

# A Heavy Top Quark and the $Zb\bar{b}$ Vertex in Non-Commuting Extended Technicolor

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We explore corrections to electroweak parameters in the context of Extended Technicolor (ETC) models in which the ETC gauge-boson which generates the top-quark mass carries weak  $SU(2)$  charge. For  $m_t \sim 150$  GeV there exist potentially large corrections to the  $Z$  decay width to  $b$ -quarks. Interestingly, in contrast to the situation in ETC models where the gauge-boson which generates the top-quark mass is a weak  $SU(2)$  singlet, the corrections may *increase* the  $Z \rightarrow b\bar{b}$  branching ratio.

## 1. Introduction

It has recently been shown that corrections to the  $Zb\bar{b}$  vertex can place stringent constraints on models of dynamical electroweak symmetry breaking [1]. In particular, for extended technicolor (ETC) models<sup>1</sup> in which the ETC gauge-boson giving rise to the top-quark mass does not carry weak  $SU(2)$  charge, it has been found that the exchange of this gauge-boson alters the  $Zb\bar{b}$  vertex by an amount proportional to  $m_t$ . These corrections decrease the  $Z \rightarrow b\bar{b}$  branching ratio and are large enough, even in walking technicolor models [5], that current LEP data [6] appear to exclude such ETC models if the top mass is 100 GeV or greater (see ref. [7]).

Finding similar constraints on models in which the ETC gauge-boson which generates the top-quark mass *does* carry weak  $SU(2)$  charge is more complicated. As noted in [1] both the size and sign of the effect on the  $Zb\bar{b}$  vertex are model-dependent because there may be contributions from two different sets of operators (those involving weak doublet and weak triplet currents).

In this letter, we make a detailed analysis of ETC-related effects on the  $Zb\bar{b}$  vertex in the context of such “non-commuting” models. We show that the two competing contributions come from the physics of top-quark mass generation and from weak gauge boson mixing. We determine the magnitudes of the effects in terms of the top-quark mass and the strength of the ETC coupling. Furthermore, we show that each of these contributions is of fixed sign – and that the signs of the two effects are opposite. Therefore, unlike the case of commuting ETC models, we cannot make a model-independent statement of the overall size or sign of the change in the  $Z \rightarrow b\bar{b}$  branching ratio: the overall effect may be small and may even *increase* the  $Z \rightarrow b\bar{b}$  branching ratio.

The starting point of our analysis is the general symmetry-breaking pattern which non-commuting ETC groups must exhibit in order to produce phenomenology consistent with both a heavy top-quark and approximate Cabibbo universality. A heavy top-quark must receive its mass from ETC dynamics at low energy scales; if the ETC bosons responsible for  $m_t$  are weak-charged, the weak group  $SU(2)_{heavy}$  under which  $(t, b)_L$  is a doublet must be embedded in the low-scale ETC group. Conversely, the light quarks and leptons cannot be

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<sup>1</sup> We confine our attention to models in which the ETC interactions do not participate directly in electroweak symmetry breaking, *i.e.* we will not consider “strong-ETC” models [2]. The fine-tuning required in strong-ETC models will give rise to additional light scalar degrees of freedom [3], and the corrections to the  $Zb\bar{b}$  vertex can be much smaller [1][4].

charged under the low-scale ETC group lest they also receive large contributions to their masses; hence the weak  $SU(2)_{light}$  group for the light quarks and leptons must be distinct from  $SU(2)_{heavy}$ . To approximately preserve low-energy Cabibbo universality the two weak  $SU(2)$ s must break to their diagonal subgroup before technicolor dynamically breaks the remaining electroweak symmetry to electromagnetism. The resulting symmetry-breaking pattern is<sup>2</sup>:

$$\begin{aligned}
& ETC \times SU(2)_{light} \times U(1)' \\
& \quad \downarrow \quad f \\
& TC \times SU(2)_{heavy} \times SU(2)_{light} \times U(1)_Y \\
& \quad \downarrow \quad u \\
& TC \times SU(2)_{weak} \times U(1)_Y \\
& \quad \downarrow \quad v \\
& TC \times U(1)_{EM},
\end{aligned} \tag{1.1}$$

where  $ETC$  and  $TC$  stand for the extended technicolor and technicolor gauge groups respectively, and  $f$ ,  $u$ , and  $v \approx 246$  GeV are scales of the expectation values of the order parameters for the three different symmetry breakings (*i.e.* the analogs of  $F_\pi$  for chiral symmetry breaking in QCD). Note that, since we are interested in the physics associated with top-quark mass generation, only  $t_L$ ,  $b_L$  and  $t_R$  need transform non-trivially under  $ETC$ . Thus  $(t, b)_L$  is a doublet under  $SU(2)_{heavy}$  but a singlet under  $SU(2)_{light}$ , while all other left-handed ordinary fermions have the opposite  $SU(2)$  assignment.

In section 2, we consider how the ETC dynamics responsible for generating the top-quark mass affects the  $Zb\bar{b}$  vertex. Section 3 examines the electroweak structure of the model and the impact of weak gauge-boson mixing on the  $Z \rightarrow b\bar{b}$  branching ratio. In section 4, we give a scenario for a theory with a non-commuting ETC group. Section 5 summarizes our conclusions.

## 2. Effects from ETC dynamics responsible for $m_t$

One effect on the  $Z \rightarrow b\bar{b}$  branching ratio comes directly from the dynamics related to the generation of the mass of the top-quark. In ETC models where the gauge-boson

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<sup>2</sup> The hypercharge group,  $U(1)_Y$ , is embedded partly in the ETC group, and therefore  $U(1)' \neq U(1)_Y$ .

whose exchange gives rise to the top-quark mass carries weak  $SU(2)$  quantum numbers, that boson transforms as a weak doublet. At energies below the scale  $f$  of ETC breaking, the effective four-fermion operator giving rise to the top-quark mass is

$$-\frac{2}{f^2} \left( \xi \bar{\psi}_L \gamma^\mu U_L + \frac{1}{\xi} \bar{t}_R \gamma^\mu T_R \right) \left( \xi \bar{U}_L \gamma_\mu \psi_L + \frac{1}{\xi} \bar{T}_R \gamma_\mu t_R \right) \quad (2.1)$$

where the left-handed heavy quarks and right-handed technifermions,  $\psi_L = (t, b)_L$  and  $T_R = (U, D)_R$ , are doublets under  $SU(2)_{heavy}$  while the left-handed technifermions are  $SU(2)_{heavy}$  singlets. Note that, since  $\bar{\psi}_L \gamma^\mu U_L$  and  $\bar{t}_R \gamma^\mu T_R$  must transform in the same way, if  $\psi_L$  is a 2 of  $SU(2)_{heavy}$  then  $T_R$  is a 2\* instead. The operator (2.1) is normalized using the conventional definition of  $f$  with  $M_{ETC} = g_{ETC} f/2$  and the conventional factor of  $1/\sqrt{2}$  for an ‘‘off-diagonal’’ gauge-boson coupling. The parameter  $\xi$  in equation (2.1) is a model-dependent Clebsch and is equal to 1 in the class of models outlined in section 4.

The piece of (2.1) contributing to  $m_t$  is the product of left-handed and right-handed currents. Fierzing this into the product of technicolor-singlet densities gives

$$\frac{4}{f^2} (\bar{\psi}_L t_R) (\bar{T}_R U_L) + h.c. \quad (2.2)$$

When the technifermions condense (we estimate the size of the condensate using dimensional analysis [8],  $\langle \bar{U}U \rangle \approx 4\pi v^3$ ) the top-quark receives a mass

$$m_t \approx \frac{8\pi v^3}{f^2}. \quad (2.3)$$

Hence the relationship between the ETC and TC scales is linear in the top-quark mass

$$\frac{v^2}{f^2} \approx \frac{m_t}{8\pi v}. \quad (2.4)$$

The purely left-handed piece of operator (2.1) affects the  $Zb\bar{b}$  vertex [1]. To see how this occurs, we Fierz the left-left operator into the product of technicolor-singlet currents

$$-\frac{2\xi^2}{f^2} (\bar{\psi}_L \gamma^\mu \psi_L) (\bar{U}_L \gamma_\mu U_L). \quad (2.5)$$

Adopting an effective chiral Lagrangian description appropriate below the technicolor chiral symmetry breaking scale, we may replace the technifermion current by a sigma-model current [9]:

$$(\bar{T}_L \gamma_\mu \tau^a T_L) = \frac{v^2}{2} Tr (\Sigma^\dagger \tau^a i D_\mu \Sigma) , \quad (2.6)$$

where  $\Sigma = \exp(2i\tilde{\pi}/v)$  transforms as  $\Sigma \rightarrow L\Sigma R^\dagger$  under  $SU(2)_L \times SU(2)_R$ . To evaluate  $D^\mu \Sigma$ , recall that since  $\psi_L$  is a 2 and  $T_R$  is a  $2^*$  of  $SU(2)_L$

$$\begin{aligned} D_\mu \psi_L &= \partial_\mu \psi_L + i \frac{e}{\sin \theta \cos \theta} Z_\mu \left( \frac{1}{2} \tau_3 - s^2 Q \right) \psi_L + \dots \\ D_\mu T_R &= \partial_\mu T_R + i \frac{e}{\sin \theta \cos \theta} Z_\mu \left( \frac{1}{2} (-\tau_3^*) - s^2 Q \right) T_R + \dots \end{aligned} \quad (2.7)$$

so that

$$D_\mu \Sigma = \partial_\mu \Sigma - \frac{ie}{\sin \theta \cos \theta} Z_\mu \left( \frac{1}{2} \Sigma \tau_3^* + \sin^2 \theta [\Sigma, Q] \right) + \dots \quad (2.8)$$

Going to unitary gauge ( $\Sigma = 1$ ) we find

$$\bar{U}_L \gamma_\mu U_L = - \frac{e}{\sin \theta \cos \theta} \frac{v^2}{4} Z_\mu. \quad (2.9)$$

Combining this with (2.5) gives the ETC-induced coupling between the  $Z$  and the t-b doublet

$$\xi^2 \frac{e}{\sin \theta \cos \theta} \frac{v^2}{2f^2} [\bar{\psi}_L \gamma^\mu Z_\mu \psi_L]. \quad (2.10)$$

This additional coupling between the  $Z$  and left-handed  $b$  quarks changes the tree-level  $Z b_L \bar{b}_L$  coupling by an amount<sup>3</sup>

$$\delta g_L = - \frac{e}{\sin \theta \cos \theta} \frac{\xi^2 v^2}{2f^2} \approx - \frac{\xi^2}{4} \frac{e}{\sin \theta \cos \theta} \frac{m_t}{4\pi v}. \quad (2.11)$$

Since the tree-level  $Z b_L \bar{b}_L$  coupling is also negative, the ETC-induced change tends to **increase** the coupling – and thereby the  $Z$  decay width to  $b$  quarks ( $\Gamma_b$ ) :

$$\begin{aligned} \frac{\delta \Gamma_b}{\Gamma_b} &= 2 \frac{g_L \delta g_L}{g_L^2 + g_R^2} \approx \frac{2 \delta g_L}{g_L} \\ &\approx \frac{\xi^2}{1 - \frac{2}{3} \sin^2 \theta} \frac{m_t}{4\pi v} \\ &\approx +5.6\% \xi^2 \left( \frac{m_t}{150 \text{ GeV}} \right). \end{aligned} \quad (2.12)$$

While this result is of the same magnitude as the change in  $\Gamma_b$  induced by top-mass generation in models where the ETC and weak  $SU(2)$  groups commute [1], because the technifermions transform as a  $2^*$  the result is of the opposite sign.

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<sup>3</sup> In eqn. (2.3) we have scaled from QCD, implicitly assuming that technicolor is a non-walking  $SU(N)$  gauge group with  $N = 3$ . We may use large- $N$  results [10] to estimate that the size of the effect is proportional to  $\sqrt{N}$ .

In order to compare these results to experiment, we consider the ratio

$$R_b = \frac{\Gamma_b}{\Gamma_{hadrons}} . \quad (2.13)$$

Both oblique effects and the leading QCD corrections cancel in this ratio along with some experimental systematic effects. We find that eqn. (2.12) results in a change to  $R_b$  of

$$\frac{\delta R_b}{R_b} \approx +4.4\% \xi^2 \left( \frac{m_t}{150\text{GeV}} \right) . \quad (2.14)$$

This is particularly interesting since recent results [6] from the LEP Electroweak Working Group find that the measured value of  $R_b$

$$R_b = 0.2207 \pm 0.0009 \pm 0.0020 , \quad (2.15)$$

is *larger* than the Standard Model prediction of approximately 0.216 for a 150 GeV top quark mass. Furthermore, the Standard Model prediction *decreases* with increasing top mass.

### 3. Effects from weak gauge boson mixing

Next we explore the effect of the weak gauge boson mixing in non-commuting ETC models on the  $Z \rightarrow b\bar{b}$  branching ratio. The electroweak group structure  $SU(2)_{heavy} \times SU(2)_{light} \times U(1)_Y$  implies the existence of an extra set of massive  $W$  and  $Z$  gauge bosons. Hence the light  $W$  and  $Z$  mass eigenstates will differ from the standard  $W$  and  $Z$ .

The electroweak symmetry breaking pattern is similar to that of the un-unified standard model [11], and our notation is chosen to facilitate this comparison. The gauge bosons associated with the electroweak group  $SU(2)_{heavy} \times SU(2)_{light} \times U(1)_Y$  are denoted  $W_l^\mu$ ,  $W_h^\mu$  and  $X^\mu$ . The charges of the ordinary fermions are

$$\begin{aligned} \text{LH heavy (t, b) quarks} & : (2, 1)_{1/6} \\ \text{LH light quarks} & : (1, 2)_{1/6} \\ \text{LH leptons} & : (1, 2)_{-1/2} \\ \text{RH quarks and leptons} & : (1, 1)_Q \end{aligned} \quad (3.1)$$

where  $Q$  is the electric charge of the right-handed fermion <sup>4</sup>. The  $U(1)_{EM}$  to which the electroweak group breaks is generated by

$$Q = T_{3l} + T_{3h} + Y. \quad (3.2)$$

The photon eigenstate can be written in terms of two weak mixing angles,

$$A^\mu = \sin\theta \sin\phi W_{3l}^\mu + \sin\theta \cos\phi W_{3h}^\mu + \cos\theta X^\mu \quad (3.3)$$

where  $\theta$  is the usual weak angle and  $\phi$  is an additional one. Eqns. (3.2) and (3.3) imply that the gauge couplings are

$$g_l = \frac{e}{s \sin\theta}, \quad g_h = \frac{e}{c \sin\theta}, \quad g' = \frac{e}{\cos\theta}, \quad (3.4)$$

where  $s \equiv \sin\phi$  and  $c \equiv \cos\phi$ .

The order parameters of the spontaneous symmetry breaking transform as

$$\langle\varphi\rangle \sim (1, 2)_{1/2}, \quad \langle\sigma\rangle \sim (2, 2)_0. \quad (3.5)$$

The breaking pattern shown in (1.1) results when they acquire values

$$\langle\varphi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad \langle\sigma\rangle = \begin{pmatrix} u & 0 \\ 0 & u \end{pmatrix}. \quad (3.6)$$

In the scenario we are considering, the role of  $\langle\varphi\rangle$  is played by the TC condensate, while the physical origin of  $\langle\sigma\rangle$  is model-dependent.

While the standard electroweak model is defined in reference to three experimental inputs,  $\alpha_{EM}$ ,  $G_F$  and  $M_Z$ , this extended model requires two additional parameters (a mass scale and a coupling) for a complete definition. Taking these to be  $u$  and  $c^2$ , we can immediately place limits on them. Equation (1.1) implies that the mass scale  $u$  is bounded from above by  $f$ , which we have already related to the size of the top-quark mass. Using relation (2.4) and defining  $x \equiv u^2/v^2$  we find

$$x \approx \left(\frac{u}{f}\right)^2 \frac{8\pi v}{m_t} \quad (3.7)$$

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<sup>4</sup> Cancellation of the  $SU(2)^2 \times U(1)$  anomalies of these fermion representations is model-dependent and will not be addressed here

so that a top-quark mass of order 150 GeV implies  $x$  less than or of order 40. The mixing angle  $c^2$ , on the other hand, is easily bounded from below. Because  $g_h$  is essentially the value of the technicolor coupling at a scale of order a TeV (the low ETC scale), we expect it to be large compared to the weak coupling; hence we expect from relation (3.4) that  $c^2$  will be relatively small. However if  $c^2$  is too small, then  $g_h$  will exceed the “critical” value at which technifermion chiral symmetry breaking occurs. As an estimate of this value, we use the results of the gap-equation analysis of chiral symmetry breaking in the “rainbow” approximation [12]. Then we require

$$c^2 \gtrsim \frac{\alpha_{EM}}{\sin^2 \theta} \frac{3C_2}{\pi} = .03 \left( \frac{N^2 - 1}{2N} \right) \quad (3.8)$$

where  $C_2$  is the quadratic Casimir of the technifermion representation of the technicolor group and the last equality uses  $C_2$  for the fundamental of  $SU(N)$  technicolor .

To calculate the effect of gauge-boson mixing on the  $Z \rightarrow b\bar{b}$  branching ratio, we must examine the light  $W$  and  $Z$  states. It is most convenient to discuss the mass eigenstates in the rotated basis

$$W_1^{\pm\mu} = s W_l^{\pm\mu} + c W_h^{\pm\mu}, \quad W_2^{\pm\mu} = c W_l^{\pm\mu} - s W_h^{\pm\mu} \quad (3.9)$$

$$Z_1^\mu = \cos \theta (s W_{3l}^\mu + c W_{3h}^\mu) - \sin \theta X^\mu, \quad Z_2^\mu = c W_{3l}^\mu - s W_{3h}^\mu, \quad (3.10)$$

in which the gauge covariant derivatives separate neatly into standard and non-standard pieces

$$\partial^\mu + ig (T_l^\pm + T_h^\pm) W_1^{\pm\mu} + ig \left( \frac{c}{s} T_l^\pm - \frac{s}{c} T_h^\pm \right) W_2^{\pm\mu}, \quad (3.11)$$

$$\partial^\mu + i \frac{g}{\cos \theta} (T_{3l} + T_{3h} - \sin^2 \theta Q) Z_1^\mu + ig \left( \frac{c}{s} T_{3l} - \frac{s}{c} T_{3h} \right) Z_2^\mu. \quad (3.12)$$

where  $g = \frac{e}{\sin \theta}$ . The mass-squared matrix for the  $Z_1$  and  $Z_2$  is as follows (because the photon is massless, the  $3 \times 3$  matrix for the neutral bosons can be reduced to a  $2 \times 2$  matrix [13].)

$$M_Z^2 = \left( \frac{ev}{2 \sin \theta} \right)^2 \begin{pmatrix} \frac{1}{\cos^2 \theta} & \frac{-s}{c \cos \theta} \\ \frac{-s}{c \cos \theta} & \frac{x}{s^2 c^2} + \frac{s^2}{c^2} \end{pmatrix}. \quad (3.13)$$

The mass-squared matrix for the  $W_1$  and  $W_2$  may be obtained by simply setting  $\cos \theta = 1$  in the above matrix.

We expect  $x$  to be reasonably large and we diagonalize the  $W$  and  $Z$  mass matrices in the limit of large  $x$ ; we will find that this is self-consistent. The perturbative expressions we obtain for the light  $W$  and  $Z$  masses and eigenstates to leading order in  $\frac{1}{x}$  are<sup>5</sup>

$$(M_W^L)^2 \approx \left( \frac{\pi\alpha_{EM}}{\sqrt{2}G_F \sin^2 \theta} \right) \left( 1 + \frac{1}{x}(1 - s^4) \right), \quad W^L \approx W_1 + \frac{cs^3}{x} W_2 \quad (3.14)$$

$$(M_Z^L)^2 \approx \left( \frac{\pi\alpha_{EM}}{\sqrt{2}G_F \sin^2 \theta \cos^2 \theta} \right) \left( 1 + \frac{1}{x}(1 - s^4) \right), \quad Z^L \approx Z_1 + \frac{cs^3}{x \cos \theta} Z_2 \quad (3.15)$$

Note that as  $c^2 \rightarrow 0$  the light  $W$  and  $Z$  approximate the  $W_1$  and  $Z_1$  states which have standard couplings to fermions.

Now we can compute the effect of  $Z_1 - Z_2$  mixing on  $R_b$ . From (3.12) and (3.15) we find a change in the coupling of the  $Z$  to  $b$ -quarks

$$\delta g_L \approx \frac{e}{\sin \theta \cos \theta} \frac{s^4}{2x} . \quad (3.16)$$

This results in the following change in  $\Gamma_b$  due to the gauge boson mixing

$$\frac{\delta \Gamma_b}{\Gamma_b} = \frac{2g_L^b \delta g_L^b}{(g_L^b)^2 + (g_R^b)^2} \approx -2.3 \frac{s^4}{x} . \quad (3.17)$$

The effect on all quarks other than the  $b$  may be computed similarly. Equations (3.15) and (3.12) imply a shift in the left-handed couplings of these quarks in the amount

$$\delta g_L = \frac{e}{\sin \theta \cos \theta} \frac{c^2 s^2}{x} T_3 , \quad (3.18)$$

where  $T_3$  is  $+\frac{1}{2}$  for up-quarks and  $-\frac{1}{2}$  for down-quarks. This results in a change in the  $Z$  width to hadrons other than the  $b$  by

$$\frac{\delta \Gamma_{h \neq b}}{\Gamma_{h \neq b}} \approx +2.3 \frac{s^2 c^2}{x} . \quad (3.19)$$

Combining (3.17) and (3.19) and using (3.7), we find that the effect of gauge-boson mixing on  $R_b$  is

$$\frac{\delta R_b}{R_b} \approx (1 - R_b) \left( \frac{\delta \Gamma_b}{\Gamma_b} - \frac{\delta \Gamma_{h \neq b}}{\Gamma_{h \neq b}} \right) \approx -4.4\% s^2 \left( \frac{f}{u} \right)^2 \left( \frac{m_t}{150 \text{GeV}} \right) . \quad (3.20)$$

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<sup>5</sup> An analysis of the low-energy charged currents, analogous to that carried out in ref. [11], shows that  $\sqrt{2}G_F = \frac{1}{v^2} + \frac{1}{u^2}$ .

We see that the effect of gauge-boson mixing on the  $Z \rightarrow b\bar{b}$  decay width is of the same order of magnitude, but opposite sign, to the effect due to the ETC dynamics of top-quark mass generation, eqn. (2.14). Unlike the case of commuting ETC models, we cannot make a model-independent statement of the overall size or sign of the change in the  $Z \rightarrow b\bar{b}$  branching ratio: the overall effect may be small and may even *increase* the  $Zb\bar{b}$  branching ratio.

Finally, we should mention that gauge-boson mixing leads to a number of other potentially large effects as well. For example, because the  $Z$  mass is used as an input in fits of LEP results to the standard model, the shift in the tree-level  $Z$  mass in eqn. (3.15) results in a redefinition of  $\sin^2 \theta$ . Both this effect and the direct shift in the  $W$  mass (eqn. (3.14)) affect  $M_W$ . Furthermore, there are also changes in the coupling of the  $Z$  to leptons of the same form as those in eqn. (3.18). None of these effects change our computation of  $R_b$ , and all vanish in the limit that  $c^2 \rightarrow 0$ . However, when combined with the model-dependent radiative corrections due to the technicolor sector [14], they will provide further constraints [15] on  $c^2$  and  $f/u$ .

#### 4. A Non-Commuting Extended Technicolor Scenario

So far we have discussed the possibility of the TC and weak gauge groups' being embedded in the ETC gauge group, but we have avoided any mention of how the fermion representations of these subgroups are embedded in representations of ETC. This is a complicated problem, and we will satisfy ourselves here with a sketch of how this might be done: we will only discuss the generation of the mass of the top quark and will ignore questions about anomalies. What we have in mind is a scenario where an  $SU(N+2)_{ETC} \otimes SU(2)_{light}$  gauge group breaks to  $SU(N)_{TC} \otimes SU(2)_{heavy} \otimes SU(2)_{light}$ . We imagine that the ETC model contains the following representations (among others) of  $SU(N+2)_{ETC} \otimes SU(3)_C \otimes SU(2)_{light}$ :

$$\begin{aligned} &(\mathbf{N} + \mathbf{2}, \mathbf{3}, \mathbf{1}), \\ &(\overline{\mathbf{A}_{N+2}}, \overline{\mathbf{3}}, \mathbf{1}), \end{aligned} \tag{4.1}$$

where  $\mathbf{A}_{N+2}$  is the antisymmetric tensor representation of  $SU(N+2)$  with dimension

$$d(\mathbf{A}_{N+2}) = \frac{(N+1)(N+2)}{2}. \tag{4.2}$$

When  $SU(N+2)_{ETC}$  breaks, we have the following representations under  $SU(N)_{TC} \otimes SU(2)_{heavy} \otimes SU(3)_C \otimes SU(2)_{light}$ :

$$\begin{aligned}
& (\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{1}) , (\mathbf{N}, \mathbf{1}, \mathbf{3}, \mathbf{1}) \\
& (t, b)_L \quad , \quad U_L \\
& (\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1}) , (\bar{\mathbf{A}}_{\mathbf{N}}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1}) , (\bar{\mathbf{N}}, \mathbf{2}, \bar{\mathbf{3}}, \mathbf{1}) \\
& t_R^c \quad , \quad A_R^c \quad , \quad (U, D)_R^c \quad .
\end{aligned} \tag{4.3}$$

After  $SU(2)_{heavy}$  and  $SU(2)_{light}$  mix, we have the following representations under  $SU(N)_{TC} \otimes SU(3)_C \otimes SU(2)_L$ :

$$\begin{aligned}
& (\mathbf{1}, \mathbf{3}, \mathbf{2}) , (\mathbf{N}, \mathbf{3}, \mathbf{1}) \\
& (t, b)_L \quad , \quad U_L \\
& (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1}) , (\bar{\mathbf{A}}_{\mathbf{N}}, \bar{\mathbf{3}}, \mathbf{1}) , (\bar{\mathbf{N}}, \bar{\mathbf{3}}, \mathbf{2}) \\
& t_R^c \quad , \quad A_R^c \quad , \quad (U, D)_R^c \quad .
\end{aligned} \tag{4.4}$$

When  $SU(N)_{TC}$  gets strong the  $(U, D)_R^c$  doublet condenses with the  $U_L$  (and with a  $D_L$  which we have not described), breaking  $SU(2)_L$ . The ETC gauge bosons with masses of order  $g_h f/2$  in this scenario have the following quantum numbers under  $SU(N)_{TC} \otimes SU(3)_C \otimes SU(2)_L$ :

$$(\mathbf{N}, \mathbf{1}, \mathbf{2}) , (\bar{\mathbf{N}}, \mathbf{1}, \mathbf{2}) , (\mathbf{1}, \mathbf{1}, \mathbf{1}) \quad . \tag{4.5}$$

Exchange of the weak-doublet ETC gauge bosons gives rise to the operator (2.1) with  $\xi = 1$ .

## 5. Conclusions

We have explored the possible effects of a new type of ETC model on the  $Z$  partial decay width into  $b\bar{b}$ . Two types of effects were uncovered: a direct vertex correction due to exchange of the ETC gauge boson responsible for top-quark mass generation, and a correction due to mixing of the  $Z$  with a techni-neutral ETC gauge boson ( $Z_2$ ). The former effect increases the  $Z$  partial width, while the latter decreases it.

Unlike the case of commuting ETC models, we cannot make a model-independent statement of the overall size or sign of the change in the  $Zb\bar{b}$  branching ratio: the overall effect may be small and may even *increase* the  $Zb\bar{b}$  branching ratio. It will be important

to explore this class of models further, since experiments at LEP find that the  $Zb\bar{b}$  width lies slightly above the standard model prediction.

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