Introduction to Inflation: Single-field Inflation

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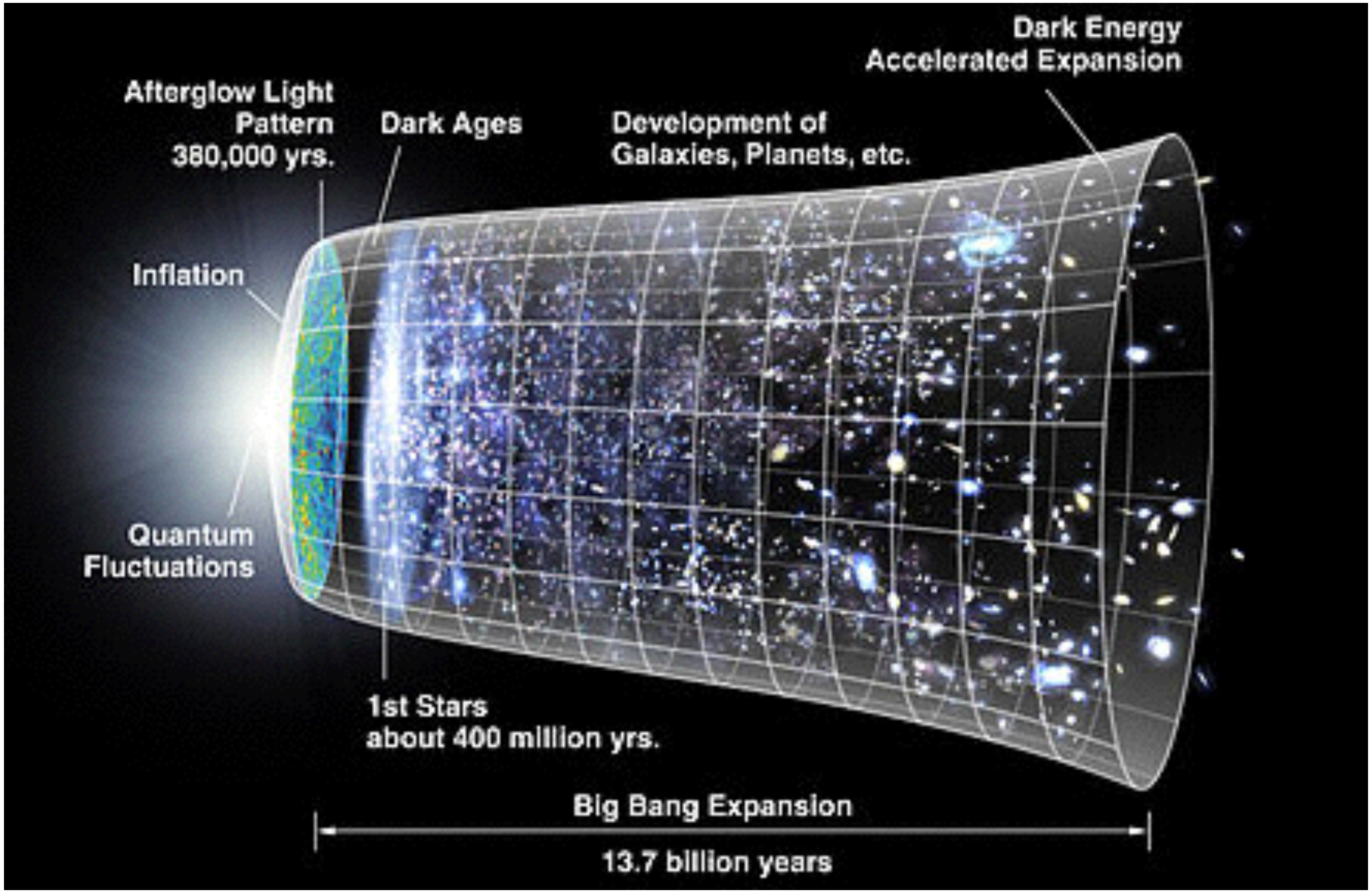
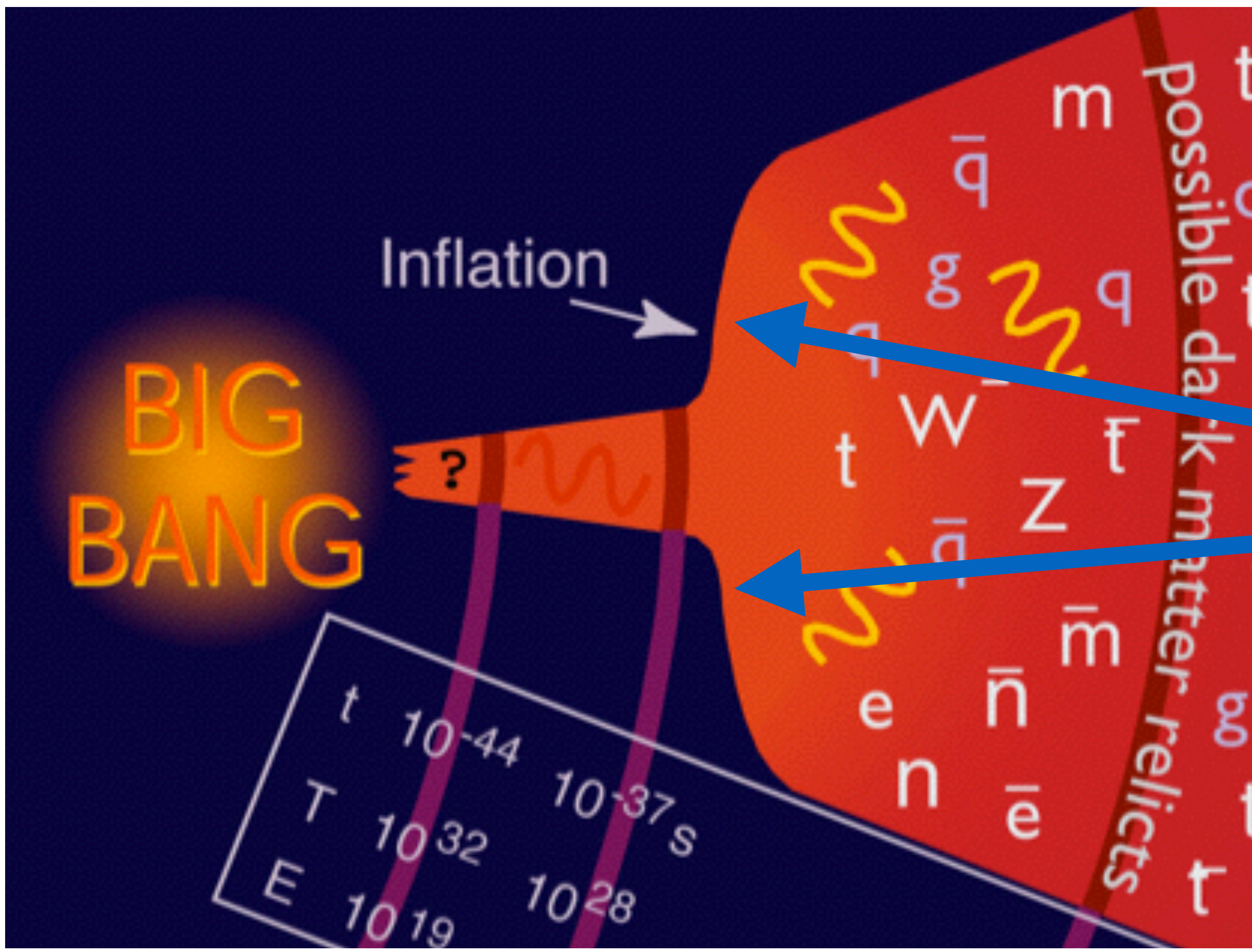
Overview

Inflation is the earliest “known” period in our history



Overview

Period of exponential growth of the scale factor

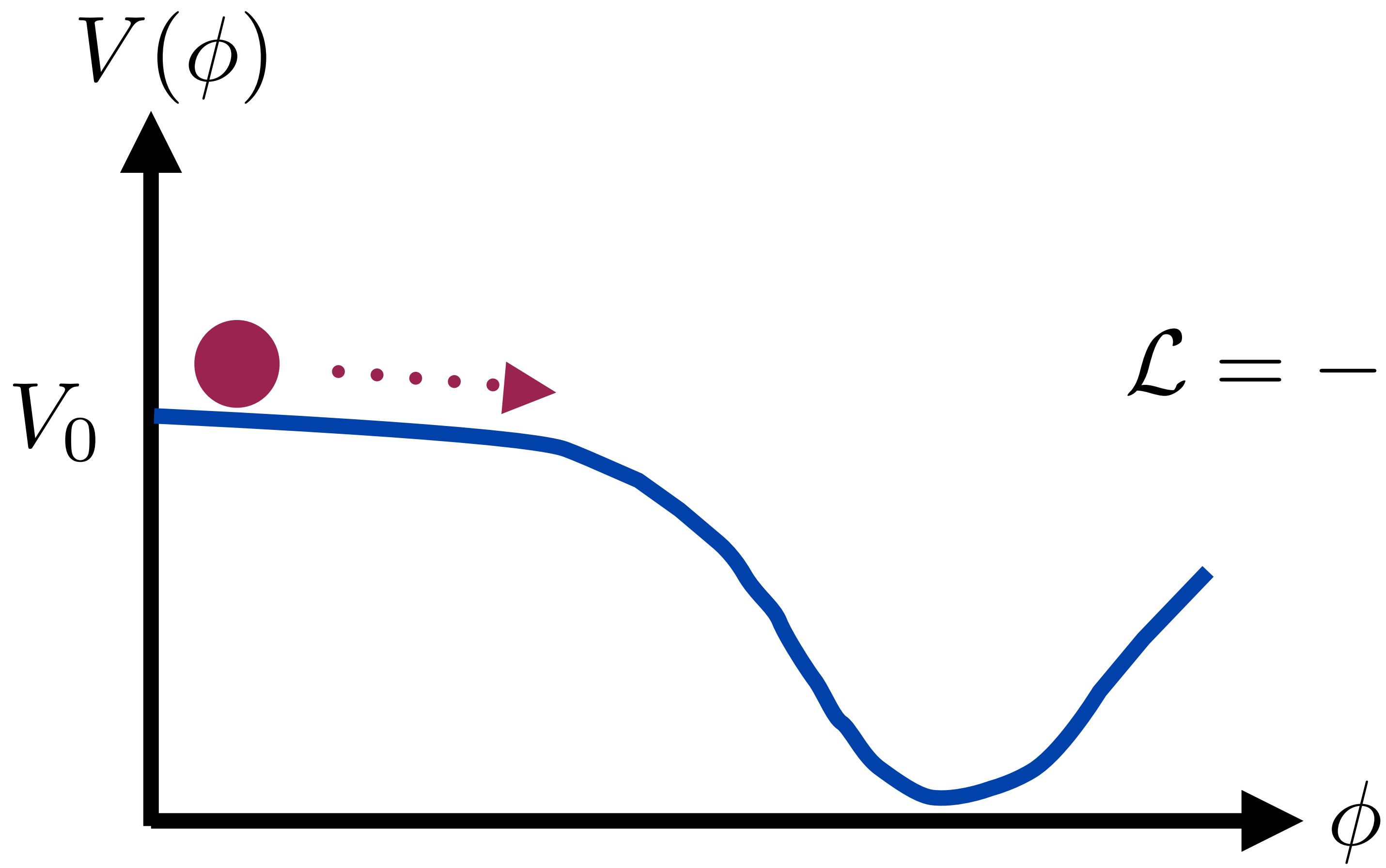


$$a(t) \simeq e^{Ht}$$

Initial seeds of structure were formed at this time

Overview

Conventional picture is something like this



$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

Potential energy dominated = Exponential growth

Overview

I will take a slightly unusual approach to inflation

Lecture 1: Single-field inflation ⟷ EWSB

- Spontaneously broken gauge theory (gravity)
- Goldstone boson equivalence theorem
- Precision tests from effective operators
- Weakly coupled models = light scalar field

Overview

I will take a slightly unusual approach to inflation

Lecture 2: Beyond Single-field phenomenology

- Single-field consistency conditions
- Violations = new particles during inflation
- Prospects for observations

Overview

I will take a slightly unusual approach to inflation

<u>Overall goal</u>: Highlight similarities with BSM physics

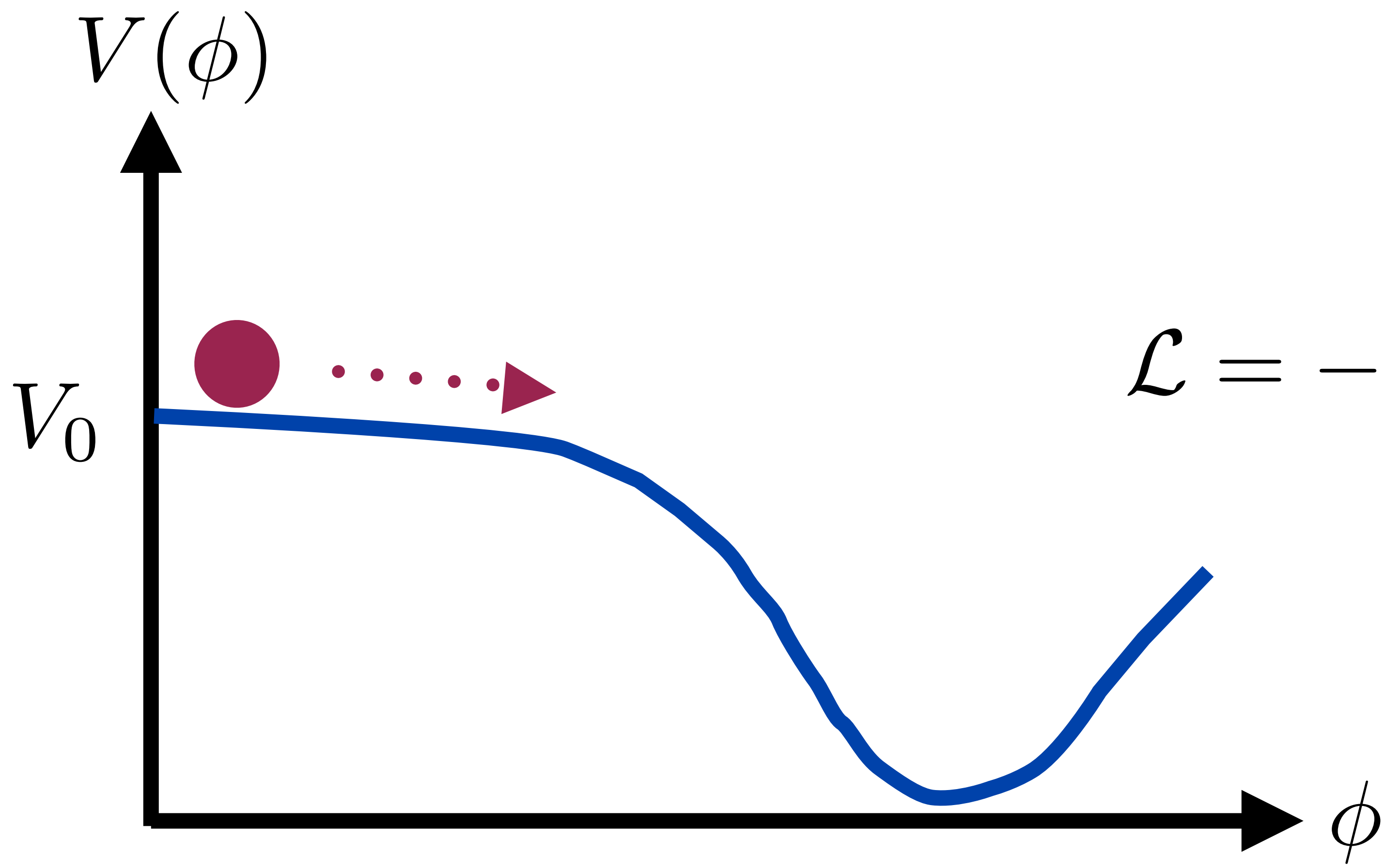
- Particle physicists have the tools to make interesting contributions to the subject

- Language makes it easier to understand cosmological observations / results

- Opportunities for interactions between the fields

Overview

I will spend little time on model building

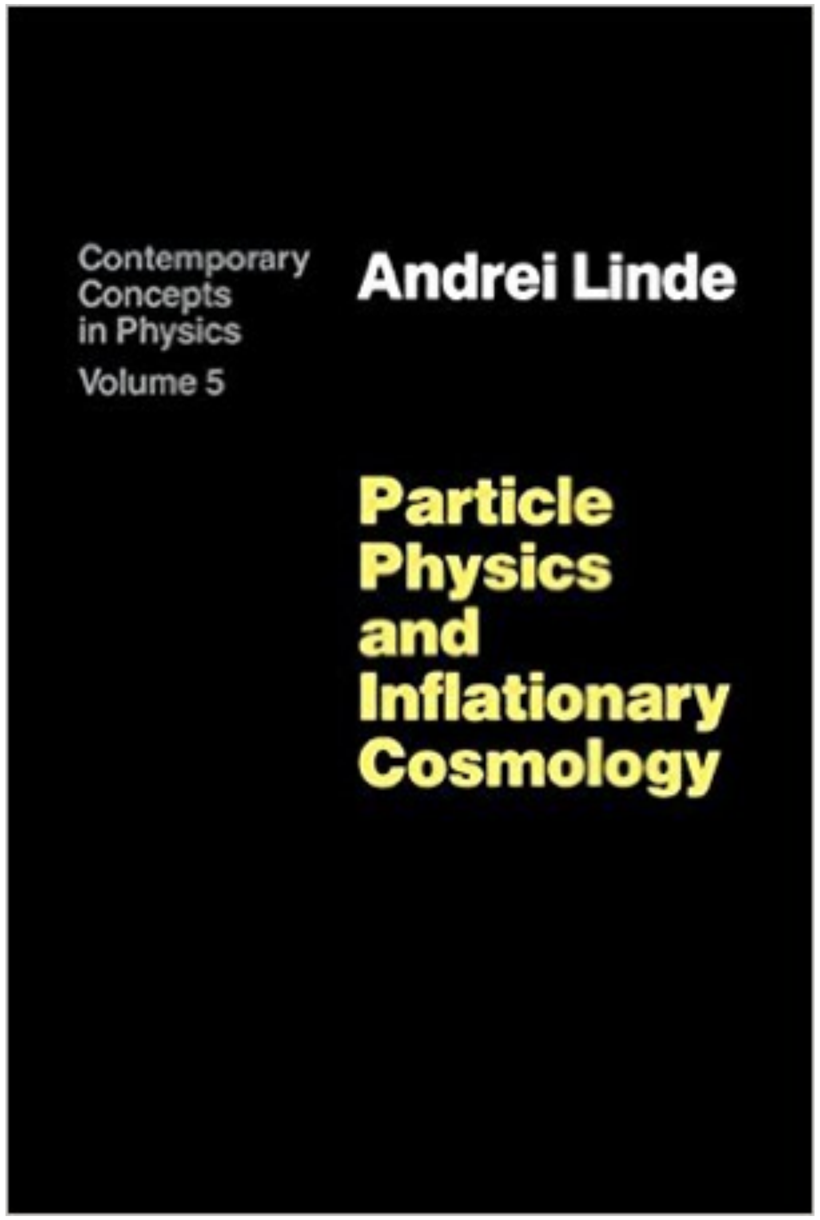


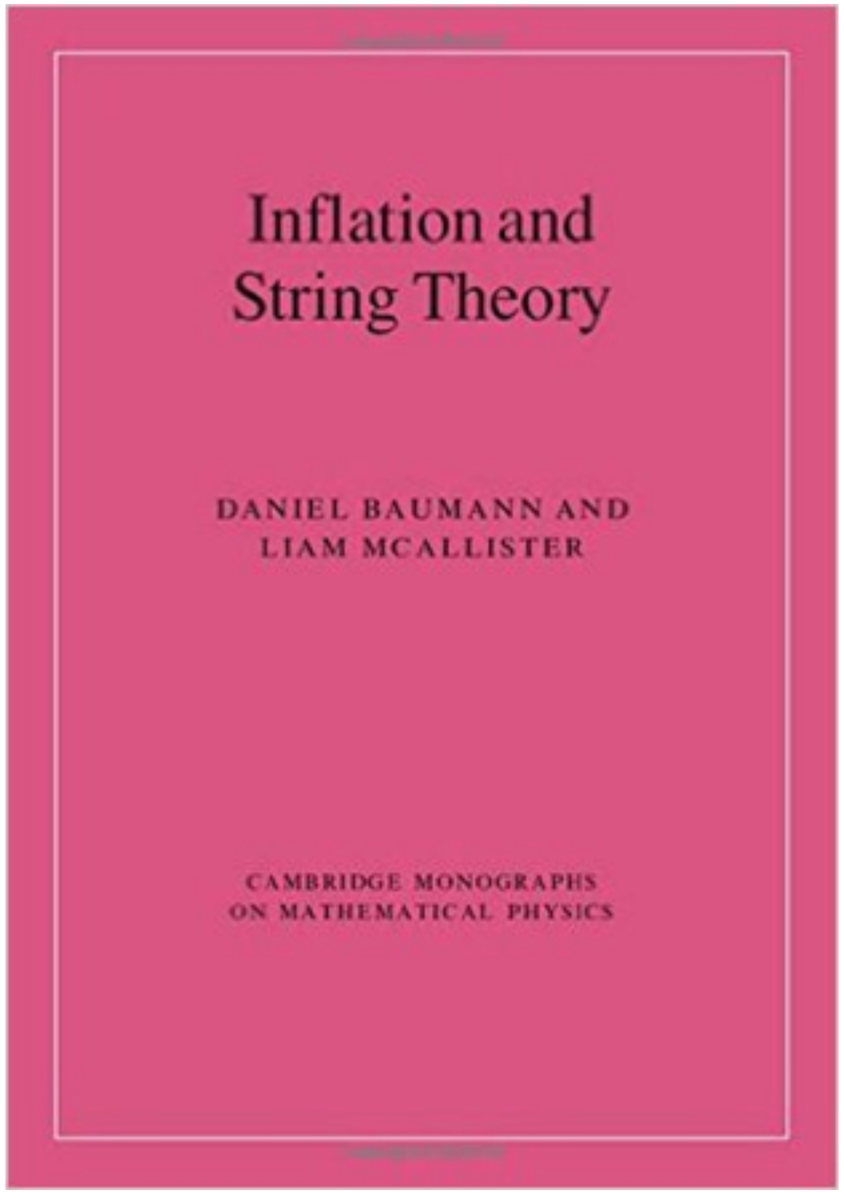
$$\mathcal{L} = -\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi)$$

E.g. technical naturalness, Planck-scale corrections, embedding in String theory or SUGRA

Overview

Reason: subject is very well covered elsewhere





Overview

Reason: subject is very well covered elsewhere

TASI Lectures on Inflation

Daniel Baumann

Department of Physics, Harvard University, Cambridge, MA 02138, USA
School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540, USA

Abstract

In a series of five lectures I review inflationary cosmology. I begin with a description of the initial conditions problems of the Friedmann-Robertson-Walker (FRW) cosmology and then explain how inflation, an early period of accelerated expansion, solves these problems. Next, I describe how inflation transforms microscopic quantum fluctuations into macroscopic seeds for cosmological structure formation. I present in full detail the famous calculation for the primordial spectra of scalar and

Baumann : 0907.5424

Overview

Reason: subject is very well covered elsewhere

Les Houches lectures on inflationary observables

and string theory

Eva Silverstein

Stanford Institute for Theoretical Physics, Stanford University, Stanford, CA 94306, USA

SLAC National Accelerator Laboratory, 2575 Sand Hill, Menlo Park, CA 94025

Abstract

These lectures cover the theoretical structure and phenomenology of some basic mechanisms for inflation. A full treatment of the problem requires 'ultraviolet completion' because of the sensitivity of inflation to quantum gravity effects, while the observables are elegantly parameterized using low energy field theory. String theory provides novel mechanisms for inflation, some subject

Silverstein : 1311.2312

Overview

Reason: subject is very well covered elsewhere

Lectures from TASI:

http://physicslearning2.colorado.edu/tasi/

E.g. Silverstein (2015), Senatore (2015,2012)

Lectures from PITP 2011:

https://video.ias.edu/pitp-2011

E.g. Creminelli, Silverstein

Outline : Lecture 1

Why Inflation?

The EFT of Inflation

Life in de Sitter Space

Energy Scales and Observations

References : Lecture 1

The Effective Field Theory of Inflation

Cheung, Creminelli, Fitzpatrick, Kaplan, & Senatore

arXiv : 0709.0293

Equilateral Non-Gaussanity and
New Physics on the Horizon

Baumann & DG

arXiv: 1102.5343

See Baumann's TASI lectures for more background

Conventions

FRW metric: $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

$$H(t) \equiv \frac{\dot{a}}{a} \qquad\qquad 3M_{\rm pl}^2 H^2 = \sum_i \rho_i(t)$$

Throughout these lectures there will be waves:

$$\zeta(x,t) \sim A_{\vec{k}}(t)\cos(\vec{k}\cdot x)$$

Physical momenta are $k_p = \frac{k}{a}$

Conventions

Dan's many laws of cosmology

A mode is "outside the horizon" when

$$k_p \equiv \frac{k}{a} < H \qquad \lambda_p \sim \frac{a}{k} > H^{-1} = R_{\text{horizon}}$$

$$k < aH$$

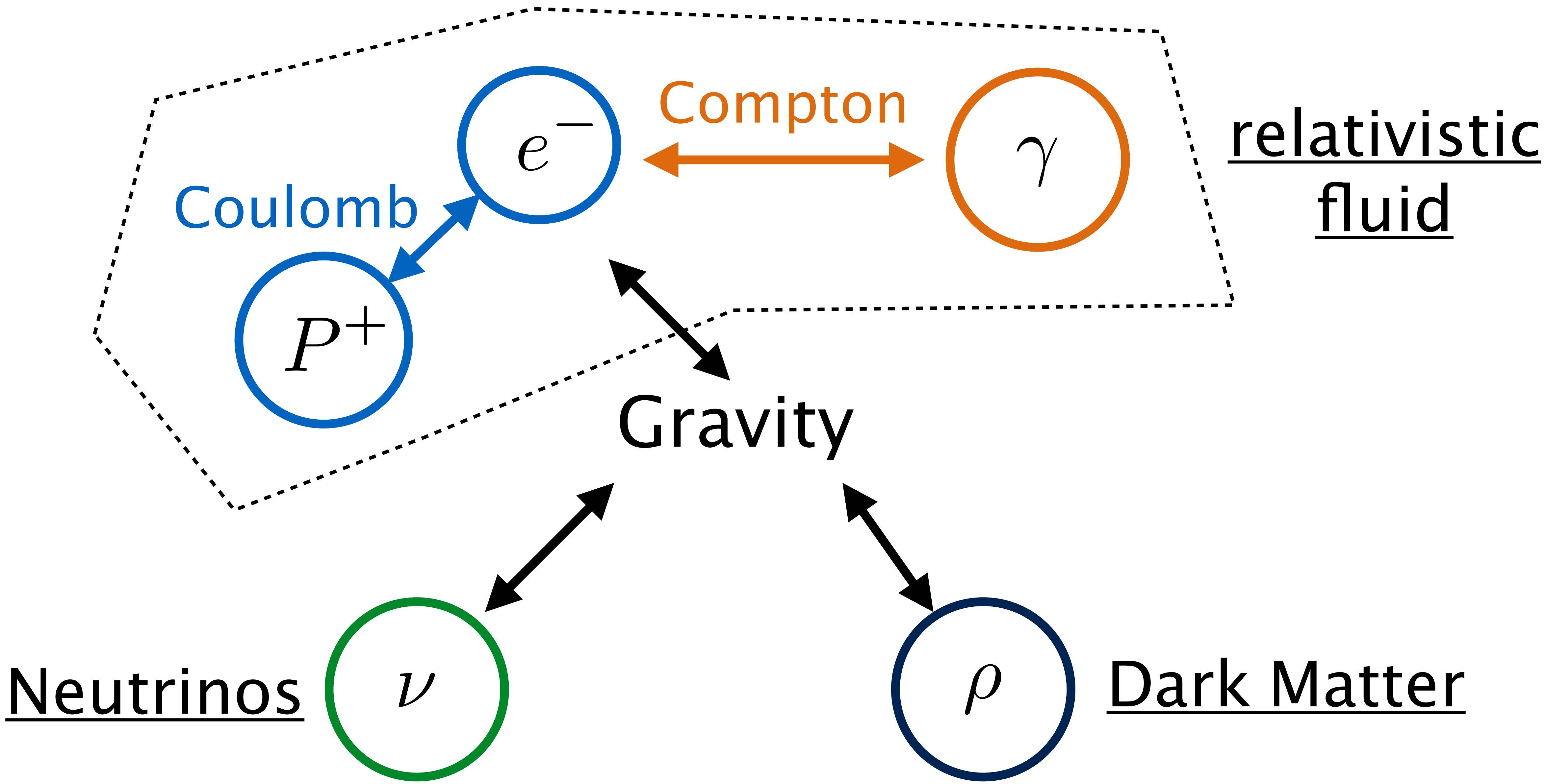
$$\frac{k}{aH} < 1$$

(remember that k does not change in time)

Why Inflation?

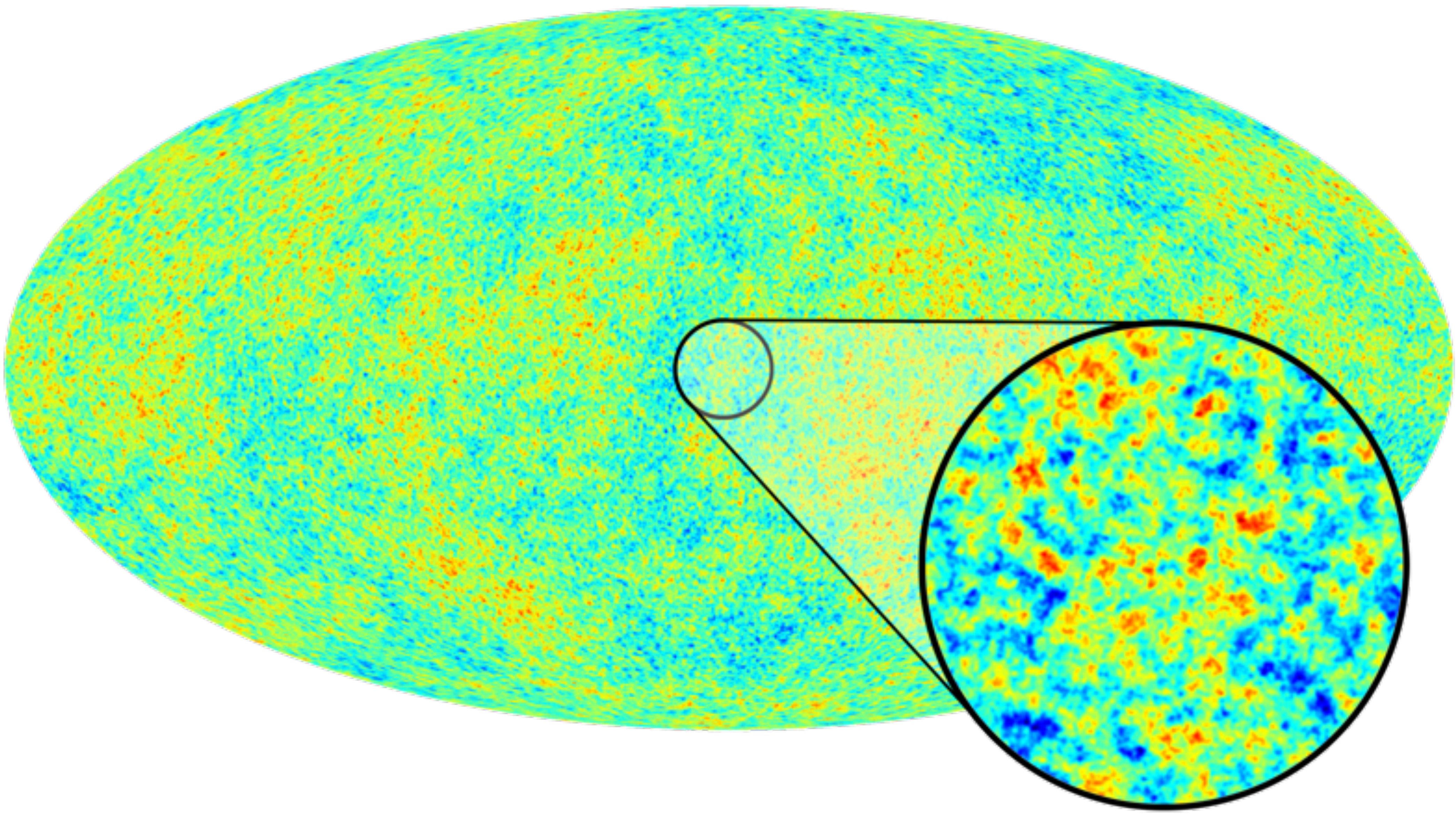
Inflation and the CMB

Before CMB, photons-baryons effectively one fluid



Inflation and the CMB

We see a snap-shot of the sound waves in this fluid



Inflation and the CMB

We decompose this map in spherical harmonics

$$T(\hat{n}) = \sum_{m,\ell} a_{\ell,m} Y_{\ell,m}(\hat{n})$$

We then average over the m index

$$C_\ell = \frac{1}{2\ell+1} \sum_m a_{\ell,m} a_{\ell,-m}$$

This is a average over waves with same wavelength

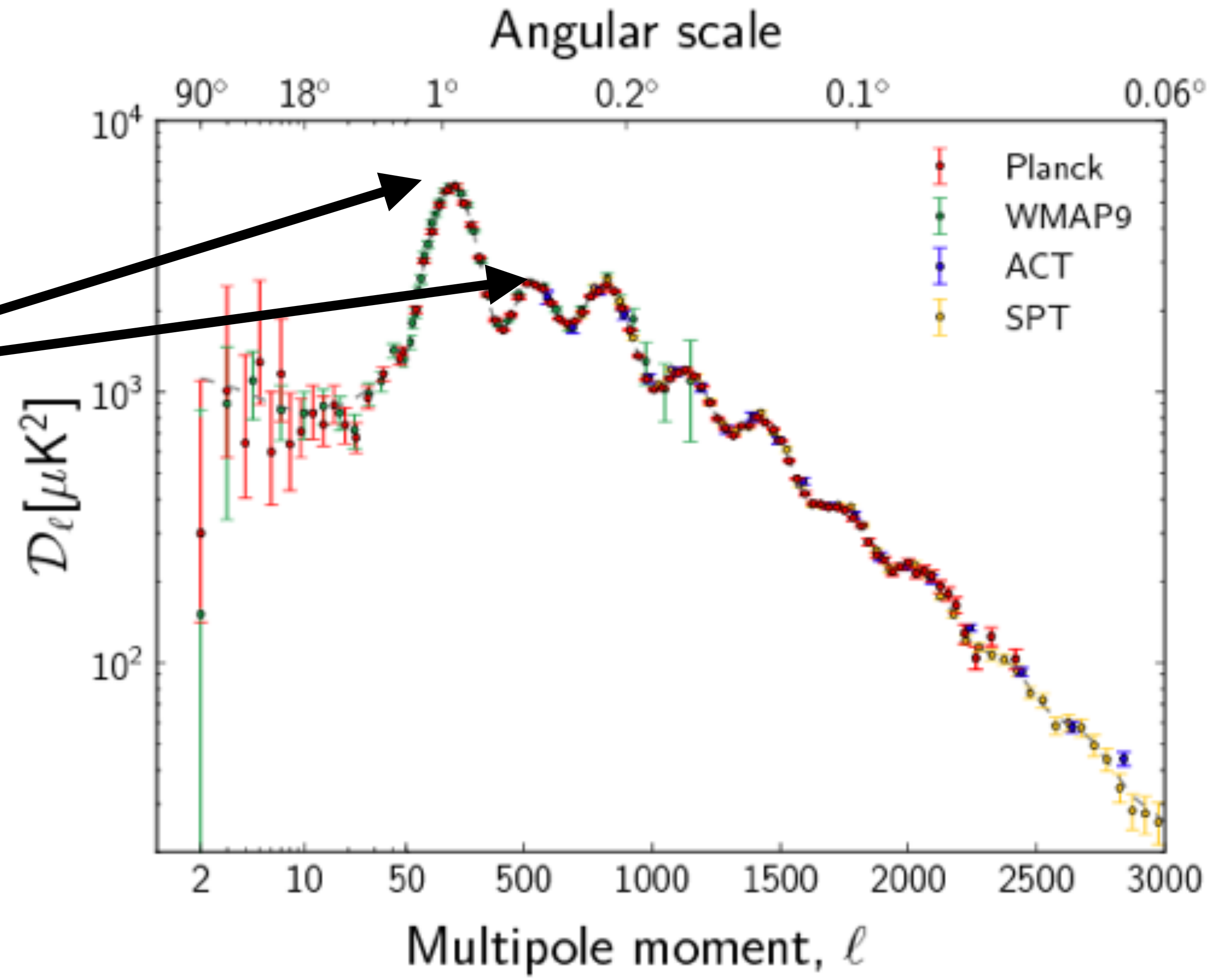
$$T \sim A_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} \longrightarrow C_\ell \sim \sum_{|\vec{k}|=\ell} |A_{\vec{k}}|^2$$

Inflation and the CMB

These waves oscillate in time

$$\frac{\delta T}{T} \sim A_{\vec{k}} \cos(k r_s)$$

$$r_s = \int^{a_\star} \frac{da}{a^2 H} \frac{1}{\sqrt{3(1 + R_b(a))}}$$



Acoustic peaks show that they are in-phase

Inflation and the CMB

Phase coherence is a stringent requirement of CMB

Any local source will have arbitrary phases

$$\frac{\delta T}{T} \sim A_k \cos kr_s + B_k \sin kr_s$$

Observed power spectrum requires $B_k = 0$

However, if mode existed outside the “horizon”

$$\frac{k}{aH} \ll 1 \to B_k \propto a^{-3} \to 0$$

Inflation and the CMB

In a universe with matter and/or radiation

$$3M_{\rm pl}^2 H^2 = \frac{\rho_{\gamma,0}}{a^4} + \frac{\rho_{m,0}}{a^3} \longrightarrow H \propto a^{-2,-3/2}$$

If a mode was outside the horizon at time of CMB

$$\frac{k}{a_{\rm CMB}} \sim H_{\rm CMB} \longrightarrow \frac{k}{a} < H \quad \text{for} \quad a < a_{\rm CMB}$$

Initial conditions set on “super-horizon” scales

Inflation and the CMB

Challenge is to explain the origin of the fluctuations

Ordinary matter + locality will not work

Two options:

(1) Non-local production of fluctuations

(2) Change the matter content of the universe

Non-locality and Quantum Gravity

Option 1 is intimately tied to singularity resolution

Problem: we can't resolve cosmological singularities

Recent progress inspired by dS/CFT and inflation

Maldacena; Maldacena & Pimentel;
McFadden, Skenderis et al.; Trivedi et al.

Would be a legitimate alternative to inflation, but not mature enough to test (yet)

What is Inflation?

Option 2 has many options (in principle)

For equation of state $p = w\rho$

$$3M_{\rm pl}^2 H^2 = \frac{\rho_{w,0}}{a^{3(1+w)}} \qquad H \propto a^{-3(1+w)/2}$$

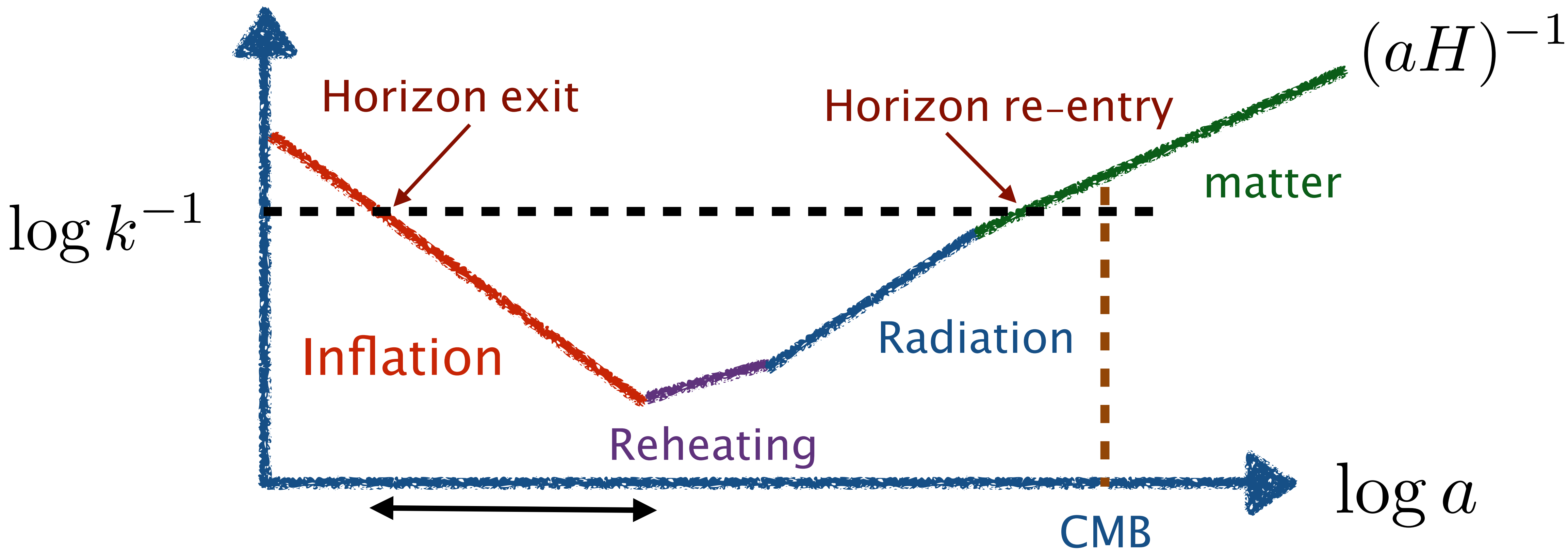
Solves our problem for any $w < -\frac{1}{3}$

Alternatively, the universe could bounce at $a = 0$

Most of these options don't work in detail

What is Inflation?

Allow modes to go inside to outside first



$$N_e = \int_{k=(aH)_{\rm inf}}^{t_{\rm end}} H dt \sim 50 - 60$$

Period needs to be long enough to match scales

What is Inflation?

A definition:

1. A period of quasi-dS expansion Guth

$$\frac{\dot{H}}{H^2} \ll 1$$

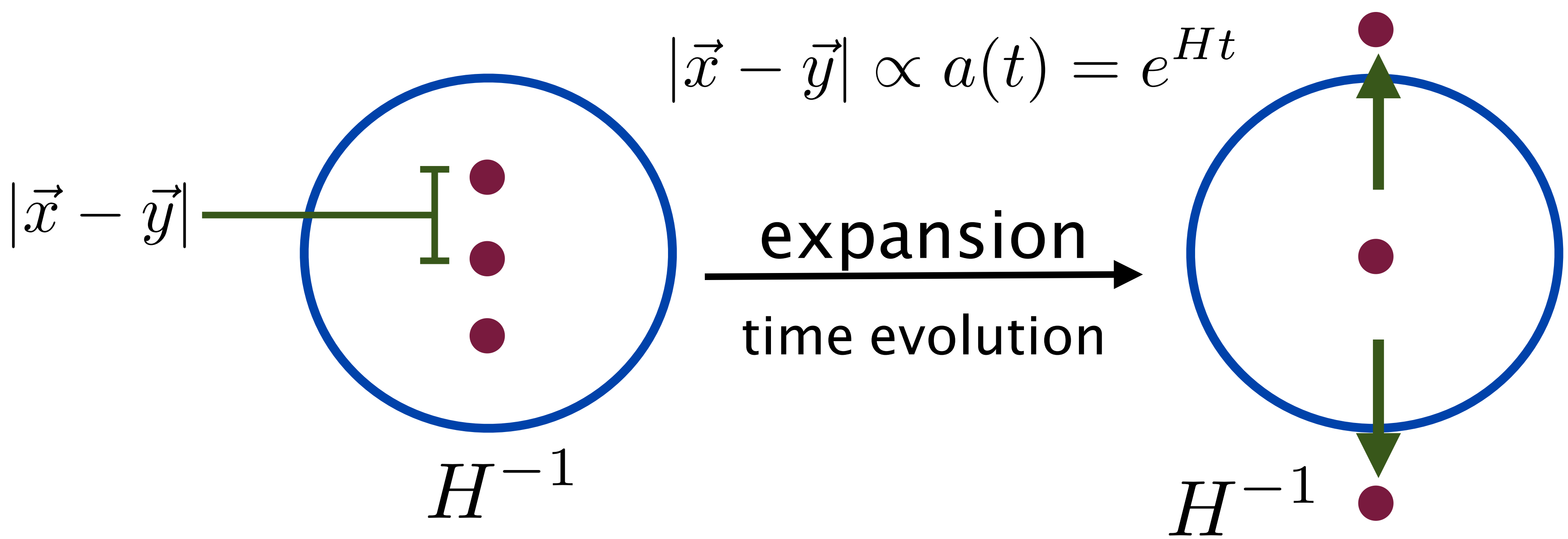
During inflation, fluctuations stretched

$$a \sim a_0 e^{Ht} \qquad \frac{k}{aH} \to 0$$

Long wavelengths evolve from short wavelengths

What is Inflation?

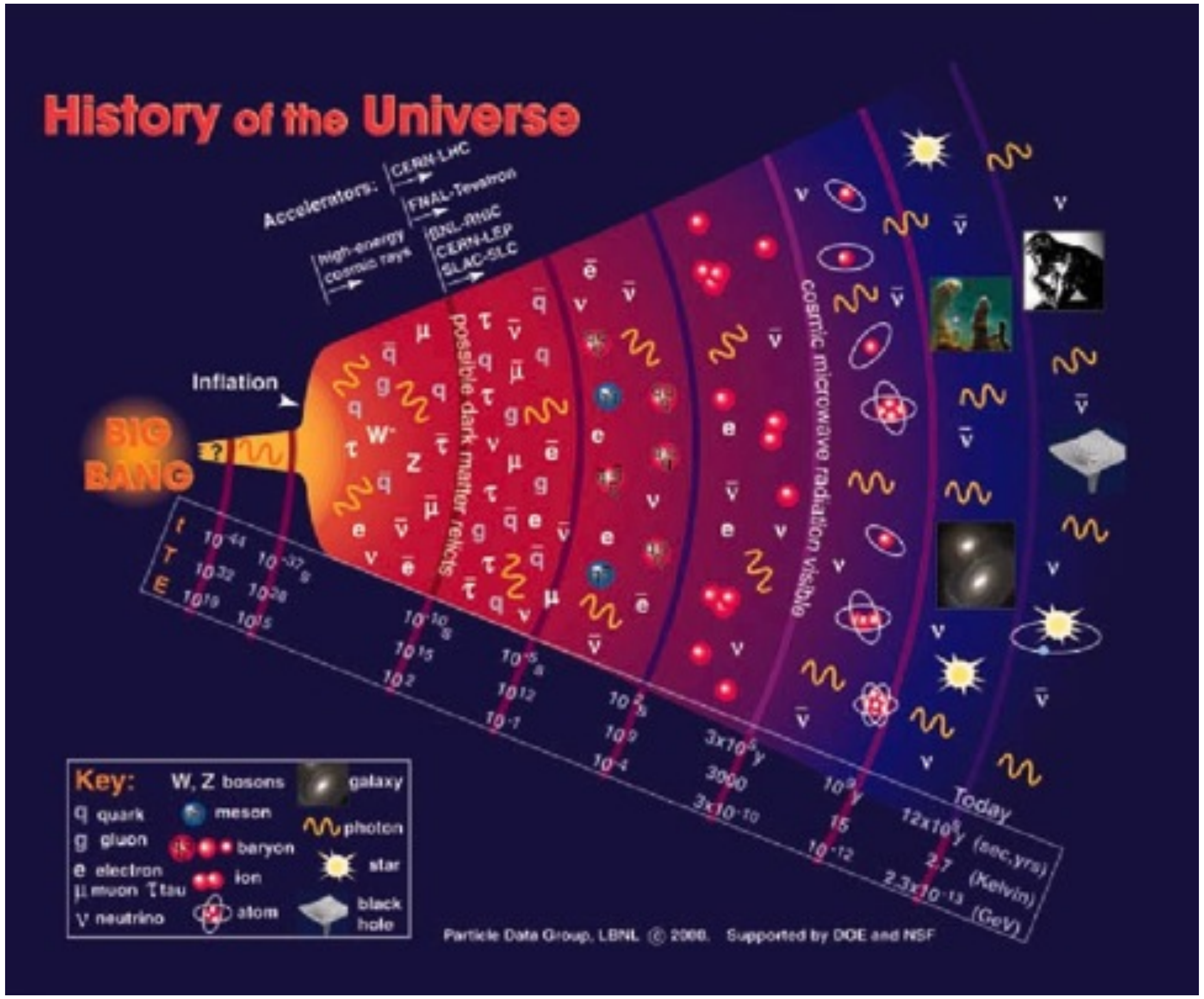
Production of fluctuations can be local



This is a property of de Sitter space

What is Inflation?

Inflation also requires that the phase ends



We must get the hot “big bang” eventually

What is Inflation?

A definition:

2. A physical clock

Linde; Albrecht & Steinhardt

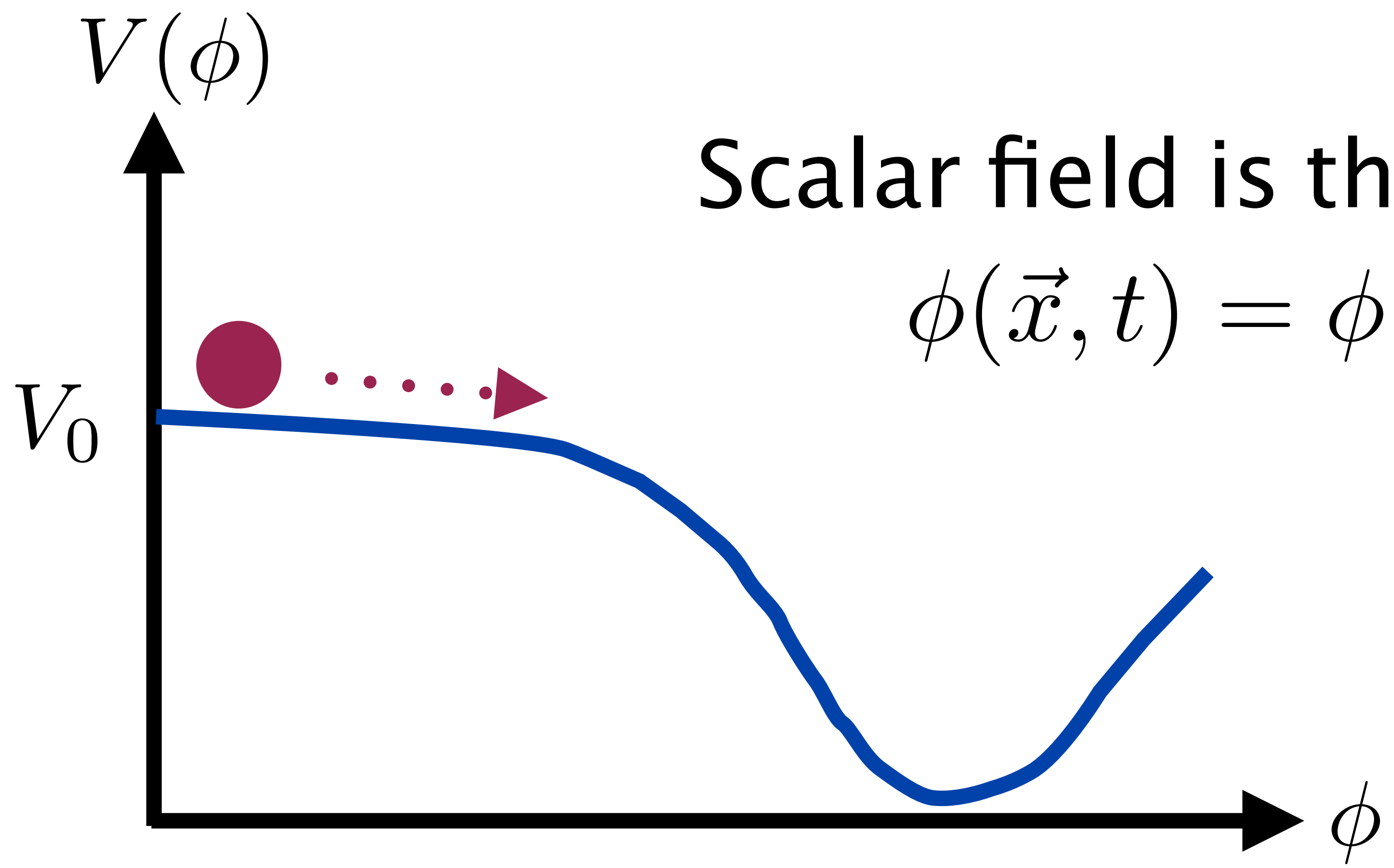
"End of inflation" needs a physical definition

Inflation must end everywhere at the same "time"

Different regions synched their clocks in the past

What is Inflation?

Slow-roll Inflation



Scalar field is the clock

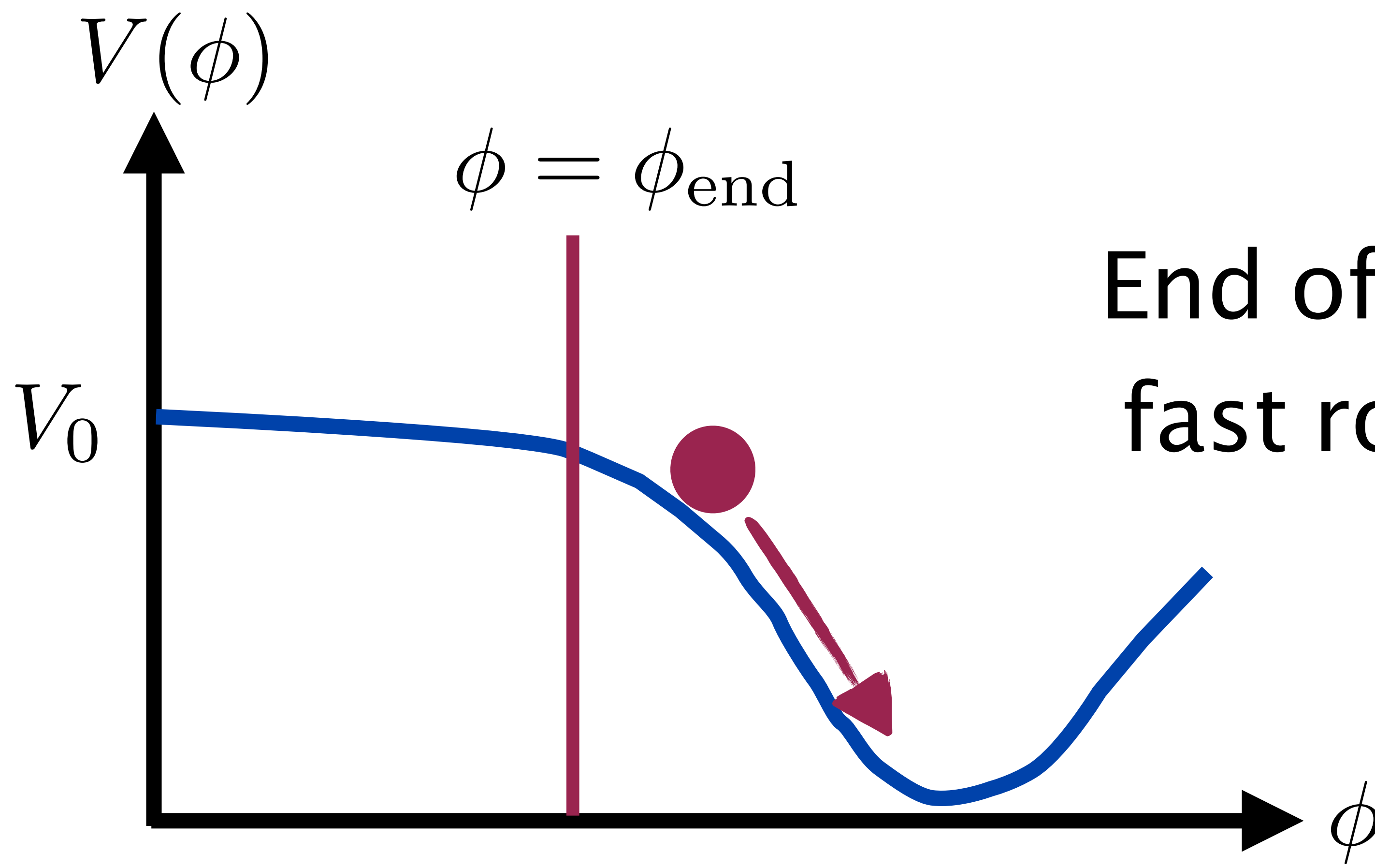
$$\phi(\vec{x}, t) = \phi(t)$$

$$H^2 \propto \frac{1}{2}\dot{\phi}^2 + V(\phi) \sim V_0$$

Slow-roll = Potential energy dominates

What is Inflation?

Slow-roll Inflation



End of inflation: fast roll begins

End of inflation defined by value of field

Afterwards, energy converted to radiation

Origin of structure

The origin of density fluctuations emerges naturally

Reason: No clock is perfect (uncertainty principle)

The amount of inflation will vary from place to place:

$$\zeta(x) \sim \frac{\delta a(x)}{a} \sim \frac{\dot{a}\delta t(x)}{a} \equiv H\delta t$$

Energy diluted differently = density fluctuations

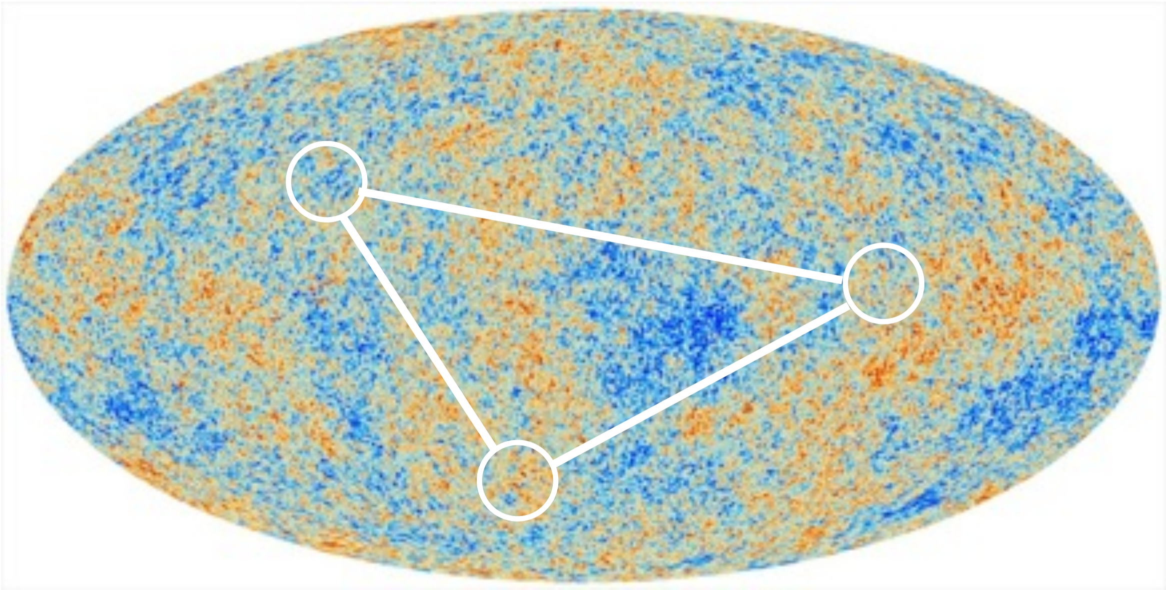
Origin of structure

Determine CMB temperature fluctuations

$$\frac{\delta T(\mathbf{n})}{T} \sim \int d^3k F(\mathbf{k}\cdot\mathbf{n}, k)\zeta_{\mathbf{k}}$$

Think of

$$\langle \zeta(x_1)..\zeta(x_n)\rangle \rightarrow$$



The EFT of Inflation

EFT of a Clock

Flat space / UV physics is time translation symmetric

A clock spontaneously breaks time translations

Goldstone boson: $t \to t - c \qquad \pi \to \pi + c$

Construct action from "linear field" $U \equiv t + \pi$

$$\mathcal{L} = F(U, \partial_\mu U \partial^\mu U, \ldots)$$

Most general action of the goldstone boson

EFT of Inflation

Gravity gauges time translations (time diffeomorphisms)

We can couple our goldstone to gravity minimally

$$S = \int dt d^3x \sqrt{-g} \left[\frac{M_{\rm pl}^2}{2} \mathcal{R} + F(U, g^{\mu\nu}\partial_\mu U \partial_\nu U, \ldots) \right]$$

We want this action to have a classical solution

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

$$|\dot{H}(t)| \ll H^2(t)$$

$$\pi(\vec{x}, t) = 0$$

EFT of Inflation

First we need to cancel tadpoles for π

Useful to realize that $(\partial_\mu U \partial^\mu U + 1)^N = \mathcal{O}(\pi^N)$

Expanding the action in π

$$S_\pi = \int dt d^3x \sqrt{-g} \left[M_{\rm pl}^2 \dot{H} \partial_\mu U \partial^\mu U - M_{\rm pl}^2 \left(3H(U)^2 + \dot{H}(U)\right) + \mathcal{O}(\pi^2) \right]$$

We used Einstein's equations to get $H(t) = \dfrac{\dot{a}}{a}$

Connection to Slow-Roll

Slow-roll inflation recovered via Einstein's equations

$$M_{\rm pl}^2 \dot{H} = -\frac{1}{2}\dot{\phi}^2$$
$$3M_{\rm pl}^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

Ignoring the fluctuations $U = t$

$$\mathcal{L} = -M_{\rm pl}^2 \dot{H} - M_{\rm pl}^2 (3H^2 + \dot{H}) = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Slow roll is captured by the universal terms

Higher order terms

There are many extra terms we could add

$$\mathcal{L} = M_{\rm pl}^2 \dot{H} \partial_\mu U \partial^\mu U - M_{\rm pl}^2 (H^2 + \dot{H}) + \sum_{n \geq 2} M_n^4(U)(\partial_\mu U \partial^\mu U + 1)^n + \mathcal{O}(\nabla_\mu \nabla_\nu U)$$

All coupling "constants" are functions of time

These terms are non-universal / model dependent

Canonical slow-roll sets all of these terms to zero

How do we use it?

This is very general, but is it practical?

Goldstone boson equivalence makes it useful

Decoupling limit

$$M_{\rm pl}^2 \to \infty \qquad \dot{H} \to 0 \qquad M_{\rm pl}^2 \dot{H} = \text{Const.}$$

Decouples Gravity (↑ $M_{\rm pl}^2 \to \infty$)

Fixes SSB / Goldstone (↑ $M_{\rm pl}^2 \dot{H} = \text{Const.}$)

How do we use it?

In the decoupling limit, e.g.

$$\mathcal{L}_\pi = \frac{M_{\rm pl}^2 |\dot H|}{c_s^2}\left[(\dot\pi^2 - c_s^2\partial_i\pi\partial^i\pi) - (1-c_s^2)(\dot\pi\partial_i\pi\partial^i\pi - \dot\pi^3)\right]$$

Here we have include the first non-universal term

$$M_2^4(\partial_\mu U\partial^\mu U + 1)^2 = \frac{M_{\rm pl}^2|\dot H|(1-c_s^2)}{c_s^2}[\dot\pi^2 + \ldots]$$

Higher orders do not affect the quadratic action at this order in derivatives

How do we use it?

The point is that π is the fluctuation of the clock

Decoupling limit is a short cut to its dynamics

Knowing the statistics of the clock lets us compute

$$\zeta \sim -H\pi$$

This is the “gauge invariant” scalar fluctuation

How do we use it?

In any FRW, ζ is a constant outside the horizon

Thermal history after inflation doesn't alter statistics

After inflation modes start coming inside horizon

We use ζ_k as the ICs not long before time of CMB

$$\frac{\delta T(\mathbf{n})}{T} \sim \int d^3k F(\mathbf{k}\cdot\mathbf{n}, k)\zeta_{\mathbf{k}}$$

Life in de Sitter Space

Massless scalar in dS

Most of our intuition comes from de Sitter space

$$ds^2 = a^2(\tau)\left(-d\tau^2 + d\vec{x}^2\right)$$

Hubble is constant is de Sitter: $a = a_0 e^{Ht}$

$$dt = a d\tau \rightarrow a = -\frac{1}{\tau H}$$

In de Sitter, future infinity is $\tau = 0$

Horizon crossing is $k\tau = -1$

Massless scalar in dS

Most of our intuition comes from scalar field in dS

$$\mathcal{L} = \int d\tau d^3\vec{x}\sqrt{-g}\,\left(-\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi\right) \to \int \frac{d\tau d^3\vec{x}}{H^2\tau^2}\,\tfrac{1}{2}(\phi'^2 - \partial_i\phi\partial^i\phi)$$

Equations of motion $\quad \phi'' - \dfrac{2}{\tau}\phi' + k^2\phi = 0$

Solution $\quad \phi = C_1(1 - ik\tau)e^{ik\tau} + C_2(1 + ik\tau)e^{-ik\tau}$

Solving is the easy part - need to understand it

Massless scalar in dS

We have our solution to the classical equations:

$$\phi = C_1(1 - ik\tau)e^{ik\tau} + C_2(1 + ik\tau)e^{-ik\tau}$$

Now we want to quantize $\hat{\phi}_{\vec{k}}(\tau) = \phi_{\vec{k}}(\tau)\hat{a}^\dagger + \mathrm{h.c.}$

At very early times, mode inside horizon $-k\tau \gg 1$

Create a positive frequency mode $C_2 = 0$

Should be normalized like in flat space

Massless scalar in dS

Fix the normalization from commutator

$$\phi(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \hat{\phi}_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} \qquad [\phi(\vec{x}), \dot{\phi}(\vec{x}')] = \frac{i}{a^3}\delta(\vec{x}-\vec{x}')$$

Using $\dot{\phi} = -\tau H \phi'$

$$\hat{\phi}_{\vec{k}}(\tau) = \frac{H}{\sqrt{2k^3}}(1-ik\tau)e^{ik\tau}\hat{a}^\dagger + \text{h.c.}$$

Modes freeze-out $\lim_{\tau\to 0} \hat{\phi}_{\vec{k}}(\tau) = \frac{H}{\sqrt{2k^3}}(\hat{a}^\dagger + \hat{a})$

Real time intuition

Much easier to get intuition in real time

$$\ddot{\phi} + 3H\dot{\phi} + \frac{k^2}{a^2}\phi = 0$$

At early times, gradients dominate. Use WKB

$$\phi \sim \frac{1}{\sqrt{2\omega}} e^{i\int^t dt'\,\omega(t')} \qquad i\omega = -\frac{3}{2}H + i\frac{k}{a} + \mathcal{O}\left(\frac{Ha}{k}\right)$$

$$\phi \sim \frac{1}{a(\tau)\sqrt{2k}} e^{ik\tau} = -\frac{H\tau k}{\sqrt{2k^3}} e^{ik\tau}$$

Inside the horizon, just like flat space

Real time intuition

Much easier to get intuition in real time

$$\ddot{\phi} + 3H\dot{\phi} + \frac{k^2}{a^2}\phi = 0$$

At late times, we can ignore the gradients

$$\ddot{\phi} + 3H\dot{\phi} = 0 \longrightarrow \phi = C_1 + C_2 a^{-3} + \mathcal{O}\left(\frac{k^2}{a^2 H^2}\right)$$

Outside the horizon, constant solutions are classical

Real time intuition

Much easier to get intuition in real time

$$\ddot{\phi} + 3H\dot{\phi} + \frac{k^2}{a^2}\phi = 0$$

All we are doing is matching at $-k\tau = \frac{k}{aH} = 1$

classical fluctuations emerge from vacuum

Expansion produces particles of energy H

de Sitter Summary

Some important things to note:

Classical density fluctuations produced at $k = aH$

Amplitude set by Hubble

Time translations symmetry = Scale invariant

$$\langle \phi_{\vec{k}} \phi_{\vec{k}'} \rangle = \frac{H^2}{2k^3} \delta(\vec{k} + \vec{k}')$$

Back to the EFT

What matters in dS is what happens when $\omega \sim H$

(like a collider with center of mass energy H)

Our decoupling limit was $M_{\rm pl}^2 \to \infty, \dot{H} \to 0$

The error we are making is the $\mathcal{O}\left(\frac{\omega^2}{M_{\rm pl}^2}, \frac{\dot{H}}{\omega^2}\right)$

Controlled by definition $M_{\rm pl}^2 \gg \omega^2 \sim H^2 \gg \dot{H}$

Inflation is necessarily in the decoupling limit

Back to the EFT

There are some important differences in inflation

$$\mathcal{L}_\pi = \frac{M_{\rm pl}^2|\dot{H}|}{c_s^2}\left[(\dot{\pi}^2 - c_s^2\partial_i\pi\partial^i\pi) + (1-c_s^2)(\dot{\pi}\partial_i\pi\partial^i\pi - \dot{\pi}^3)\right]$$

1. Non-relativistic when $c_s^2 < 1$
2. Interaction linked to speed of sound
3. Explicit time dependence

Back to the EFT

There are some important differences in inflation

$$\mathcal{L}_\pi = \frac{M_{\rm pl}^2|\dot{H}|}{c_s^2}\left[(\dot{\pi}^2 - c_s^2\partial_i\pi\partial^i\pi) - (1-c_s^2)(\dot{\pi}\partial_i\pi\partial^i\pi - \dot{\pi}^3)\right]$$

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Back to the EFT

There are some important differences in inflation

$$\mathcal{L}_\pi = \frac{M_{\rm pl}^2 |\dot{H}(t)|}{c_s^2(t)} \left[(\dot{\pi}^2 - c_s^2 \partial_i \pi \partial^i \pi) - (1 - c_s^2)(\dot{\pi} \partial_i \pi \partial^i \pi - \dot{\pi}^3) \right]$$

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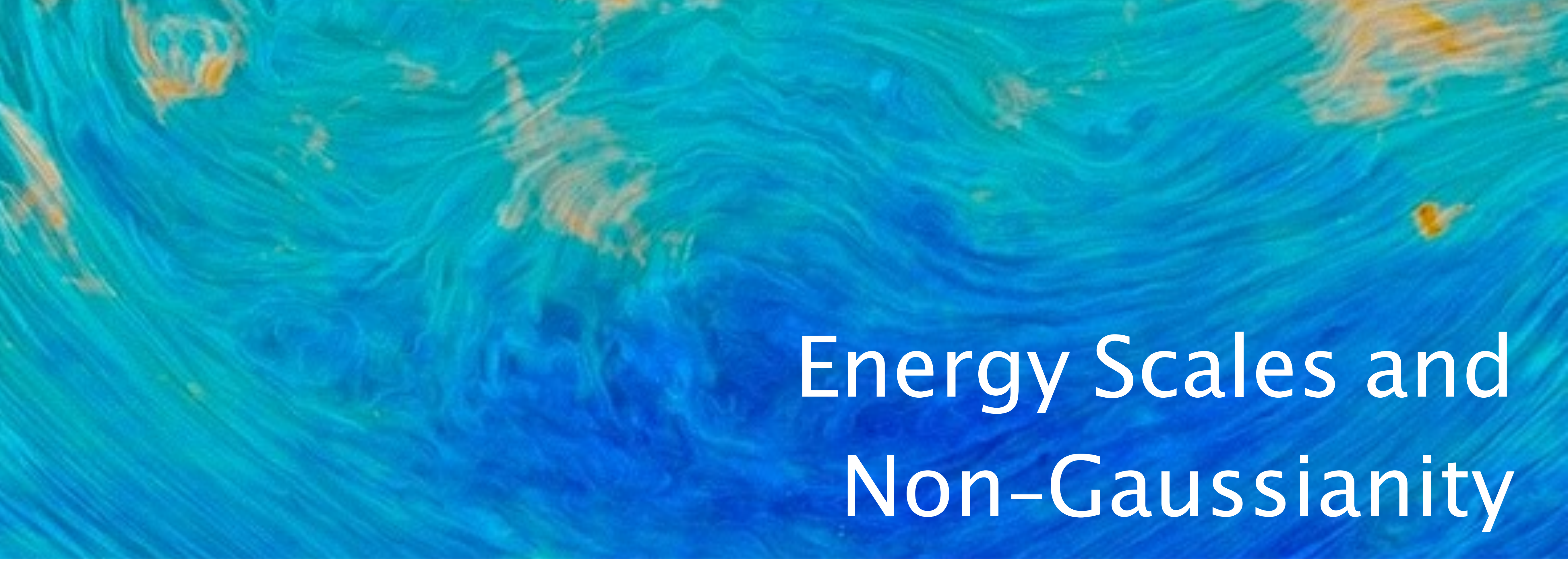
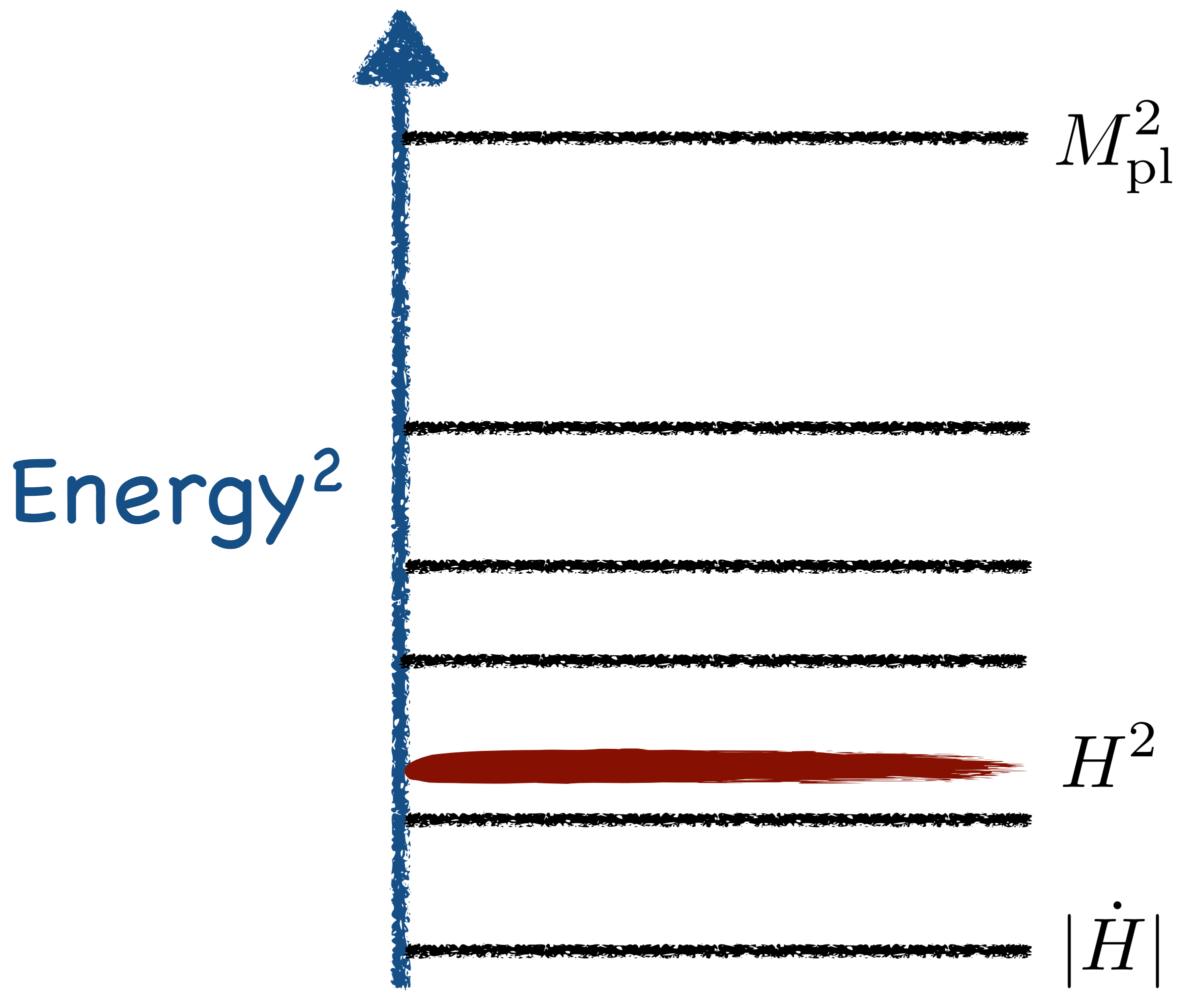


Energy Scales and Non-Gaussianity

Energy Scales

Hierarchies of scales is what gives EFT power



We still need to fill in this picture from our EFT

Global Symmetries

Spontaneously Broken global symmetries

Current always exists $\partial_\mu j^\mu = 0$

Charge not well-defined $Q = \int d^3x j^0(x) \to \infty$

Goldstone = IR divergence $j^\mu = f_\pi^2 \partial^\mu \pi + \mathcal{O}(\pi^2)$

SSB defined by $f_\pi^2 > 0$

The current defines f_π^2 unambiguously

Space-time symmetry

Spontaneously Broken time translations

Current always exists $\partial_\mu T^{0\mu} = 0$

Charge not well-defined $P^0 = \int d^3x T^{00} \to \infty$

Goldstone = IR divergence $T^{00} = \frac{M_{\rm pl}^2 |\dot{H}|}{c_s^2} \dot{\pi} + \mathcal{O}(\pi^2)$

SSB defined by $\frac{M_{\rm pl}^2 |\dot{H}|}{c_s^2} > 0$

Is this the analogue of f_π^2 ?

Symmetry breaking scale

We need to be careful to define <u>energy</u> scales

From kinetic terms: $\left[\frac{2M_{\rm pl}^2|\dot{H}|}{c_s^2}\right] = [\omega][k^3]$

We will define our scales in units of energy

$$f_\pi^4 \equiv 2M_{\rm pl}^2|\dot{H}|c_s$$

With this definition $[f_\pi] = [\omega]$

Symmetry breaking scale

Does this make sense?

For slow-roll $c_s^2 = 1$ and Einstein's equations give

$$f_\pi^4 = 2M_{\rm pl}^2|\dot{H}| = \dot{\phi}^2$$

Time translations broken by time dependent vev

Makes sense that this is fixed in decoupling limit

Strong Coupling Scale

Action contains irrelevant operators

$$\mathcal{L}_3 = \frac{M_{\rm pl}^2 |\dot{H}|(1-c_s^2)}{c_s^2}[\dot{\pi}\partial_i\pi\partial^i\pi - \dot{\pi}^3]$$

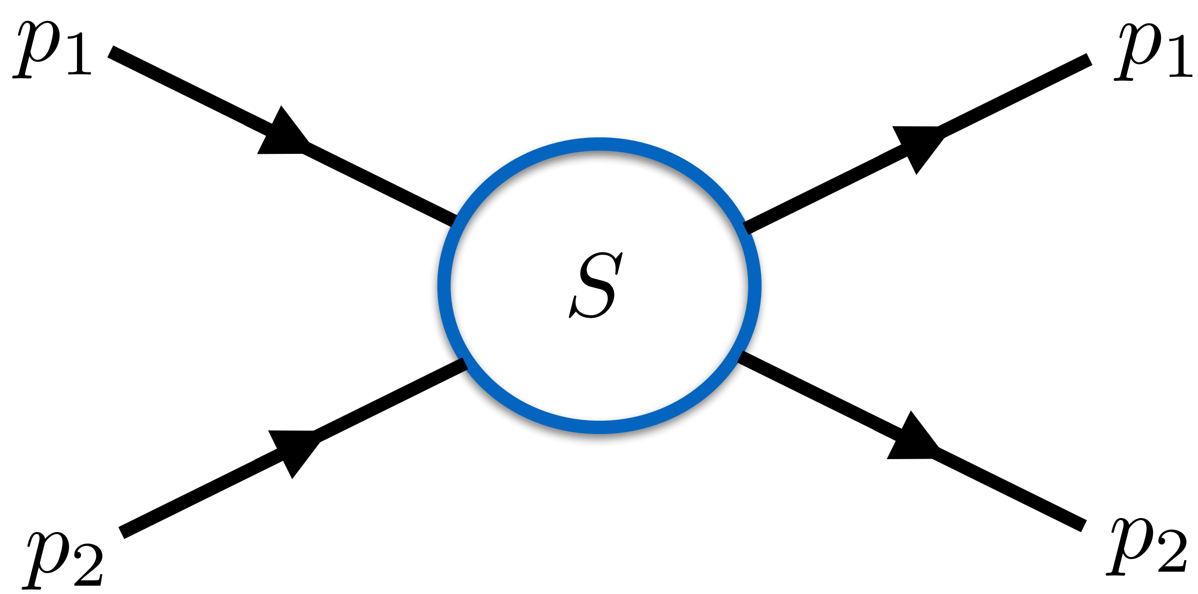
Canonically normalize and rescale $x = c_s\tilde{x}$

$$\tilde{\mathcal{L}}_3 = \frac{1}{\Lambda^2}\left[\dot{\pi}\tilde{\partial}_i\pi\tilde{\partial}^i\pi - c_s^2\dot{\pi}^2\right] \qquad \Lambda^4 = 2M_{\rm pl}^2|\dot{H}|\frac{c_s^5}{(1-c_s^2)^2}$$

Guess that strong coupling at $\omega > \Lambda$

Strong Coupling Scale

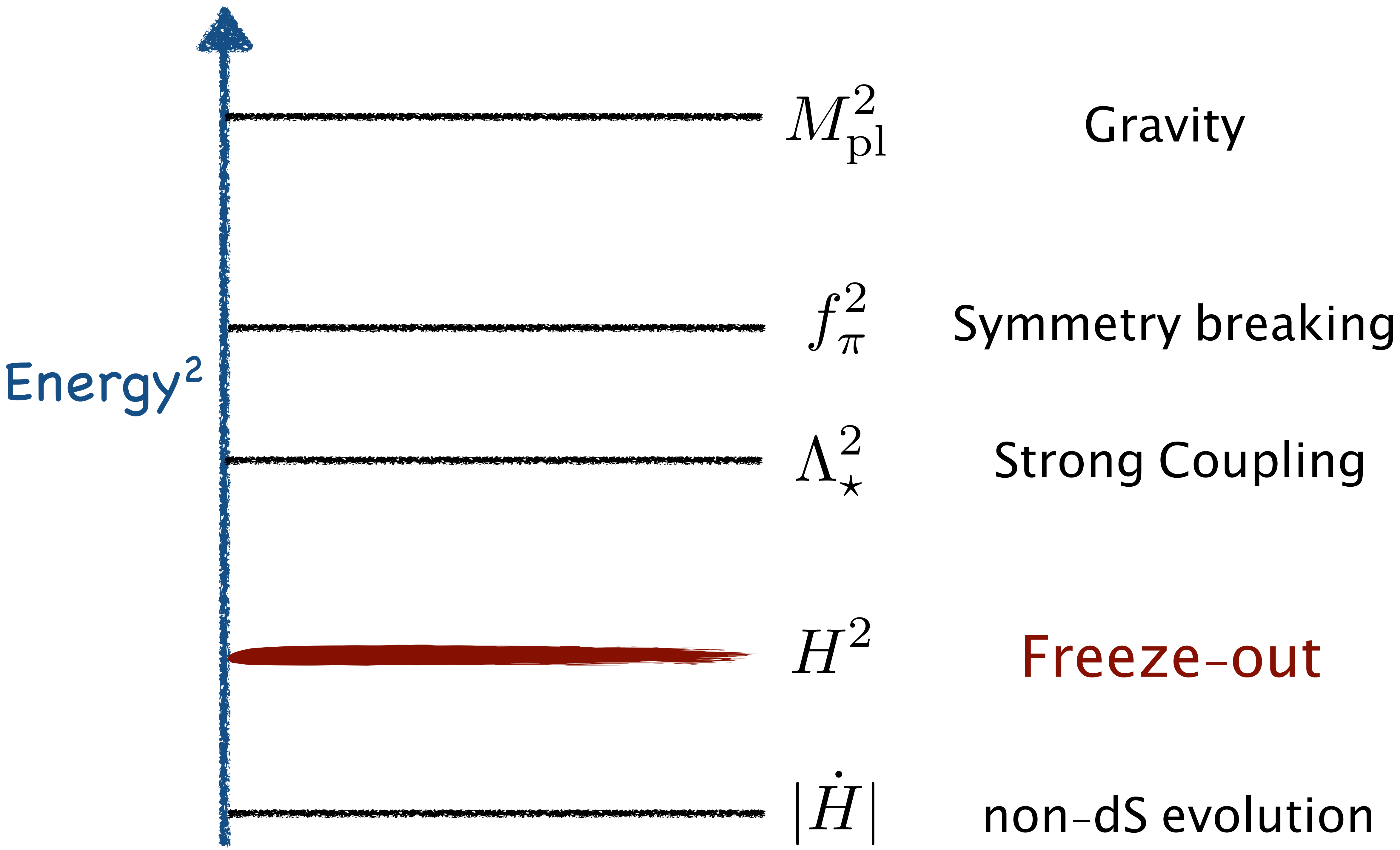
A more precise measure is perturbative unitarity



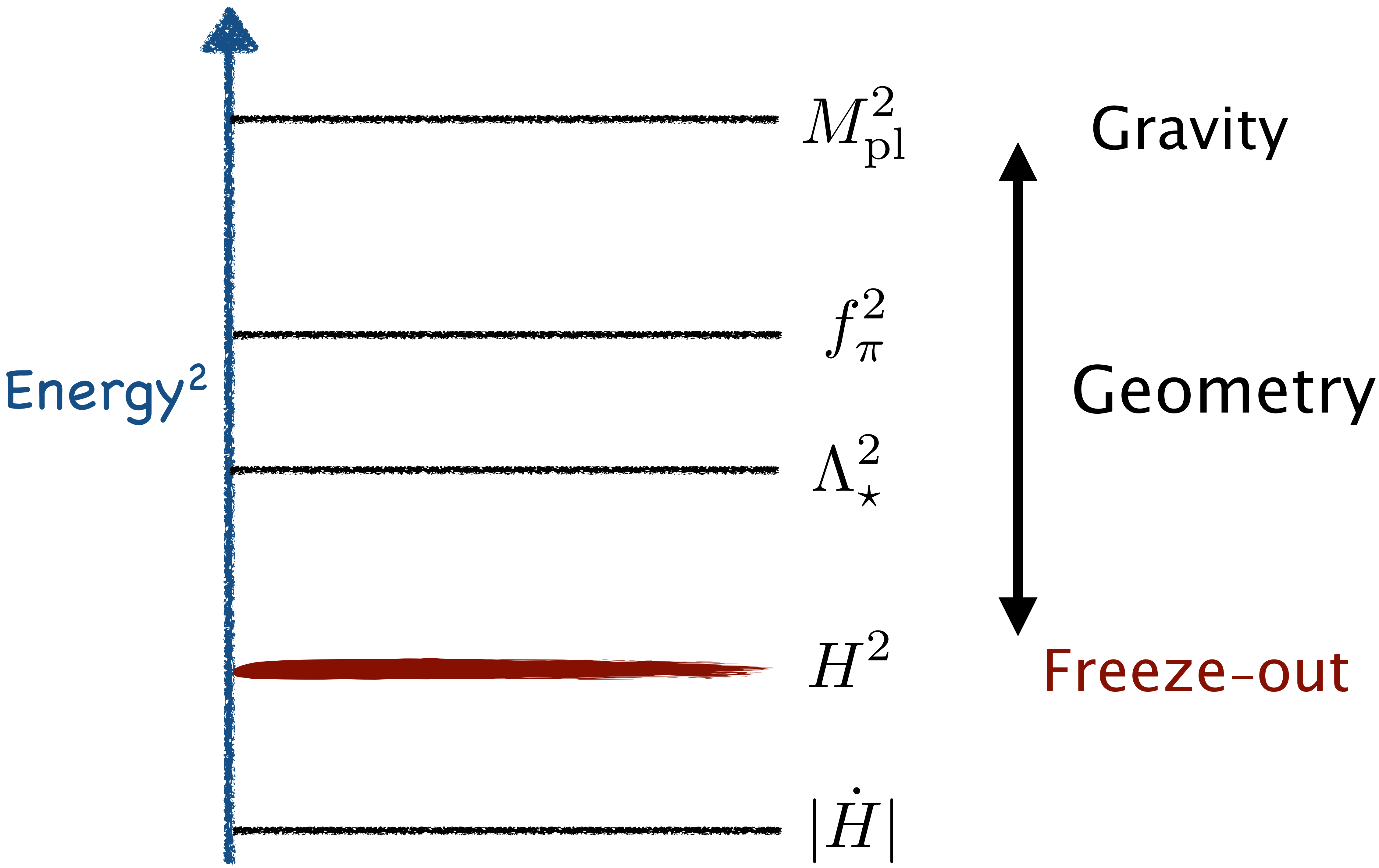
Unitarity requires partial wave amplitude $|a_\ell| \leq \frac{1}{2}$

Violated for $\omega^4 \geq 30\pi \frac{f_\pi^4 c_s^4}{1 - c_s^2} = 30\pi(1 - c_s^2)\Lambda^4 \equiv \Lambda_\star^4$

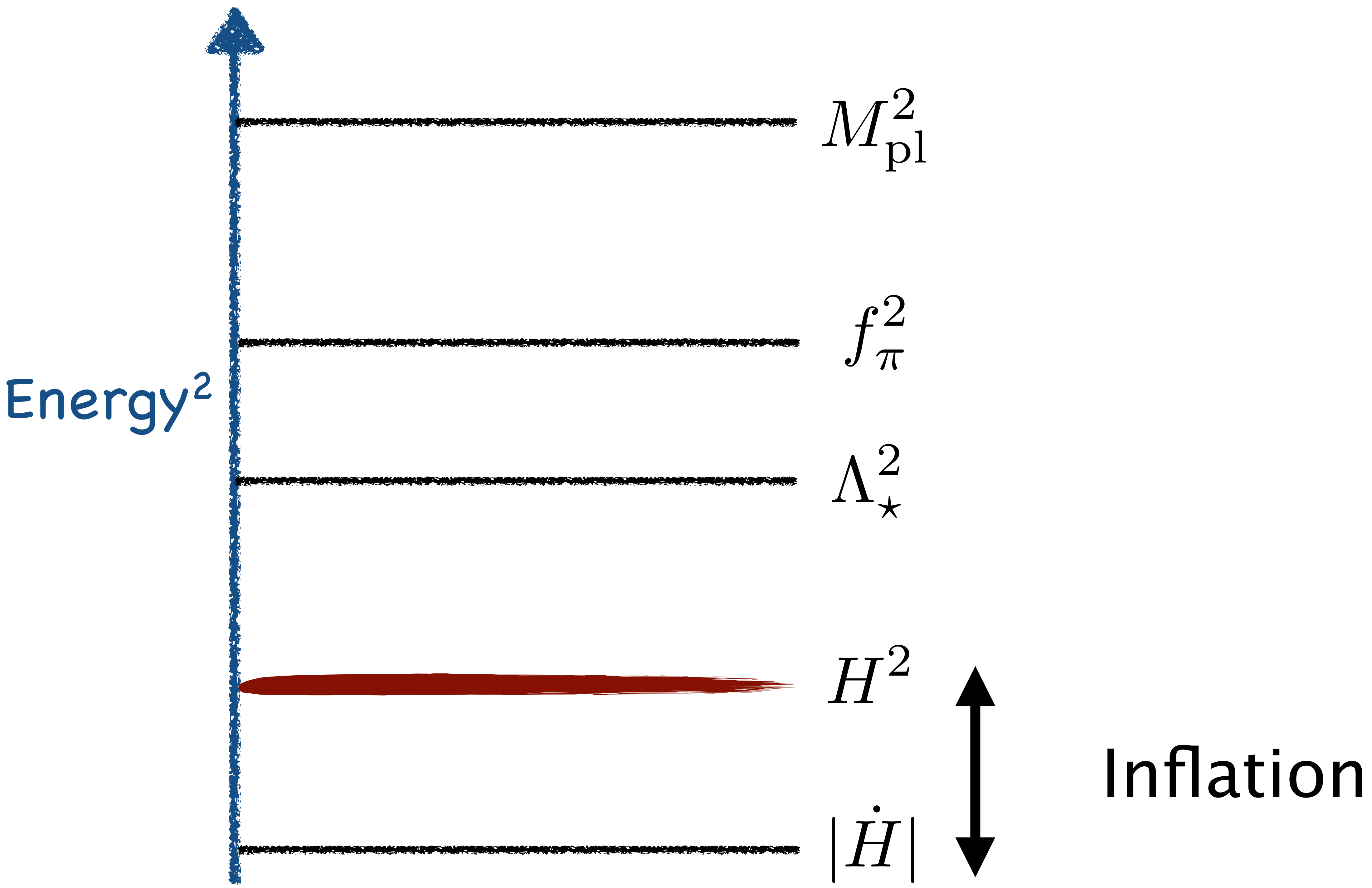
Energy Scales



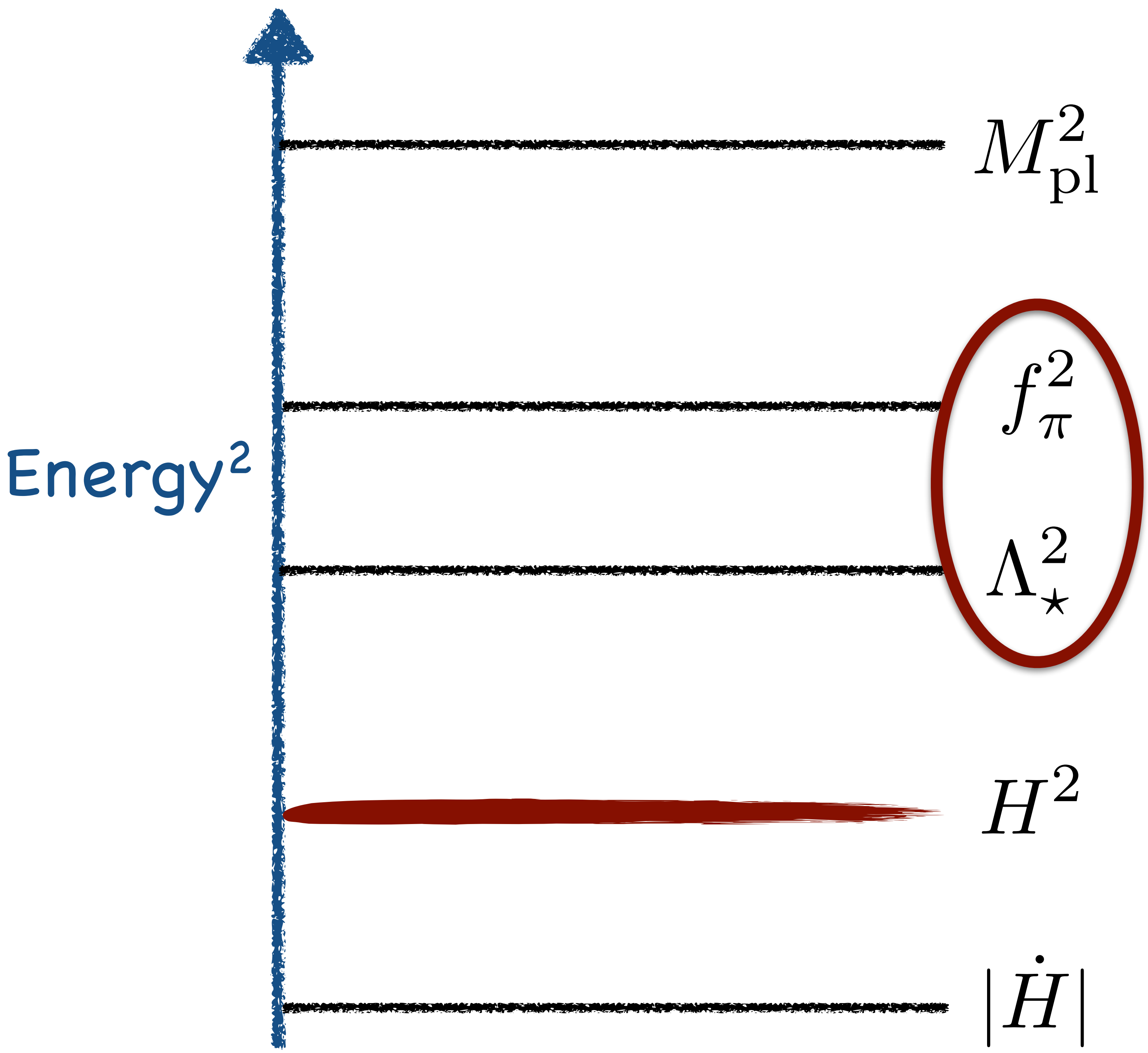
Energy Scales



Energy Scales



Energy Scales



How do these fit in?

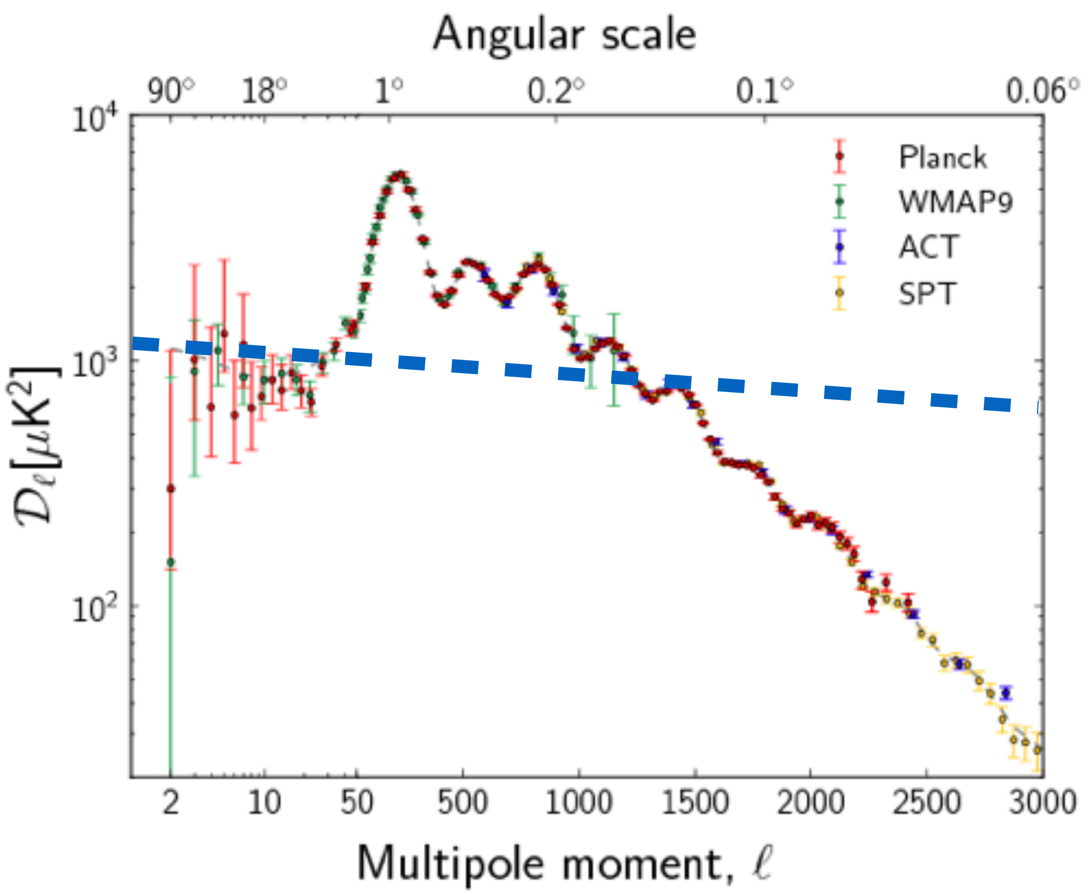
Scalar Power Spectrum

We know the amplitude for scalar fluctuations

$$\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \rangle = \frac{2\pi^2 \Delta_\zeta^2}{k^{3-(n_s-1)}} \delta(\vec{k} + \vec{k}')$$

$$\Delta_\zeta^2 = 2.2 \times 10^{-9}$$

$$n_s = 0.9653 \pm 0.0048$$



This must tell us something since $\langle \zeta^2 \rangle \sim H^2 \langle \pi^2 \rangle$

Scalar Power Spectrum

We computed the answer in dS with $c_s = 1$

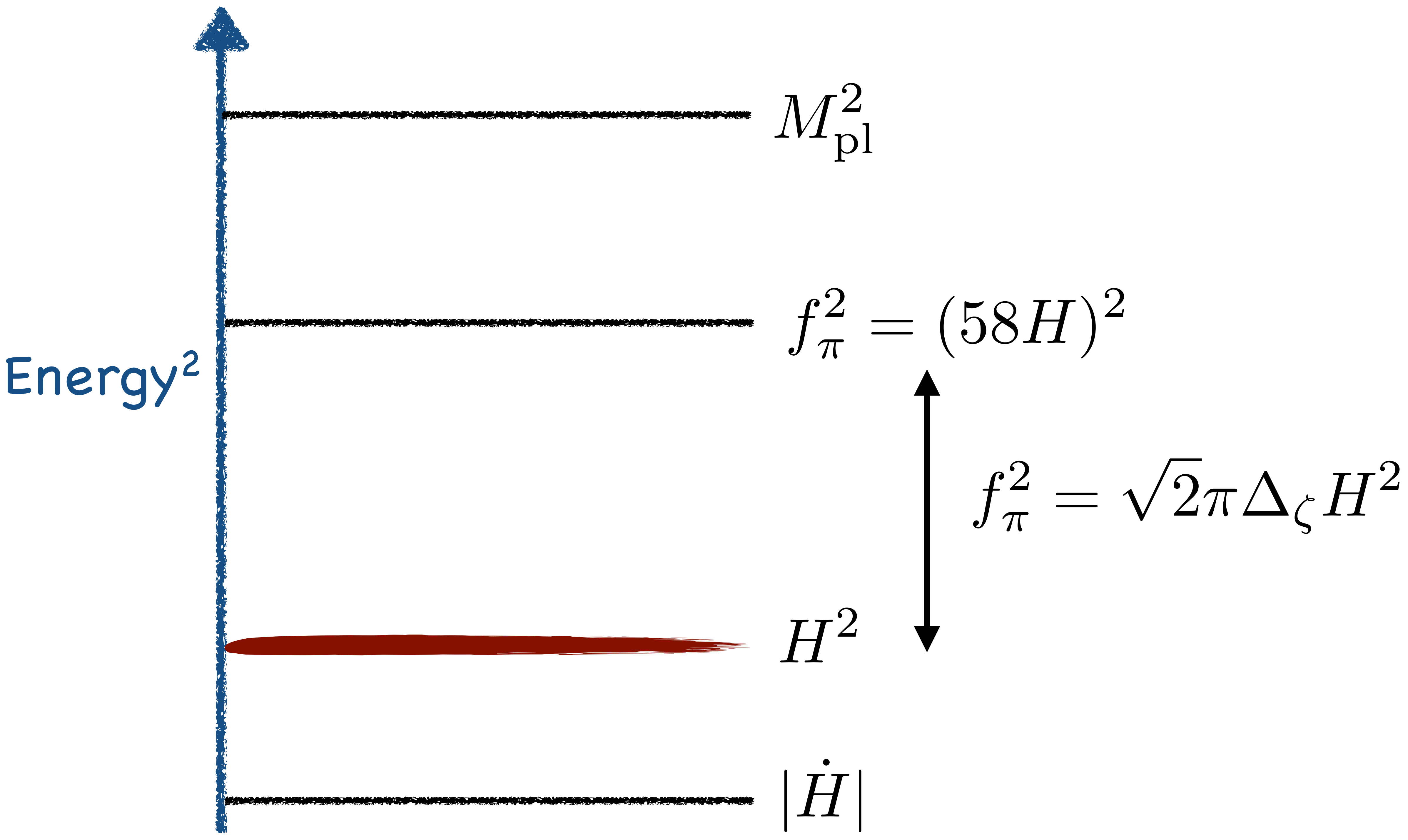
By rescaling, we get a canonical action

$$\pi_c = \frac{f_\pi^2}{c_s}\pi \qquad \tilde{k} = c_s k$$

$$\langle \pi_c^2 \rangle = \frac{H^2}{2\tilde{k}^3} \longrightarrow \langle \zeta^2 \rangle = \frac{H^4}{f_\pi^4}\frac{1}{2k^3}$$

Almost could have guessed by dimensional analysis

Scalar Power Spectrum



Scalar Power Spectrum

Again, think about error in length of inflation

$$a(t,x) = a(t)e^{\zeta(x)} \rightarrow \zeta \sim H\delta t = H\frac{\delta\phi}{\dot{\phi}} = -H\pi$$

Amplitude of fluctuations set by $\delta\phi \sim H$

Faster rolling = fluctuations have a smaller impact

$$\langle (H\delta t)^2 \rangle \sim \frac{H^4}{\dot{\phi}^2} = \frac{H^4}{f_\pi^2}$$

Scalar Power Spectrum

Time dependence is what generates the tilt

$$\langle \zeta^2 \rangle = \frac{H^4(t)}{M_{\rm pl}^2 |\dot{H}(t)| c_s(t)} \Big|_{a(t_\star)H(t_\star)=k}$$

We evaluate time dependent functions at freeze-out

$$d \log k \simeq H dt$$

$$n_s - 1 = \frac{\partial \log \langle \zeta^2 \rangle}{\partial \log k} = \frac{4\dot{H}}{H^2} - \frac{\ddot{H}}{H|\dot{H}|} - \frac{\dot{c}_s}{c_s H}$$

Tensor Power Spectrum

Just like scalar, gravitons are also excited

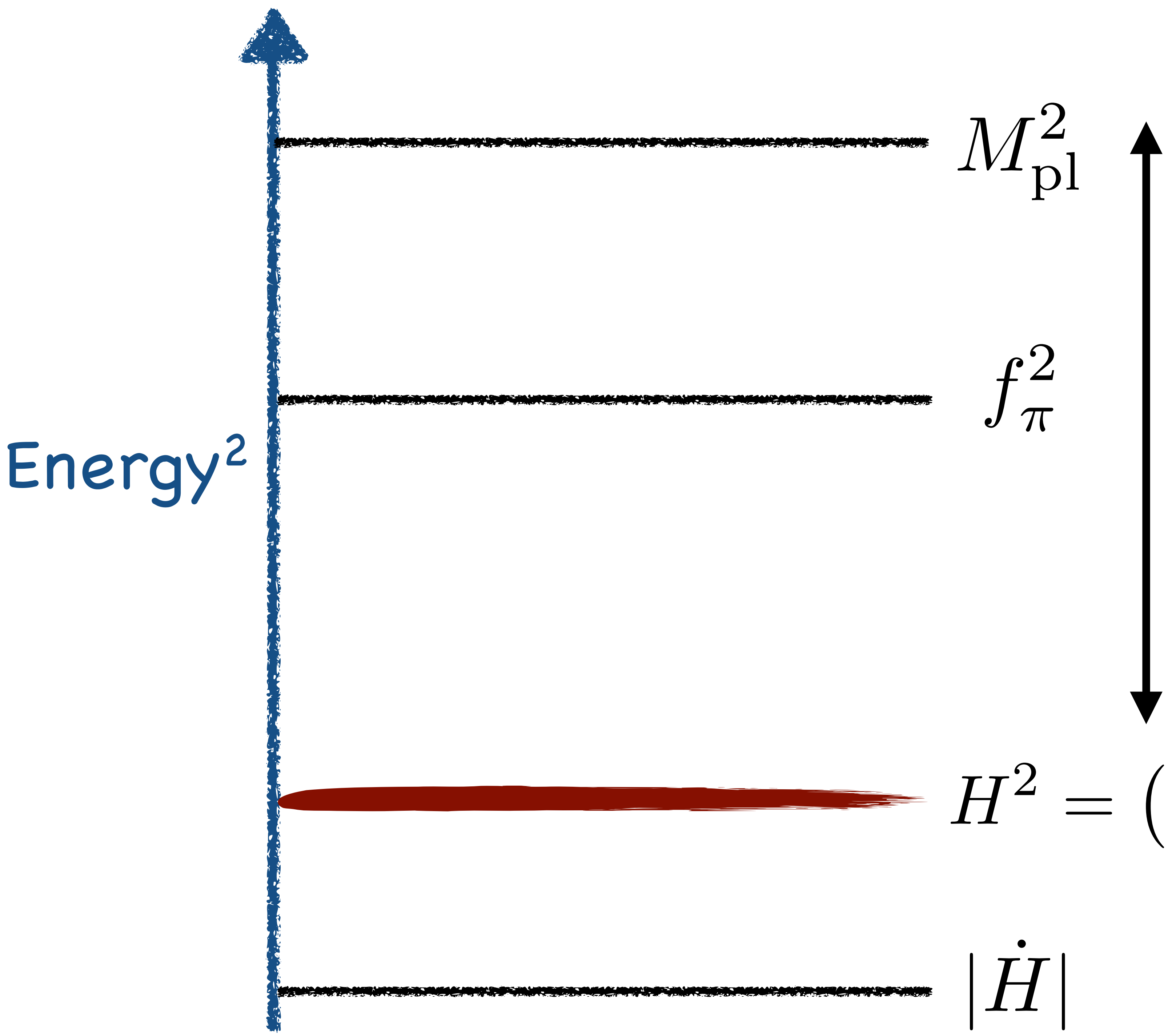
$$\langle h_{ij}h^{ij}\rangle = 4\frac{H^2}{M_{\rm pl}^2} = 2\pi^2\Delta_h^2$$

Typically expressed as $r = \frac{\Delta_h^2}{\Delta_\zeta^2} = 16\frac{|\dot{H}|}{H^2}c_s$

Current limit $r < 0.09\,(95\%)$

If detectable, all the scales were VERY high

Tensor Power Spectrum

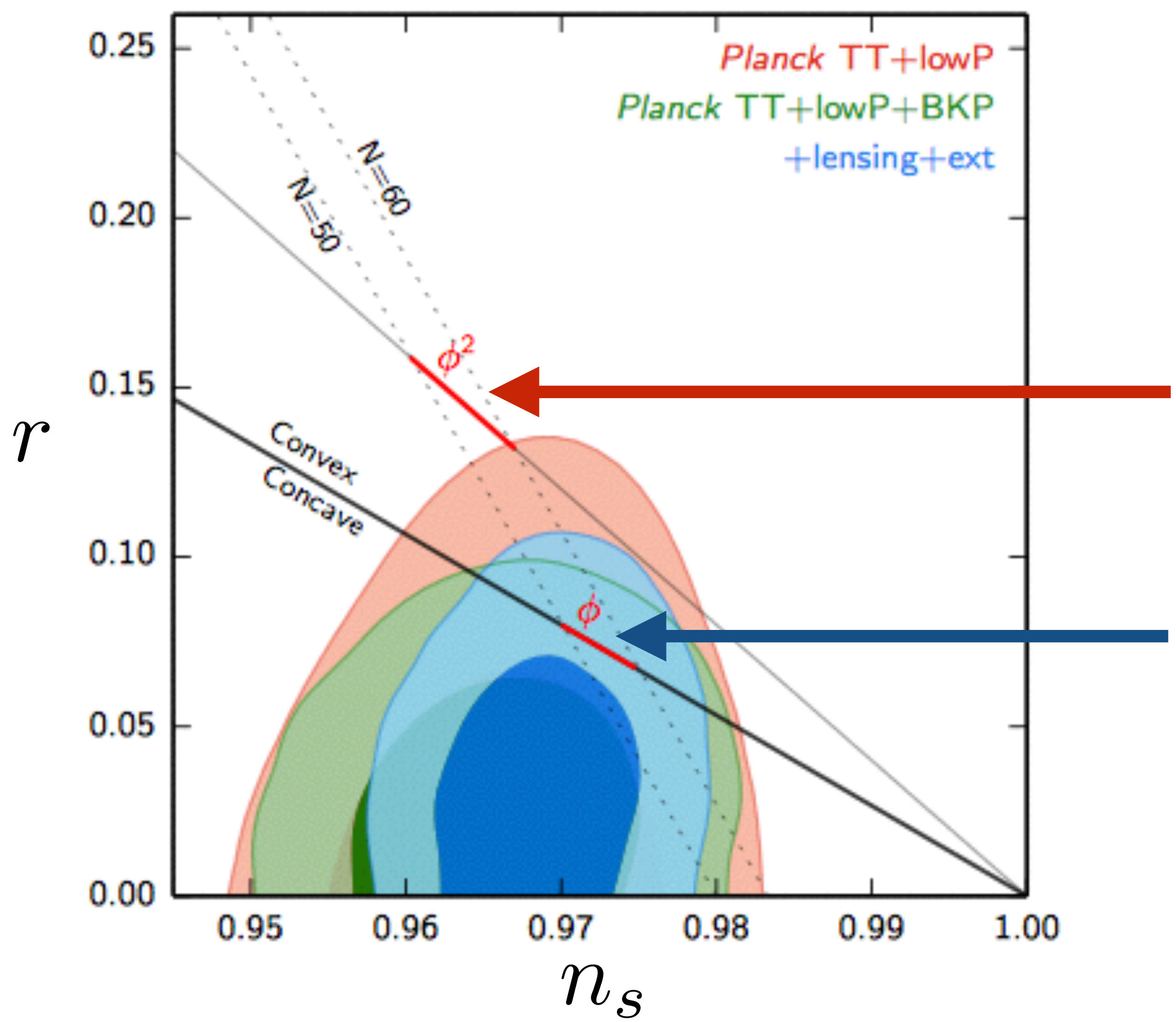


$$\frac{H^2}{M_{\rm pl}^2} = r\,\frac{2\pi^2\Delta_\zeta^2}{8}$$

$$H^2 = \left(\frac{r}{0.1}\right)\left(5.7\times 10^{13}\,\mathrm{GeV}\right)^2$$

Tensor Power Spectrum

Often these get combine into one plot



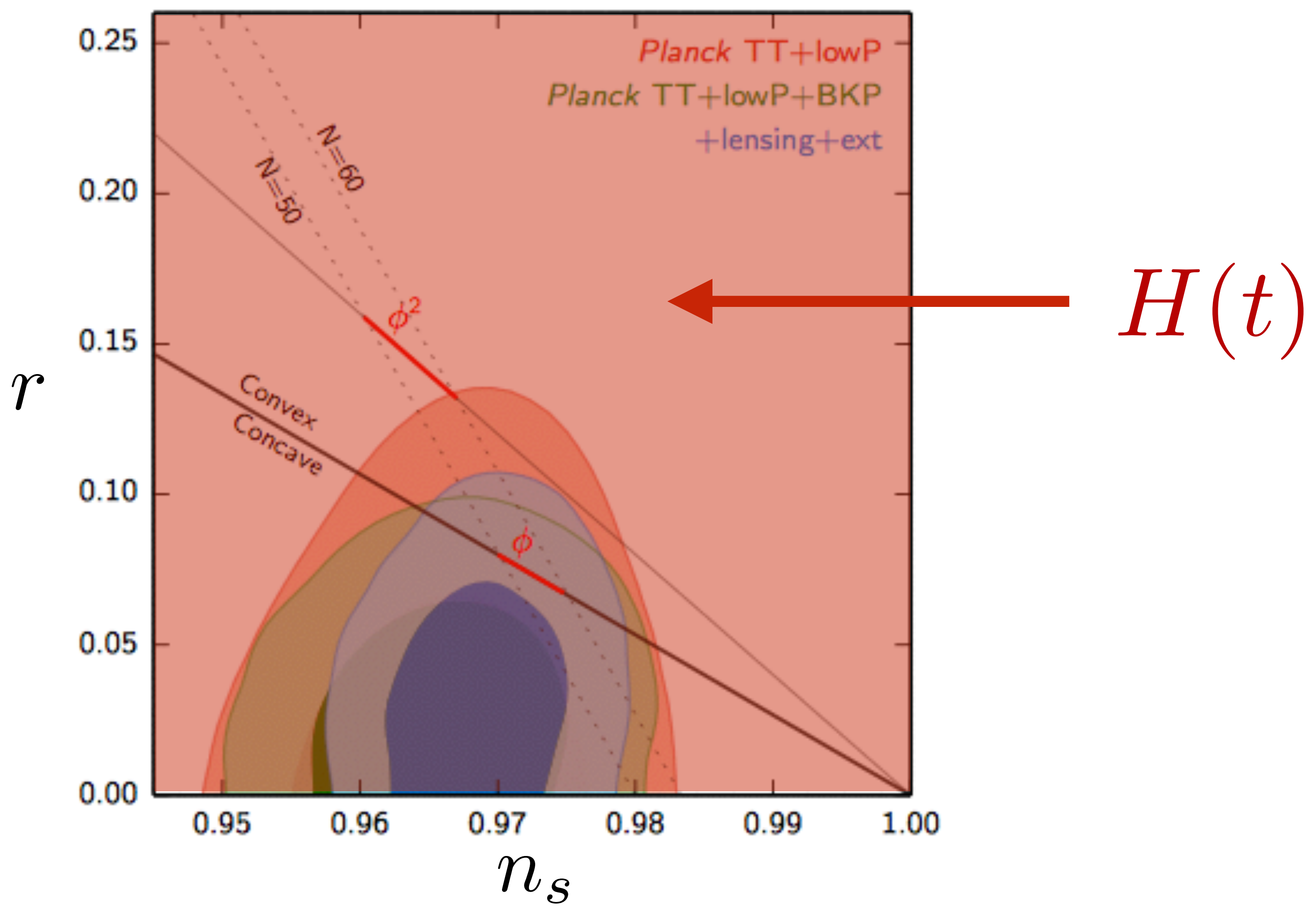
$$V(\phi) = \frac{1}{2}m^2\phi^2$$

$$V(\phi) = \mu^3\phi$$

This plot is very important for simple models

Tensor Power Spectrum

Every point is possible in EFT of inflation



Less useful from a general perspective

Non-Gaussanity

Quadratic action leads to gaussian statistics

(i.e. 2-pt function determines everything)

We saw that interactions are allowed

$$\tilde{\mathcal{L}}_3 = \frac{1}{\Lambda^2}\left[\dot{\pi}\tilde{\partial}_i\pi\tilde{\partial}^i\pi - c_s^2\dot{\pi}^2\right]$$

At horizon crossing, size of interaction $\frac{\omega^2 \sim H^2}{\Lambda^2}$

This effect is naturally small (irrelevant)

Non-Gaussanity

Using perturbation theory, we compute bispectrum

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^3 B(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)$$

The precise function is called the “shape”

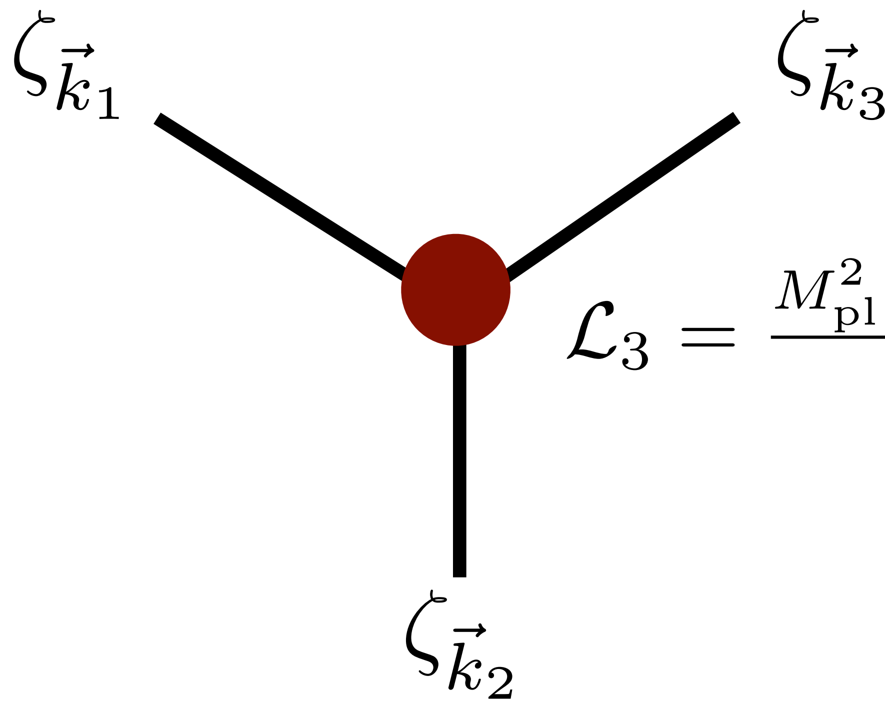
The amplitude is typically given in terms of

$$f_{\rm NL} = \frac{5}{18} \frac{B(k,k,k)}{P_\zeta^2(k)}$$

Order one non-gaussian means $f_{\rm NL} \sim 10^5$

Non-Gaussanity

Intuitively, we should expect $f_{\rm NL}\Delta_\zeta \sim \frac{H^2}{\Lambda^2}$



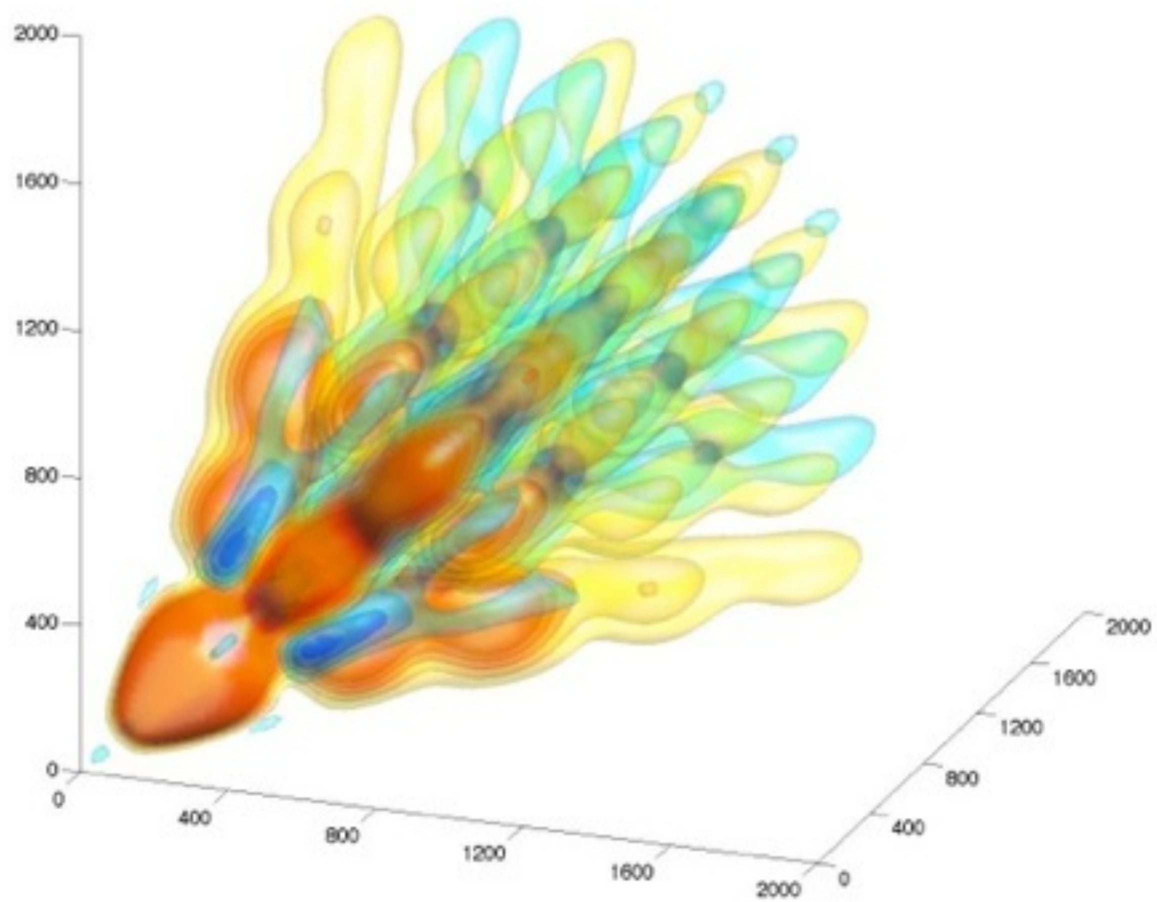
$$\mathcal{L}_3 = \frac{M_{\rm pl}^2 H(1-c_s^2)}{c_s^2}\dot{\pi}(\partial\pi)^2$$

For the speed of sound term, we get

$$f_{\rm NL}^{\rm equilateral} = -\frac{85}{325}\frac{H^2}{\Lambda^2}(2\pi\Delta_\zeta)^{-1} = -\frac{85}{325}\frac{f_\pi^2}{\Lambda^2}$$

Non-Gaussanity

Planck looks for this shape in the data



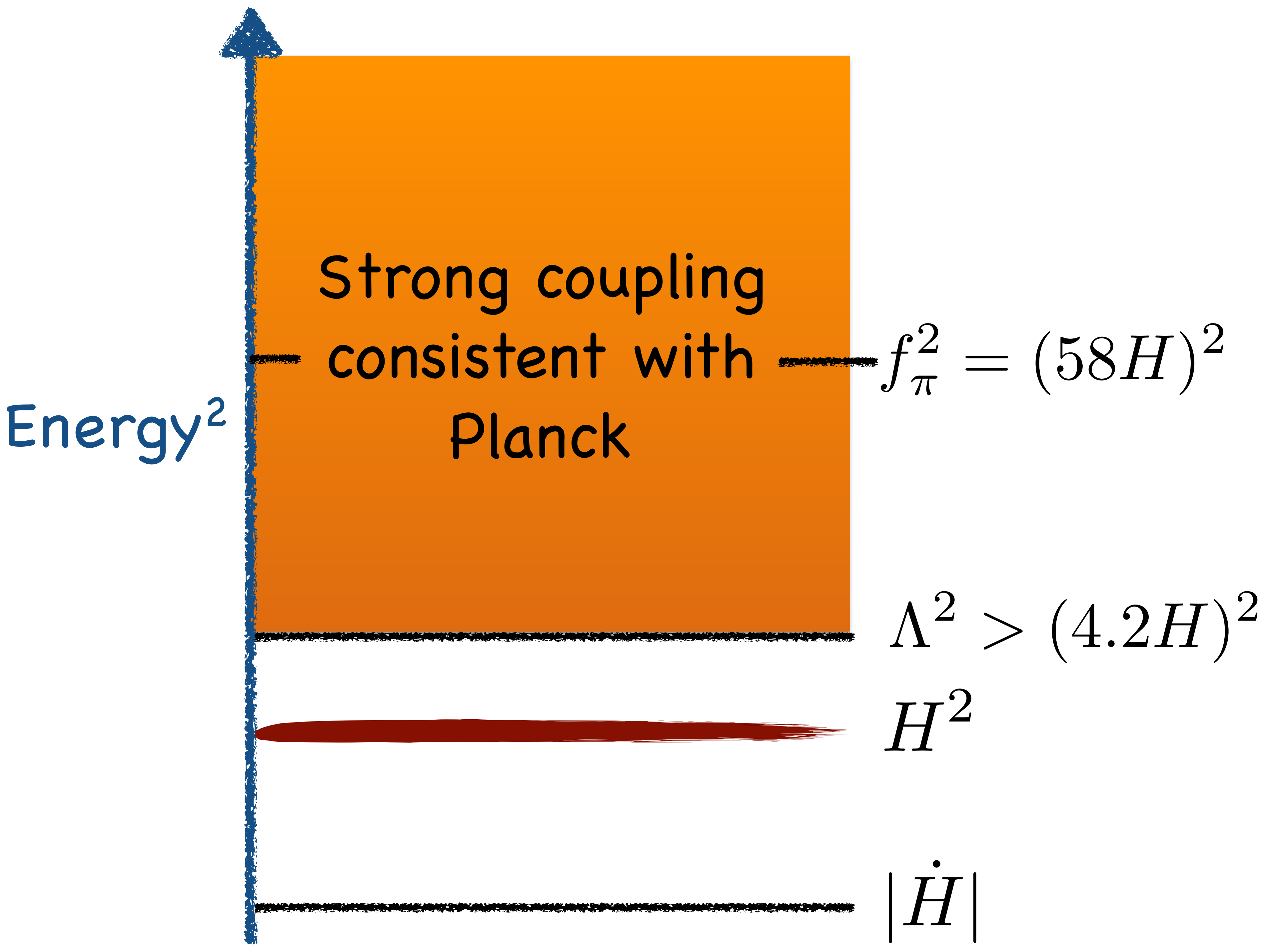
Equilateral

Peaked at:

$$k_1 = k_2 = k_3$$

$$f_{\rm NL}^{\rm equil.} = -4 \pm 43 \quad \text{(68\% C.I.)}$$

Non-Gaussanity



Non-Gaussanity

What would we expect from slow-roll ?

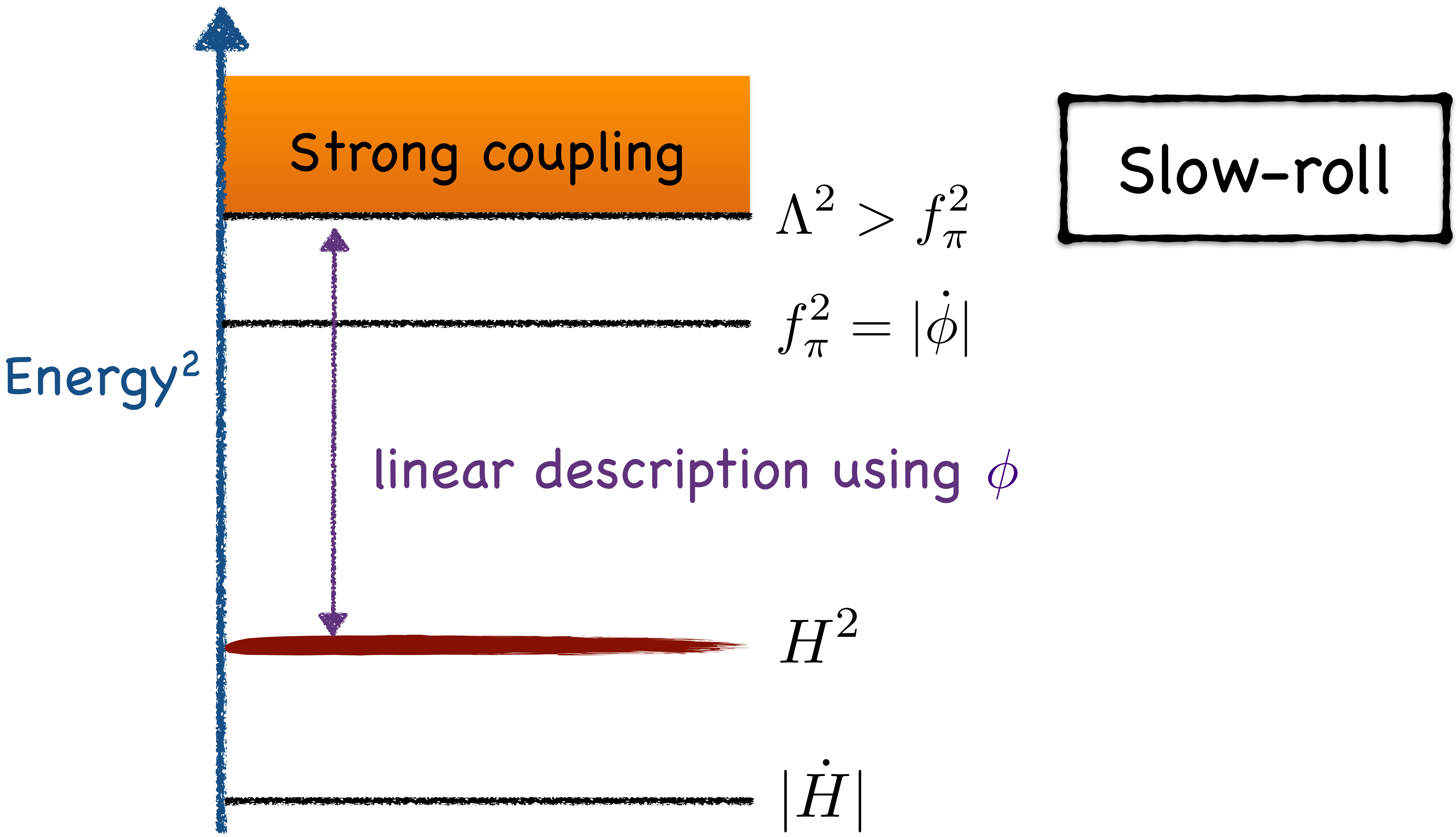
$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{\tilde{\Lambda}^4}(\partial_\mu\phi\partial^\mu\phi)^2$$

Control of background requires $\tilde{\Lambda}^2 > \dot{\phi}$

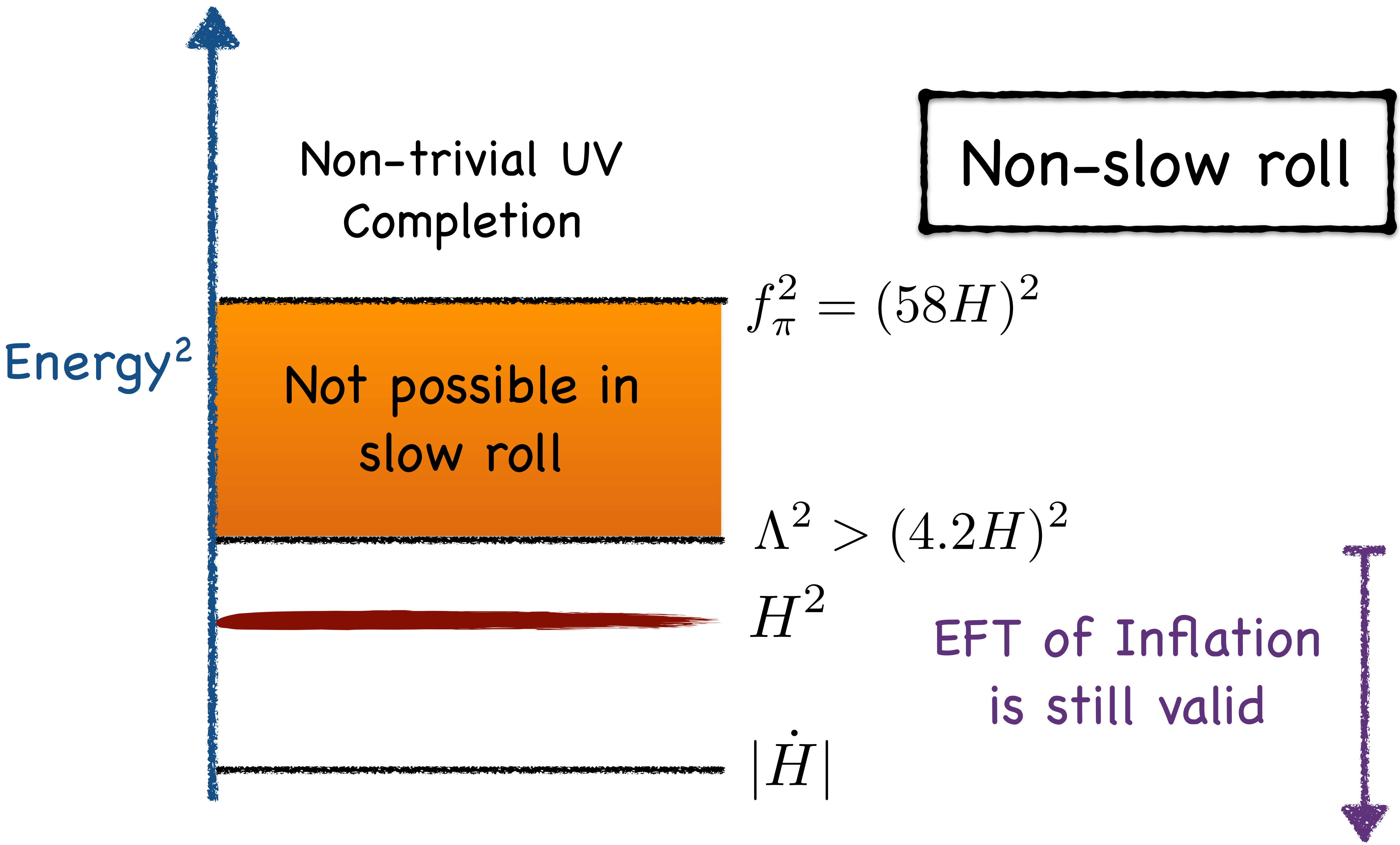
Expand in fluctuations to find bispectrum

$$\mathcal{L}_3 \sim \frac{\dot{\phi}}{\tilde{\Lambda}^4}\delta\dot{\phi}(\partial\delta\phi)^2 \qquad f_{\rm NL} \sim \frac{f_\pi^2\dot{\phi}}{\tilde{\Lambda}^4} = \frac{\dot{\phi}^2}{\tilde{\Lambda}^4} \ll 1$$

Non-Gaussanity



Non-Gaussanity



Non-Gaussanity

General take-aways:

NG is like precision EW tests of the Standard model

NG is an IR probe of higher dim. effective operators

Current constraints are $\Lambda > \mathcal{O}(5)H$

Current tests are not sensitive enough to suggest inflation is weakly coupled (i.e. slow-roll)

Summary: Lecture 1

Summary

Single-field inflation is well describe by SSB

A field gets a time dependent vev around dS

The goldstone boson is produced during inflation

Goldstone boson eaten by the metric $\zeta = -H\pi$

Current data does not point to a UV completion

Summary

These few assumptions explain:

- Gaussianity of fluctuations (irrelevant interactions)
- Small amplitude of fluctuations (hierarchy of scales)
- Near scale invariance (near de Sitter background)
- Small tensor amplitude (scale of inflation)

These features are generic (but can be violated with work)