Exercises for Chap. 8

1. Consider from the superpotential

$$W = \frac{1}{2} M \phi_H^2 + \frac{\lambda}{2} \phi_H \phi^2 + \frac{\delta}{3} \phi_H \phi^3 + \frac{y}{6} \phi_H^3 \ ,$$

   find the exact effective superpotential after integrating out ϕ_H. Can you argue from holomorphy what the general form of the effective superpotential has to be?

2. Using holomorphy and symmetries show that the superpotential

$$W = \mu_1 \phi + \mu_2 \phi^2 + \ldots + \mu_n \phi^n + \ldots$$

   is not renormalized.

3. Starting from the theory with superpotential

$$W = M_{ij} \phi^i \phi^j + \lambda_{ijk} \phi^i \phi^j \phi^k \ ,$$

   show that this superpotential is not renormalized.

4. Consider the theory with superpotential

$$W = \frac{M}{2} \phi_1^2 + y \phi_1 \phi_2^2 + \lambda \phi_2^3 + \frac{1}{\mu} \phi_1 \phi_2^3 \ ,$$

   Using holomorphy and symmetries find the form of the the low-energy effective superpotential that arises when the heavy field ϕ_1 is integrated out of the theory.