The Unhiggs

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Abstract

We examine a scenario where the Higgs is part of an approximate conformal field theory, and has a scaling dimension greater than one. Such an unparticle Higgs (or Unhiggs) can still break electroweak symmetry and unitarize WW scattering, but its gauge couplings are suppressed. An Unhiggs model has a reduced sensitivity of the weak scale to the cutoff, and can thus provide a solution to the little hierarchy problem.

1 Introduction

Recently Georgi [1,2] has introduced a new way of studying conformal sectors that couple to the standard model using two-point functions of operators with scaling dimension between one and two. Formally the phase space corresponding to the spectral density of this two-point function resembles the phase space for a fractional number of particles, hence the name "unparticles." In the context of electroweak symmetry breaking these ideas have been applied in models where the Higgs couples to an approximately conformal sector and can mix with an unparticle [3–5]. Here however we are interested in models where the Higgs itself emerges from an approximately conformal sector, or in other words the Higgs itself is an unparticle (see ref. [6] for work on related ideas).

A model with an unparticle Higgs (aka Unhiggs) requires that we be able to gauge the kinetic term of an unparticle effective action, but this can be done in a fairly straightforward way [7]. We also know that new excitations that couple to weak interactions cannot be arbitrarily light, since we would have seen such states in low-energy experiments. So the conformal symmetry of the Higgs sector must be broken not too far below the weak scale. Fortunately there is a simple way to account for such an threshold as well [4,7,8]. In the limit where the scaling dimension of the unparticle approaches its canonical value, the threshold becomes an ordinary mass. This threshold, by itself, is not enough to account for a vacuum expectation value (VEV) for the Unhiggs. However, coupling an Unhiggs to the standard model (SM) fields will force additional conformal symmetry breaking effects at loop level and induce a potential for the Unhiggs. Experience with the SM and its extensions suggests that top quark loops would tend to produce the largest effects (due to the large top Yukawa coupling) and that top loops also tend to destabilize the symmetric vacuum with a vanishing VEV. For this paper we will simply assume that a suitable effective potential can be arranged and then explore the consequences of the resulting VEV for the electroweak gauge bosons.

Effective actions for unparticles are somewhat unfamiliar since they must be non-local in position space, but this is precisely in accord with the requirements of the anti-de Sitter/conformal field theory (AdS/CFT) correspondence [8]. It has even been checked that for fermions such non-local actions reproduce the required scaling dimension independence of anomaly factors [9]. These cross checks give us some confidence that unparticle actions provide a consistent effective field theory for sufficiently small scaling dimensions.

The ideas we are exploring in this paper are closely related to conformal technicolor models [10] where the operator that breaks electroweak symmetry has a dimension larger than one and the square of this operator (the analogue of the Higgs mass term) is assumed to be larger than four. Here we will only consider weakly coupled effective actions which restricts us to having an Unhiggs mass operator dimension that is roughly twice as big as the Unhiggs scaling dimension. The Unhiggs model is even more closely related to gaugephobic Higgs models [11] where a five dimensional AdS description is set up with a bulk Higgs that corresponds to a state with an arbitrary scaling dimension. Taking the limit where the Higgs scaling dimension goes to infinity just gives a Randall-Sundrum model. In the gaugephobic Higgs analysis only scaling dimensions larger than two were considered, since this ensures that the hierarchy problem is solved. Here we will be content with only addressing the

little hierarchy problem (that is, why the weak scale is small compared to 10 TeV), and so we can consider scaling dimensions less than two. This is the regime where the unparticle description is useful. We expect that a five dimensional description would yield equivalent results, but the unparticle analysis is much easier to perform. We expect that this will be even more of an advantage when one tries to calculate loops containing Unhiggs propagators.

The outline of the paper is as follows. We will review the inclusion of gauge interactions for an Unhiggs in the next section. Then we will examine WW scattering and see how it is unitarized in such a model. We will then consider the phenomenological implications for LEP bounds on the Higgs mass, where we will find that an Unhiggs can be much lighter than an SM Higgs. Next we will address the problem of the top Yukawa coupling possibly becoming non-perturbative below the cutoff scale. We will then comment on the little hierarchy problem and how an Unhiggs can reduce the sensitivity of the weak scale to the cutoff. We will also comment on the effects of additional loop induced kinetic terms and finally present our conclusions.

2   Gauge Interactions of the Unhiggs

Consider the momentum space effective action for an unparticle field H with scaling dimension d and an infrared cutoff[1] scale μ:

$$S_0 = - \int \frac{d^4 p}{(2\pi)^4} \, H^\dagger \left(-p^2 + \mu^2\right)^{2-d} H \ . \tag{2.1}$$

The field H thus has an unparticle propagator with a threshold at μ:

$$\Delta_H(p) = \frac{-i}{\left(-p^2 + \mu^2 - i\epsilon\right)^{2-d}} \ , \tag{2.2}$$

which approaches the usual particle propagator as $d \to 1$. If we now include a gauge coupling[2] of this field to the standard model electroweak gauge group and a Yukawa coupling to the top quark with a cutoff scale Λ we have

$$S = \int d^4 x \, - H^\dagger \left(D^2 + \mu^2\right)^{2-d} H - \lambda_t \bar{t}_R \frac{H^\dagger}{\Lambda^{d-1}} \left(\begin{array}{c} t \\ b \end{array} \right)_L + \text{ h.c. } , \tag{2.3}$$

where the Unhiggs field transforms under the electroweak gauge group $SU(2)_L \times U(1)_Y$ as a $\mathbf{2}_{1/2}$ and D is a gauge covariant derivative [7]. The Λ dependence in the Yukawa coupling means that H is scaled so that its engineering dimension matches its scaling dimension. Loop corrections involving these standard model couplings will break the conformal symmetry and give additional terms:

$$S_{loop} = \int d^4 x \, \frac{C}{\Lambda^{2d-2}} \, D_\mu H^\dagger D^\mu H - \lambda \left(\frac{H^\dagger H}{\Lambda^{2d-2}} - \frac{V^2}{2} \right)^2 \ . \tag{2.4}$$

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[1] For a discussion of the infrared cutoff, or threshold, see [8].

[2] For a discussion of the spin-1 resonances in the conformal sector with gauge boson quantum numbers, see [8].

The renormalized action, $S + S_{loop}$, includes two types of masses and two types of kinetic terms. The loop induced potential term allows for a nontrivial VEV, while the loop induced kinetic term does not lead to any qualitatively new behavior, so we save our comments on this term for section 7. For $d \to 1$ this model just reduces to the SM Higgs sector.

As in the SM the instability in the potential terms forces a non-zero vacuum expectation value (VEV) for the Unhiggs:

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{\sqrt{\lambda V^2 \Lambda^{2d-2} - \mu^{4-2d} \Lambda^{4d-4}}}{\sqrt{2\lambda}} \end{pmatrix} \equiv \begin{pmatrix} 0 \\ \frac{v^d}{\sqrt{2}} \end{pmatrix} . \tag{2.5}$$

Decomposing the Unhiggs into physical and Goldstone modes we can write

$$H = \frac{1}{\sqrt{2}} e^{iT^a \pi^a / v^d} \begin{pmatrix} 0 \\ v^d + h \end{pmatrix} \tag{2.6}$$

$$= \langle H \rangle + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h \end{pmatrix} + \Pi + \dots \tag{2.7}$$

where

$$\Pi = \begin{pmatrix} \pi^+ \\ \frac{i\pi^3}{\sqrt{2}} \end{pmatrix} . \tag{2.8}$$

To understand the gauge interactions it may be simpler to first write the pure derivative terms as a non-local theory in position space

$$S = \int d^4x \, d^4y \, H^\dagger(y) F(x - y) H(x) , \tag{2.9}$$

and then ensure gauge invariance by using Mandelstam's method [12] of introducing a path-ordered exponential of the gauge field, i.e. a Wilson line,

$$W(x, y) = P \exp \left[-ig\tau^a \int_x^y A_\mu^a dw^\mu \right] , \tag{2.10}$$

between the two unparticle fields evaluated at x and y as in [12]. Applying this method to the electroweak interactions of the Unhiggs allows us to calculate the Feynman vertex (Figure 1) for a gauge boson (with gauge generator T^a) coupled to two Unhiggs fields. The result [7] using Eq. (2.3) is

$$\begin{aligned} ig\Gamma^{a\alpha}(p, q) &\equiv \frac{i\delta^3 S}{\delta A^{a\alpha}(q) \delta\phi^\dagger(p+q) \delta\phi(p)} \\ &= -igT^a \frac{2p^\alpha + q^\alpha}{2p \cdot q + q^2} \left[\left(\mu^2 - (p+q)^2\right)^{2-d} - \left(\mu^2 - p^2\right)^{2-d} \right] . \end{aligned} \tag{2.11}$$

Note that this vertex satisfies the Ward-Takahashi identity [13] which relates it to the propagator $\Delta(p)$:

$$iq_\alpha \Gamma^{a\alpha} = \Delta^{-1}(p+q)T^a - T^a \Delta^{-1}(p) . \tag{2.12}$$
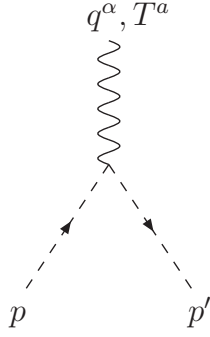
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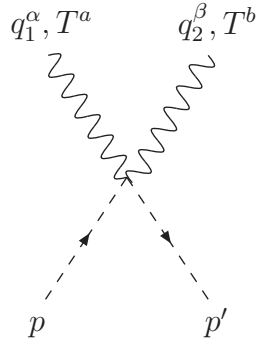Figure 1: The one gauge boson two Unhiggs Feynman diagram (Eq. 2.11)

Figure 2: The two gauge boson two Unhiggs Feynman diagram (Eq. 2.13)

The path-ordered exponential includes arbitrarily high powers of the gauge field, so there are vertices with arbitrary numbers of gauge bosons. The two gauge boson two Unhiggs vertex (Figure 2) is [7]

$$
\begin{aligned}
ig^2\Gamma^{ab\alpha\beta}(p, q_1, q_2) \;=\; ig^2\Bigg\{ & \left(T^aT^b + T^bT^a\right) g^{\alpha\beta}\mathcal{F}(p, q_1 + q_2) \\
& +T^aT^b\frac{(2p + q_2)^{\beta}(2p + 2q_2 + q_1)^{\alpha}}{q_1^2 + 2(p + q_2)\cdot q_1}\left[\mathcal{F}(p, q_1 + q_2) - \mathcal{F}(p, q_2)\right] \\
& +T^bT^a\frac{(2p + q_1)^{\alpha}(2p + 2q_1 + q_2)^{\beta}}{q_2^2 + 2(p + q_1)\cdot q_2}\left[\mathcal{F}(p, q_1 + q_2) - \mathcal{F}(p, q_1)\right]\Bigg\} \,,
\end{aligned}
\tag{2.13}
$$

where

$$
\mathcal{F}(p, q) = -\,\frac{\left(\mu^2 - (p + q)^2\right)^{2-d} - \left(\mu^2 - p^2\right)^{2-d}}{q^2 + 2p\cdot q}\,.
\tag{2.14}
$$

From Eq. (2.13) we infer that the momentum space action includes a quadratic term for

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gauge bosons

$$-g^2 A^a_\alpha A^b_\beta \langle H^\dagger \rangle T^a T^b \langle H \rangle \tag{2.15}$$

$$\left\{ g^{\alpha\beta}(d-2)\mu^{2-2d} - \frac{q^\alpha q^\beta}{q^2}\left[(d-2)\mu^{2-2d} - \frac{(\mu^2-q^2)^{2-d}-(\mu^2)^{2-d}}{q^2}\right]\right\} .$$

From Eq. (2.11) we can infer that the action also includes a gauge boson-Goldstone boson mixing term:

$$\mathcal{L}_{mix} = g\left(\langle H^\dagger\rangle A^a_\alpha T^a \Pi - \Pi^\dagger A^a_\alpha T^a \langle H\rangle\right)\left[(\mu^2-q^2)^{2-d} - (\mu^2)^{2-d}\right]q^\alpha/q^2 . \tag{2.16}$$

The last term in Eq. (2.15) is not gauge invariant, but note that the contribution to the vacuum polarization from mixing with the Goldstone bosons is given by

$$\Pi^{ab\alpha\beta}(q) = -g^2\langle H^\dagger\rangle T^a T^b \langle H\rangle \frac{q^\alpha q^\beta}{q^4}\left[(\mu^2-q^2)^{2-d} - (\mu^2)^{2-d}\right]^2 \Delta_{GB}(q). \tag{2.17}$$

With the Goldstone boson propagator

$$\Delta_{GB}(q) \;=\; -\frac{i}{(\mu^2-q^2-i\epsilon)^{2-d} - \mu^{4-2d}} \;, \tag{2.18}$$

the mixing contribution (2.17) cancels the gauge non-invariant term.

To discuss propagators and to perform non-trivial calculations it is convenient to remove the mixing terms by including gauge fixing terms:

$$\mathcal{L}_{GF} = \frac{-1}{2\xi q^2 \mu^{2-2d}}\left[q^\alpha q^\beta W^a_\alpha W^a_\beta + 2g\xi\mu^{2-2d}q^\alpha W^a_\alpha\left(\Pi^\dagger T^a \langle H\rangle - \langle H^\dagger\rangle T^a \Pi\right)\right. \tag{2.19}$$

$$- g^2\xi^2\mu^{4-4d}\left(\Pi^\dagger T^a \langle H\rangle - \langle H^\dagger\rangle T^a \Pi\right)^2$$

$$+ q^\alpha q^\beta B_\alpha B_\beta + g'\xi\mu^{2-2d}q^\alpha B_\alpha\left(\Pi^\dagger\langle H\rangle - \langle H^\dagger\rangle\Pi\right)$$

$$\left. - \frac{1}{4}g'^2\xi^2\mu^{4-4d}\left(\Pi^\dagger\langle H\rangle - \langle H^\dagger\rangle\Pi\right)^2\right]K(q^2) \;,$$

with

$$K(q^2) = \mu^{4-2d} - (\mu^2-q^2)^{2-d} \;. \tag{2.20}$$

Collecting all the coefficients of the quadratic terms for the $W^\pm$ gauge bosons we have

$$S_{W^+W^-} = \int \frac{d^4q}{(2\pi)^4}\, W^+_\alpha(q) W^-_\beta(q) I^{\alpha\beta} \tag{2.21}$$

where

$$I^{\alpha\beta} \;=\; \left(-q^2 + M_W^2\right)g^{\alpha\beta} \tag{2.22}$$

$$+ \left[q^2 - M_W^2 - \frac{(\mu^2)^{2-d} - (\mu^2-q^2)^{2-d}}{\mu^{2-2d}}\left(\frac{1}{\xi} - \frac{M_W^2}{(2-d)q^2}\right)\right]\frac{q^\alpha q^\beta}{q^2} \;.$$

and

$$M_W^2 \quad = \quad \frac{g^2(2-d)\mu^{2-2d}v^{2d}}{4} \quad .$$ (2.23)

The propagators for the gauge bosons are then very different from in the SM:

$$\Delta_W(q) \quad = \quad \frac{-i}{q^2 - M_W^2 + i\epsilon}$$ (2.24)

$$\left(g_{\alpha\beta} + \frac{\xi\left(q^2 - M_W^2\right)\mu^{2-2d} - \left(\mu^{4-2d} - (\mu^2 - q^2)^{2-d}\right)\left(1 - \frac{\xi M_W^2}{(2-d)q^2}\right)}{\left(\mu^{4-2d} - (\mu^2 - q^2)^{2-d}\right)\left(q^2 - \frac{\xi M_W^2}{2-d}\right)} q_\alpha q_\beta \right) \quad .$$

The propagators for the Goldstone bosons are then:

$$\Delta_{\pi^\pm}(q) \quad = \quad -\frac{i}{(\mu^2 - q^2 - i\epsilon)^{2-d} - \mu^{4-2d} - \xi\frac{M_W^2}{2-d}\left[(\mu^2 - q^2)^{2-d} - (\mu^2)^{2-d}\right]/q^2}$$ (2.25)

$$\Delta_{\pi^3}(q) \quad = \quad -\frac{i}{(\mu^2 - q^2 - i\epsilon)^{2-d} - \mu^{4-2d} - \xi\frac{M_Z^2}{2-d}\left[(\mu^2 - q^2)^{2-d} - (\mu^2)^{2-d}\right]/q^2} \quad , \quad (2.26)$$

while for the physical Unhiggs mode we have

$$\Delta_h(q) \quad = \quad -\frac{i}{m^{4-2d} - \mu^{4-2d} + (\mu^2 - q^2 - i\epsilon)^{2-d}}$$ (2.27)

where

$$m^{4-2d} \quad = \quad \frac{2\lambda v^{2d}}{\Lambda^{4d-4}} \quad .$$ (2.28)

From (2.27), the location of the Unhiggs resonance is given by

$$M_{Unh}^2 = \mu^2 - (\mu^{4-2d} - m^{4-2d})^{\frac{1}{2-d}} \quad .$$ (2.29)

Note that the physical Unhiggs has a width if $m > \mu$ and it may also have a tachyonic mass for $m > 2\mu$, depending on the value of d. To avoid these complications, we will assume $\mu > m$ in the rest of this paper.

3 WW Scattering and Unitarity

The effects of unparticles on unitarity constraints have been studied in the case of WW scattering [14] and in the case of Higgs-Higgs scattering [15]. In both of these cases, the Higgs boson was assumed to be an ordinary particle, and the unparticle belonged to a non-SM sector. The case of the Unhiggs is very different because an unparticle is replacing the SM Higgs. Thus it is important to determine whether the Unhiggs can perform the same role
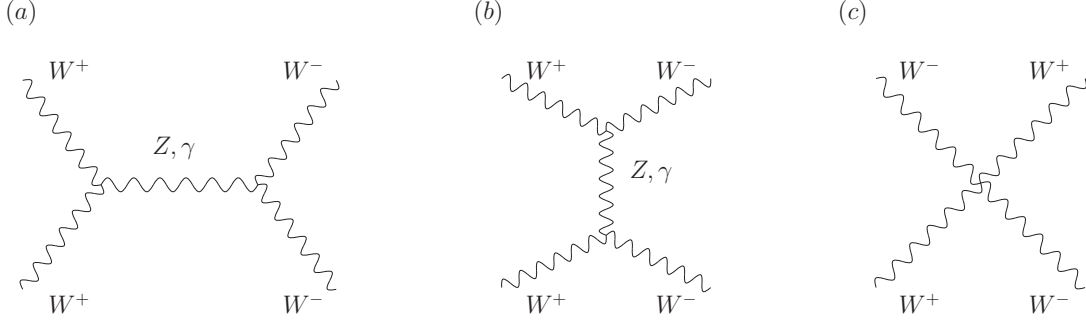
Figure 3: WW scattering diagrams containing no Unhiggs vertices or propagators

as the SM Higgs. One of the most important functions of the Higgs in the Standard Model is that it unitarizes longitudinal WW scattering as the incoming energy becomes large. As we will show, although the Unhiggs case is more complicated and necessitates the use of a non-SM vertex, the Unhiggs is also sufficient to unitarize WW scattering at high energies.

To calculate the WW scattering diagrams, it will be easiest to use the Landau gauge, $\xi = 0$, where

$$\Delta_W(q) = \frac{-i}{q^2 - M_W^2 + i\epsilon}\left(g_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2}\right) \ , \tag{3.1}$$

$$\Delta_{\pi^\pm}(q) = \Delta_{\pi^3}(q) = -\frac{i}{\left(\mu^2 - q^2 - i\epsilon\right)^{2-d} - \mu^{4-2d}} \ . \tag{3.2}$$

Since in this gauge the gauge boson propagators are the same as in the SM Landau gauge, it is easy to see that the WW scattering diagrams given in Figure 3, which contain no Unhiggs propagators or vertices, are the same in the Unhiggs model as in the SM. In the high energy limit, $s, t \gg M_W^2, M_Z^2$, the contributions of these diagrams to the WW scattering amplitude are [16]

$$\mathcal{M}_{Gauge,SM} = \frac{1}{4}\frac{ig^2}{M_W^2}(s + t) \ . \tag{3.3}$$

The WW scattering diagrams in Figure 4 are analogues of similar SM diagrams, but now contain the two gauge boson one Unhiggs vertex and an Unhiggs propagator. From (2.13)
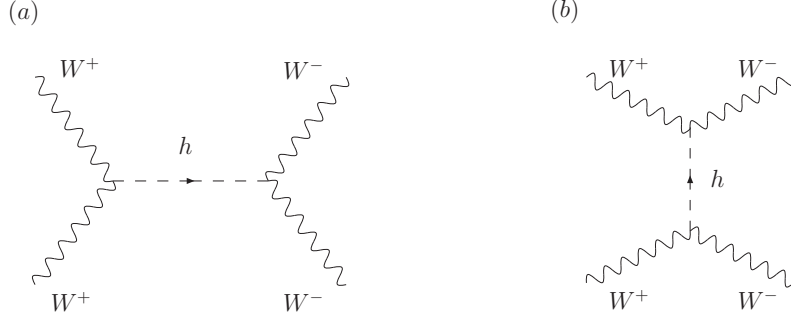
Figure 4: WW scattering diagrams containing two gauge boson one Unhiggs vertices.

we see that the two gauge boson one Unhiggs vertex is given by:

$$g^2\Gamma_1^{ab\alpha\beta}(q_1,q_2) \;=\; g^2\Bigg\{ \left(T^aT^b + T^bT^a\right)\langle H\rangle g^{\alpha\beta}\mathcal{F}(0,q_1+q_2) \tag{3.4}$$

$$+ T^aT^b\langle H\rangle\frac{q_2^\beta(2q_2+q_1)^\alpha}{q_1^2+2q_2\cdot q_1}\left[\mathcal{F}(0,q_1+q_2)-\mathcal{F}(0,q_2)\right]$$

$$+ T^bT^a\langle H\rangle\frac{q_1^\alpha(2q_1+q_2)^\beta}{q_2^2+2q_1\cdot q_2}\left[\mathcal{F}(0,q_1+q_2)-\mathcal{F}(0,q_1)\right]\Bigg\}$$

$$+ g^2\Bigg\{ \langle H^\dagger\rangle\left(T^aT^b+T^bT^a\right)g^{\alpha\beta}\mathcal{F}(-q_1-q_2,q_1+q_2)$$

$$- \langle H^\dagger\rangle T^aT^b\frac{q_1^\alpha(2q_1+q_2)^\beta}{q_1^2}\left[\mathcal{F}(-q_1-q_2,q_1+q_2)-\mathcal{F}(-q_1-q_2,q_2)\right]$$

$$- \langle H^\dagger\rangle T^bT^a\frac{q_2^\beta(2q_2+q_1)^\alpha}{q_2^2}\left[\mathcal{F}(-q_1-q_2,q_1+q_2)-\mathcal{F}(-q_1-q_2,q_1)\right]\Bigg\},$$

where q_1 and q_2 are the incoming momenta of the gauge bosons. Since the polarization vectors satisfy $\epsilon(q_i)\cdot q_i = 0$ and using $\mathcal{F}(0,q_1+q_2) = \mathcal{F}(-q_1-q_2,q_1+q_2)$ we have

$$g^2\epsilon_\alpha(q_1)\epsilon_\beta(q_2)\Gamma_1^{ab\alpha\beta}(q_1,q_2) \;=\; g^2\left(T^aT^b + T^bT^a\right)\langle H\rangle\,\epsilon(q_1)\cdot\epsilon(q_2)\mathcal{F}(0,q_1+q_2) \tag{3.5}$$

$$+ g^2\langle H^\dagger\rangle\left(T^aT^b + T^bT^a\right)\epsilon(q_1)\cdot\epsilon(q_2)\mathcal{F}(0,q_1+q_2) . \tag{3.6}$$

Therefore, in terms of W^+, W^- and the physical Unhiggs, we have

$$ig^2\epsilon_\alpha(q_1)\epsilon_\beta(q_2)\Gamma_1^{+-\alpha\beta}(q_1,q_2) \;=\; i\frac{g^2v^d}{2}\epsilon(q_1)\cdot\epsilon(q_2)\mathcal{F}(0,q_1+q_2) . \tag{3.7}$$

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So for the s-channel Unhiggs exchange contribution (Figure 4b) to WW scattering we find

$$\mathcal{M}_h(s) = -g^2 \frac{M_W^2}{(2-d)\mu^{2-2d}} \left(\epsilon(q_1) \cdot \epsilon(q_2) \frac{(\mu^2 - s)^{2-d} - (\mu^2)^{2-d}}{s} \right) \Delta_h(s) \qquad (3.8)$$

$$\cdot \left(\epsilon(q_3) \cdot \epsilon(q_4) \frac{(\mu^2 - s)^{2-d} - (\mu^2)^{2-d}}{s} \right)$$

$$= -g^2 \frac{M_W^2}{(2-d)\mu^{2-2d}} \left(\left(1 - \frac{s}{2M_W^2} \right) \frac{(\mu^2 - s)^{2-d} - (\mu^2)^{2-d}}{s} \right)^2 \Delta_h(s) . \qquad (3.9)$$

When $s \gg M_W^2, \mu^2, m^2$ we have

$$\mathcal{M}_h(s) = i\frac{g^2}{4M_W^2(2-d)\mu^{2-2d}} (-s)^{2-d} . \qquad (3.10)$$

Similarly, the t-channel Unhiggs exchange contribution (Figure 4a) to WW scattering will be, for $t \gg M_W^2, \mu^2, m^2$

$$\mathcal{M}_h(t) = i\frac{g^2}{4M_W^2(2-d)\mu^{2-2d}} (-t)^{2-d} . \qquad (3.11)$$

Combining the three amplitudes in (3.3), (3.10) and (3.11), we see that in the $d \to 1$ limit, the terms which grow with energy disappear, and WW scattering is unitarized. However, in the case of interest, $d \neq 1$, the dangerous high energy terms do not cancel. So, unlike in the case of the SM, the Unhiggs exchange diagrams in Figure 4 are insufficient to unitarize WW scattering.

As noted earlier, however, the Unhiggs action allows for vertices with arbitrary numbers of gauge bosons. Therefore, there is another Unhiggs contribution to WW scattering from the four gauge boson two Unhiggs vertex, given by

$$ig^4\Gamma^{abcd\mu\nu\alpha\beta}(p, q_1, q_2, q_3, q_4) \equiv \frac{i\delta^6 S}{\delta A^{a\mu}(q_1)\delta A^{b\nu}(q_2)\delta A^{c\alpha}(q_3)\delta A^{d\beta}(q_4)\delta\phi^\dagger(p')\delta\phi(p)} \qquad (3.12)$$

with

$$p' = p + q_1 + q_2 + q_3 + q_4 . \qquad (3.13)$$

The four gauge boson two Unhiggs vertex which contributes to WW scattering is shown in Figure 5. The crosses on the Unhiggs lines denote that we have taken $p = p' = 0$ and set the Unhiggs to its VEV. Using the methods outlined in Section 2, we derive an expression for the four gauge boson two Unhiggs vertex which we can then apply to WW scattering. The piece of the four gauge boson two Unhiggs vertex with no uncontracted momenta is given by

$$ig^4\Gamma^{abcd\mu\nu\alpha\beta}(p, q_1, q_2, q_3, q_4) \ni -ig^4\langle H^\dagger \rangle \{ \mathcal{R}^{cd}\mathcal{R}^{ab}g^{\mu\nu}g^{\alpha\beta}\mathcal{G}(p', q_1, q_2, p) \qquad (3.14)$$

$$+ \mathcal{R}^{bd}\mathcal{R}^{ac}g^{\mu\alpha}g^{\nu\beta}\mathcal{G}(p', q_1, q_3, p) + \mathcal{R}^{ac}\mathcal{R}^{bd}g^{\nu\beta}g^{\alpha\mu}\mathcal{G}(p', q_2, q_4, p)$$

$$+ \mathcal{R}^{ad}\mathcal{R}^{bc}g^{\nu\alpha}g^{\mu\beta}\mathcal{G}(p', q_2, q_3, p) + \mathcal{R}^{ab}\mathcal{R}^{cd}g^{\mu\nu}g^{\alpha\beta}\mathcal{G}(p', q_3, q_4, p)$$

$$+ \mathcal{R}^{bc}\mathcal{R}^{ad}g^{\mu\beta}g^{\nu\alpha}\mathcal{G}(p', q_1, q_4, p) \} \langle H \rangle$$
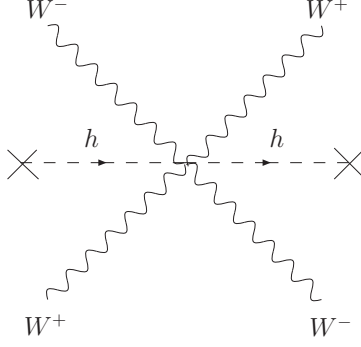
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Figure 5: The four gauge boson two Unhiggs contribution to WW scattering

where

$$\mathcal{R}^{ab} \equiv (T^a T^b + T^b T^a) \tag{3.15}$$

and

$$\mathcal{G}(p', q_i, q_j, p) \equiv \left\{ \frac{(\mu^2 - p'^2)^{2-d}}{(p'^2 - (p + q_i + q_j)^2)(p'^2 - p^2)} \right. \tag{3.16}$$
$$\left. + \frac{(\mu^2 - (p + q_i + q_j)^2)^{2-d}}{((p + q_i + q_j)^2 - p^2)((p + q_i + q_j)^2 - p'^2)} \right\}.$$

Taking the limit $p \to 0$, $p' \to 0$, we have

$$\mathcal{G}(0, q_i, q_j, 0) = \frac{(2-d)(\mu^{2-2d})}{(q_i + q_j)^2} + \frac{(\mu^2 - (q_i + q_j)^2)^{2-d}}{(q_i + q_j)^4} - \frac{\mu^{4-2d}}{(q_i + q_j)^4}. \tag{3.17}$$

It turns out that to evaluate the WW scattering amplitude arising from the four gauge boson two Unhiggs vertex we only need to consider the piece of the vertex with no uncontracted momenta, as given in (3.14). We can see this by looking at the Lorentz structure of the other terms in the vertex. The terms in the vertex with no $g^{\mu\nu}$ factors have a Lorentz structure given by

$$(2p + q_1)^\mu (2p + 2q_1 + q_2)^\nu (2p + 2q_2 + q_3)^\alpha (2p + 2q_2 + 2q_3 + q_4)^\beta + \text{permutations.} \tag{3.18}$$

Upon setting $p = p' = 0$, and contracting with $\epsilon(q_1)_\mu \epsilon(q_2)_\nu \epsilon(q_3)_\alpha \epsilon(q_4)_\beta$, we find that the contribution to the amplitude is zero from this term, again using the fact that $\epsilon(q_i) \cdot q_i = 0$. There are also terms in the vertex which contain one factor of $g^{\mu\nu}$. These terms have a Lorentz structure given by

$$(2p + q_1)^\mu (2p + 2q_1 + q_2)^\nu g^{\alpha\beta} + g^{\nu\alpha}(2p + q_1)^\mu (2p + 2q_1 + 2q_2 + 2q_3 + q_4)^\beta \tag{3.19}$$
$$+ g^{\mu\nu}(2p + 2q_1 + 2q_2 + q_3)^\alpha (2p + 2q_1 + 2q_2 + 2q_3 + q_4)^\beta + \text{permutations.}$$

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Upon taking $p \to 0$ and contracting with $\epsilon(q_1)_\mu \epsilon(q_2)_\nu \epsilon(q_3)_\alpha \epsilon(q_4)_\beta$, the first two terms in (3.20) clearly go to zero. The third term in (3.20) also goes to zero upon contracting with the polarization vectors because $2p + 2q_1 + 2q_2 + 2q_3 + q_4 = 2p' - q_4 \to -q_4$ when taking $p' \to 0$.

So the only contribution from the four gauge boson vertex to the WW scattering amplitude comes from (3.14) and is given by

$$\mathcal{M}_{2h}(s) = \frac{-ig^4 v^{2d}}{4} \left(\frac{(2-d)(\mu^{2-2d})}{s} + \frac{(\mu^2 - s)^{2-d}}{s^2} - \frac{\mu^{4-2d}}{s^2} \right) [\epsilon(q_1) \cdot \epsilon(q_2)]^2 \quad (3.20)$$

and

$$\mathcal{M}_{2h}(t) = \frac{-ig^4 v^{2d}}{4} \left(\frac{(2-d)(\mu^{2-2d})}{t} + \frac{(\mu^2 - t)^{2-d}}{t^2} - \frac{\mu^{4-2d}}{t^2} \right) [\epsilon(q_1) \cdot \epsilon(q_3)]^2 \ . \quad (3.21)$$

Taking $s \gg M_W^2, \mu^2, m^2$ and $t \gg M_W^2, \mu^2, m^2$, we have

$$\mathcal{M}_{2h}(s) = \frac{-ig^4 v^{2d}}{16 M_W^4} \left[(2-d)(\mu^{2-2d})s + (-s)^{2-d} \right] \quad (3.22)$$

$$= -i \frac{g^2}{4 M_W^2} \left[s + \frac{(-s)^{2-d}}{(2-d)(\mu^{2-2d})} \right]$$

and

$$\mathcal{M}_{2h}(t) = \frac{-ig^4 v^{2d}}{16 M_W^4} \left[(2-d)(\mu^{2-2d})t + (-t)^{2-d} \right] \quad (3.23)$$

$$= -i \frac{g^2}{4 M_W^2} \left[t + \frac{(-t)^{2-d}}{(2-d)(\mu^{2-2d})} \right] \ .$$

The total WW scattering amplitude is given by

$$\mathcal{M} = \mathcal{M}_{Gauge,SM} + \mathcal{M}_h + \mathcal{M}_{2h} \ . \quad (3.24)$$

Combining Eqs. (3.3), (3.10), (3.11), (3.22) and (3.23), we get

$$\mathcal{M} = \frac{1}{4} \frac{ig^2}{M_W^2}(s+t) + i \frac{g^2}{4 M_W^2 (2-d)\mu^{2-2d}}(-s)^{2-d} + i \frac{g^2}{4 M_W^2 (2-d)\mu^{2-2d}}(-t)^{2-d} \quad (3.25)$$

$$- i \frac{g^2}{4 M_W^2} \left[s + \frac{(-s)^{2-d}}{(2-d)\mu^{2-2d}} \right] - i \frac{g^2}{4 M_W^2} \left[t + \frac{(-t)^{2-d}}{(2-d)\mu^{2-2d}} \right] + \mathcal{O}(s^0) + \mathcal{O}(t^0) \ .$$

All of the terms that scale with positive powers of energy cancel in the full amplitude, and therefore the Unhiggs does unitarize WW scattering.

However, partial wave unitarity can still be violated for certain values of the parameters μ, m, and d. Therefore, we now examine the finite terms for $s, t, \mu^2, m^2 \gg M_W^2, M_Z^2$. Following [17], we note that there is a bound on the coefficient of the first partial wave such that
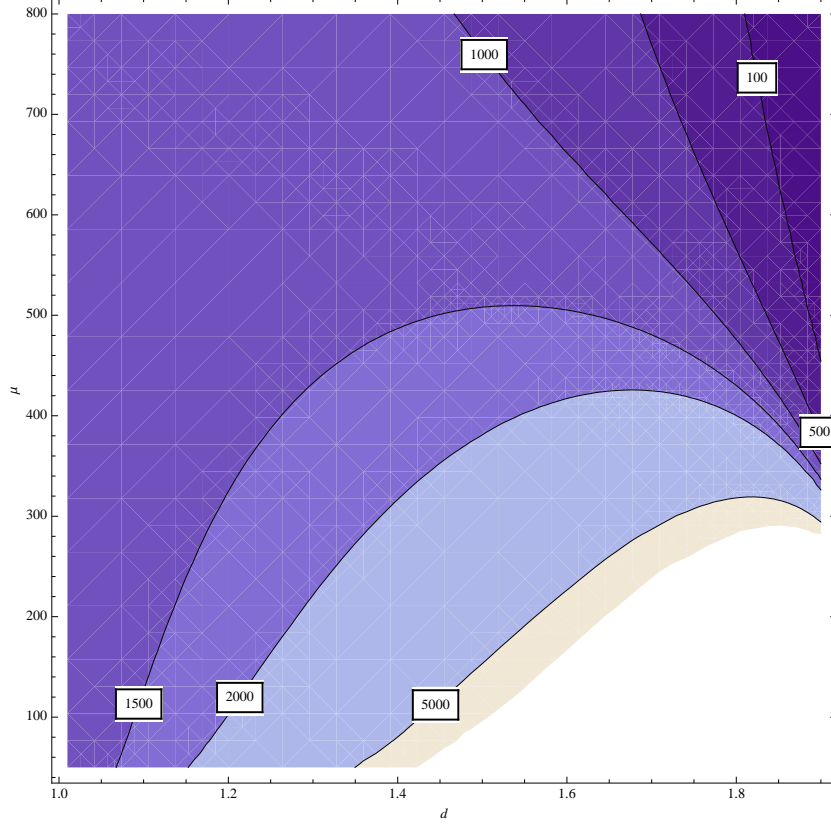
$$|a_0| \leq 1 \quad (3.26)$$

Figure 6: Contour plots of the bound on m in the d-μ plane. The darkest regions have the lowest upper bound on m, Contour lines are shown for 100, 500, 1000, 1500, 2000, and 5000 GeV.

where a_l is defined by the equation

$$\mathcal{M}(s,t) = 16\pi \sum_{l=0} (2l+1)a_l(s)P_l(\cos\theta) \tag{3.27}$$

and $P_l(\cos\theta)$ is a Legendre polynomial. Projecting out the zeroth partial wave and using $t = 2M_W^2 - \frac{s}{2}(1-\cos\theta) \approx -\frac{s}{2}(1-\cos\theta)$, we have

$$a_0(s) = \frac{1}{16\pi s}\int_{-s}^{0} \mathcal{M}(s,t)dt \ . \tag{3.28}$$

In the limit $s,t,\mu^2,m^2 \gg M_W^2, M_Z^2$, the full scattering amplitude is given by

$$
\begin{aligned}
\mathcal{M}(s,t) &= i\frac{g^2 m^{4-2d}}{4M_W^2(2-d)\mu^{2-2d}}\left\{\frac{\mu^{4-2d}-(\mu^2-s)^{2-d}}{m^{4-2d}-\mu^{4-2d}+(\mu^2-s)^{2-d}}\right.\\
&\left.+\frac{\mu^{4-2d}-(\mu^2-t)^{2-d}}{m^{4-2d}-\mu^{4-2d}+(\mu^2-t)^{2-d}}\right\} \ .
\end{aligned}
\tag{3.29}
$$

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Inserting $\mathcal{M}(s,t)$ into (3.28) and performing the integration, we find

$$a_0(s) \;=\; i\frac{G_F m^{4-2d}}{8\pi\sqrt{2}(2-d)\mu^{2-2d}}\left\{\frac{\mu^{4-2d}-(\mu^2-s)^{2-d}}{m^{4-2d}-\mu^{4-2d}+(\mu^2-s)^{2-d}}\right.$$
$$\left.-1+\frac{1}{s}\left(\frac{\mu^{4-2d}}{m^{4-2d}-\mu^{4-2d}}+1\right)\left[(\mu^2+s)Q_d(\mu^2+s)-\mu^2 Q_d(\mu^2)\right]\right\} \qquad (3.30)$$

where $G_F = 1.166\times10^{-5}$ GeV^{-2} is the Fermi constant and $Q_d(z)$ is defined in terms of the hypergeometric function as

$$Q_d(z)\equiv {}_2F_1\left(\frac{1}{2-d},1;1+\frac{1}{2-d};-\frac{z^{2-d}}{m^{4-2d}-\mu^{4-2d}}\right) . \qquad (3.31)$$

For large s this gives the bound

$$G_F\, m^{4-2d} < 4\pi\sqrt{2}(2-d)\mu^{2-2d} . \qquad (3.32)$$

The resulting bound in the d-μ plane is shown in Figure 6. The bound is only stringent for large $d\sim 2$ and large μ.

4 Lowering the LEP bounds on the Unhiggs

The LEP experiment put a lower bound on the Standard Model Higgs mass of 114.4 GeV [18]. In the case of a non-SM Higgs, such as the Unhiggs, there are two effects which can change the lower bound. One way for the bound to be invalid is to have a branching ratio of $H \to b\bar{b}$ that is different than in the SM, eg. ref. [19]. The other way is to suppress the cross section for Higgs production. At LEP, the Higgs is produced by the "Higgs-strahlung" process, $e^+e^- \to Z^* \to HZ$. If the cross section for this process is suppressed relative to the SM, then the lower bound will be reduced. In the case of the Unhiggs, we expect such a suppression as $d \to 2$, because in this limit the gauge covariant derivative in Eq. (2.3) disappears. The suppression is also clear from noting that the Unhiggs-gauge boson couplings in Eqs. (2.11), (2.13) and (3.4) go to zero in the $d \to 2$ limit. The Unhiggs becomes gaugephobic [11] when its scaling dimension is near 2 or larger [8]. We can quantify the suppression with the definition

$$\xi^2 \equiv \frac{\sigma_{Unh}(e^+e^- \to HZ)}{\sigma_{SM}(e^+e^- \to HZ)} . \qquad (4.1)$$

We find that, upon ignoring terms proportional to the electron mass, there is a simple relation between the amplitude squared in the Unhiggs model and the amplitude squared in the Standard Model.

$$|\mathcal{M}_{Unh}(e^+e^- \to HZ)|^2 = \frac{|\mathcal{P}|^2}{(2-d)\mu^{2-2d}}|\mathcal{M}_{SM}(e^+e^- \to HZ)|^2 \qquad (4.2)$$
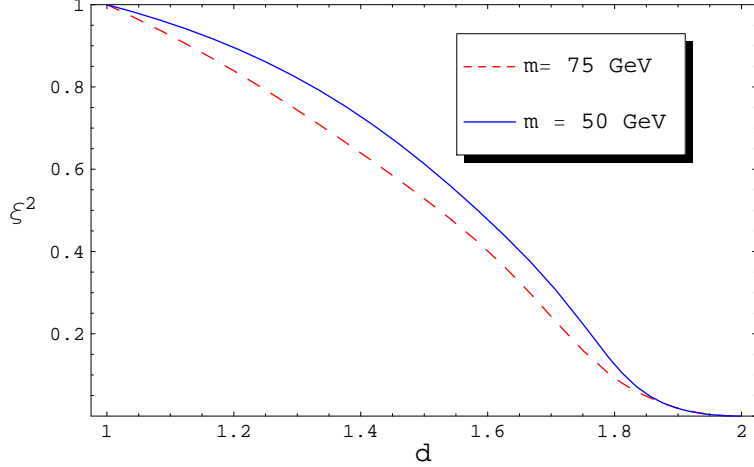
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Figure 7: ξ^2 as a function of d for $\mu = 100$ GeV, $m = 75$ GeV and $\mu = 100$ GeV, $m = 50$ GeV

where

$$\mathcal{P} = \frac{(\mu^2)^{2-d} - (\mu^2 - p_h^2)^{2-d}}{p_h^2} \ , \tag{4.3}$$

p_h is the momentum 4-vector of the outgoing Unhiggs and

$$|\mathcal{M}_{SM}(e^+e^- \to HZ)|^2 \propto \left[1 + \frac{E_Z^2}{M_Z^2} + (1 - \frac{E_Z^2}{M_Z^2})\cos^2\theta\right] \ . \tag{4.4}$$

The cross section for $2 \to 2$ scattering is given in general by

$$\sigma = \int \frac{|\mathcal{M}|^2}{2s} d\Phi \tag{4.5}$$

where $d\Phi$ is the phase space factor associated with the outgoing particles.

The phase space factor associated with the Unhiggs contains both a continuum above the IR cutoff μ, and a pole at $q^2 = M_{Unh}^2$, where M_{Unh} is given by Eq. (2.29). If $m < \mu$, then the pole is necessarily below the IR cutoff and the phase space takes a relatively simple form and is given by

$$\begin{aligned}
d\Phi_h(q^2) &= \frac{A_d\theta(q^0)\theta(q^2 - \mu^2)(q^2 - \mu^2)^{2-d}}{(\mu^{4-2d} - m^{4-2d})^2 + (q^2 - \mu^2)^{4-2d} - 2(\mu^{4-2d} - m^{4-2d})(q^2 - \mu^2)^{2-d}\cos(d\pi)} \\
&+ \theta(q^0)\frac{-\pi A_d}{\sin(d\pi)}\frac{(\mu^{4-2d} - m^{4-2d})^{\frac{d-1}{2-d}}}{(2 - d)}\delta(q^2 - M_{Unh}^2) \ , \tag{4.6}
\end{aligned}$$

where A_d is a normalization factor as in [1]. The first line in Eq. (4.6) contains the continuum while the second line contains the pole. Note that for $\mu > m$, the continuum part of the

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phase space goes to zero as $d \to 1$ because of the fact that $A_{d=1} = 0$. Also, the part of the phase space containing the pole has the following $d \to 1$ limit:

$$\lim_{d \to 1} d\Phi_{h,pole}(q^2) = 2\pi\theta(q^0)\delta(q^2 - m^2) . \tag{4.7}$$

Thus, for $\mu > m$, the Unhiggs phase space in Eq. (4.6) does indeed reduce to the Standard Model Higgs result.

Using Eqs. (4.2), (4.5) and (4.6), ξ^2 can be calculated numerically as a function of d for any values of the parameters μ and m. A plot of ξ^2 vs. d for two pairs of μ and m is shown in Figure (7). As expected, ξ^2 falls as d gets larger and is approximately zero for $d \to 2$. This shows that for moderate to high values of d, the suppression of the Unhiggs-Gauge couplings allows for an Unhiggs lighter than 114 GeV to have evaded detection at LEP.

5 Running of the top Yukawa coupling

Consider the top Yukawa coupling given in Eq. (2.3) which leads to an $h t\bar{t}$ interaction term

$$\mathcal{L}_Y = \frac{1}{\sqrt{2}} \frac{\lambda_t}{\Lambda^{d-1}} h t\bar{t} . \tag{5.1}$$

After expanding around the Unhiggs VEV, the top Yukawa coupling is given at tree level by

$$\frac{\lambda_t}{\Lambda^{d-1}} \frac{v^d}{\sqrt{2}} = m_t \tag{5.2}$$

which, after writing v^d in terms of M_W leads to

$$\lambda_t = \frac{\sqrt{2}}{2} \frac{m_t}{M_W} g\sqrt{2-d} \left(\frac{\Lambda}{\mu}\right)^{d-1} . \tag{5.3}$$

In the Standard Model, $\lambda_t = \frac{\sqrt{2}}{2} \frac{m_t}{M_W} g \approx 1$ at the electroweak scale. Therefore, with a cutoff $\Lambda = 10$ TeV, λ_t may be significantly greater than one at the electroweak scale, depending on the values of μ and d. In addition, we know that λ_t grows in the UV due to quantum corrections associated with the top Yukawa coupling. Thus we need to calculate the running of λ_t to make sure that it does not become non-perturbative before the cutoff at around 10 TeV. We will start by defining the value of λ_t at the electroweak scale $s_0 \approx 100$ GeV where M_W and m_t are measured, so that

$$\lambda_t(s_0) = \frac{\sqrt{2}}{2} \frac{m_t}{M_W} g\sqrt{2-d} \left(\frac{\Lambda}{\mu}\right)^{d-1} . \tag{5.4}$$

To explain how we compute the running of λ_t, we start by noting that in the SM, the top Yukawa beta function gets a positive contribution from terms proportional to λ_t^3 and a negative contribution from terms proportional to the gauge couplings. Since the $U(1)_Y$
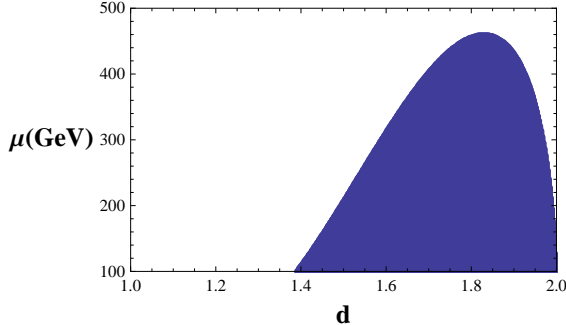
Figure 8: The white region is the region in μ-d parameter space where λ_t remains perturbative up to 10 TeV, while the dark region is the region where λ_t is non-perturbative at or below 10 TeV.

and $SU(2)_L$ gauge couplings are small compared to the top Yukawa coupling and the QCD coupling, we will ignore diagrams proportional to g and g'. The gluon contribution will be the same in the Unhiggs model as in the SM, so it will not be necessary to perform that calculation in this paper. However, we must calculate the contribution from diagrams proportional to λ_t^3, as these will in general be different than in the SM. Considering only diagrams proportional to λ_t^3, the correction to the top quark propagator as well as the correction to the proper Yukawa vertex both contain Unhiggs propagators in the loop, whereas the correction to the Unhiggs propagator is due to a top loop and thus does not contain an unparticle propagator in the loop. This is important, due to the fact that since the unparticle propagator has a reduced power of p^2 compared to a regular particle propagator, any loop which does not contain an unparticle propagator will generate a subleading Unhiggs kinetic term[3] compared to a loop with an unparticle propagator. For this reason, the correction to the Unhiggs propagator is subleading and we have thus only included the correction to the top quark propagator and the correction to the proper Yukawa vertex.

Using the procedure described above and ignoring the gluon contribution for now, we find that the top Yukawa coupling at an energy scale s is given by

$$\lambda_t(s) = \lambda_t(s_0) \left[1 - \frac{3\left(\lambda_t(s_0)\right)^2}{64\pi^2} \frac{1}{d-1} \left(1 - \left(\frac{s_0^2}{s^2} \right)^{d-1} \right) \right]^{-1} . \tag{5.5}$$

The contour plot shown in Figure 8 represents the value of the top Yukawa coupling at the cutoff scale $\Lambda = 10$ TeV as a function of μ and d. We require $\lambda_t(\Lambda) \leq 2\pi$ so that the coupling remains perturbative up to the cutoff. The white region of the plot is the region for which $\lambda_t(\Lambda)$ does indeed remain less than 2π. It is important to note that since we did not include the gluon loops, which, as described above, causes λ_t to decrease in the UV, the plot is a very conservative estimate of the allowed values of μ and d.

From Figure 8, we see that these conservative estimates require that to have moderate to high values of d, we must choose μ to be somewhat above the electroweak scale. For

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[3]These kinetic terms are further discussed in Section 7.

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instance, with a value of $d = 1.7$, we need to choose $\mu \gtrsim 400$ GeV.

6 Yukawa Couplings and the UV cutoff

The top Yukawa coupling is also important in estimating the maximum value of the cutoff allowed in the theory by using fine tuning arguments. From the usual top loop correction to the quadratic Unhiggs term in the action we find

$$\delta m_h^{4-2d} = \frac{3|\lambda_t|^2}{8\pi^2}\Lambda^{4-2d} \ . \tag{6.1}$$

Qualitatively, we want the correction to the Unhiggs mass term to be at most of the order of the tree level term to avoid excessive fine-tuning. This means we want

$$\delta m_h^{4-2d} \approx \frac{3|\lambda_t|^2}{8\pi^2}\Lambda^{4-2d} < \mu^{4-2d} \ . \tag{6.2}$$

Thus, Eq. (6.2) leads to a larger value of the maximum cutoff, Λ_{max}, for larger values of d, as in Figure 9.

In this plot, we have chosen a fixed value of μ and λ_t consistent with the bounds required by Figure 8 at $d = 1.7$. We have also normalized so that the maximum cutoff in the Standard Model ($d = 1$) is 1 TeV. The plot clearly shows that we can push the UV scale past the usual SM value of ~ 1 TeV for values of d greater than 1. For example, the cutoff can be near 10 TeV without much fine-tuning for $d \sim 1.7$.
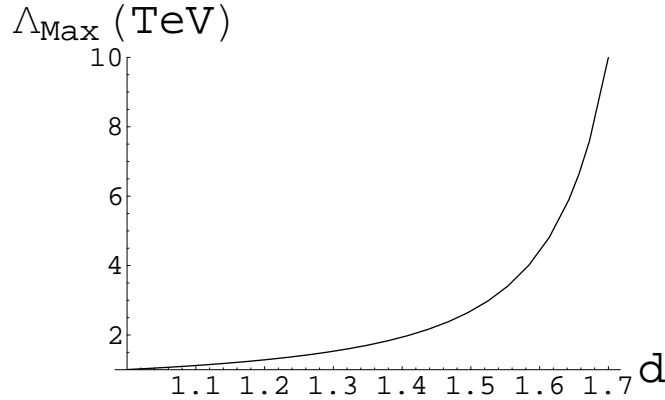

Figure 9: Qualitative behavior of Λ_{max} as a function of d for fixed μ and λ_t

7 Loop Induced Kinetic Term

As we mentioned in Section 2, loop effects will also induce terms in the Lagrangian of the form

$$\mathcal{L}_{kin} = -\frac{C}{\Lambda^{2d-2}}H^\dagger D^2 H \tag{7.1}$$

where C is a dimensionless coefficient. Qualitatively, our analysis above is not affected by this term. However, we can estimate its quantitative effect by comparing it with the kinetic term in the original Lagrangian (2.3). The ratio, R, of the momentum scales between the two terms is:

$$R = \frac{\frac{C}{\Lambda^{2d-2}}p^2}{p^{2(2-d)}} = C \left(\frac{p^2}{\Lambda^2} \right)^{d-1} . \tag{7.2}$$

Since we are considering values for an Unhiggs threshold around ≈ 100 GeV, we take $p \approx 100$ GeV. Inserting our previous value of $\Lambda = 10$ TeV, we find

$$R = C(.0001)^{d-1} . \tag{7.3}$$

We expect $C < 1$ since it is a loop suppressed coefficient. For values of d near one, $R \approx C$ and the loop induced term will have a relatively small quantitative effect. However, for moderate values of d, R becomes extremely small and the term in Eq. (7.1) will have no appreciable effect on the results of the previous sections. This loop induced term will affect the region near $d = 2$ where a pure unparticle is highly gaugephobic [8] since it provides an additional contribution to gauge couplings.

However, since the model should be valid up to at least a few TeV, it is important to show that the loop induced kinetic term does not qualitatively change the longitudinal WW scattering analysis of Section 3, even for large p^2. After expanding Eq. (7.1) around the VEV, the loop induced kinetic term contributes to the Lagrangian three terms relevant for WW scattering.

$$\mathcal{L} \ni \frac{C}{\Lambda^{2d-2}} \left(\frac{g^2}{4} W^+ W^- v^{2d} + \frac{g^2}{2} W^+ W^- h v^d + \frac{1}{2} \partial_\mu h \partial^\mu h \right) . \tag{7.4}$$

This causes a modification to the W mass, which is now given by

$$M_W^2 = \frac{g^2(2-d)\mu^{2-2d}v^{2d}}{4} + \frac{C}{\Lambda^{2d-2}} \frac{g^2 v^{2d}}{4} . \tag{7.5}$$

The Unhiggs propagator and the hW^+W^- vertex will also be modified by Eq. (7.4). The pure gauge contribution (Fig. 3) to the WW scattering amplitude will retain the same form as in Eq. (3.3), but with the modified W mass. Also, the four gauge boson two Unhiggs contribution (Fig. 5) is still given by Eq. (3.22), again with the modified W mass. The contribution from the two gauge boson one Unhiggs vertex (Fig. 4) contains both the modified hW^+W^- vertex as well as the modified Unhiggs propagator. The s-channel contribution to the longitudinal WW scattering amplitude is now given by

$$\mathcal{M}_{1h} = \frac{-ig^4 v^{2d}}{4} \left(1 - \frac{s}{2M_W^2} \right) \frac{\left(\frac{C}{\Lambda^{2d-2}} + \frac{\mu^{4-2d} - (\mu^2 - s)^{2-d}}{s} \right)^2}{\mu^{4-2d} - m^{4-2d} - (\mu^2 - s)^{2-d} + \frac{C}{\Lambda^{2d-2}} s} . \tag{7.6}$$

Upon taking $s \gg M_W, \mu, m$, we get

$$\mathcal{M}_{1h} = \frac{-ig^4 v^{2d}}{16 M_W^4} \left[\frac{C}{\Lambda^{2d-2}} s - (-s)^{2-d} \right] . \tag{7.7}$$

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Combining the contributions from Eqs. (3.3), (3.22) and (7.7) yields the following for the s-channel contribution to the WW scattering amplitdue:

$$\mathcal{M}_{s-channel} = \frac{-ig^4 v^{2d}}{16 M_W^4} \frac{C}{\Lambda^{2d-2}} s + \frac{ig^2}{4 M_W^2} \left[1 - \frac{g^2 v^{2d}}{4 M_W^2} (2-d) \mu^{2-2d} \right] s \qquad (7.8)$$

$$= \frac{ig^2}{4 M_W^2} \left[1 - \frac{g^2 v^{2d}}{4 M_W^2} \left(\frac{C}{\Lambda^{2d-2}} + (2-d) \mu^{2-2d} \right) \right] s \; .$$

Using the modified formula for the W mass in Eq. (7.5), we find

$$\mathcal{M}_{s-channel} = \frac{ig^2}{4 M_W^2} \left(1 - \frac{g^2 v^{2d}}{4 M_W^2} \frac{4 M_W^2}{g^2 v^{2d}} \right) s \qquad (7.9)$$

$$= \frac{ig^2}{4 M_W^2} (1-1) s = 0 \; .$$

The analysis of the t-channel contribution is exactly analogous. Thus, we have shown that for $s, t \to \infty$, the loop induced kinetic term does not affect the unitarity of longitudinal WW scattering. Of course, the loop induced kinetic term will affect the quantitative analysis of the allowed values of the parameters in Figure 6, but since we expect C to be loop suppressed, the loop induced kinetic term should not drastically affect the results.

8 Conclusions

We have explored the possibility that the Higgs arises from an approximately conformal sector and is described by an unparticle. We have found that such an Unhiggs can still break electroweak symmetry and unitarize WW scattering, but that the lower bounds on the mass threshold from LEP are much weaker than for a SM Higgs. Raising the scaling dimension of the Unhiggs mass term serves to weaken the little hierarchy problem since the power dependence on the cutoff is reduced. This is as one would expect, since breaking electroweak symmetry by an operator with dimension greater than two and thus a mass term operator dimension greater than four (at least at weak coupling) provides a solution to the full hierarchy problem. This is essentially what happens in Randall-Sundrum models (and other technicolor-like models): the scaling dimension of the (analogue) of the Higgs mass operator is very large and thus safe from any divergences. For an Unhiggs the mass term operator dimension is between two and four, an thus the mass term divergence scales as the cutoff to a power between two and zero. It would also be very interesting to explore how Unhiggs loops affect precision electroweak measurements.

Acknowledgments

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