

Exercises for Chapter 4

1. Calculate the D-term contribution, δ_f , to the squark or slepton mass squared matrix for a sfermion with diagonal $SU(2)_L$ charge T_f^3 and electric charge Q_f .
2. Taking the couplings $\alpha_2 = 0.0322$ and $\alpha_3 = 0.0895$ (renormalized at 1 TeV) use one-loop running to estimate the MSSM unification scale and the value of the unified coupling at that scale.
3. Write out all the quartic interactions in the Lagrangian that corresponds to the superpotential

$$W_{\text{Higgs}} = y_t(\bar{t}tH_u^0 - \bar{t}bH_u^+) - y_b(\bar{b}tH_d^- - \bar{b}bH_d^0) - y_\tau(\bar{\tau}\nu_\tau H_d^- - \bar{\tau}\tau H_d^0) + \mu(H_u^+ H_d^- - H_u^0 H_d^0) . \quad (1)$$

4. Derive the D-term quartic potential in eqn (4.21) using $H_u = (H_u^+, H_u^0)$, and $H_d = (H_d^0, H_d^-)$.
5. Calculate the D-term contribution

$$\delta_f = -gT_f^3 \langle D^3 \rangle - g'Y_f \langle D' \rangle = (T_f^3 - Q_f s_W^2) \cos 2\beta M_Z^2 , \quad (2)$$

to the squark or slepton mass squared matrix for a sfermion with diagonal $SU(2)_L$ charge T_f^3 and electric charge Q_f .

6. Use the equations of motion for the neutral Higgs fields to derive Eqs. (4.34) and (4.35).
7. Show that the tree-level mass of the light neutral Higgs, given in Eq. (4.46), is maximized at $m_A = \infty$, and derive the upper bound on m_h .
8. Estimate the A -term contribution to the $\mu \rightarrow e\gamma$ width. Assuming A_{e22} is proportional to the μ Yukawa coupling, $\mathbf{A}_{e22} = a_\mu M_{\text{SUSY}} y_\mu$, what is the bound on a_μ ?
9. Consider the Higgs-Higgsino-Bino coupling y'

$$\mathcal{L}_{\text{int}} = \frac{\sqrt{2}y'}{2} H_u^{0*} \tilde{H}_u^0 \tilde{B} + h.c. . \quad (3)$$

At tree-level this coupling is equal to the $U(1)_Y$ gauge coupling (i.e. $y' = g'$). Using a renormalization group argument estimate the $\mathcal{O}(y_t^2)$ splitting of y' and g' due to the splitting of the top squark masses and the top quark mass. You can take the left and right top squark masses to be equal.

10. Starting with the superpotential

$$W = y_t H_u Q_3 \bar{u}_3 ,$$

verify that in the SUSY limit the conditions given in the first lecture to cancel the divergent contributions to the Higgs mass are indeed satisfied.