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The Effective Lagrangian in the Randall-Sundrum Model and Electroweak Physics

Csaba Csáki[a], **Joshua Erlich**[b], **and John Terning**[b]

[a] *Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853, USA*
[b] *Theory Division T-8, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*
Email: csaki@mail.lns.cornell.edu, erlich@lanl.gov, terning@lanl.gov

Abstract: We consider the two-brane Randall-Sundrum (RS) model with bulk gauge fields. We carefully match the bulk theory to a 4D low-energy effective Lagrangian. In addition to the four-fermion operators induced by KK exchange we find that large negative S and T parameters are induced in the effective theory. This is a tree-level effect and is a consequence of the shapes of the W and Z wave functions in the bulk. Such effects are generic in extra dimensional theories where the standard model (SM) gauge bosons have non-uniform wave functions along the extra dimension. The corrections to precision electroweak observables in the RS model are mostly dominated by S. We fit the parameters of the RS model to the experimental data and find somewhat stronger bounds than previously obtained; however, the standard model bound on the Higgs mass from precision measurements can only be slightly relaxed in this theory.

1. Introduction

Theories with extra dimensions might explain some of the outstanding problems of particle physics [1–3]. In particular some of these models could shed light on why gravity is so much weaker than the other three forces. One of the prominent proposals of this sort is the Randall-Sundrum (RS) model [2,3], where the strong warping of the extra dimensions introduces an exponential hierarchy between the Planck and the weak scales. There are several variants of this model, depending on whether the extra dimension is finite (RS1) or infinite (RS2), and whether or not the gauge fields are in the bulk. Each of these models can be interesting for slightly different motivations. Here we will concentrate on the case where the extra dimension is finite (so that it solves the hierarchy problem), and where the gauge fields are in the bulk. This model could possibly yield unification of gauge couplings [4], and also may have a simple physical origin [5,6] via the AdS/CFT correspondence [7]. The holographic dual of this theory should be a broken conformal field theory, which becomes strongly interacting at low energies and spontaneously breaks the weakly gauged $SU(2) \times U(1)$ electroweak symmetries. Thus this holographic dual of the RS model with the gauge bosons in the bulk is in essence a technicolor-like theory, where the broken CFT replaces the technicolor group [5], and the KK modes of the gauge fields and gravitons would be interpreted as bound states of the CFT resulting in the technimesons, analogously to the glueball states appearing in the case of ordinary AdS/CFT [8]. In QCD-like technicolor theories the new strong interactions introduced to solve the hierarchy problem always generate large contributions to the electroweak precision observables [9], in particular there are large contributions to the S-parameter. However the phenomenological importance of (non-QCD-like) approximate conformal symmetry has long been emphasized in the technicolor literature [10], where the slowly running gauge coupling is refered to as "walking". The difficulty of estimating the value of S in these walking theories is also well known [11] since it involves non-perturbative, non-supersymmetric gauge dynamics near a non-trivial fixed point. Therefore it is interesting to find out whether there is a non-vanishing S-parameter in the RS model since it provides us with the first approximately conformal ("walking") model of electroweak symmetry breaking where such a calculation can be performed. However, in the 5D gravity theory (that is the RS model) the S parameter should not be the effect of quantum loops, but rather a purely tree-level effect. The purpose of this paper is to carefully match the RS model to an effective 4D description and find the value of S in the effective Lagrangian describing electroweak physics in this model. Indeed we find that the wave functions of the W and Z bosons are distorted due to the Higgs expectation values on the TeV brane, resulting in different wave function and mass renormalizations of the W and Z. The physical consequence of this effect is non-vanishing S and T parameters, which we calculate. Our method of finding the low-energy effective 4D theory is general, and we expect that similar effects will appear in any extra dimensional theory where the SM gauge bosons have non-uniform wavefunctions. In addition to these parameters the well-known effect of the four-fermion operators generated by the exchange of Kaluza-Klein gauge bosons has to be included. The coefficient of these four-fermion operators has been called V in [12, 13]. We use a global fit to the most recent precision electroweak data to place a bound on the size of the extra dimension in the RS model, and find bounds that are somewhat stronger than those previously obtained.

The paper is organized as follows: in Section 2 we review the results on gauge propagators and wave functions in the RS model that will be necessary to calculate the effective Lagrangian. In Section 3, we match the higher dimensional theory to an effective 4D Lagrangian, and evaluate the S and T parameters. In Section 4 we first calculate the V parameter and then use these results for constraining the parameters of the RS model via a global fit to the electroweak precision measurements. We conclude in Section 5, while Appendix A contains the detailed expressions of the electroweak observables in terms of S, T and V and the SM input and experimental values used for our fit.

2. The Gauge Propagator and Wavefunctions in the RS Model

In this section we review the results on gauge propagators and wave functions in the RS model [4,12-17] that will be necessary for us to calculate the effective low-energy theory.

The 5D metric of the RS model can be written in the form

$$ds^2 = \left(\frac{R}{z}\right)^2 \left(-dz^2 + \eta_{\mu\nu} dx^\mu \, dx^\nu\right), \tag{2.1}$$

for $R < z < R'$. Here R represents the radius of curvature of the AdS space. There is a Planck brane at $z = R$ and a TeV brane at $z = R'$ which cutoff the space with $\mathbb{Z}_2$ orbifold boundary conditions.

The 5D action for the bulk $SU(2) \times U(1)$ gauge bosons is given by

$$\begin{aligned} S_{5D} = \int d^4x \int_R^{R'} dz \sqrt{-G} \Big[& -\frac{1}{4g_5^2} G^{MP} G^{NQ} W^a_{MN} W^a_{PQ} - \frac{1}{4{g'_5}^2} G^{MP} G^{NQ} B_{MN} B_{PQ} \\ & + \frac{v^2}{8} \frac{\delta(z-R')}{\sqrt{G_{55}}} G^{MP} \left(W^1_M W^1_P + W^2_M W^2_P + (W^3_M - B_M)(W^3_P - B_P)\right) \Big], \end{aligned} \tag{2.2}$$

where the δ-function mass terms arise from a localized Higgs expectation value $\langle H \rangle = v/2$. Since δ-functions on boundaries require special care, we will take the definition of this term to be the limit of having the Higgs localized at a point which approaches the brane. This amounts to a factor of two difference in the definition of v^2 from taking the Higgs directly on the brane, but has the advantage of making comparison with the calculation via the AdS/CFT correspondence simpler [18].

We define the weak mixing angle through s, which represents the bare value of the $\sin\theta_W$:

$$s = \frac{g_5'}{\sqrt{g_5^2 + g_5'^2}}, \qquad c = \frac{g_5}{\sqrt{g_5^2 + g_5'^2}} \ . \tag{2.3}$$

We can diagonalize the action by performing a field redefinition:

$$W^3_\mu = c^2 Z_\mu + A_\mu \, , \qquad B_\mu = -s^2 Z_\mu + A_\mu \ . \tag{2.4}$$

The reason behind this unusual form for the field redefintions is that none of the fields W^3, B, Z or A are canonically normalized, but it is equivalent to the standard redefinition in the canonical basis. In our new basis we obtain the Lagrangian:

$$\begin{aligned} S_{5D} = \int d^4x \int_R^{R'} dz \frac{R}{z} \Big[&-\frac{1}{2g_5^2} W^+_{MN} W^{-MN} - \frac{1}{4}\left(\frac{1}{g_5^2} + \frac{1}{g_5'^2}\right) F_{MN}F^{MN} \\ &- \frac{1}{4(g_5^2 + g_5'^2)} Z_{MN}Z^{MN} + \frac{v^2}{4}\delta(z-R')\frac{R}{z} W^+_M W^{-M} + \frac{v^2}{8}\delta(z-R')\frac{R}{z} Z_M Z^M \Big] \end{aligned} \tag{2.5}$$

In the R_ξ gauge where $W_5 = Z_5 = A_5 = 0$, the propagator for the bulk W gauge boson is given by [4],

$$\Delta_W^{\mu\nu} = (\eta^{\mu\nu} - q^\mu q^\nu/q^2)\, \Delta_W(q, z, z') + \Delta_{W,\xi}\, q^\mu q^\nu/q^2, \tag{2.6}$$

where

$$\Delta_{W,\xi} = \Delta_W(q/\sqrt{\xi}, z, z'), \tag{2.7}$$

and Δ_W satisfies

$$g_5^2 \frac{z}{R}\delta(z-z') = (\partial_z^2 - \frac{1}{z}\partial_z + q^2 - \frac{1}{4}v^2 g_5^2 \delta(z-R')\frac{R}{R'})\Delta_W(q,z,z') \tag{2.8}$$

with boundary conditions

$$\partial_z \Delta_W|_{z=R} = 0 \ , \qquad \partial_z \Delta_W|_{z=R'} = -\frac{1}{4} g_5^2 v^2 \frac{R}{R'} \Delta_W \ . \tag{2.9}$$

W^5 also propagates in a generic R_ξ gauge, but the coupling to fermions is pseudoscalar, and vanishes at zero fermion mass. In addition, because W^5 is fixed to be odd under the $\mathbb{Z}_2$ (consider the $\mathbb{Z}_2$ behavior of $W_{5\mu}$), it vanishes on the TeV brane and hence does not couple to matter on the TeV brane. For our purposes we will therefore be able to neglect W^5.

For $R \leq (z, z') \leq R'$ the Green's function can be constructed by patching together the solutions of the corresponding homogeneous equation with $z < z'$ and $z > z'$, which we refer to as $\Delta_{W<}$ and $\Delta_{W>}$ respectively:

$$\Delta_W = \theta(z-z')\Delta_{W>} + \theta(z'-z)\Delta_{W<} \tag{2.10}$$

Plugging the patched solution into Eq. (2.8) for $z' \neq R$, $z' \neq R'$ yields:

$$\begin{aligned} \Delta_{W<}|_{z=z'} &= \Delta_{W>}|_{z=z'} \\ \partial_z(\Delta_{W>} - \Delta_{W<})|_{z=z'} &= g_5^2 \frac{z'}{R} \end{aligned} \tag{2.11}$$

Setting $z' = R'$ in Eq. (2.11) and combining with Eq. (2.9) yields the IR ($z = R'$) boundary condition for the propagator with a source on the TeV brane:

$$\partial_z \Delta_{W<}|_{z=z'=R'} = -\frac{R'}{R} g_5^2 - \frac{1}{4} g_5^2 v^2 \frac{R}{R'} \Delta_{W<}|_{z=z'=R'} \tag{2.12}$$

The solution has the form

$$\Delta_W(q, R', z) = \Delta_{W<}|_{z'=R'} = \frac{z}{R} \left(\alpha_W J_1(qz) + \beta_W Y_1(qz)\right) \tag{2.13}$$

where the coefficients are given by

$$\alpha_W = \frac{4 g_5^2 R' Y_0(qR)}{D(q)}, \qquad \beta_W = \frac{-4 g_5^2 R' J_0(qR)}{D(q)} \tag{2.14}$$

and the denominator is

$$D(q) = J_0(qR)\left(4qR' Y_0(qR') + g_5^2 v^2 R Y_1(qR')\right) - Y_0(qR)\left(4qR' J_0(qR') + g_5^2 v^2 R J_1(qR')\right). \tag{2.15}$$

Setting $z = R'$ gives the propagator on the TeV brane. The coefficients α_W and β_W have the same denominators, and the roots of these denominators determine the 4D poles of the propagator (when the numerators do not vanish concurrently). The n^{th} pole corresponds to the n^{th} W eigenmode , $W^{(n)}$, with mass $M_W^{(n)}$. We will label the lowest mode by $n = 0$. It is this lowest mode that we would like to identify with the observed W gauge boson, so we will write $M_W^{(0)} = M_W$. For $vR \ll 1$ this is an "almost zero mode" and we can find the pole analytically by expanding the Bessel functions in $qR < qR' \ll 1$. To leading order in the coupling the pole is at:

$$M_W^2 \approx \frac{g_5^2}{R \log(R'/R)} \frac{R^2 v^2}{4R'^2} \tag{2.16}$$

The bulk Z propagator is obtained from Δ_W by taking $g_5^2 \to g_5^2 + g_5'^2$, which gives:

$$M_Z^2 \approx \frac{g_5^2 + g_5'^2}{R \log(R'/R)} \frac{R^2 v^2}{4R'^2} \tag{2.17}$$

Similarly we can find the wavefunction, $\psi_W^{(n)}$, of the n^{th} eigenmode. The wavefunctions are given by:

$$\psi_W^{(n)}(z) = \frac{z}{R'} \frac{J_1(M_W^{(n)} z) Y_0(M_W^{(n)} R) - Y_1(M_W^{(n)} z) J_0(M_W^{(n)} R)}{J_1(M_W^{(n)} R') Y_0(M_W^{(n)} R) - Y_1(M_W^{(n)} R') J_0(M_W^{(n)} R)} \tag{2.18}$$

where we have normalized the wavefunction by $\psi^{(n)}(R') = 1$.

For $vR \ll 1$ the $n = 0$ mode is an "almost zero mode" and we can find a simple expression for the wavefunction by expanding the Bessel functions in $M_W R \ll 1$, $M_W R' \ll 1$. To order M_W^2 we obtain:

$$\psi_W^{(0)}(z) \approx 1 + \frac{M_W^2}{4}\left[z^2 - R'^2 - 2z^2\log(z/R) + 2R'^2\log(R'/R)\right]. \tag{2.19}$$

The wavefunction for the Z can be obtained by $M_W \to M_Z$, while the photon remains massless and its wavefunction is simply $\psi_\gamma^{(0)}(z) = 1$.

3. The 4D Effective Lagrangian of the RS Model

Given the wavefunctions we can find the 4D effective action by integrating the 5D action over z. We want to match the RS calculation onto an effective 4D theory. After integrating out the Higgs, the most general Lagrangian for the electroweak gauge bosons (with operators of dimension 4 or less) can be written as [9, 19, 20]:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2g^2}Z_W W^+_{\mu\nu}W^{-\mu\nu} - \frac{1}{4(g^2+g'^2)}Z_Z Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4e^2}Z_\gamma F_{\mu\nu}F^{\mu\nu} + \frac{sc}{2e^2}\Pi'_{\gamma Z}F_{\mu\nu}Z^{\mu\nu}\\ &+\left(\frac{f^2}{4} + \frac{1}{g^2}\Pi_{WW}(0)\right)W^+_\mu W^{-\mu} + \frac{1}{2}\left(\frac{f^2}{4} + \frac{1}{(g^2+g'^2)}\Pi_{ZZ}(0)\right)Z_\mu Z^\mu\end{aligned} \tag{3.1}$$

where $Z_\gamma \equiv 1 - \Pi'_{\gamma\gamma}$, $Z_W \equiv 1 - \Pi'_{WW}$, $Z_Z \equiv 1 - \Pi'_{ZZ}$, $\Pi'_{\gamma Z}$, $\Pi_{WW}(0)$, and $\Pi_{ZZ}(0)$ incorporate the effects of new (oblique) physics beyond the standard model. Although we are only doing a tree-level matching calculation we have adopted the standard notation for vacuum polarizations to represent the wavefunction renormalizations that arise from classical 5D physics. With our conventions $f \approx 246$ GeV.

Using the 5D wavefunctions (to second order in masses) we can easily calculate the coefficients of the kinetic and mass terms:

$$\begin{aligned}
\frac{1}{e^2} Z_\gamma &\equiv \left(\frac{1}{g_5^2} + \frac{1}{g_5'^2}\right) \int_R^{R'} |\psi_\gamma^{(0)}(z)|^2 \frac{R dz}{z} = \left(\frac{1}{g_5^2} + \frac{1}{g_5'^2}\right) R \log(R'/R) \\
\frac{1}{g^2} Z_W &\equiv \frac{1}{g_5^2} \int_R^{R'} |\psi_W^{(0)}(z)|^2 \frac{R dz}{z} = \frac{1}{g_5^2} R \log(R'/R) - \Pi'_{11} \\
\frac{1}{g^2 + g'^2} Z_Z &\equiv \frac{1}{g_5^2 + g_5'^2} \int_R^{R'} |\psi_Z^{(0)}(z)|^2 \frac{R dz}{z} = \frac{1}{g_5^2 + g_5'^2} R \log(R'/R) - \Pi'_{33} \\
\Pi'_{\gamma Z} &= 0 \\
\frac{f^2}{4} + \frac{1}{g^2} \Pi_{WW}(0) &\equiv \frac{R^2}{4R'^2} v^2 + \frac{1}{g_5^2} \int_R^{R'} |\partial_z \psi_W^{(0)}(z)|^2 \frac{R dz}{z} = \frac{R^2}{4R'^2} v^2 + \Pi_{11}(0) \\
\frac{f^2}{4} + \frac{1}{(g^2 + g'^2)} \Pi_{ZZ}(0) &\equiv \frac{R^2}{4R'^2} v^2 + \frac{1}{g_5^2 + g_5'^2} \int_R^{R'} |\partial_z \psi_Z^{(0)}(z)|^2 \frac{R dz}{z} = \frac{R^2}{4R'^2} v^2 + \Pi_{33}(0) \quad (3.2)
\end{aligned}$$

where

$$\begin{aligned}
\Pi'_{11} = \Pi'_{33} &= -\frac{R^2 v^2}{8R'^2} \left(2R'^2 \log(R'/R) - 2R'^2 + \frac{R'^2 - R^2}{\log(R'/R)}\right) \\
\Pi_{11}(0) &= \frac{g_5^2 R^3 v^4}{64 R'^4} \left(2R'^2 - \frac{2R'^2}{\log(R'/R)} + \frac{R'^2 - R^2}{\log(R'/R)^2}\right) = -\frac{M_W^2}{2} \Pi'_{11} \\
\Pi_{33}(0) &= \frac{(g_5^2 + g_5'^2) R^3 v^4}{64 R'^4} \left(2R'^2 - \frac{2R'^2}{\log(R'/R)} + \frac{R'^2 - R^2}{\log(R'/R)^2}\right) = -\frac{M_Z^2}{2} \Pi'_{11} \ . \quad (3.3)
\end{aligned}$$

Here we have used the leading order results for M_W and M_Z, Eqs. (2.16). The corrections to the wave function renormalization Π'_{11} and Π'_{33} arise from integrating the W and Z wave functions, while the contributions to the mass renormalizations Π_{11} and Π_{33} appear from the 5D kinetic terms of W and Z, where a z derivative acts on the wave functions.

Even though there are no $1/(16\pi^2)$ loop suppression factors, for $vR \ll 1$, Π_{11} and Π_{33} can be treated as small perturbations of the leading terms. Thus a simple convention is to identify the 4D bare gauge couplings with the leading terms and the $Z_i - 1$ with the subleading Π' terms. This is a convenient choice because we want to separate out the new 5D physics from radiative corrections by loops of standard model particles. It also ensures that there are no additional corrections to the couplings of the W and Z to quarks and leptons. In other words, with this convention the mixing angles determined by diagonalizing the 5D action are identical to the bare 4D mixing angles:

$$s = \frac{g'}{\sqrt{g^2 + g'^2}}, \qquad c = \frac{g}{\sqrt{g^2 + g'^2}} \ . \quad (3.4)$$

It is these mixing angles that appear in the quark and lepton gauge couplings. Other conventions are possible, but physical observables are independent of convention. Thus in our convention

$$\begin{aligned}
\frac{1}{e^2} &\equiv \left(\frac{1}{g_5^2}+\frac{1}{g_5'^2}\right) R\log(R'/R)\\
\frac{1}{g^2} &\equiv \frac{1}{g_5^2} R\log(R'/R)\\
\frac{1}{g^2+g'^2} &\equiv \frac{1}{g_5^2+g_5'^2} R\log(R'/R).
\end{aligned} \tag{3.5}$$

Using this identification of the 4D couplings we then obtain for the other parameters of the effective Lagrangian:

$$\begin{array}{lll}
Z_\gamma = 1\ , & Z_W = 1-g^2\Pi'_{11}\ , & Z_Z = 1-(g^2+g'^2)\Pi'_{33}\ ,\\
f^2 = \frac{R^2}{R'^2}v^2\ , & \Pi_{WW}(0)=g^2\Pi_{11}(0)\ , & \Pi_{ZZ}(0)=(g^2+g'^2)\Pi_{33}(0)\ .
\end{array} \tag{3.6}$$

Note that since the photon is massless it receives no 5D renormalization, so $\Pi_{\gamma\gamma} = e^2\Pi_{QQ} = 0$. Furthermore since this is a tree-level calculation no new $Z-\gamma$ mixing can be induced so $\Pi_{3Q} = 0$. We can now use the standard definitions [9, 21] for the oblique parameters:

$$\begin{aligned}
S &\equiv 16\pi(\Pi'_{33}-\Pi'_{3Q})\\
T &\equiv \frac{4\pi}{s^2c^2M_Z^2}(\Pi_{11}(0)-\Pi_{33}(0))\\
U &\equiv 16\pi(\Pi'_{11}-\Pi'_{33})
\end{aligned} \tag{3.7}$$

Plugging in our results yields:

$$\begin{aligned}
S &\approx -4\pi f^2 R'^2\log(R'/R)\\
T &\approx -\frac{\pi}{2c^2} f^2 R'^2\log(R'/R)\ ,\\
U &= 0\ ,
\end{aligned} \tag{3.8}$$

where we have dropped terms which are suppressed by powers of $\log(R'/R)$. Note that both S and T are negative and large (i.e. log enhanced relative to a naive dimensional analysis estimate).

We can check these results by examining the poles of the propagators directly by expanding the denominators for $qR \ll 1$. At leading order the W pole is determined by

$$0 = -\frac{1}{4}\frac{R^2}{R'^2}v^2 + q^2\frac{R\log(R'/R)}{g_5^2}\ . \tag{3.9}$$

At next to leading order we must keep terms that are suppressed by $q^2R'^2$ relative to the leading terms. The pole is then determined by

$$0 = -\frac{1}{4}\frac{R^2}{R'^2}v^2 + q^2\frac{R\log(R'/R)}{g_5^2} + Aq^2 + Bq^4 \tag{3.10}$$

where

$$
\begin{aligned}
A &= \frac{R^2 v^2}{16 R'^2}\left(2R'^2 \log(R'/R) + R^2 - R'^2\right) \ , \\
B &= -\frac{R}{4g_5^2}\left((R'^2 + R^2)\log(R'/R) + R^2 - R'^2\right) \ .
\end{aligned} \tag{3.11}
$$

Thus to sub-leading order the pole is at:

$$
\begin{aligned}
M_W^2 &\simeq \frac{g_5^2}{R\log(R'/R)}\frac{R^2 v^2}{4R'^2} - \frac{g_5^4 R^2 v^4}{64 R'^4 \log(R'/R)^3}\left(2R'^2\log(R'/R)^2 - 2R'^2\log(R'/R) + R'^2\right) \\
&\simeq \frac{g^2 f^2}{4} - \frac{g^4 f^4}{64}\left(2R'^2\log(R'/R)^2 - 2R'^2\log(R'/R) + R'^2\right),
\end{aligned} \tag{3.12}
$$

which agrees at this order with the effective Lagrangian calculation

$$
M_W^2 \simeq g^2\left(\frac{f^2}{4} + \Pi_{11}(0)\right)\left(1 + g^2\Pi'_{11}\right) \simeq g^2\frac{f^2}{4}\left(1 + \frac{g^2}{2}\Pi'_{11}\right) \ . \tag{3.13}
$$

4. The Comparison of RS to Data

In addition to the oblique corrections we have described, once quarks and leptons are included in the theory with couplings to the bulk gauge bosons they will have additional four-fermion interactions beyond those in the standard model due to the exchange of the gauge boson resonances. The effect of these corrections has been parameterized in ref. [12, 13] by a correction to G_F denoted by V. Recall that the contribution to G_F from W exchange is [21]:

$$
4\sqrt{2}G_{F,W} = \frac{1}{\frac{f^2}{4} + \Pi_{11}(0)} \ . \tag{4.1}
$$

To include the effect of resonances we can write the bulk W propagator as a sum over poles:

$$
\Delta_W(q, R', R') = g_5^2 \sum_{n=0}^{\infty} \frac{\psi_W^{(n)}(R')^2}{N_n(q^2 - M_W^{(n)2})} \tag{4.2}
$$

where N_n is determined by

$$
N_n = \int_R^{R'} dz |\psi_W^{(n)}(z)|^2 \ , \tag{4.3}
$$

and hence

$$
N_0 = Z_W R \log(R'/R) \ . \tag{4.4}
$$

Since we chose $\psi_W^{(n)}(R') = 1$, we then have

$$
\Delta_W(q, R', R') = \frac{1}{Z_W}\frac{g^2}{q^2 - M_W^2} + g^2 \sum_{n=1}^{\infty} \frac{N_0 \psi_W^{(n)}(R')^2}{Z_W N_n (q^2 - M_W^{(n)2})}. \tag{4.5}
$$

At zero momentum the first term on the right hand side is just $-4\sqrt{2}G_{F,W}$, and the remaining terms are the additional corrections not coming from the W pole. If we write the correction to G_F as $G_F = G_{F,W}(1+V)$ then we have

$$\begin{aligned} V &= -\left(\Delta_W(q=0,R',R') + \frac{1}{\frac{f^2}{4}+\Pi_{11}(0)}\right)\left(\frac{f^2}{4}+\Pi_{11}(0)\right) \\ &\simeq \frac{g_5^2 R v^2}{16R'^2}\left(2R'^2 - \frac{2R'^2}{\log(R'/R)} + \frac{R'^2-R^2}{\log(R'/R)^2}\right) \simeq \frac{g^2}{8} f^2 R'^2 \log(R'/R)\ , \end{aligned} \tag{4.6}$$

where in the last line we have again dropped terms suppressed by powers of $\log(R'/R)$.

Thus we see that there are three types of corrections to precision electroweak observables in RS models: S, T, and V. Note that some of these corrections have also been considered in Refs. [15,17]. To relate our parameters to observables we use the standard definition of $\sin\theta_0$ from the Z pole,

$$\sin^2\theta_0\cos^2\theta_0 = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}G_F M_Z^2}\ , \tag{4.7}$$

$$\sin^2\theta_0 = 0.23105 \pm 0.00008 \tag{4.8}$$

where [22] $\alpha(M_Z^2)^{-1} = 128.92 \pm 0.03$ is the running SM fine-structure constant at M_Z. We can relate this measured value with the bare value in this class of models,

$$\sin^2\theta_0 = s^2 + \frac{s^2c^2}{c^2-s^2}\left(-\frac{\alpha}{4s^2c^2}S + \alpha T - V\right), \tag{4.9}$$

which is obtained by considering all corrections to (4.7) in the usual way (see [21]). Also, in the RS model we have the simple result that with only the tree-level 5D renormalizations the running couplings defined by Kennedy and Lynn [23] which appear in Z-pole asymmetries are the same as the bare couplings:

$$s_*^2(q^2) = s^2, \quad e_*^2(q^2) = e^2\ . \tag{4.10}$$

In addition to the contribution from T, there are further corrections to the low-energy ratio of charged- to neutral-current interactions coming from resonance exchange. We will absorb this effect into the parameter ρ_*:

$$\rho_* = \frac{\frac{f^2}{4}+\Pi_{11}(0)}{\frac{f^2}{4}+\Pi_{33}(0)}\left(\frac{1+V/c^2}{1+V}\right) \approx 1 + \alpha T + \frac{s^2}{c^2}V\ . \tag{4.11}$$

Curiously in the RS model we find that the contributions from T and V cancel, and $\rho_* = 1$. We will however present the general results for precision observables in Appendix A without assuming a relation between T and V, so that our results can be used for more general models.

With Eqs. (4.9) and (4.11) it is straightforward to calculate the corrections in a general model to precision electroweak observables in terms of S, T, and V. The expressions for the various observables together with the SM predictions and experimental results are given in Appendix A.

The result of a global fit to the 23 observables listed in Table 1 is that for $M_{\rm Higgs} = 115$ GeV

$$R'(\log(R'/R))^{1/2} < 0.50\,\mathrm{TeV}^{-1} \tag{4.12}$$

at the 95 % confidence level. Taking $\log(R'/R) = 32$ (as is often done to naturally explain the hierarchy between the Planck and weak scales [2–4]) we have:

$$1/R' > 11\,\mathrm{TeV}\ . \tag{4.13}$$

For a value of R' which saturates this bound we have

$$\begin{aligned} S &= -0.19 \\ T &= -0.03 \\ V &= 0.00082 \end{aligned} \tag{4.14}$$

Since the T and V contributions to ρ_* cancel and for these values the contribution of S to $(s^2 - \sin^2\theta_0)$ is about 8.6 times larger than that of T and 2.6 times larger than that of V, it is the S parameter constraint that dominantly determines the bound on R'.

It is interesting to note that for the Z-pole observables, which do not depend on ρ_*, one can absorb the contribution from V into an effective T:

$$T_{\rm eff} = T - \frac{V}{\alpha}\ . \tag{4.15}$$

Thus in the RS model, $T_{\rm eff}$ is even more negative than T. One can then use the bounds on S and T to estimate the bounds on R', which yields results similar to (4.13).

In the RS model there is also a light radion that contributes to precision electroweak observables at the loop level (together with loops of the KK gravitons and gauge bosons). These contributions have been calculated separately [24] and are small unless an extra Higgs-radion coupling [25] is introduced. With this additional coupling, the radion corrections tend to make S more negative and hence only tighten the bound on R'.

In the SM, the fit to data gets significantly worse when the Higgs mass is raised. The reason is that the Higgs contributes positively to S, while the data prefers a small or negative S. However, in our case S is negative, so one might think that a larger Higgs mass can be accommodated. Unfortunately at the same time the Higgs also contributes negatively to T, as can be seen from the approximate expressions [21]

$$S_{Higgs} \approx \frac{1}{12\pi}\log\left(\frac{m_H^2}{m_{H,ref}^2}\right), \quad T_{Higgs} \approx -\frac{3}{12\pi c^2}\log\left(\frac{m_H^2}{m_{H,ref}^2}\right). \tag{4.16}$$

Therefore even though the agreement with S can be improved by raising the Higgs mass, T will start to deviate even more. In order to see if the fit can be improved we have repeated it for the SM results evaluated at $M_H = 300$ GeV and 600 GeV. In the case of the SM, χ^2 for the 23 observables listed in Table 1 in the Appendix increases by about 11 as M_H increases from 115 to 300 GeV. If we now turn on the corrections from the RS model, the difference between the minimal χ^2's for the 300 GeV and 115 GeV Higgs reduces to about 7.6, slightly improving the fit, but still outside the 95% confidence region ($\Delta\chi^2 = 6.2$) of a two parameter fit (our parameters being the Higgs mass and $X = f^2 R'^2 \log(R'/R)$). Hence, the Higgs mass bound is slightly relaxed but not significantly. Assuming a 300 GeV Higgs in turn would relax the limit for R' to $1/R' \geq 9.0$ TeV (again assuming $\log R'/R = 32$). For the case of the 600 GeV Higgs the increase in minimum χ^2 is 19, and is clearly excluded by a wide margin in the two parameter fit. Assuming a 600 GeV Higgs the bound on R' becomes 8.2 TeV $< 1/R' <$ 22 TeV. These results are illustrated in Fig. 1. If one

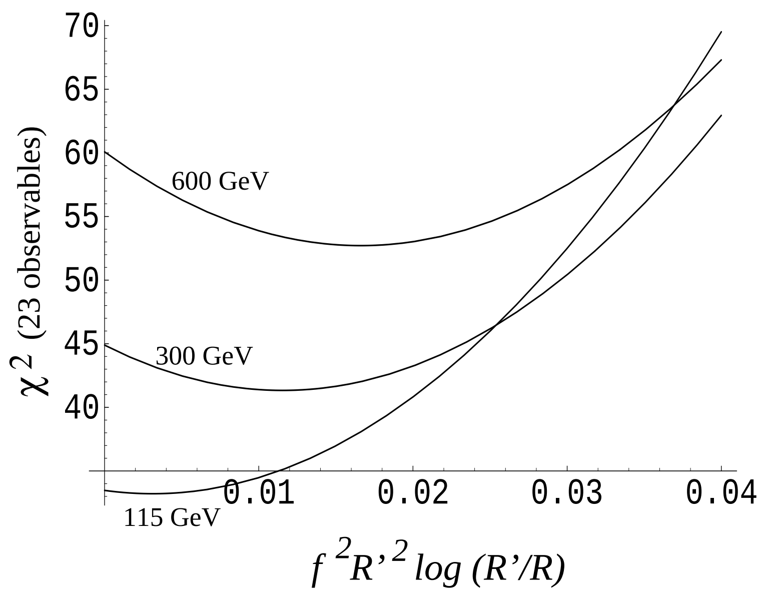

Figure 1: The change in χ^2 in the RS model as a function of R' and R, for three different values of $M_H = 115, 300,$ and 600 GeV.

were intent on having a heavy Higgs one could add additional new physics to the model that give positive contributions to T [27], but this seems completely ad hoc in the present context.

The bound (4.13) on R' pushes the W gauge boson resonance masses up to around 27 TeV. The graviton KK masses are always heavier than the gauge boson modes [13], for example here the lowest possible value is around 46 TeV. Thus in the RS model with bulk gauge bosons there will be no possible signatures from resonances in WW scattering to observe at the LHC. The focus would have to be on the Higgs-radion sector [26].

To the extent that the RS model corresponds to a technicolor like model we find these models can be consistent with experiment as long as the resonance masses are dialed up. Unfortunately for technicolor models there was no parameter which changed the resonance mass independently of the electroweak breaking scale f.

5. Conclusions

We have matched the 5D RS model onto a 4D low-energy effective theory and found large (logarithmically enhanced) negative contributions to S and T. It is interesting to note that this is the first model to naturally produce a large negative value for S [27]; however this is mitigated by the fact that the resulting bound on R' from precision electroweak measurements forces a seemingly unnatural hierarchy of about a factor of 50 between the Higgs vev and the nominal electroweak scale of the RS model $1/R' \approx 11$ TeV. This pushes the gauge boson resonances (and the graviton KK modes) far beyond the reach of the LHC.

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Appendix

A. Predictions for Electroweak Observables

In this appendix we give the predictions of a general model model with contributions to S, T, and V for the electroweak precision observables. We also give in Table (1) the experimental data [22, 28] and the SM predictions used for our fit in Section 4. Using the results given in [21, 29] as well as the low-energy νe couplings:

$$g_{eV}(\nu e \to \nu e) = 2\rho_*(s^2 - \frac{1}{4}) \ , \quad g_{eA}(\nu e \to \nu e) = -\frac{\rho_*}{2} \ . \tag{A.1}$$

we find the following results:

$$
\begin{aligned}
&\Gamma_Z = (\Gamma_Z)_{SM}\left(1 - 3.8\times 10^{-3}S + 0.011T - 1.4V\right)\\
&R_e = (R_e)_{SM}\left(1 - 2.9\times 10^{-3}S + 2.0\times 10^{-3}T - 0.26V\right)\\
&R_\mu = (R_\mu)_{SM}\left(1 - 2.9\times 10^{-3}S + 2.0\times 10^{-3}T - 0.26V\right)\\
&R_\tau = (R_\tau)_{SM}\left(1 - 2.9\times 10^{-3}S + 2.0\times 10^{-3}T - 0.26V\right)\\
&\sigma_h = (\sigma_h)_{SM}\left(1 + 2.2\times 10^{-4}S - 1.6\times 10^{-4}T + 0.021V\right)\\
&R_b = (R_b)_{SM}\left(1 + 6.6\times 10^{-4}S - 4.0\times 10^{-4}T + 0.052V\right)\\
&R_c = (R_c)_{SM}\left(1 - 1.3\times 10^{-3}S + 1.0\times 10^{-3}T - 0.13V\right)\\
&A^e_{FB} = (A^e_{FB})_{SM} - 6.8\times 10^{-3}S + 4.8\times 10^{-3}T - 0.62V\\
&A^\mu_{FB} = (A^\mu_{FB})_{SM} - 6.8\times 10^{-3}S + 4.8\times 10^{-3}T - 0.62V\\
&A^\tau_{FB} = (A^\tau_{FB})_{SM} - 6.8\times 10^{-3}S + 4.8\times 10^{-3}T - 0.62V\\
&A_\tau(P_\tau) = (A_\tau(P_\tau))_{SM} - 0.028S + 0.020T - 2.6V\\
&A_e(P_\tau) = (A_e(P_\tau))_{SM} - 0.028S + 0.020T - 2.6V\\
&A^b_{FB} = \left(A^b_{FB}\right)_{SM} - 0.020S + 0.014T - 1.8V\\
&A^c_{FB} = (A^c_{FB})_{SM} - 0.016S + 0.011T - 1.4V\\
&A_{LR} = (A_{LR})_{SM} - 0.028S + 0.020T - 2.6V\\
&M_W = (M_W)_{SM}\left(1 - 3.6\times 10^{-3}S + 5.5\times 10^{-3}T - 0.71V\right)\\
&M_W/M_Z = (M_W/M_Z)_{SM}\left(1 - 3.6\times 10^{-3}S + 5.5\times 10^{-3}T - 0.71V\right)\\
&g_L^2(\nu N\to\nu X) = \left(g_L^2(\nu N\to\nu X)\right)_{SM} - 2.7\times 10^{-3}S + 6.5\times 10^{-3}T - 0.066V\\
&g_R^2(\nu N\to\nu X) = \left(g_R^2(\nu N\to\nu X)\right)_{SM} + 9.3\times 10^{-4}S - 2.0\times 10^{-4}T + 0.10V\\
&g_{eV}(\nu e\to\nu e) = (g_{eV}(\nu e\to\nu e))_{SM} + 7.2\times 10^{-3}S - 5.4\times 10^{-3}T + 0.65V\\
&g_{eA}(\nu e\to\nu e) = (g_{eA}(\nu e\to\nu e))_{SM} - 3.9\times 10^{-3}T - 0.15V\\
&Q_W(Cs) = (Q_W(Cs))_{SM} - 0.793S - 0.0090T - 95V
\end{aligned}
\tag{A.2}
$$

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Quantity	Experiment	SM(115 GeV)	SM(300 GeV)	SM(600 GeV)
Γ_Z	2.4952 $\pm$ 0.0023	2.4965	2.4963	2.4954
R_e	20.8040 $\pm$ 0.0500	20.7425	20.7403	20.7332
R_μ	20.7850 $\pm$ 0.0330	20.7426	20.7405	20.7334
R_τ	20.7640 $\pm$ 0.0450	20.7879	20.7857	20.7786
σ_h	41.5410 $\pm$ 0.0370	41.4800	41.4774	41.4814
R_b	0.2165 $\pm$ 0.00065	0.2157	0.2154	0.2151
R_c	0.1719 $\pm$ 0.0031	0.1723	0.1724	0.1725
A^e_{FB}	0.0145 $\pm$ 0.0025	0.0163	0.0159	0.0157
A^μ_{FB}	0.0169 $\pm$ 0.0013	0.0163	0.0159	0.0157
A^τ_{FB}	0.0188 $\pm$ 0.0017	0.0163	0.0159	0.0157
$A_\tau(P_\tau)$	0.1439 $\pm$ 0.0041	0.1475	0.1457	0.1446
$A_e(P_\tau)$	0.15138 $\pm$ 0.0022	0.1475	0.1457	0.1446
A^b_{FB}	0.0990 $\pm$ 0.0017	0.1034	0.1021	0.1013
A^c_{FB}	0.0685 $\pm$ 0.0034	0.0739	0.0729	0.0723
A_{LR}	0.1513 $\pm$ 0.0021	0.1475	0.1457	0.1446
M_W	80.450 $\pm$ 0.039	80.3890	80.3775	80.3672
M_W/M_Z	0.8822 $\pm$ 0.0006	0.8816	0.8815	0.8813
$g_L^2(\nu N \to \nu X)$	0.3020 $\pm$ 0.0019	0.3039	0.3038	0.3017
$g_R^2(\nu N \to \nu X)$	0.0315 $\pm$ 0.0016	0.0301	0.0301	0.0302
$g_{eA}(\nu e \to \nu e)$	-0.5070 $\pm$ 0.014	-0.5065	-0.5065	-0.5065
$g_{eV}(\nu e \to \nu e)$	-0.040 $\pm$ 0.015	-0.0397	-0.0393	-0.0390
$Q_W(Cs)$	-72.65 $\pm$ 0.44	-73.11	-73.17	-73.20
m_{top}	174.3 $\pm$ 5.1	176.3	185	192

Table 1: The experimental results [22, 28] and the SM predictions for the various electroweak precision observables used for the fit. The SM predictions are for $M_{\rm Higgs} = 115$, 300, 600 GeV and $\alpha_s = 0.120$ and calculated [30] using GAPP [31].

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