Exercises for Chapter 2

1. Check that

$$S = \int d^4x \, (-\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + i\lambda^{\dagger a} \overline{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a) \tag{1}$$

is a SUSY invariant using eqns (2.91)–(2.94). After doing the SUSY transformations you can go to a gauge where at the point of interest, x_0^μ, the gauge field vanishes ($A^a_\nu(x_0) = 0$). You will need to use the Bianchi identity $\epsilon^{\mu\nu\alpha\beta}(D_\nu F_{\alpha\beta})^a = 0$.

2. Check that the commutator of two SUSY transformations closes:

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})X = -i(\epsilon_1 \sigma^\mu \epsilon_2^\dagger - \epsilon_2 \sigma^\mu \epsilon_1^\dagger) D_\mu X^a \ , \tag{2}$$

for $X^a = F^a_{\mu\nu}$, λ^a, $\lambda^{\dagger a}$, D^a. These calculations requires the identities:

$$\xi \sigma^\mu \overline{\sigma}^\nu \chi = \chi \sigma^\nu \overline{\sigma}^\mu \xi = (\chi^\dagger \overline{\sigma}^\nu \sigma^\mu \xi^\dagger)^* = (\xi^\dagger \overline{\sigma}^\mu \sigma^\nu \chi^\dagger)^* \ , \tag{3}$$

$$\overline{\sigma}^\mu \sigma^\nu \overline{\sigma}^\rho = -\eta^{\mu\rho}\overline{\sigma}^\nu + \eta^{\nu\rho}\overline{\sigma}^\mu + \eta^{\mu\nu}\overline{\sigma}^\rho + i\epsilon^{\mu\nu\rho\kappa}\overline{\sigma}_\kappa \ , \tag{4}$$

$$\sigma^\mu_{\alpha\dot{\alpha}} \overline{\sigma}^{\dot{\beta}\beta}_\mu = 2\delta^\beta_\alpha \delta^{\dot{\beta}}_{\dot{\alpha}} \ . \tag{5}$$

3. Derive the supercurrent

$$\begin{aligned} J^\mu_\alpha \;=\; & (\sigma^\nu \overline{\sigma}^\mu \psi_i)_\alpha \, D_\nu \phi^{*i} - i(\sigma^\mu \psi^{\dagger i})_\alpha \, W_i^* \\ & - \frac{1}{2\sqrt{2}} (\sigma^\nu \overline{\sigma}^\rho \sigma^\mu \lambda^{\dagger a})_\alpha \, F^a_{\nu\rho} - \frac{i}{\sqrt{2}} g \phi^* T^a \phi \, (\sigma^\mu \lambda^{\dagger a})_\alpha . \end{aligned} \tag{6}$$

The first term was done in Section 2.3. The second term comes from the total derivative in eqn (2.67).

4. Consider a Wess-Zumino-like model with the superpotential

$$W = \frac{y}{3}\phi^3 + \frac{\lambda}{4M}\phi^4 \ . \tag{7}$$

What are the off-shell SUSY transformations of the the scalar ϕ and its superpartner fermion ψ expressed only in terms of ϕ and ψ?

5. For the superpotential given in (7) what is the corresponding Lagrangian in terms of ϕ and ψ?

6. Schematically (include the parametric dependence on the couplings) what are the Feynman rules for the cubic and quartic interactions?

7. Check that the SUSY transformation of the gauge field A^a_μ in

$$\mathcal{L}_{\psi\ gauge\ int.} = -\psi^\dagger \overline{\sigma}^\mu g A^a_\mu T^a \psi \ , \tag{8}$$

cancels against the SUSY transformation of ϕ and ϕ^* in

$$\mathcal{L}_{Yukawa} = -\sqrt{2} g \left[(\phi^* T^a \psi)\lambda^a + \lambda^{\dagger a}(\psi^\dagger T^a \phi) \right] . \tag{9}$$

You will need to use the generalized Pauli identity (A.27). Note that the cancellation relates two terms where two fields are replaced by superpartners.

8. For the superpotential

$$W = m\,\phi_2 \phi_3 + \frac{y}{2}\phi_1 \phi_3^2 \ ; \tag{10}$$

a) calculate the scalar potential;
b) show that the SUSY vacua are given by $\langle\phi_1\rangle = v$; $\langle\phi_2\rangle = 0$, $\langle\phi_3\rangle = 0$, for arbitray v;
c) expanding around the vacua $\phi_1 = v + \tilde{\phi}_1$, and writing the scalar potential out to quadratic order, find the mass squared matrix, M^2_ϕ, for the scalars;
d) find the fermion mass matrix M_ψ and verify that $M_\psi M^\dagger_\psi = M^2_\phi$.

9. Using

$$V^a = \theta\sigma^\mu\bar{\theta} A^a_\mu + \theta^2\bar{\theta}\lambda^{\dagger a} + \bar{\theta}^2\theta\lambda^a + \frac{1}{2}\theta^2\bar{\theta}^2 D^a \ , \tag{11}$$

perform the superspace integration for the Lagrangian

$$\mathcal{L} = \int d^4\theta\ \Phi^\dagger e^{2gT^a V^a} \Phi \tag{12}$$

keeping only terms of order g or higher. (We did the g independent terms in class.)

10. For a general SUSY gauge theory with renormalizable interactions find the soft SUSY breaking terms that are produced by giving θ^2 spurion components to the background chiral superfields corresponding to the mass and Yukawa coupling, and the coefficient of $W_\alpha W^\alpha$ as well as the wavefunction renormalization:

$$Z = 1 + b\,\theta^2 + b^*\bar{\theta}^2 + c\,\theta^2\bar{\theta}^2 \ . \tag{13}$$