

More Seiberg duality

$SO(N)$ gauge theory

with F quarks in the vector representation

	$SO(N)$	$SU(F)$	$U(1)_R$
Q	\square	\square	$\frac{F+2-N}{F}$

discrete for $N > 3$, axial Z_{2F} symmetry

$$Q \rightarrow e^{2\pi i/2F} Q$$

for $N = 3$ there is a discrete axial Z_{4F} symmetry

one-loop β function coefficient, for $N > 4$ is

$$b = 3(N - 2) - F$$

no dynamical spinors, static spinor sources cannot be screened
distinction between area-law confining and Higgs phases

$SO(N)$ group theory

adjoint of $SO(N)$ is two-index antisymmetric tensor

odd N , there is one spinor representation

even N there are two inequivalent spinors

for $N = 4k$ the spinors are self-conjugate

for $N = 4k + 2$ the two spinors are complex conjugates

$SO(N)$ group theory

$SO(2N + 1)$		
Irrep \mathbf{r}	$d(\mathbf{r})$	$2T(\mathbf{r})$
\square	$2N + 1$	2
S	2^N	2^{N-2}
$\begin{array}{ c } \hline \square \\ \hline \end{array}$	$N(2N + 1)$	$4N - 2$
$\begin{array}{ c c } \hline \square & \square \\ \hline \end{array}$	$(N + 1)(2N + 1) - 1$	$4N + 6$

$SO(2N)$		
Irrep \mathbf{r}	$d(\mathbf{r})$	$2T(\mathbf{r})$
\square	$2N$	2
S, \bar{S}	2^{N-1}	2^{N-3}
$\begin{array}{ c } \hline \square \\ \hline \end{array}$	$N(2N - 1)$	$4N - 4$
$\begin{array}{ c c } \hline \square & \square \\ \hline \end{array}$	$N(2N + 1) - 1$	$4N + 4$

S denotes a spinor, and \bar{S} denotes the conjugate spinor

The $SO(N)$ moduli space $F < N$

D-flatness conditions (up to flavor transformations):

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & & & \\ & \ddots & & \\ & & v_F & \\ 0 & \dots & 0 & \\ \vdots & & \vdots & \\ 0 & \dots & 0 & \end{pmatrix}$$

generic point in the classical moduli space $SO(N) \rightarrow SO(N - F)$
 $NF - N(N - 1) + (N - F)(N - F - 1)$ massless chiral supermultiplets

The $SO(N)$ moduli space $F \geq N$

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & & 0 & \dots & 0 \\ & \ddots & \vdots & & \vdots \\ & & v_N & 0 & \dots & 0 \end{pmatrix}$$

generic point in the moduli space the $SO(N)$ broken completely
 $NF - N(N - 1)$ massless chiral supermultiplets.

describe light degrees of freedom by “meson” and (for $F \geq N$)
“baryon” fields:

$$\begin{aligned} M_{ji} &= \Phi_j \Phi_i \\ B_{[i_1, \dots, i_N]} &= \Phi_{[i_1} \dots \Phi_{i_N]} \end{aligned}$$

The $SO(N)$ $F < N - 2$

	$U(1)_A$	$U(1)_R$
W^a	0	1
Λ^b	$2F$	0
$\det M$	$2F$	$2(F + 2 - N)$

ADS superpotential:

$$W_{\text{dyn}} = c_{N,F} \left(\frac{\Lambda^b}{\det M} \right)^{1/(N-2-F)}$$

Duality for $SO(N)$

$F \geq 3(N - 2)$ lose asymptotic freedom
 F just below $3(N - 2)$ we have an IR fixed point
 solution to the anomaly matching for $F > N - 2$, is given by:

	$SO(F - N + 4)$	$SU(F)$	$U(1)_R$
q	\square	$\bar{\square}$	$\frac{N-2}{F}$
M	$\mathbf{1}$	$\square\square$	$\frac{2(F+2-N)}{F}$

For $F > N - 1$, $N > 3$ unique superpotential

$$W = \frac{M_{ji}}{2\mu} \phi^j \phi^i$$

dual baryon operators:

$$\tilde{B}^{[i_1, \dots, i_{\tilde{N}}]} = \phi^{[i_1} \dots \phi^{i_{\tilde{N}}]}$$

Hybrid “Baryon” Operators

since adjoint is an antisymmetric tensor. In $SO(N)$ we have:

$$\begin{aligned} h_{[i_1, \dots, i_{N-4}]} &= W_\alpha^2 \Phi_{[i_1} \dots \Phi_{i_{N-4}]} \\ H_{[i_1, \dots, i_{N-2}]\alpha} &= W_\alpha \Phi_{[i_1} \dots \Phi_{i_{N-4}]} \end{aligned}$$

While in the dual theory we have:

$$\begin{aligned} \tilde{h}^{[i_1, \dots, i_{N-4}]} &= \tilde{W}_\alpha^2 \phi^{[i_1} \dots \phi^{i_{N-4}]} \\ \tilde{H}_\alpha^{[i_1, \dots, i_{N-2}]} &= \tilde{W}_\alpha \phi^{[i_1} \dots \phi^{i_{N-4}]} \end{aligned}$$

The two theories thus have a mapping of mesons, baryons, and hybrids:

$$\begin{aligned} M &\leftrightarrow M , \\ B_{i_1, \dots, i_N} &\leftrightarrow \epsilon_{i_1, \dots, i_F} \tilde{h}^{i_1, \dots, i_{N-4}} \\ h_{i_1, \dots, i_{N-4}} &\leftrightarrow \epsilon_{i_1, \dots, i_F} \tilde{B}^{i_1, \dots, i_N} \\ H_\alpha^{[i_1, \dots, i_{N-2}]} &\leftrightarrow \epsilon_{i_1, \dots, i_F} \tilde{H}_\alpha^{[i_1, \dots, i_{N-2}]} \end{aligned}$$

Dual one-loop β function

$$\beta(\tilde{g}) \propto -\tilde{g}^3(3(\tilde{N} - 2) - F) = -\tilde{g}^3(2F - 3(N - 2))$$

lose asymptotic freedom when $F \leq 3(N - 2)/2$

When

$$F = 3(\tilde{N} - 2) - \epsilon\tilde{N}$$

perturbative IR fixed point in the dual theory

$SO(N)$ with F vectors has an **interacting IR fixed point** for $3(N - 2)/2 < F < 3(N - 2)$

$N - 2 \leq F \leq 3(N - 2)/2$ **IR free massless composite gauge bosons, quarks, mesons, and their superpartners**

Special case: $F \leq N - 5$

$$SO(N) \rightarrow SO(N - F) \supset SO(5)$$

gaugino condensation, dynamical superpotential:

$$W_{\text{dyn}} \propto \langle \lambda\lambda \rangle \propto \left(\frac{16\Lambda^{3(N-2)-F}}{\det M} \right)^{1/(N-2-F)}$$

runaway vacua

Special case: $F = N - 4$

$$SO(N) \rightarrow SO(4) \sim SU(2)_L \times SU(2)_R$$

two gaugino condensates

$$W_{\text{cond.}} = 2\langle\lambda\lambda\rangle_L + 2\langle\lambda\lambda\rangle_R = \frac{1}{2}(\epsilon_L + \epsilon_R) \left(\frac{16\Lambda^{2N-1}}{\det M} \right)^{1/2}$$

$$\epsilon_{L,R} = \pm 1$$

two physically distinct branches: $(\epsilon_L + \epsilon_R) = \pm 2$ and $(\epsilon_L + \epsilon_R) = 0$
first branch has runaway vacua, second has a quantum moduli space.

at $M = 0$, M satisfies the 't Hooft anomaly matching

confinement without chiral symmetry breaking, no baryons

Integrating out a flavor on first branch gives runaway ($F = N - 5$)

second branch no SUSY vacua

Special case: $F = N - 3$

$SO(N) \rightarrow SO(4) \sim SU(2)_L \times SU(2)_R \rightarrow SU(2)_d \sim SO(3)$
instanton effects ($\Pi_3(G/H) = \Pi_3(SU(2)) = \mathbb{Z}$)
and gaugino condensation

$$W_{\text{inst.}+\text{cond.}} = 4(1 + \epsilon) \frac{\Lambda^{2N-3}}{\det M}$$

two phases of the gaugino condensate

two physically distinct branches: $\epsilon = 1$ and with $\epsilon = -1$

first has runaway vacua, while the second has a **quantum moduli space**

Integrating out a flavor, we would need to find two branches again

so $W \neq 0$ even on the second branch

Special case: $F = N - 3$

must have some other fields
anomaly matching given by:

	$SU(F)$	$U(1)_R$
q	$\bar{\square}$	$\frac{N-2}{F}$
M	$\square\square$	$\frac{2(F+2-N)}{F}$

most general superpotential

$$W = \frac{1}{2\mu} M q q f \left(\frac{\det M M q q}{\Lambda^{2N-2}} \right)$$

where $f(t)$ is an unknown function
adding a mass term gives

$$q_F = \pm i v$$

which gives correct number of ground states

$$q \leftrightarrow h = Q^{N-4} W_\alpha W^\alpha$$

confinement without chiral symmetry breaking with hybrids

Special case: $F = N - 1$

Starting with the $F = N$ dual which has an $SO(4)$ gauge group, and integrating out a flavor there will be instanton effects when we break to $SO(3)$

dual superpotential is modified in the case $F = N - 1$ to be:

$$W = \frac{M_{ji}}{2\mu} \phi^j \phi^i - \frac{1}{64\Lambda^{2N-5}} \det M$$

Special case: $F = N - 2$

both descriptions generically break to $SO(2) \sim U(1)$

monopoles

SUSY $Sp(2N)$

An $Sp(2N)$ gauge theory with $2F$ quarks (F flavors) in the fundamental representation has a global $SU(2F) \times U(1)_R$ symmetry as follows:

	$Sp(2N)$	$SU(2F)$	$U(1)_R$
Q	\square	\square	$\frac{F-1-N}{F}$

adjoint of $Sp(2N)$ is the two-index symmetric tensor

$Sp(2N)$ Representations

$Sp(2N)$		
Irrep \mathbf{r}	$d(\mathbf{r})$	$T(\mathbf{r})$
\square	$2N$	1
$\begin{array}{c} \square \\ \square \end{array}$	$N(2N - 1) - 1$	$2N - 2$
$\begin{array}{cc} \square & \square \end{array}$	$N(2N + 1)$	$2N + 2$
$\begin{array}{c} \square \\ \square \\ \square \end{array}$	$\frac{N(2N-1)(2N-2)}{3} - 2N$	$\frac{(2N-3)(2N-2)}{2} - 1$
$\begin{array}{ccc} \square & \square & \square \end{array}$	$\frac{N(2N+1)(2N+2)}{3}$	$\frac{(2N+2)(2N+3)}{2}$
$\begin{array}{cc} \square & \square \\ \square & \end{array}$	$\frac{2N(2N-1)(2N+1)}{3} - 2N$	$(2N)^2 - 4$

dimension $\begin{array}{c} \square \\ \square \end{array}$ smaller by -1 than naive expectation
 invariant tensor of $Sp(2N)$ is ϵ_{ij}
 representation formed with two antisymmetric indices is reducible

SUSY $Sp(2N)$

one-loop β function for $N > 4$ is

$$b = 3(2N + 2) - 2F$$

moduli space is parameterized by a “meson”

$$M_{ji} = \Phi_j \Phi_i$$

antisymmetric in the flavor indices i, j

holomorphic intrinsic scale considered as a spurion field
Pfaffian of a $2F \times 2F$ matrix M is given by

$$\text{Pf}M = \epsilon^{i_1 \dots i_{2F}} M_{i_1 i_2} \dots M_{i_{2F-1} i_{2F}}$$

	$U(1)_A$	$U(1)_R$
$\Lambda^{b/2}$	$2F$	0
$\text{Pf}M$	$2F$	$2(F - 1 - N)$

SUSY $Sp(2N)$

for $F < N + 1$ possible to generate a dynamical superpotential

$$W_{\text{dyn}} \propto \left(\frac{\Lambda^{\frac{b}{2}}}{\text{Pf}M} \right)^{1/(N+1-F)}$$

For $F = N + 1$ one finds confinement with chiral symmetry breaking

$$\text{Pf}M = \Lambda^{2(N+1)}$$

For $F = N + 2$ one finds s-confinement with a superpotential:

$$W = \text{Pf}M$$

Duality for $Sp(2N)$

solution to the anomaly matching for $F > N - 2$:

	$Sp(2(F - N - 2))$	$SU(2F)$	$U(1)_R$
q	\square	$\bar{\square}$	$\frac{N+1}{F}$
M	$\mathbf{1}$	\square	$\frac{2(F-1-N)}{F}$

a unique superpotential:

$$W = \frac{M_{ji}}{\mu} \phi^j \phi^i$$

For $3(N + 1)/2 < F < 3(N + 1)$ we have an **IR fixed point**

For $N + 3 \leq F \leq 3(N + 1)/2$ the dual is **IR free**

Why chiral gauge theories are interesting

- vector-like theory we can give masses to all the matter fields
- pure YM, gaugino condensation but no SUSY breaking
- Witten's index argument: number of bosonic minus fermionic vacua does not change
- If taking the mass to zero does not move some vacua in from or out to infinity, then the massless theory has unbroken SUSY

first example of a chiral gauge theory

	$SU(N)$	$SU(N + 4)$
\bar{Q}	$\bar{\square}$	\square
T	$\square \square$	$\mathbf{1}$

dual to

	$SO(8)$	$SU(N + 4)$
q	\square	$\bar{\square}$
p	\mathbf{S}	$\mathbf{1}$
$U \sim \det T$	$\mathbf{1}$	$\mathbf{1}$
$M \sim \bar{Q}T\bar{Q}$	$\mathbf{1}$	$\square \square$

with a superpotential

$$W = Mqq + Upp$$

This dual theory is vector-like!

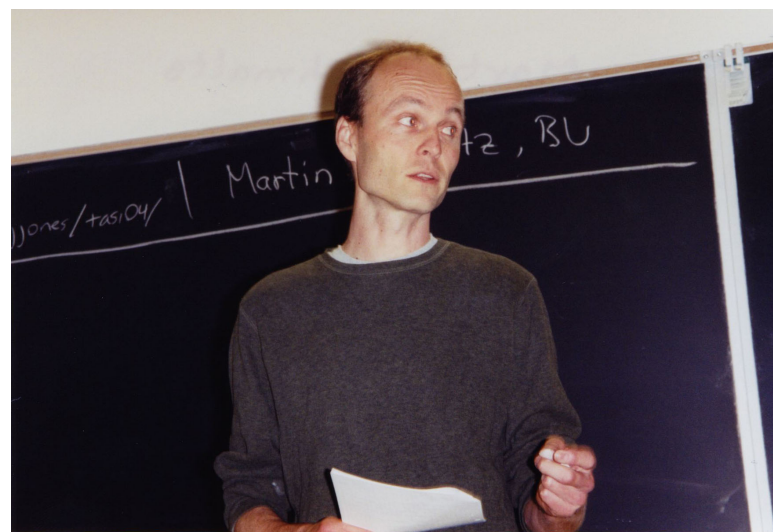
chiral dual of vector theory

The dual β function coefficient is:

$$b = 3(8 - 2) - (N + 4) - 1 = 13 - N$$

So the dual is IR free for $N > 13$

Csaki, Schmaltz, Skiba



S-Confinement

$SU(N)$ with $N + 1$ flavors.

$$W = \frac{1}{\Lambda^{2N-1}} (\det M - BM\bar{B})$$

meson–baryon description was valid over the whole moduli space
smooth description with no phase transitions
theory has complementarity, static source screened by squarks

To generalize:

need fields that are fundamentals of SU or Sp and spinors of SO
only consider theories with superpotential in the confined description

Theories that satisfy these conditions are called s-confining

S-Confinement

single gauge group G , choose $U(1)_R$ such that

	G	$U(1)_R$
ϕ_i	\mathbf{r}_i	q
$\phi_{j \neq i}$	\mathbf{r}_j	0

q is determined by anomaly cancellation:

$$\begin{aligned}
 0 &= (q - 1)T(\mathbf{r}_i) + T(Ad) - \sum_{j \neq i} T(\mathbf{r}_j) \\
 &= qT(\mathbf{r}_i) + T(Ad) - \sum_j T(\mathbf{r}_j)
 \end{aligned}$$

can do this for any field, and for each choice the superpotential has R -charge 2, we have

$$W \propto \Lambda^3 \left[\prod_i \left(\frac{\phi_i}{\Lambda} \right)^{T(\mathbf{r}_i)} \right]^{2/(\sum_j T(\mathbf{r}_j) - T(Ad))}$$

in general, a sum of terms with different contractions of gauge indices

S-Confinement

Requiring superpotential be holomorphic at the origin \Rightarrow integer powers of the composites \Rightarrow integer powers of the fundamental fields

Unless all the $T(r_i)$ have a common divisor must have

$$\begin{aligned}\sum_j T(r_j) - T(Ad) &= 1 \text{ or } 2, \text{ for } SO \text{ or } Sp \\ 2(\sum_j T(r_j) - T(Ad)) &= 1 \text{ or } 2, \text{ for } SU\end{aligned}$$

cases from different conventions for normalizing generators, for SO and Sp $T(\square) = 1$, while for SU $T(\square) = 1/2$

Anomaly cancellation for SU and Sp require that the left-hand side be even.

condition is necessary for s-confinement, but not sufficient

S-Confinement

check explicitly (by exploring the moduli space) that for SO none of the candidate theories where the sum is 2 turn out to be s-confining

$$\sum_j T(r_j) - T(Ad) = \begin{cases} 1, & \text{for } SU \text{ or } SO \\ 2, & \text{for } Sp \end{cases}$$

gives finite list of candidate s-confining theories

check candidate theories by going out in moduli space
generically break to theories with smaller gauge groups
if the sub-group theory not s-confining the original theory not s-confining

S-Confinement

For SU one finds that the following theories are s-confinings:

$$\begin{array}{l|l}
 SU(N) & (N+1)(\square + \bar{\square}); \quad \square + N\bar{\square} + 4\square; \quad \square + \bar{\square} + 3(\square + \bar{\square}) \\
 SU(5) & 3(\square + \bar{\square}); \quad 2\square + 2\square + 4\bar{\square} \\
 SU(6) & 2\square + 5\bar{\square} + \square; \quad \square + 4(\square + \bar{\square}) \\
 SU(7) & 2(\square + 3\bar{\square})
 \end{array}$$

$SU(N)$ with \boxplus , $F = 4$

	$SU(2N + 1)$	$SU(4)$	$SU(2N + 1)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	\boxplus	1	1	0	$2N + 5$	0
\overline{Q}	$\overline{\square}$	1	\square	4	$-2N + 1$	0
Q	\square	\square	1	$-2N - 1$	$-2N + 1$	$\frac{1}{2}$

confined description:

	$SU(4)$	$SU(2N + 1)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$(Q\overline{Q})$	\square	\square	$3 - 2N$	$-4N + 2$	$\frac{1}{2}$
$(A\overline{Q}^2)$	1	\boxplus	8	$-2N + 7$	0
$(A^N Q)$	\square	1	$-2N - 1$	$2N^2 + 3N + 1$	$\frac{1}{2}$
$(A^{N-1} Q^3)$	$\overline{\square}$	1	$-6N - 3$	$2N^2 - 3N - 2$	$\frac{3}{2}$
(\overline{Q}^{2N+1})	1	1	$4(2N + 1)$	$-4N^2 + 1$	0

$SU(N)$ with \boxplus , $F = 4$

s-confinement superpotential:

$$W = \frac{1}{\Lambda^{2N}} \left[(A^N Q)(Q\bar{Q})^3 (A\bar{Q}^2)^{N-1} + (A^{N-1} Q^3)(Q\bar{Q})(A\bar{Q}^2)^N + (\bar{Q}^{2N+1})(A^N Q)(A^{N-1} Q^3) \right]$$

equations of motion reproduce classical constraints

integrating out a flavor gives confinement with chiral symmetry breaking

Deconfinement

Consider $SU(N)$ with \square for odd N with $F \geq 5$ ($\bar{F} \equiv N + F - 4$):

	$SU(N)$	$SU(F)$	$SU(\bar{F})$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	\square	$\mathbf{1}$	$\mathbf{1}$	0	$-2F$	$\frac{-12}{N}$
Q	\square	\square	$\mathbf{1}$	1	$N - F$	$2 - \frac{6}{N}$
\bar{Q}	$\bar{\square}$	$\mathbf{1}$	\square	$\frac{-F}{N+F-4}$	F	$\frac{6}{N}$

take A to be a composite meson of a s-confining Sp theory

	$SU(N)$	$Sp(N-3)$	$SU(F)$	$SU(N+F-4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
Y	\square	\square	$\mathbf{1}$	$\mathbf{1}$	0	$-F$	$\frac{-6}{N}$
Z	$\mathbf{1}$	\square	$\mathbf{1}$	$\mathbf{1}$	0	FN	8
\bar{P}	$\bar{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	$F - FN$	$6 - \frac{6}{N}$
Q	\square	$\mathbf{1}$	\square	$\mathbf{1}$	1	$N - F$	$2 - \frac{6}{N}$
\bar{Q}	$\bar{\square}$	$\mathbf{1}$	$\mathbf{1}$	\square	$\frac{-F}{N+F-4}$	F	$\frac{6}{N}$

Deconfinement

superpotential

$$W = YZ\bar{P}$$

eqm for P sets the meson $(YZ) = 0$, also sets Pf $M = 0$ in dynamical superpotential to zero

$SU(N)$ gauge group of this new description has $N + F - 3$ flavors, use **SUSY QCD duality to find another dual**

Deconfinement

	$SU(F - 3)$	$Sp(N - 3)$	$SU(F)$	$SU(\bar{F})$
y	\square	\square	$\mathbf{1}$	$\mathbf{1}$
\bar{p}	$\bar{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
q	\square	$\mathbf{1}$	$\bar{\square}$	$\mathbf{1}$
\bar{q}	$\bar{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\bar{\square}$
M	$\mathbf{1}$	$\mathbf{1}$	\square	\square
L	$\mathbf{1}$	\square	$\mathbf{1}$	\square
B_1	$\mathbf{1}$	$\mathbf{1}$	\square	$\mathbf{1}$

with a superpotential

$$W = Mq\bar{q} + B_1q\bar{p} + Ly\bar{q}$$

$Sp(N - 3)$ with $N + 2F - 7$ fundamentals has an $Sp(2F - 8)$ dual

Deconfinement

	$SU(F - 3)$	$Sp(2F - 8)$	$SU(F)$	$SU(\bar{F})$
\tilde{y}	$\bar{\square}$	\square	$\mathbf{1}$	$\mathbf{1}$
\bar{p}	$\bar{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
q	\square	$\mathbf{1}$	$\bar{\square}$	$\mathbf{1}$
\bar{q}	$\bar{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\bar{\square}$
M	$\mathbf{1}$	$\mathbf{1}$	\square	\square
l	$\mathbf{1}$	\square	$\mathbf{1}$	$\bar{\square}$
B_1	$\mathbf{1}$	$\mathbf{1}$	\square	$\mathbf{1}$
a	$\begin{array}{ c } \hline \square \\ \hline \end{array}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
H	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\begin{array}{ c } \hline \square \\ \hline \end{array}$
(Ly)	\square	$\mathbf{1}$	$\mathbf{1}$	\square

with

$$W = a\tilde{y}\tilde{y} + Hll + (Ly)l\tilde{y} + Mq\bar{q} + B_1q\bar{p} + (Ly)\bar{q}$$

Deconfinement

after integrating out (Ly) and \bar{q} becomes

$$W = a\tilde{y}\tilde{y} + Hll + Mql\tilde{y} + B_1q\bar{p}$$

With $F = 5$ we have a gauge group $SU(2) \times SU(2)$ and one can show (using $D[\text{scalar}] \geq 1$) that for $N > 11$ this theory has an **IR fixed point** also show that some of the fields (eg. H and l) are **IR-free**

integrating out one flavor completely breaks the gauge group
light degrees of freedom are just the composites of the s-confinement for
 $F = 4$