathbf{m}^{\mathbf{2}}_{\widetilde{\mathbf{d}}} = \begin{pmatrix} \mathbf{m_d}^\dagger \mathbf{m_d} + \delta_d \, \mathbf{I} & \mathbf{\Delta_d} \\ \mathbf{\Delta_d}^\dagger & \mathbf{m_d} \mathbf{m_d}^\dagger + \delta_{\overline{d}} \, \mathbf{I} \end{pmatrix},$$

where $\mathbf{m_u}$ and $\mathbf{m_d}$ are the 3×3 quark mass matrices, $\mathbf{I}$ is the identity matrix Note that $\delta_u + \delta_{\overline{u}} + \delta_d + \delta_{\overline{d}} = 0$, so at least one $\delta_f \leq 0$ Suppose $\delta_u \leq 0$, let $\vec{\gamma}$ be an eigenvector with the smallest eigenvalue,

$$\mathbf{m_u} \vec{\gamma} = m_u \vec{\gamma} \ ,$$

squark mass$^2 > 0$, upper bound on the smallest squark eigenvalue, m^2_{min}

$$m^2_{min} \leq (\vec{\gamma}^T, 0) \mathbf{m}^{\mathbf{2}}_{\widetilde{\mathbf{u}}} \begin{pmatrix} \vec{\gamma} \\ 0 \end{pmatrix} \leq m^2_u$$

So there would be a squark lighter than the u quark

Chargino spectrum

In the basis $\psi = (\widetilde{W}^+, \widetilde{H}_u^+, \widetilde{W}^-, \widetilde{H}_d^-)$, the chargino mass terms are

$$\mathcal{L}_{\text{chargino}} = -\tfrac{1}{2}\psi^T \mathbf{M}_{\widetilde{C}} \psi + hc$$

where

$$\mathbf{M}_{\widetilde{C}} = \begin{pmatrix} \mathbf{0} & \mathbf{M}^T \\ \mathbf{M} & \mathbf{0} \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} M_2 & \sqrt{2} s_\beta\, M_W \\ \sqrt{2} c_\beta\, M_W & \mu \end{pmatrix}$$

mixing comes from the wino–higgsino–Higgs coupling
can be diagonalized by a singular value decomposition:

$$\mathbf{L}^* \mathbf{M} \mathbf{R}^{-1} = \begin{pmatrix} m_{\widetilde{C}_1} & 0 \\ 0 & m_{\widetilde{C}_2} \end{pmatrix},$$

with mass eigenstates given by

$$\begin{pmatrix} \widetilde{C}_1^+ \\ \widetilde{C}_2^+ \end{pmatrix} = \mathbf{R} \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix}, \begin{pmatrix} \widetilde{C}_1^- \\ \widetilde{C}_2^- \end{pmatrix} = \mathbf{L} \begin{pmatrix} \widetilde{W}^- \\ \widetilde{H}_d^- \end{pmatrix},$$

Chargino spectrum

After diagonalization the elements of **L** and **R** appear in the interaction vertices for chargino mass eigenstates

$$m^2_{\widetilde{C}_1,\widetilde{C}_2} = \frac{1}{2}\Big[(|M_2|^2 + |\mu|^2 + 2M_W^2) \mp \sqrt{(|M_2|^2 + |\mu|^2 + 2M_W^2)^2 - 4|\mu M_2 - M_W^2 \sin 2\beta|^2}\Big]$$

In the limit that $||\mu| \pm M_2| \gg M_W$ the charginos are approximately a wino and a higgsino with masses $|M_2|$ and $|\mu|$

Neutralino spectrum

$\psi^0 = (\widetilde{B}, \widetilde{W}^3, \widetilde{H}_d^0, \widetilde{H}_u^0)$, mass terms in the Lagrangian are

$$\mathcal{L}_{\text{neutralino}} - \frac{1}{2}(\psi^0)^T \mathbf{M}_{\widetilde{N}} \psi^0 + hc$$

where

$$\mathbf{M}_{\widetilde{N}} = \begin{pmatrix} M_1 & 0 & -c_\beta\, s_W\, M_Z & s_\beta\, s_W\, M_Z \\ 0 & M_2 & c_\beta\, c_W\, M_Z & -s_\beta\, c_W\, M_Z \\ -c_\beta\, s_W\, M_Z & c_\beta\, c_W\, M_Z & 0 & -\mu \\ s_\beta\, s_W\, M_Z & -s_\beta\, c_W\, M_Z & -\mu & 0 \end{pmatrix}$$

mixing terms come from the wino–higgsino–Higgs and bino–higgsino–Higgs couplings

Since $\mathbf{M}_{\widetilde{N}}$ is a symmetric complex matrix it can be diagonalized by a Takagi factorization using a unitary matrix $\mathbf{U}$

$$\mathbf{M}_{\widetilde{N}}^{\text{diag}} = \mathbf{U}^* \mathbf{M}_{\widetilde{N}} \mathbf{U}^{-1} \ .$$

Neutralino spectrum

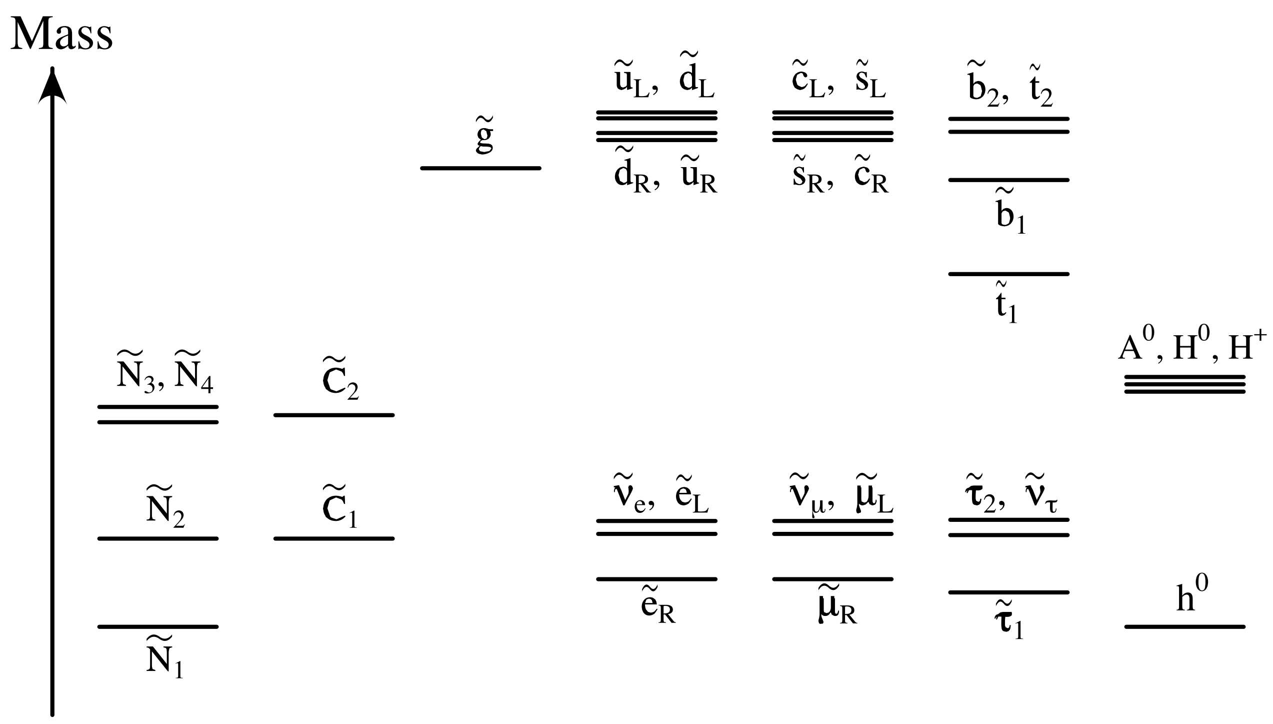
In the region of parameter space where

$$M_Z \ll |\mu \pm M_1|, |\mu \pm M_2|$$

then the neutralino mass eigenstates are very nearly $\widetilde{B}$, $\widetilde{W}^0$, $(\widetilde{H}_u^0 \pm \widetilde{H}_d^0)/\sqrt{2}$, with masses: $(|M_1|, |M_2|, |\mu|, |\mu|)$.

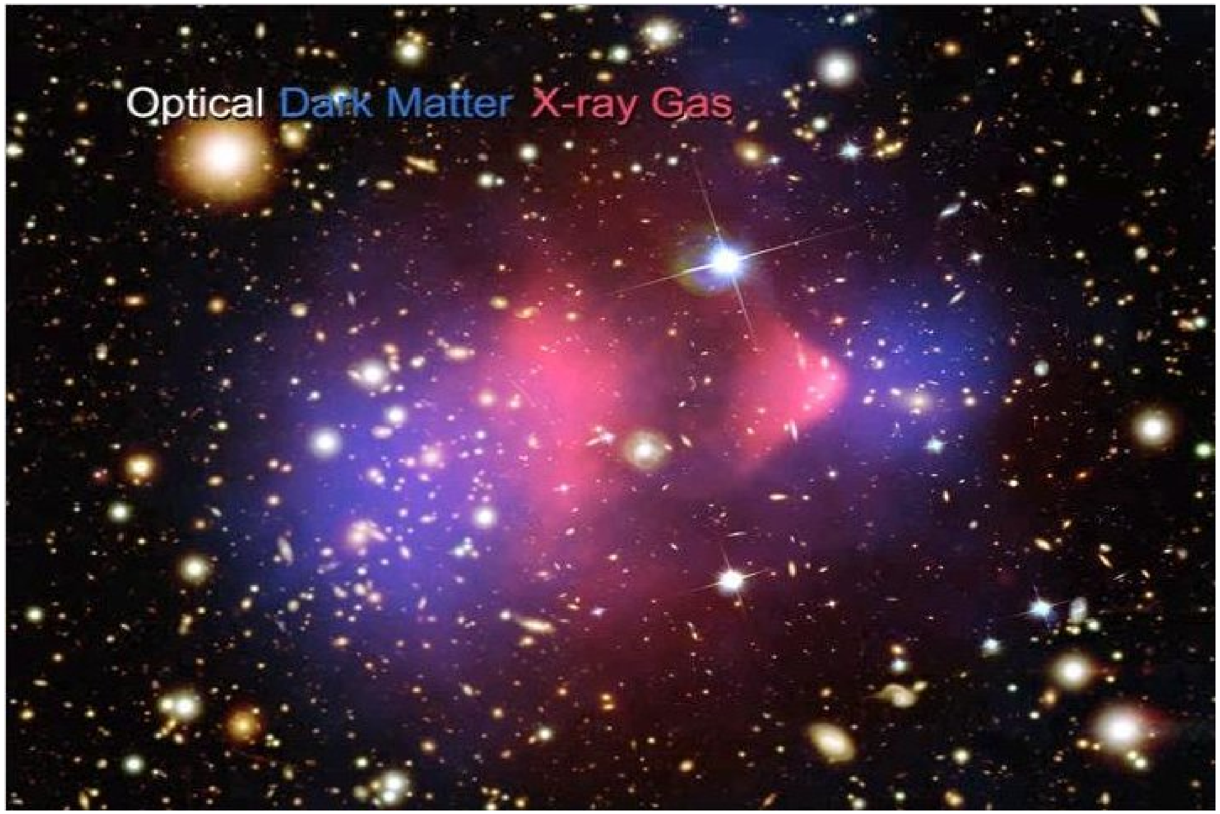
A "bino-like" LSP can make a good dark matter candidate, N_1 is often arranged to be the LSP

Spectrum



Martin, hep-ph/9709356

Dark Matter



astro-ph/0608407

Dark Matter Relic Abundance

Robertson-Walker metric and scale factor R

$$ds^2 = -dt^2 + R(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$$

Friedman equation

$$H^2 \equiv \left(\frac{\dot{R}}{R} \right)^2 = \frac{8}{3}\pi G\rho - \frac{k}{R^2} + \dots \ ,$$

relates the Hubble parameter H to Newton's constant, G, times the energy density, ρ, the critical density is for $k = 0$ is

$$\rho_c = \frac{3H^2}{8\pi G} \approx 10^{-29}\,\mathrm{g/cm^3} \approx 3 \times 10^{-47}\,\mathrm{GeV^4} \ .$$

Dark Matter Relic Abundance

Energy conservation

$$
\begin{aligned}
R^3\left(\frac{dp}{dt}\right) &= \frac{d}{dt}\left[R^3\left(\rho+p\right)\right] \\
\frac{dp}{dt} &= -3\frac{\dot{R}}{R}\left(\rho+p\right)
\end{aligned}
$$

for $p = a\rho$

$$\rho \propto R^{-3(1+a)}$$

radiation	$a = 1/3$	$\rho \propto R^{-4}$
matter	$a = 0$	$\rho \propto R^{-3}$
curvature	$a = 0$	$\rho \propto R^{-2}$
vacuum energy	$a = -1$	$\rho \propto R^{0}$

Dark Matter Relic Abundance

a stable weakly interacting dark matter particle X is held in equilibrium by annihilations

$$XX \leftrightarrow p_i\bar{p}_i$$

eventually the expansion of the Universe dilutes the particles so they are too sparse to maintain equilibrium

equilibrium number density, n_{eq}, thermal average of the annihilation cross section times the relative velocity $\langle\sigma v\rangle$

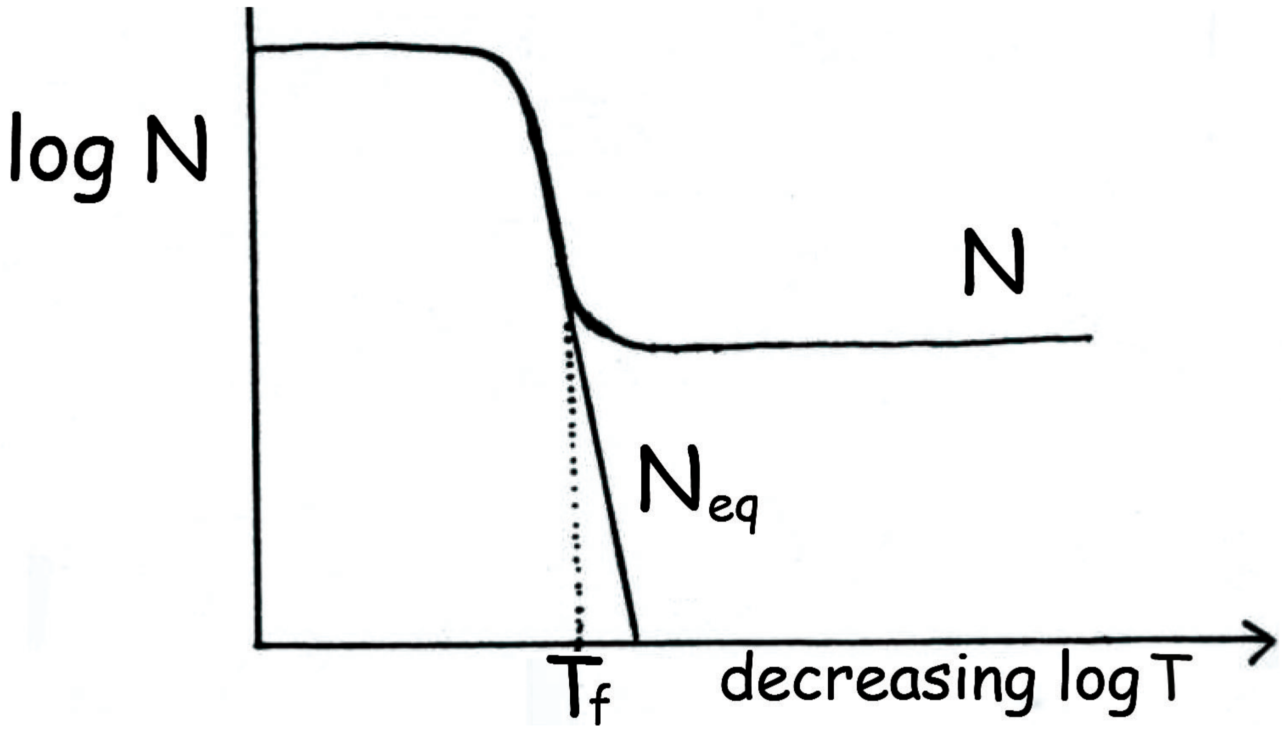
$\dot{n}_{\text{annihilations}} \sim \langle\sigma v\rangle n_{eq}^2$

$\dot{n}_{\text{expansion}} \sim 3Hn_{eq}$

when $\dot{n}_{\text{annihilations}} \approx \dot{n}_{\text{expansion}}$ dark matter 'freezes out"

after freeze out, number of dark matter particles per comoving volume $N \equiv n/T^3$ remains constant

Freeze Out



Quantum Stat. Mech.

Bose-Einstein and Fermi-Dirac

$$
\begin{aligned}
b(E) &= \frac{1}{e^{(E-\mu)/T}-1} \\
f(E) &= \frac{1}{e^{(E-\mu)/T}+1}
\end{aligned}
$$

assume chemical potential $\mu = 0$ and relativistic

$$
\begin{aligned}
N_b &= \frac{g_s}{2\pi^2}\int_0^\infty dp \frac{p^2}{e^{p/T}-1} \\
N_f &= \frac{g_s}{2\pi^2}\int_0^\infty dp \frac{p^2}{e^{p/T}+1}
\end{aligned}
$$

scalar	$g_s = 1$
Dirac	$g_s = 2 \times 2 = 4$
Majorana	$g_s = 2$
photon	$g_s = 2$
Z	$g_s = 3$
W	$g_s = 2 \times 3 = 6$

Quantum Stat. Mech.

$$\int_0^\infty dx \frac{x^{\nu-1}}{e^{ax}-1} = a^{-\nu}\,\Gamma(\nu)\zeta(\nu)$$
$$\int_0^\infty dx \frac{x^{\nu-1}}{e^{ax}+1} = (1-2^{1-\nu})\,a^{-\nu}\,\Gamma(\nu)\zeta(\nu)$$

$$N_b = \frac{g_s}{\pi^2}\zeta(3)T^3$$
$$N_f = \frac{3}{4}\frac{g_s}{\pi^2}\zeta(3)T^3$$

$$\rho_b = \frac{g_s}{2\pi^2}\int_0^\infty dp \frac{p^3}{e^{p/T}-1} = \frac{g_s \pi^2}{30} T^4$$
$$\rho_f = \frac{g_s}{2\pi^2}\int_0^\infty dp \frac{p^3}{e^{p/T}+1} = \frac{7}{8}\,\frac{g_s \pi^2}{30} T^4$$

where we used $\zeta(4) = \pi^4/90$

Quantum Stat. Mech.

assume chemical potential $\mu = 0$ and non-relativistic $m \gg T$

$$
\begin{aligned}
N_{f,b} &\approx \frac{g_s}{2\pi^2}\int_0^\infty dp\frac{p^2}{e^{m/T+p^2/(2mT)}\pm 1} \\
&\approx \frac{g_s T^3}{2\pi^2}\int_0^\infty du\frac{u^2}{e^{m/T+u^2T/m}\pm 1} \\
&\approx \frac{g_s T^3 e^{-m/T}}{2\pi^2}\int_0^\infty du\, u^2\, e^{-u^2T/m} \\
&\approx \frac{g_s T^3 e^{-m/T}}{(2\pi T/m)^{3/2}}
\end{aligned}
$$

Equilibrium

equilibrium number of nonrelativistic particles per comoving volume:

$$N_{eq} = \frac{e^{-m_X/T}}{(2\pi)^{3/2}} \left(\frac{m_X}{T}\right)^{3/2}$$

above $T \approx 1$ eV the universe is radiation-dominated

$$\rho = \frac{\pi^2}{15} N_* T^4$$

$$N_* = \frac{1}{2}\left(n_b + \frac{7}{8} n_f\right)$$

so

$$H = \sqrt{\frac{8}{3}\pi G\rho} = \sqrt{\frac{8\pi^3 N_* G}{15}}\, T^2$$

$$\langle \sigma v \rangle = \sigma_0 \left(\frac{T}{m}\right)^{\alpha} \ ,$$

$\alpha = 0$ for Dirac fermion, $\alpha = 1$ for a Majorana fermion

Cross Sections

Dirac fermion:

$$\langle \sigma v \rangle = \frac{G_F^2}{2\pi} m_X^2$$

Majorana fermions have no vector current couplings only axial current:

$$\langle \sigma v \rangle \propto \frac{G_F^2}{2\pi} p^2$$

referred to as p-wave suppression

$$\langle p^2 \rangle = \frac{3}{2} m_X T$$

Freeze Out

Equating the annihilation rate with the expansion rate at $T = T_f$

$$\begin{aligned} \langle \sigma v \rangle n_{eq}^2 &= 3 H n_{eq} \\ \sigma_0 \left(\frac{T_f}{m_X}\right)^{\alpha} \frac{e^{-m_X/T_f}}{(2\pi)^{3/2}} \left(\frac{m_X}{T_f}\right)^{3/2} T_f^3 &= 3\sqrt{\frac{8\pi^3 N_* G}{15}}\, T_f^2 \end{aligned}$$

$$e^{-m_X/T_f} = 3\sqrt{\frac{8\pi^3 N_* G}{15}}\, \frac{(2\pi)^{3/2}}{\sigma_0\, m_X} \left(\frac{m_X}{T_f}\right)^{\alpha - 1/2}$$

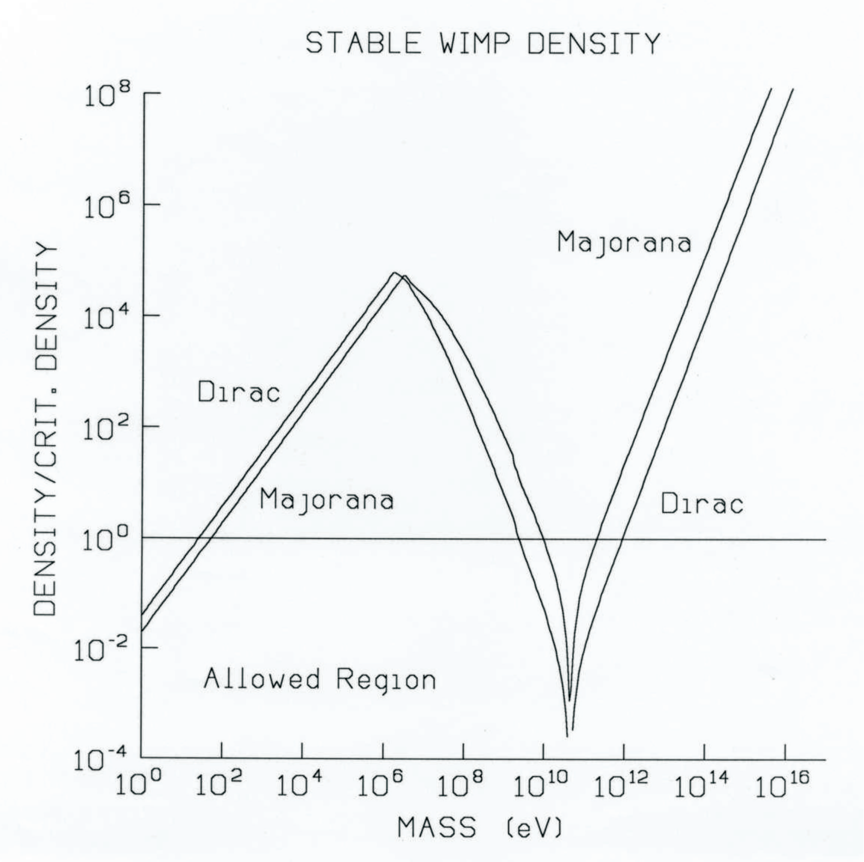
Numerically $m_X/T_f \approx 30$. So the number per comoving volume at T_f is

$$N_f = \sqrt{\frac{8\pi^3 N_* G}{15}}\, \frac{3}{\sigma_0\, m_X} \left(\frac{m_X}{T_f}\right)^{1+\alpha}$$

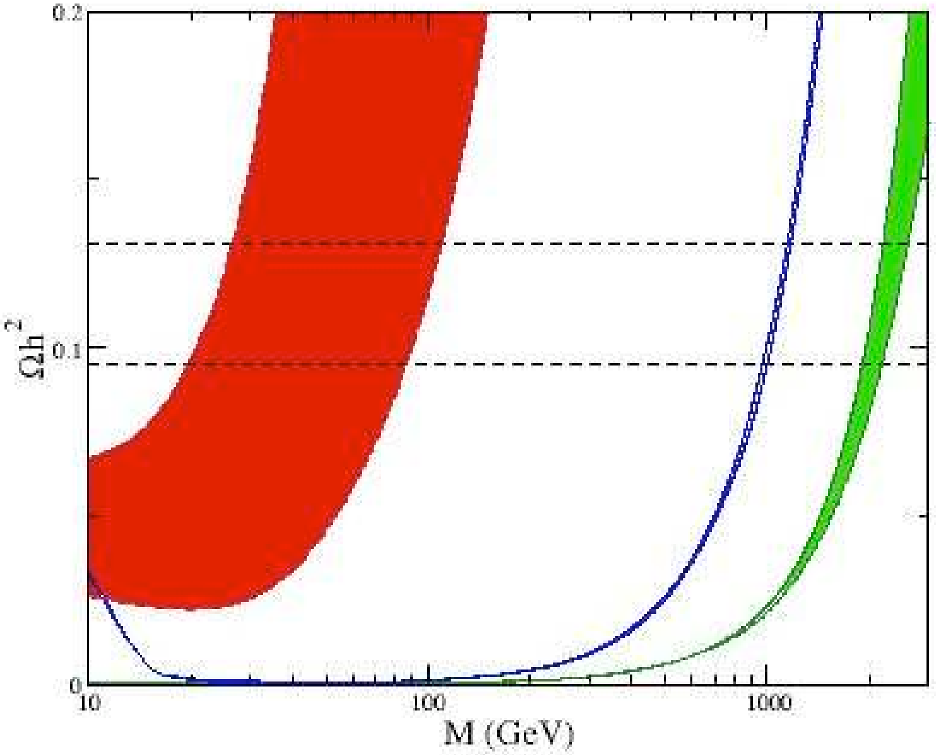
$\times T^3$ gives the number density, $\times m_X$ gives the energy density. weak annihilation cross section $\sigma_0 = N_A G_F^2 m_X^2 / 2\pi$ (where N_A counts final states) with a current temperature of $T = 2.7$ K $= 2 \times 10^{-13}$ GeV, $\alpha = 1$, $N_* = 100$, $N_A = 20$, that

$$\frac{\rho_X}{\rho_c} = 0.6 \left(\frac{100\text{GeV}}{m_X}\right)^2$$

Stable WIMPS



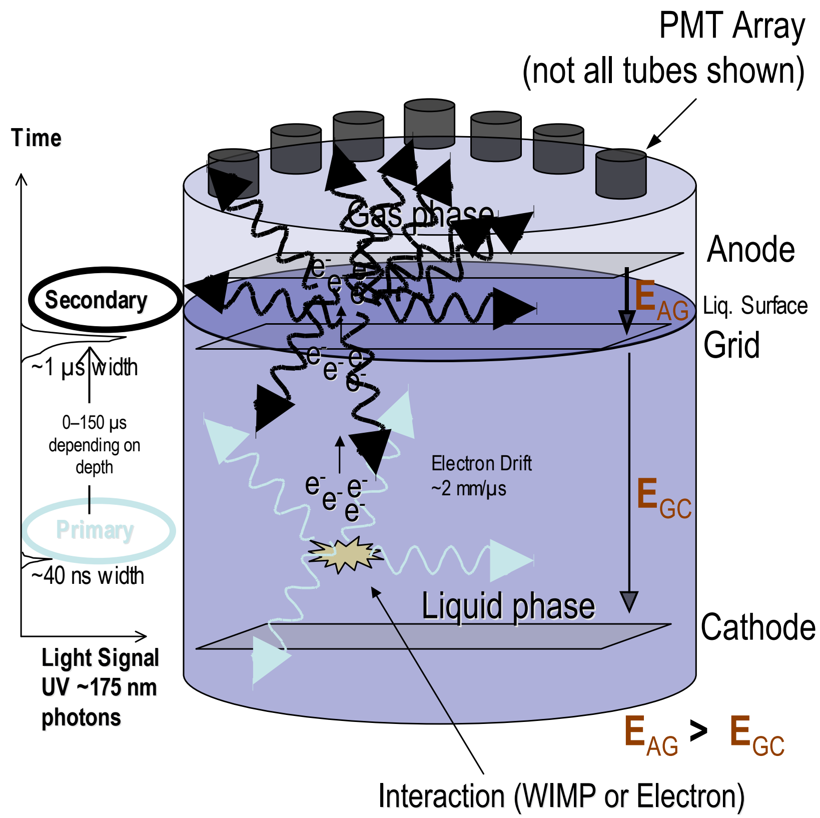
LSP Dark Matter



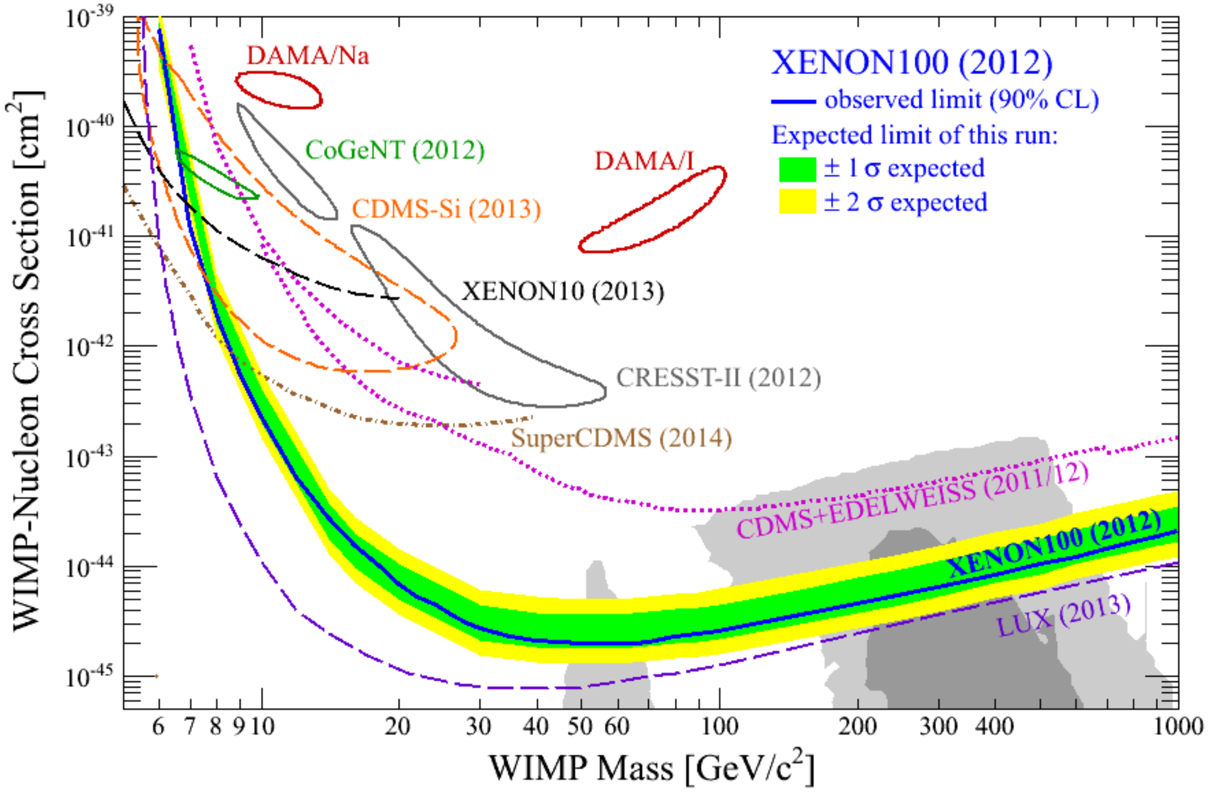
Bino, Higgsino, Wino

Arkani-Hamed, Delgado, Giudice, hep-ph/0601041

Xenon Detector

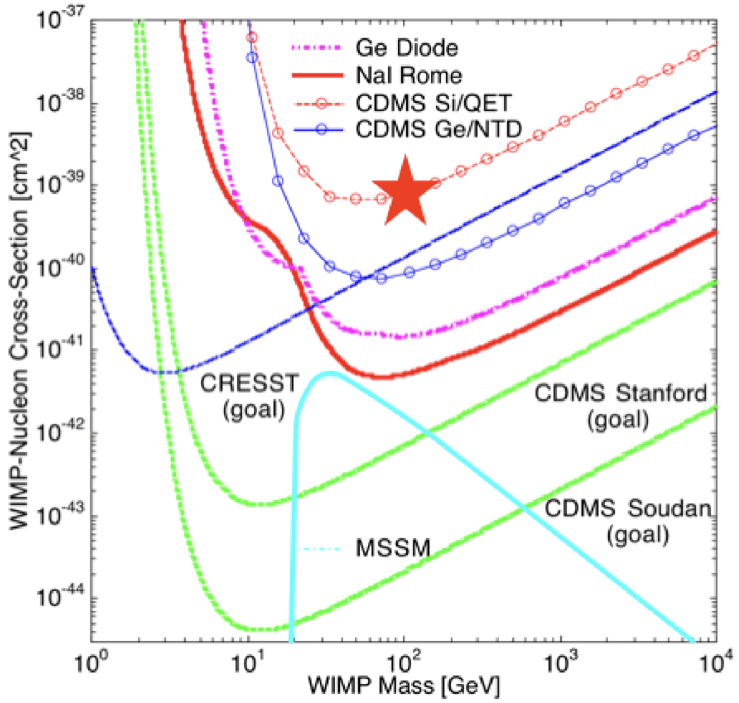


Xenon 100 and LUX



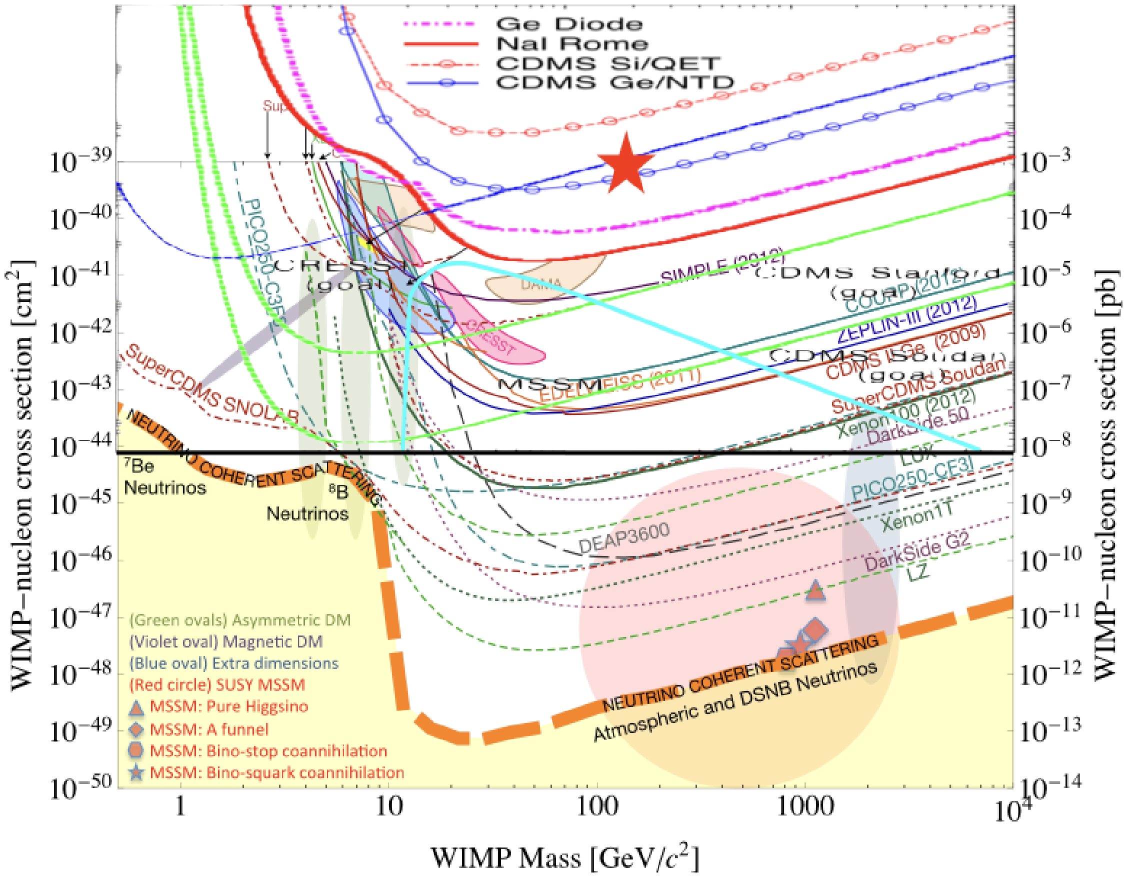
Schumann 1405.7600

Dark Matter Searches



astro-ph/9712343

Dark Matter Searches



hep-ex/1310.8327

$SU(5)$ GUT

$$SU(5) \to SU(3) \times SU(2) \times U(1)$$

$$\mathbf{5} \to (\mathbf{3},\mathbf{1})_{-1/3} + (\mathbf{1},\mathbf{2})_{1/2} \sim d_R + (e^c, -\nu^c)_L$$

$$\bar{\mathbf{5}} \to (\bar{\mathbf{3}},\mathbf{1})_{+1/3} + (\mathbf{1},\mathbf{2})_{-1/2} \sim d_R^c + (\nu, e)_L$$

$$\mathbf{5} \times \mathbf{5} = \mathbf{10}_A + \mathbf{15}_S$$

$$\begin{aligned} \mathbf{10} \quad \to \quad & (\mathbf{3},\mathbf{2})_{1/6} + (\bar{\mathbf{3}},\mathbf{1})_{-2/3} + (\mathbf{1},\mathbf{1})_1 \\ & \sim (u,d)_L + u_R^c + e^c \end{aligned}$$

$\bar{\mathbf{5}} + \mathbf{10}$ is anomaly free

$$SU(5) \to SU(3) \times SU(2) \times U(1)$$

$$\mathbf{5} \to (\mathbf{3},\mathbf{1})_{-1/3} + (\mathbf{1},\mathbf{2})_{1/2}$$

$$\begin{aligned} \mathbf{5} \times \bar{\mathbf{5}} &= \mathbf{1} + \mathbf{24} \\ &= (\mathbf{1},\mathbf{1})_0 + (\mathbf{8},\mathbf{1})_0 + (\mathbf{1},\mathbf{1})_0 + (\mathbf{1},\mathbf{3})_0 \\ &\quad +(\mathbf{3},\mathbf{2})_{-5/6} + (\bar{\mathbf{3}},\mathbf{2})_{5/6} \end{aligned}$$

$$\begin{aligned} \mathbf{24} &\to (\mathbf{8},\mathbf{1})_0 + (\mathbf{1},\mathbf{3})_0 + (\mathbf{1},\mathbf{1})_0 \\ &\quad +(\mathbf{3},\mathbf{2})_{-5/6} + (\bar{\mathbf{3}},\mathbf{2})_{5/6} \end{aligned}$$

$SU(5) \to SU(3) \times SU(2) \times U(1)$

$$G^a_\mu \leftrightarrow T^{1,\ldots,8} = \frac{1}{2}\begin{pmatrix} \lambda^{1,\ldots,8} & 0 \\ 0 & 0 \end{pmatrix}$$

$$W^a_\mu \leftrightarrow T^{9,10,11} = \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & \sigma^{1,2,3} \end{pmatrix}$$

$$X_\mu, Y_\mu \leftrightarrow T^{12,\ldots,23} = \frac{1}{2}\begin{pmatrix} 0 & x \\ x^\dagger & 0 \end{pmatrix}$$

$$B_\mu \leftrightarrow T^{24} = \frac{1}{2\sqrt{15}}\begin{pmatrix} -2 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

$$SU(5) \to SU(3) \times SU(2) \times U(1)$$

$$\mathrm{Tr}\, T^{24} T^{24} = \frac{1}{4 \cdot 15} \mathrm{Tr} \begin{pmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 9 \end{pmatrix} = \frac{1}{2}$$

$$Y = \frac{\sqrt{15}}{3} T^{24} = \sqrt{\frac{5}{3}} T^{24}$$

$$g' Y = \left(\sqrt{\frac{5}{3}} g'\right) \left(\sqrt{\frac{3}{5}} Y\right) = g_1 T^{24}$$

$$g_1 = \sqrt{\frac{5}{3}} g'$$

$$Q_1 = T^{24} = \sqrt{\frac{3}{5}} Y$$

Gauge coupling unification

for $SU(5)_{GUT}$

$$g_1 \equiv \sqrt{\tfrac{5}{3}} g' \ , \quad g_2 \equiv g, \quad g_3 \equiv g_C, \quad \alpha_i \equiv \frac{g_i^2}{4\pi}$$

The measured values of gauge couplings renormalized at M_Z are

$$\begin{aligned}
\alpha_1(M_Z) &= 0.016830 \pm 0.000007 \\
\alpha_2(M_Z) &= 0.03347 \pm 0.00003 \\
\alpha_3(M_Z) &= 0.1187 \pm 0.002
\end{aligned}$$

These couplings run at one-loop according the RG equation:

$$\mu \frac{dg_a}{d\mu} = -\frac{1}{16\pi^2} b_a g_a^3 \quad \Rightarrow \quad \mu \frac{d\alpha_a^{-1}}{d\mu} = \frac{b_a}{2\pi}$$

In the SM and MSSM the β function coefficients are

$$\begin{aligned}
b_a^{\mathrm{SM}} &= (-41/10, 19/6, 7) \\
b_a^{\mathrm{MSSM}} &= (-33/5, -1, 3)
\end{aligned}$$

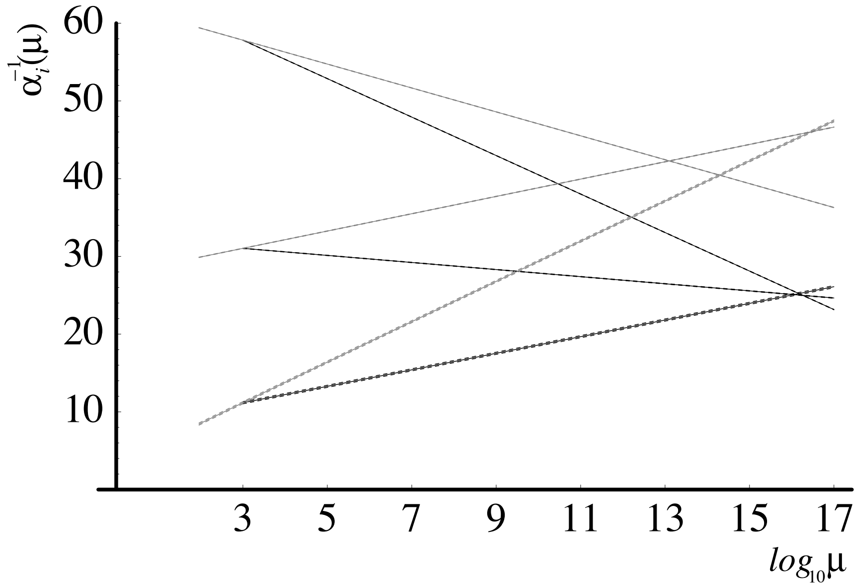
SM β-functions

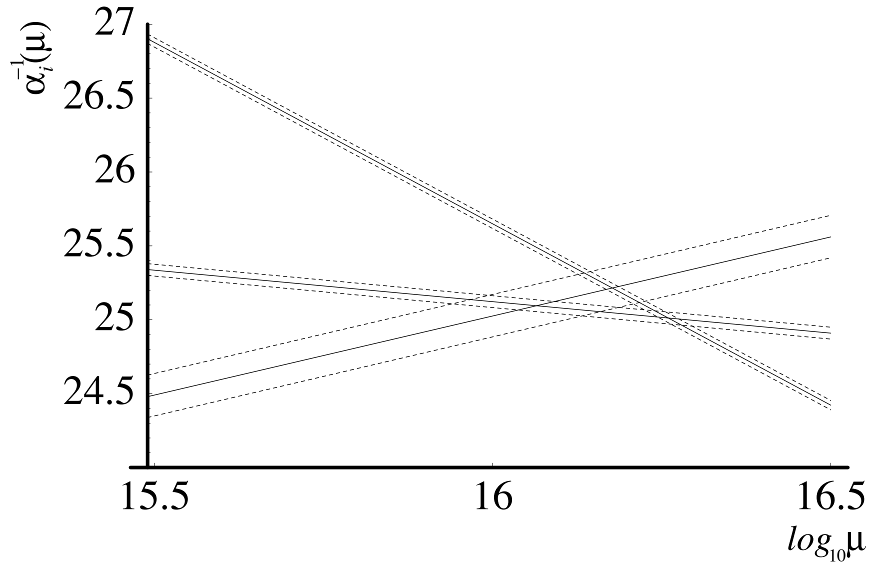
$$
\begin{aligned}
b_1 &= -\tfrac{2}{3}Q_F^2 - \tfrac{1}{3}Q_S^2 = -\tfrac{3}{5}\left(\tfrac{2}{3}Y_F^2 + \tfrac{1}{3}Y_S^2\right) \\
&= -\tfrac{3}{5}\left(\tfrac{2}{3}N_{\rm gen}\left[3\cdot 2\cdot Y_Q^2 + 3Y_u^2 + 3Y_d^2 + 2Y_L^2 + Y_e^2\right] + \tfrac{1}{3}2Y_H^2\right) \\
&= -\tfrac{1}{5}\left(2N_{\rm gen}\left[3\cdot 2\cdot\left(\tfrac{1}{6}\right)^2 + 3\left(\tfrac{2}{3}\right)^2 + 3\left(\tfrac{1}{3}\right)^2 + 2\left(\tfrac{1}{2}\right)^2 + 1^2\right] + 2\left(\tfrac{1}{2}\right)^2\right) \\
&= -\tfrac{1}{5}\left(2N_{\rm gen}\left[\tfrac{1}{6} + \tfrac{4}{3} + \tfrac{1}{3} + \tfrac{1}{2} + 1\right] + \tfrac{1}{2}\right) \\
&= -\tfrac{1}{5}\left(N_{\rm gen}\tfrac{1+8+2+9}{3} + \tfrac{1}{2}\right) = -\tfrac{1}{5}\left(N_{\rm gen}\tfrac{20}{3} + \tfrac{1}{2}\right) \\
&= -\tfrac{41}{10} \\
b_2 &= \tfrac{11}{3}\cdot N - \tfrac{2}{3}T(F) - \tfrac{1}{3}T(S) = \tfrac{22}{3} - \tfrac{2}{3}N_{\rm gen}\left(3\cdot\tfrac{1}{2} + \tfrac{1}{2}\right) - \tfrac{1}{3}\cdot\tfrac{1}{2} \\
&= \tfrac{22}{3} - \tfrac{4}{3}N_{\rm gen} - \tfrac{1}{6} = \tfrac{22}{3} - \tfrac{4}{3}N_{\rm gen} - \tfrac{1}{6} = \tfrac{20-1}{6} \\
&= \tfrac{19}{6} \\
b_3 &= \tfrac{11}{3}\cdot 3 - \tfrac{2}{3}T(F) = \tfrac{33}{3} - \tfrac{2}{3}N_{\rm gen}\left(2\cdot 2\cdot\tfrac{1}{2}\right) = \tfrac{33}{3} - \tfrac{4}{3}N_{\rm gen} \\
&= \tfrac{33-12}{3} = 7
\end{aligned}
$$

MSSM β-functions

$$
\begin{aligned}
b_1 &= -\tfrac{2}{3}Q_F^2 - \tfrac{1}{3}Q_S^2 = -Q^2 = -\tfrac{3}{5}Y^2 \\
&= -\tfrac{3}{5}\left(N_{\text{gen}}\left[3\cdot 2\cdot Y_Q^2 + 3Y_u^2 + 3Y_d^2 + 2Y_L^2 + Y_e^2\right] + 2\cdot 2Y_H^2\right) \\
&= -\tfrac{3}{5}\left(N_{\text{gen}}\left[3\cdot 2\cdot \left(\tfrac{1}{6}\right)^2 + 3\left(\tfrac{2}{3}\right)^2 + 3\left(\tfrac{1}{3}\right)^2 + 2\left(\tfrac{1}{2}\right)^2 + 1^2\right] + 4\left(\tfrac{1}{2}\right)^2\right) \\
&= -\tfrac{3}{5}\left(N_{\text{gen}}\left[\tfrac{1}{6} + \tfrac{4}{3} + \tfrac{1}{3} + \tfrac{1}{2} + 1\right] + 1\right) \\
&= -\tfrac{3}{5}\left(N_{\text{gen}}\tfrac{1+8+2+9}{6} + 1\right) = -\tfrac{3}{5}\left(N_{\text{gen}}\tfrac{20}{6} + 1\right) \\
&= -\tfrac{33}{5} \\
b_2 &= 3N - F = 3\cdot 2 - N_{\text{gen}}\left(3\cdot\tfrac{1}{2} + \tfrac{1}{2}\right) - 1 \\
&= 6 - 2N_{\text{gen}} - 1 \\
&= -1 \\
b_3 &= 3\cdot 3 - 2N_{\text{gen}} = 9 - 6 \\
&= 3
\end{aligned}
$$

Gauge coupling unification





common threshold $M_{\rm SUSY}$

$$3 \text{ GeV} < M_{\rm SUSY} < 100 \text{ TeV}.$$

$$M_U \approx 2 \times 10^{16} \text{ GeV}.$$

Radiative electroweak symmetry breaking

RG equations for the soft SUSY breaking masses of the Higgs and third-generation scalars
gaugino terms additive consider separately
consider only the running induced by y_t

$$16\pi^2 \frac{d}{dt} m^2_{H_d} = 0$$

$$16\pi^2 \frac{d}{dt} \begin{pmatrix} m^2_{H_u} \\ m^2_{\overline{u}33} \\ m^2_{Q33} \end{pmatrix} = 2|y_t|^2 \begin{pmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} m^2_{H_u} \\ m^2_{\overline{u}33} \\ m^2_{Q33} \end{pmatrix}$$

Radiative electroweak symmetry breaking

transform to an eigenbasis: $(1, -1, 0)$, $(0, 1, -1)$, and $(3, 2, 1)$
eigenvalues 0, 0 , and 6.
eigenvector $(3, 2, 1)$ scaled to zero

$m^2_{H_u} = m^2_{\overline{u}_3} = m^2_{Q_3} = m^2_0$ at high scale

decompose initial conditions:

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

in IR masses run to

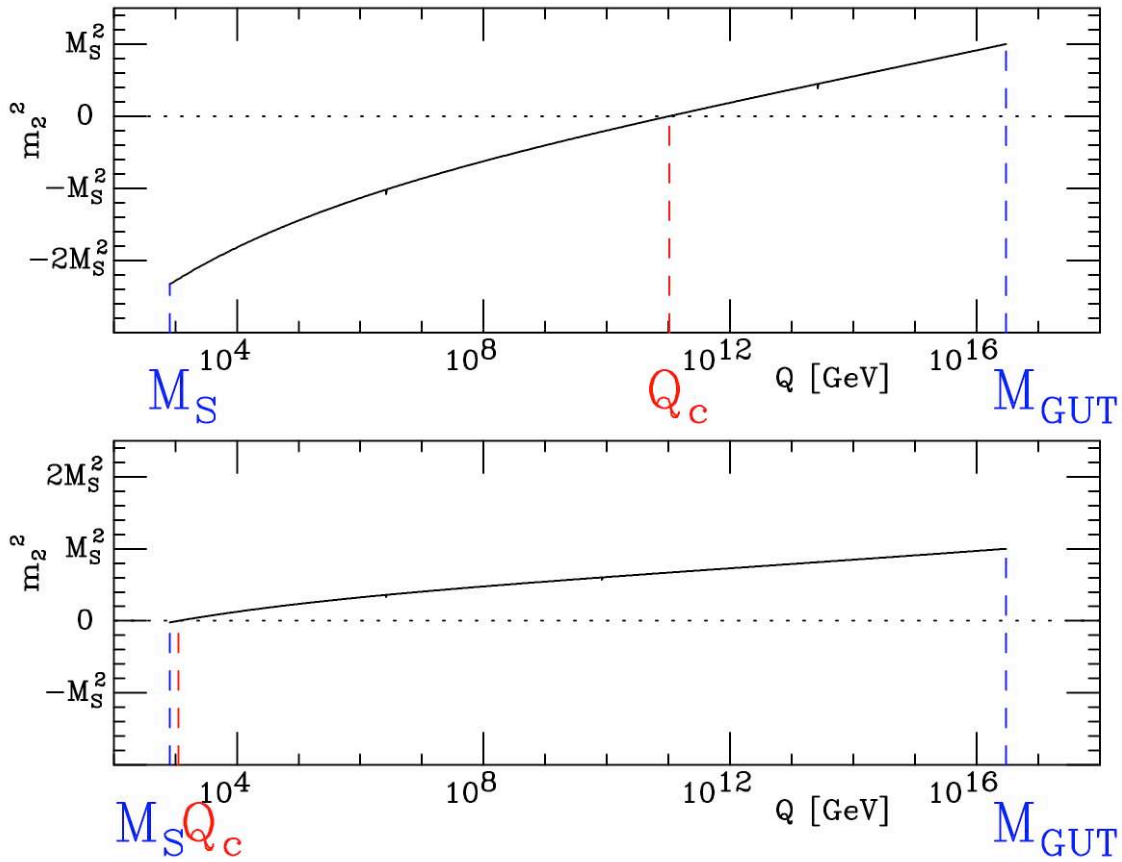
$$\frac{m_0^2}{2}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

Radiative electroweak symmetry breaking

$m^2_{H_u}$ runs negative. EWSB may or may not follow depending on the values of μ and b. claimed that this "predicted" a large top mass, but it really only required a large :

$$y_t = \frac{\sqrt{2}\ m_t}{v \sin\beta}$$

Radiative EWSB



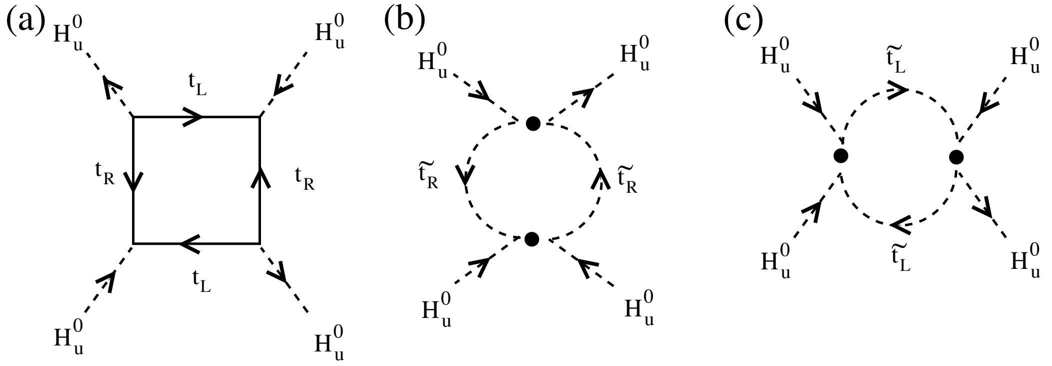
Giudice, Rattazzi, hep-ph/0606105

One-loop Higgs mass

tree-level :

$$m_h < |\cos 2\beta| M_Z = \frac{g^2+g'^2}{4}|v_d^2 - v_u^2|$$

Higgs mass is controlled by the quartic Higgs coupling
failure of the top-stop cancellation should give the leading correction



$$\begin{aligned} \lambda(m_t) &= \lambda(m_{\tilde{t}}) + \int_{m_{\tilde{t}}}^{m_t} \beta_\lambda \, d\ln\mu \\ &= \lambda_{\rm SUSY} + \frac{2N_c|y_t|^4}{16\pi^2} \ln\left(\frac{m_{\tilde{t}1} m_{\tilde{t}2}}{m_t^2}\right) \end{aligned}$$

One-loop Higgs mass

shift in the physical Higgs mass squared

$$\Delta(m_{h^0}^2) = 2\,\delta\lambda\, v_u^2 = \frac{3}{4\pi^2} v^2 y_t^4 \sin^2\beta \;\ln\left(\frac{m_{\tilde{t}1} m_{\tilde{t}2}}{m_t^2}\right)$$
$$\approx \frac{(90\ \mathrm{GeV})^2}{\sin^2\beta}$$

assuming y_t does not blowup below the unification scale:

$$m_{h^0} < 130\ \mathrm{GeV}$$

NMSSM Higgs mass

add a new singlet field N with coupling

$$W_{\text{NMSSM}} = y_N N H_u H_d$$

so the VEV of N can generate the μ-term gives a new contribution, $\mathcal{O}(y_N^2)$, to the Higgs quartic coupling
assuming that y_N remains perturbative up to the unification scale :

$$m_{h^0} < 150 \text{ GeV}$$

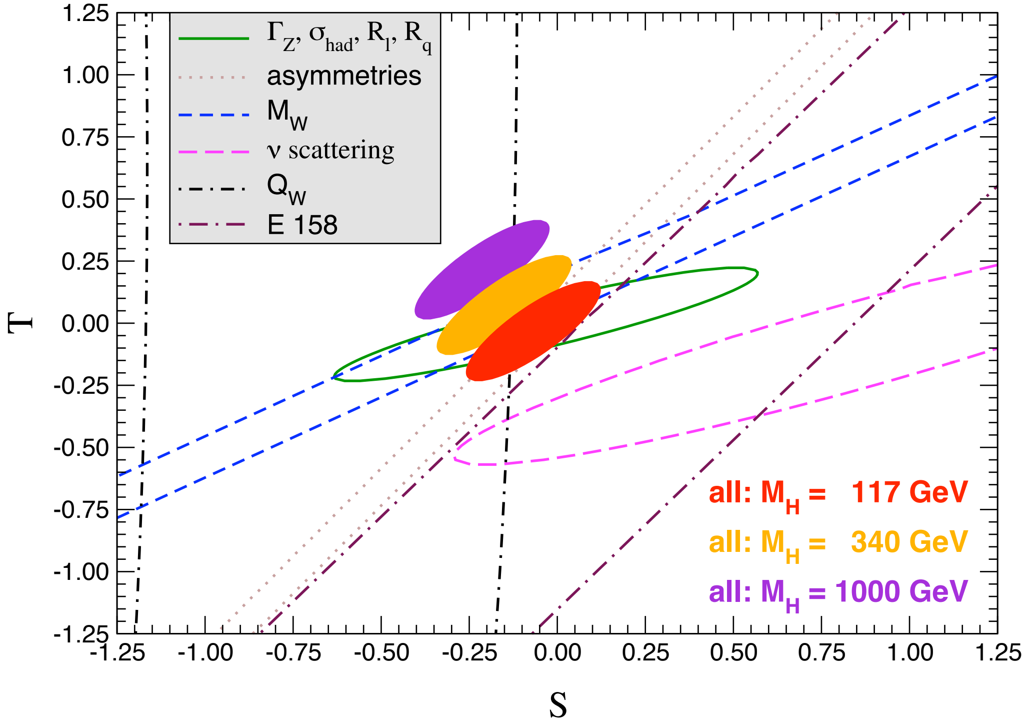
Precision electroweak measurements

Below the EWSB scale terms in the effective Lagrangian like

$$\mathcal{L}_{\text{eff}} \subset -\frac{gg'S}{16\pi} W^3_{\mu\nu} B^{\mu\nu}$$

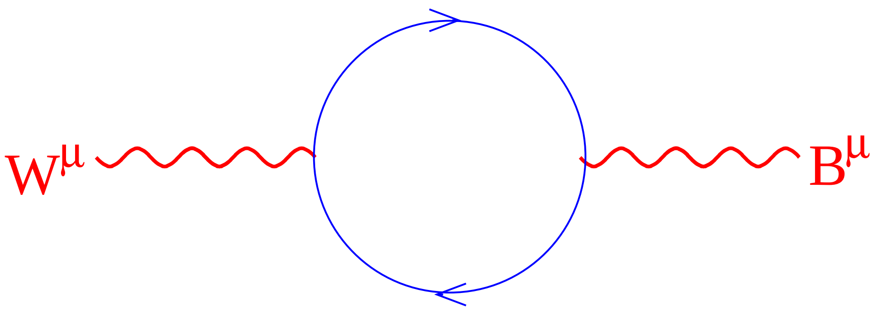
Experimentally S must be $\mathcal{O}(1/10)$

Precision electroweak measurements



Particle Data Group, `http://pdg.lbl.gov/`

Precision electroweak measurements



$SU(2)_L$ doublet fermion with N_c colors that gets a mass from EWSB contributes to vacuum polarization $\Pi^{3B}_{\mu\nu}(p^2)$ for LL gauge vertices

$$\mathrm{Tr}\, T^3_L Y = 0$$

for LR gauge vertices

$$\mathrm{Tr}\, T^3_L Y = \mathrm{Tr}\, T^3_L Q = \tfrac{1}{2}$$

by gauge invariance

$$\Pi^{3B}_{\mu\nu}(p^2) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \Pi^{3B}(p^2)$$

Precision electroweak measurements

For $m \gg M_Z$, Taylor series around $p^2 = 0$:

$$\Pi^{3B}(p^2) = m^2 \sum_{n=0}^{\infty} a_n \left(\frac{p^2}{m^2}\right)^n ,$$

contribution to S proportional to

$$\frac{d}{dp^2}\Pi^{3B}(p^2)|_{p^2=0} \propto N_c \frac{m^2}{m^2} \ .$$

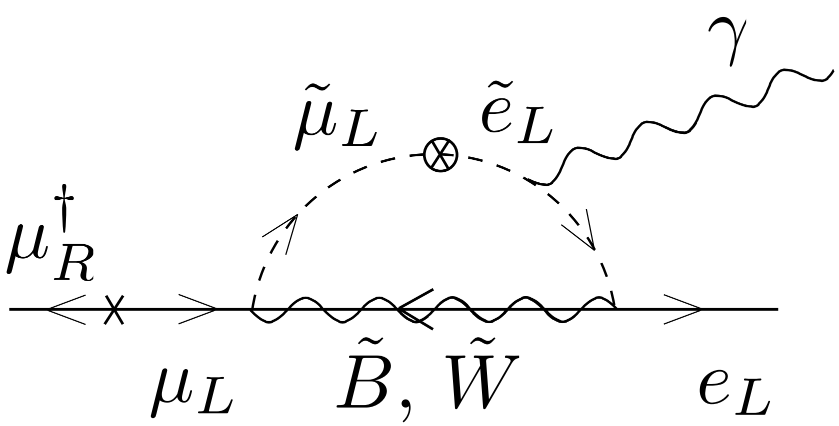
S parameter counts the number of fields in the EWSB sector

For a superpartner in the MSSM the masses are of the form $m_{\rm sp}(m_{\rm soft}, \mu, v)$. In the limit μ, $m_{\rm soft} \to \infty$ with v fixed we have $m_{\rm sp} \to \infty$, $S \propto (v/m_{\rm sp})^n$ superpartners decouple from EWSB if they are sufficiently heavy

R-parity: at low-energy superpartners only contribute at loop-level

Problems with flavor and CP

generically the mass matrices $m_{\bar{e}}^2$ and m_L^2 are not diagonal in the same basis as the lepton mass matrix. This leads to the nonobserved decay $\mu \to e\gamma$



$$\Gamma_{\mu \to e\gamma} \approx 8 \sin^2\theta_W \left(\frac{\alpha_2}{4\pi}\right)^3 \frac{\pi m_\mu^5}{M_{\rm SUSY}^4} \left(\frac{\Delta m_L^2}{M_{\rm SUSY}^2}\right)^2$$

$$\Gamma_{\mu \to e\nu\bar{\nu}} = \left(\frac{\alpha_2}{4\pi}\right)^2 \frac{\pi m_\mu^5}{64 M_W^4}$$

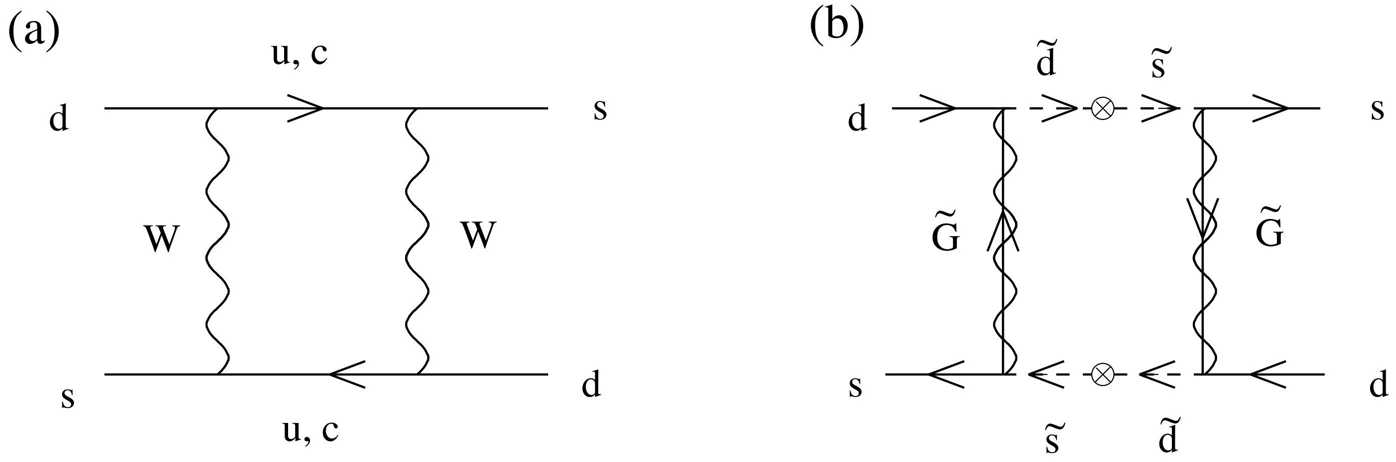
Problems with flavor and CP

$$\frac{\Gamma_{\mu\to e\gamma}}{\Gamma_{\mu\to e\nu\bar{\nu}}} \approx 3\times 10^{-4}\left(\frac{500\ \mathrm{GeV}}{M_{\mathrm{SUSY}}}\right)^4\left(\frac{\Delta m_L^2}{M_{\mathrm{SUSY}}^2}\right)^2 ,$$

experimentally less than 5×10^{-11}

FCNC's

$K\overline{K}$ mixing:



for SM in the limit $m_q \to 0$, diagram is proportional to CKM elements

after diagonalizing the up-type and down-type quark mass matrices by unitary matrices $\mathbf{U_u}$ and $\mathbf{U_d}$ the product $\mathbf{V} = \mathbf{U_d}^\dagger \mathbf{U_u}$ appear in the W couplings

$VV^\dagger = I$, so loop is proportional to

$$\left(V_{di}\, V_{is}^*\right)\left(V_{sj}^* V_{jd}\right) = \delta_{ds}\delta_{sd} = 0\ ,$$

Glashow, Iliopoulos, Maiani

leading contribution comes only at $\mathcal{O}(m^2_{\text{quark}})$ known as the GIM suppression mechanism

FCNC's

$$\mathcal{M}^{\rm SM}_{K\overline{K}} \approx \alpha_2^2 \frac{m_c^2}{M_W^4} \sin^2\theta_c \cos^2\theta_c \ ,$$

where $V_{ud} = \cos\theta_c$.

$$\mathcal{M}^{\rm MSSM}_{K\overline{K}} \approx 4\alpha_3^2 \left(\frac{\Delta m_Q^2}{M^2_{\rm SUSY}}\right)^2 \frac{1}{M^2_{\rm SUSY}} \ .$$

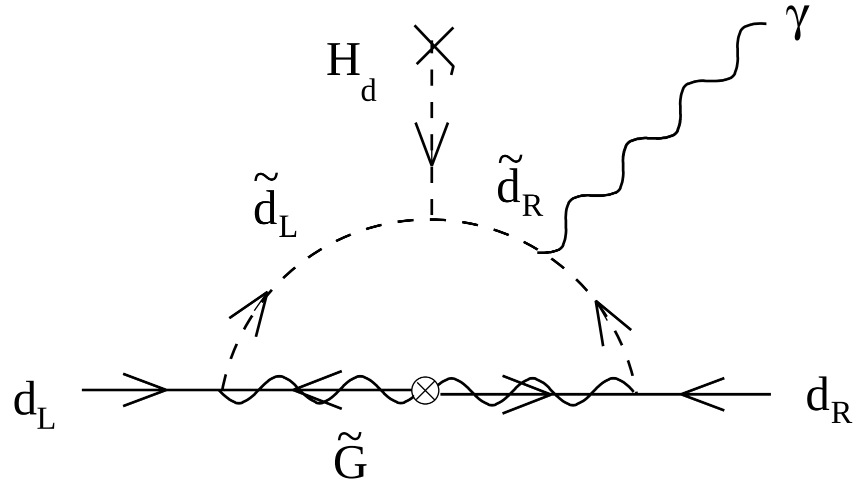
Since the SM amplitude roughly accounts for the observed K_L-K_S mass splitting, we require $\mathcal{M}^{\rm SM}_{K\overline{K}} > \mathcal{M}^{\rm MSSM}_{K\overline{K}}$, so

$$\left(\frac{\Delta m_Q^2}{M^2_{\rm SUSY}}\right) < 4\times 10^{-3} \frac{M_{\rm SUSY}}{500\ {\rm GeV}} \ .$$

observed size of CP violation in the $K\overline{K}$ leads to stringent bounds on the phases of the squark mixing matrix

EDM's

with Higgs VEV, A-terms introduce off-diagonal squark and slepton mass mixing
gives rise to an electric dipole moment (EDM) the d quark, and neutron.
dimension 5 operator in the low-energy effective theory, $d_R^\dagger \sigma^{\mu\nu} d_L F_{\mu\nu}$,



the amplitude must have an inverse mass dimension, and it must be proportional to the VEV of H_d.

EDM's

call the overall phase δ

$$\mathcal{M}_{\rm EDM} \approx \frac{\alpha_3}{4\pi}\frac{e\, v c_\beta\, A_{d11}\,\delta}{M^2_{\rm SUSY}} \ .$$

The experimentally EDM of the neutron is $< 0.97 \times 10^{-25}$ e cm, which translates into the bound:

$$c_\beta A_{d11}\delta \left(\frac{500{\rm GeV}}{M^2_{\rm SUSY}}\right)^2 < 5\times 10^{-7} \ .$$

for $\mathbf{A_d} = \mathbf{Y_d}$

$$\delta < \left(\frac{M^2_{\rm SUSY}}{500\ {\rm GeV}}\right)^2 10^{-2} \ .$$

Safe Neighborhoods

- "Soft Breaking Universality" requires the soft SUSY breaking squark and slepton masses are proportional to the identity in the same basis where quark and lepton mass matrices are diagonal, the A-term $\propto$ Yukawa , and no new nontrivial phases

- The "More Minimal Supersymmetric Model" only require the leading quadratic divergences in the Higgs mass to cancel. $\widetilde{t}_L$, $\widetilde{t}_R$, $\widetilde{b}_L$, $\widetilde{H}_u$, $\widetilde{H}_d$, $\widetilde{B}$,$\widetilde{W}$ must have masses below 1 TeV, while first- and second-generation sparticles can be as heavy as 20 TeV. possible danger: two-loop running below the heavy squark threshold

$$\frac{dm^2_{\widetilde{t}}}{dt} = \frac{8g_3^2}{16\pi^2} C_2 \left[\frac{3g_3^2}{16\pi^2} m^2_{\widetilde{u},\widetilde{d}} - M_3^2 \right] ,$$

  may drive the top squark mass squared negative, depending on gluino mass

Safe Neighborhoods

- The "Alignment" scenario requires a particular relation between squark mass matrices and Yukawa matrices

$$\begin{aligned} \mathbf{m_Q^2} &= \mathbf{Y_u^* Y_u^T} + \mathbf{Y_d^* Y_d^T} \ , \\ \mathbf{m_u^2} &= \mathbf{Y_u^\dagger Y_u} \ , \\ \mathbf{m_d^2} &= \mathbf{Y_d^\dagger Y_d} \ , \end{aligned}$$

  such that FCNC processes are suppressed.