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Holographic RG and Cosmology in Theories with Quasi-Localized Gravity

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Abstract: We study the long distance behaviour of brane theories with quasi-localized gravity. The 5D effective theory at large scales follows from a holographic renormalization group flow. As intuitively expected, the graviton is effectively four dimensional at intermediate scales and becomes five dimensional at large scales. However in the holographic effective theory the essentially 4D radion dominates at long distances and gives rise to scalar anti-gravity. The holographic description shows that at large distances the GRS model is equivalent to the model recently proposed by Dvali, Gabadadze and Porrati (DGP), where a tensionless brane is embedded into 5D Minkowski space, with an additional induced 4D Einstein-Hilbert term on the brane. In the holographic description the radion of the GRS model is automatically localized on the tensionless brane, and provides the ghost-like field necessary to cancel the extra graviton polarization of the DGP model. Thus, there is a holographic duality between these theories. This analysis provides physical insight into how the GRS model works at intermediate scales; in particular it sheds light on the size of the width of the graviton resonance, and also demonstrates how the holographic RG can be used as a practical tool for calculations.

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1. Introduction

Randall and Sundrum (RS) have recently shown that it is possible to localize gravity to a brane in five dimensional anti-de Sitter space [1, 2]. In this model the theory on the brane reproduces four dimensional Einstein gravity at large distances even though the size of the extra dimension is infinitely large. This idea has sparked a flurry of activity in this field [3–18] (see [19, 20] for related earlier work). Gregory, Rubakov and Sibiryakov (GRS) [21] proposed a modified version of the RS model in which gravity appears five dimensional both at short and at long distances (see [22] for a related idea), while at intermediate scales four dimensional gravity is reproduced. The GRS model has a positive tension brane, as in the RS model, and negative tension branes at a large proper distance away from the positive tension brane on either side of it. These negative tension branes have half the tension of the central brane. The brane tensions and the negative bulk cosmological constant are tuned such that the background is static, and the geometry between the branes is a slice of AdS_5, while beyond the negative tension branes the spacetime is ordinary 5D Minkowski space. The fact that 4D gravity is reproduced at intermediate scales has been explained in [21,23,24], and the reason for this is that in this model the 4D graviton is replaced by a resonance with a finite lifetime, which can decay into the bulk; thus gravity is only quasi-localized. It was also suggested in [24, 25] that there may be a connection in these theories to bulk supersymmetry and vanishing of the cosmological constant. It has been shown in [26, 27], that at intermediate distance scales the theory indeed reproduces the results of ordinary general relativity due to the bending of the brane in the presence of inhomogenous matter on the brane. The reason for this is that the effect of the bending of the brane exactly cancels the effects of the extra polarization in the massive graviton propagator (up to corrections that can be made arbitrary small by adjusting the width of the resonance).

We should stress that one of the essential features of these models is that they do not have a 4D low-energy effective field theory description (see also [25]). Instead of an effective 4D theory, these models have a an effective 5D theory at large distances which can be derived through a holographic renormalization group (RG) flow. This holographically renormalized theory will play the role of the low-energy effective theory. In order to simply perform a calculation at a given energy scale on the brane one performs the RG running to that scale in the theory on the brane. From the conjectured AdS/CFT correspondence the RG flow corresponds to moving the brane a finite distance into the bulk. This procedure is referred to as the holographic RG and will be the key to understanding the theory at large distances. At intermediate energies, the RG flow corresponds to moving the brane inside the AdS slice. Since an exactly AdS bulk corresponds to a conformal field theory, the brane tension of the effective theory remain unchanged. However, at distances large enough that effectively the branes have crossed, the effective low-energy theory will be that of a tensionless brane in 5D Minkowski space. This procedure can also be implemented for models which are smooth versions of the GRS model (see [23]), with the difference that there will be a continuous running in the brane tension of the theory. However, as long as the asymptotic metrics are equivalent to those of GRS the asymptotic form of the low energy effective theory will be the same as for GRS. In fact, a detailed analysis below will show that one finds that at intermediate distances (more precisely at a scale $k^{-1}e^{ky_0}$, where y_0 is the location of the negative tension brane in the GRS model) the model becomes equivalent to the model recently proposed by Dvali, Gabadaze and Porrati [49], where a tensionless brane is embedded into 5D Minkowski space, but there is an additional induced four dimensional curvature term on the brane present. The induced operator is a consequence of the holographic renormalization. In addition, the radion mode of the GRS model (which corresponds to fluctuations of the distance between the two branes) will also be localized on the tensionless brane, as predicted in [48]. This radion field will have a wrong-signed kinetic term, which is needed to cancel the effects of the extra graviton polarization in the DGP model. However, at very long distances, where the graviton mode becomes 5 dimensional, the radion will start to dominate and give rise to a peculiar 4D scalar antigravity, as discussed in [27, 48]. Due to the negative kinetic term of the radion these theories are probably not internally consistent at large scales; but from a purely phenomenological point of view it is still interesting to use these results to see how the cosmology of these models deviates from the ordinary FRW expansion of the Universe at large scales.

The paper is organized as follows: in Section 2 we explain the basic idea behind the holographic renormalization group, and calculate the effective brane tension and induced curvature term on the brane. In Section 3 we review the calculation of the induced radion kinetic term on the effective brane. In Section 4 we use the effective holographic theory obtained in Sections 2 and 3 to calculate the graviton propagator at large distances. We speculate on the cosmology of these models in Section 5, and conclude in Section 6.

2. Holographic Renormalization in Quasi-Localized Gravity Scenarios

Consider a 5D metric of the form

$$ds^2 = dy^2 + e^{-A(y)}\eta_{\mu\nu}dx^\mu\, dx^\nu \ , \tag{2.1}$$

where the warp factor $A(y)$ approaches the AdS form for $|y| \ll y_0$:

$$A(y) \to 2k|y| \ , \tag{2.2}$$

while for $|y| \gg y_0$ the metric becomes flat:

$$A(y) \to \text{constant} \ . \tag{2.3}$$

In the GRS model, [21] this is achieved by simply patching AdS_5 to flat space at some point $y = y_0$:

$$e^{-A(y)} = \begin{cases} e^{-2k|y|} & |y| \le y_0 \\ e^{-2ky_0} & |y| \ge y_0 \ . \end{cases} \tag{2.4}$$

We can also construct examples which smoothly interpolate between AdS_5 and flat space [23]:

$$e^{-A(y)} = e^{-2k|y|} + e^{-2ky_0} \ . \tag{2.5}$$

In this case the cross-over from AdS to flat space occurs over a small region $\delta y \sim k^{-1}$. In both these scenarios we need $e^{ky_0} \gg 1$ so that the cross-over to flat space occurs at a large proper distance from the origin in the transverse space. Since the metric approaches the RS metric for $|y| \ll y_0$, there is a brane located at $y = 0$ with a positive tension $V = 6k/\kappa^2$ (the "Planck brane") on which the matter fields will live. In the GRS model, there are additional branes at $y = \pm y_0$ with negative tensions $-\frac{1}{2}V$. In the smoothed version (2.5), the region of negative tension is smeared over a scale $\delta y \sim k^{-1}$. As pointed out in [23,25] such smooth backgrounds violate positivity, which will give rise to the instability discussed in [48].

In [26], the propagator for gravity on the positive tension Planck brane at $y = 0$ was found to have the form

$$G(x,x')_{\mu\nu,\rho\sigma} = \Delta_5(x,0;x',0)\big(\tfrac{1}{2}\eta_{\mu\rho}\eta_{\nu\sigma} + \tfrac{1}{2}\eta_{\mu\sigma}\eta_{\nu\rho} - \tfrac{1}{3}\eta_{\mu\nu}\eta_{\rho\sigma}\big) - \tfrac{k}{6}\Delta_4(x,x')\eta_{\mu\nu}\eta_{\rho\sigma} \ . \tag{2.6}$$

Here, the first term is what would naïvely be expected, since the tensor structure is five dimensional, and $\Delta_5(x,y;x',y')$ is the scalar Green's function for the background (2.1):

$$\big(e^A\Box^{(4)} + \partial_y^2 - 2A'\partial_y\big)\Delta_5(x,y;x',y') = e^{2A}\delta^{(4)}(x-x')\delta(y-y') \ , \tag{2.7}$$

The unexpected piece is the last term in (2.6), which occurs because matter sources on the brane actually bend the brane, and this effect modifies the braneworld propagator. Notice that the brane-bending term involves the four-dimensional massless scalar propagator defined via

$$\Box^{(4)}\Delta_4(x,x') = \delta^{(4)}(x-x') \ . \tag{2.8}$$

If the negative tension branes are sufficiently far away ($e^{ky_0} \gg 1$), then there is a large intermediate region of four-dimensional distance scales $k^{-1} \ll r \ll k^{-1}e^{3ky_0}$, where $\Delta_5(x,0;x',0)$ is dominated by a zero energy resonance and is approximately $k\Delta_4(x,x')$. In this region the two terms in (2.6) combine into the usual 4D graviton propagator:

$$G(x,x')_{\mu\nu,\rho\sigma} \to kG_4(x,x')_{\mu\nu,\rho\sigma} = k\Delta_4(x,x')\big(\tfrac{1}{2}\eta_{\mu\rho}\eta_{\nu\sigma} + \tfrac{1}{2}\eta_{\mu\sigma}\eta_{\nu\rho} - \tfrac{1}{2}\eta_{\mu\nu}\eta_{\rho\sigma}\big) \ . \tag{2.9}$$

Hence, at these intermediate distance scales conventional gravity is recovered as a consequence of the interplay between the resonance and the bending of the brane. This makes intuitive sense: suppose we send out signals (*e.g.* a gravitational wave pulse) from a point on the brane, along the brane and transverse to it. If the signal along the brane only travels a distance $r \ll k^{-1}e^{ky_0}$, in the same proper time the transverse signal only explores the AdS portion of the transverse space and we expect that the physics on the brane is unchanged from the RS scenario where the AdS space extends out to infinite y [1].

At larger distance scales on the brane the corresponding transverse signal reaches the flat portion of the transverse space and so the physics at scales $r \gg k^{-1}e^{ky_0}$ on the brane will be modified from pure 4D gravity. In fact there is a cross-over region at distance scales $r \simeq k^{-1}e^{3ky_0}$, where $\Delta_5(x, 0; , x', 0)$ becomes the massless scalar propagator in 5D Minkowski space. So at large distances the first term in (2.6) gives rise to a gravitational potential that falls off as $1/r^2$. However, the brane-bending term is always four dimensional and gives rise to a gravitational potential that falls off as $1/r$. Hence at very large distances the brane-bending term dominates and gives rise to scalar anti-gravity as noted in [27].

The same result has been also obtained by Pilo, Rattazzi and Zaffaroni (PRZ) [48], who have calculated the graviton propagator on the brane by identifying all relevant physical modes of the theory. These were shown to be the graviton resonance, and the massless radion field describing the fluctuations of the distance between the two branes, which turns out to have a negative kinetic term. The interpretation of the brane bending calculation summarized above is then that at intermediate energies, where the graviton is effectively localized, the negative kinetic term radion exactly cancels the contribution of the extra polarization in the massive graviton propagator, thus reproducing 4D Einstein gravity. For large distances however, the radion will dominate, and thus lead to the scalar antigravity as predicted in [27].

Below we will show how to use the technology known as the holographic renormalization group to obtain these results. In the meantime we clarify the meaning of holographic renormalization. This procedure corresponds to a renormalization group *coarse graining* which simply describes the effective physics on the brane. As usual, this involves integrating out degrees-of-freedom up to a certain physical length scale on the brane, which we denote as $\tilde{r}$, but which now also includes an averaging over the transverse space out to proper distances $\tilde{r}$. A proper distance r on the brane corresponds to going out to y in the transverse direction, where

$$r = \int_0^y e^{A(y')/2}\, dy' \ . \tag{2.10}$$

This can be understood in two different ways. First, one can calculate how far into the bulk light can travel, while traveling a distance r on the brane. For light $ds^2 = 0$, which yields $dr = e^{A(y)/2}dy$, thus yielding (2.10). Another way of obtaining the correspondence between distances on the brane and the bulk is to check how far into the bulk the horizon of an object with horizon size r on the brane will be penetrating. This can be obtained from examining the Gregory-Laflamme instability for a black string [13]. The result is again given by (2.10). Hence, physical scales $r \gg k^{-1}e^{ky_0}$ on the brane correspond to distances in the transverse space which reach out into the 5D Minkowski portion of the space in the GRS model.

Fortunately, recent developments in string theory [29] suggest the tools we need to do the averaging which determines the effective theory on the brane at large distances. The technology is known as the holographic RG [30–35]. The idea is that integrating out short distance degrees-of-freedom up to a four-dimensional length scale $\tilde{r}$, according to a brane observer, gives rise to an

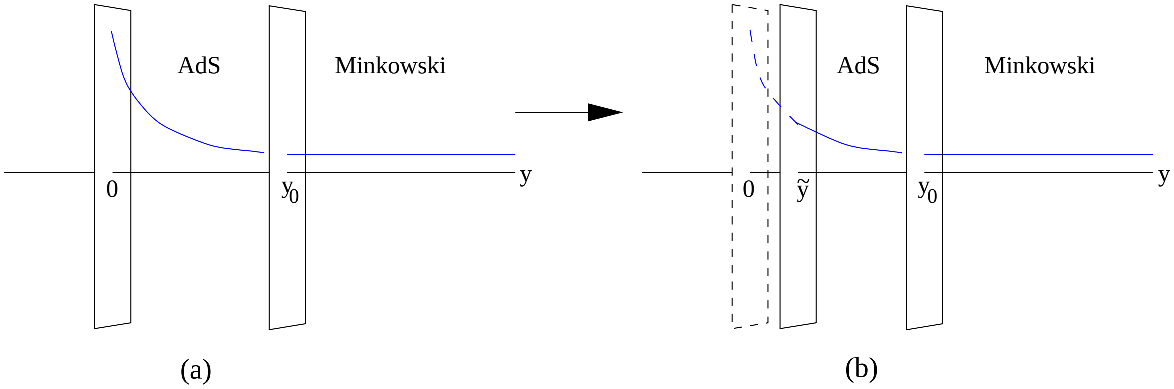

Figure 1: (a) The GRS brane set-up and warp factor. (b) Brane translation associated with the holographic RG running.

effective theory that is described by a theory with the position of the brane shifted in the transverse space to $\tilde{y}$, where

$$\tilde{r} = \int_{-\infty}^{\tilde{y}} e^{A(y)/2}\, dy \ . \tag{2.11}$$

(In the above, we are assuming that $A(y) = 2k|y|$, the AdS form, for $y < 0$, and so when the brane is at $y = 0$ the cut-off is k^{-1}.) This is illustrated in Figure 1. More precisely, we cut out the region $-\tilde{y} \leq y \leq \tilde{y}$ and re-glue the two portions as illustrated in Figure 2. The effective theory is then described by a metric

$$d\tilde{s}^2 = dy^2 + e^{-\tilde{A}(y)}\eta_{\mu\nu}dx^\mu\, dx^\nu \ , \tag{2.12}$$

where

$$\tilde{A}(|y|) = A(|y| + \tilde{y}) \ , \tag{2.13}$$

and we have shifted $|y| \to |y| - \tilde{y}$ so that the effective brane remains at $y = 0$. Notice that in the effective background (2.12) the negative tension branes of the GRS model, or the regions where they are smoothed, lie at a smaller proper distance $k^{-1}e^{k(y_0-\tilde{y})}$ from the effective (renormalized) brane. Notice also that for an AdS background, the effective metric (2.12) is identical to the original metric, expressing the fact that this situation describes a fixed-point of the RG flow.

In general, the effective brane theory will have a renormalized tension, unless the background is exactly dS_5, AdS_5 or 5 dimensional Minkowski space. In addition to the renormalized brane tension, there could be additional operators induced in the effective brane action, encoding the effects of the metric fluctuations integrated over the strip.[2] This is familiar from ordinary RG

[2]In addition to the effects we are describing there are purely quantum renormalization effects which are discussed in ref. [35].

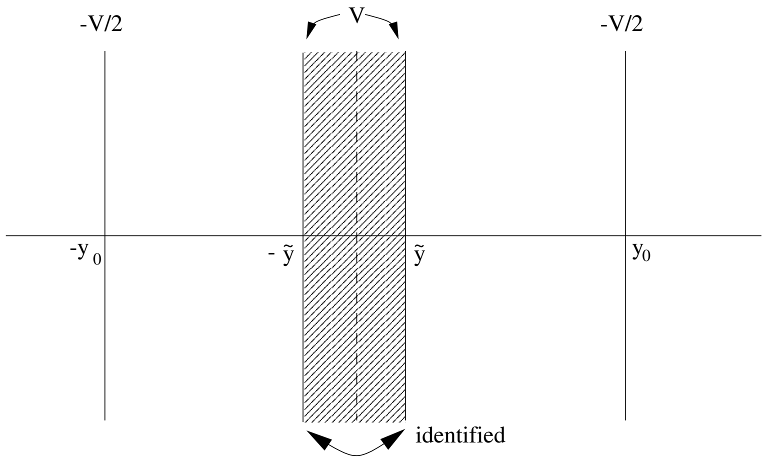

Figure 2: The holographic RG: the Planck brane is translated to $\tilde{y}$ and identified with the image brane at $-\tilde{y}$.

running:q when relevant operators are not present in the underlying (UV) theory, they can still be generated in the effective (IR) theory. The presence of these induced operators will ensure that the infrared physics is kept constant.[3] First we discuss the renormalization of the effective brane tension. This is simply obtained from the Israel junction condition at the position of the effective brane:

$$\left[\frac{\partial}{\partial y} g_{ij}\right]\Big|_{\tilde{y}} = -\frac{2\kappa^2}{3} g_{ij} \widetilde{V}(\tilde{y}) \ , \tag{2.14}$$

which implies,

$$\widetilde{V}(\tilde{y}) = \frac{3A'(\tilde{y})}{\kappa^2} \ . \tag{2.15}$$

One can obtain the same result on the effective brane tension by explicitly integrating the action over the strip between 0 and $\tilde{y}$, and requiring that the contribution of the renormalized brane tension to the effective 4D cosmological constant matches the combined contributions of the original brane tension and the curvature of the strip we have integrated over. It is easy to see explicitly how this works in the general background metric (2.1). The action is given by

$$S = -\int dy d^4x \Big[\sqrt{g}\left(\frac{1}{2\kappa^2} R + \Lambda(y)\right)\Big] - \int dy d^4x \sqrt{\tilde{g}} \Big[\mathcal{V}(y) + V\delta(y)\Big] \ , \tag{2.16}$$

where $\tilde{g}$ is the induced four dimensional metric at constant y, and $\Lambda(y) = -\frac{3}{2}\kappa^{-2}A'(y)^2$, $\mathcal{V}(y) = \frac{3}{2}\kappa^{-2}A''(y)$ (away from the branes) are the static source terms [8] needed to produce the metric (2.1). The source term $\mathcal{V}$ corresponds to a "smeared brane tension", not to be confused with the actual tension V or the effective tension $\tilde{V}(\tilde{y})$. Note that with these conventions Einstein's

[3]The first version of this paper was missing these induced operators, and thus neglected some important effects due to fluctuations of the graviton and radion modes.

equation is given by $G_{ab} = \kappa^2(\Lambda(y) g_{ab} + (\mathcal{V}(y) + V\delta(y)) g_{\mu\nu}\delta^\mu_a \delta^\nu_b)$. Our renormalization procedure corresponds to performing the integral in the action (2.16) over a slice between 0 and $\tilde{y}$, and replace this space with an effective brane with tension $\tilde{V}(\tilde{y})$. Thus the definition of the effective brane tension is

$$-\int_0^{\tilde{y}} dy \Big[\sqrt{g}\big(\frac{R}{2\kappa^2} + \Lambda(y)\big) + \sqrt{\tilde{g}}\mathcal{V}(y)\Big] + \tilde{V}(0) = \sqrt{\tilde{g}(\tilde{y})}\Big(- \frac{R^{(sing)}(\tilde{y})}{2\kappa^2} - \tilde{V}(\tilde{y})\Big), \tag{2.17}$$

where $R^{(sing)}(\tilde{y})$ is the singular piece in the curvature at the position of the effective brane. Using the expression for the curvature $R = -\frac{10}{3}\Lambda(y) - \frac{8}{3}\mathcal{V}(y) - \frac{8}{3}\tilde{V}(\tilde{y})\delta(y - \tilde{y})$, we find the formula for $\tilde{V}(\tilde{y})$ given in (2.15).

Let us now discuss the operators induced in the effective brane action. In a general background, there are a variety of terms. The simplest of these is a four-dimensional induced Einstein-Hilbert term of the form,

$$\int d^4x \sqrt{g^{ind}} R^{(4)}, \tag{2.18}$$

where g^{ind} is the induced metric at the effective brane, and $R^{(4)}$ is the curvature scalar calculated from the induced metric. One way to obtain the GRS model is as a limit of the two-strip model introduced in [48]. This model is just like the GRS model with two branes, except that the second brane tension is chosen to be smaller in magnitude than $-V/2$; therefore the space past the second brane is also AdS_5, except with a different cosmological constant. This model has a localized graviton, and is qualitatively very similar to the RS model, except that there is an additional radion mode appearing in the theory. We can calculate the effective brane action in the two-strip model, and take the limit where the second brane tension goes to $-V/2$. Following [47, 48] we write the metric in the form,

$$ds^2 = e^{-A(y)} \left(1 + B(y)\, f(x)\right) \left(\eta_{\mu\nu} + h_{\mu\nu}(x, y) + 2\epsilon(y)\partial_\mu\partial_\nu f(x)\right)\, dx^\mu\, dx^\nu + \left(1 - \frac{2B'(y)}{A'(y)} f(x)\right) dy^2. \tag{2.19}$$

Note that $B'(y)/A'(y)$ is a smooth function, even though $A'(y)$ and $B'(y)$ are discontinuous. Here, $h_{\mu\nu}$ are the four-dimensional graviton fluctuations including the zero mode of the two-strip model and the Kaluza-Klein modes, f is the radion, and in the GRS model the background warp factor $A(y) = 2k|y|$ for $|y| < y_0$ and $A(y) = 2ky_0$ for $|y| > y_0$.

In order to calculate the coefficient of the induced operator (2.18), we will approximate the the graviton fluctuation in (2.19) by the zero mode $h_{\mu\nu}(x, y) = h_{\mu\nu}(x)$. This will be a good approximation as long as the width of the resonance is small. Expanding the 5D bulk curvature term $\sqrt{g}R$ to zeroth order in f we find it contains the operator $e^{-A(y)}\sqrt{\tilde{g}}\tilde{R}^{(4)}$, where $\tilde{g}$ and $\tilde{R}^{(4)}$ correspond to the 4D metric $\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. Thus we can integrate this term over the strip from 0 to $\tilde{y}$, to obtain the coefficient of the 4D Ricci scalar part of the effective brane action, $2\int_0^{\tilde{y}} e^{-A(y)} dy$.

If we wish to write this operator in the effective brane action in terms of the induced metric g^{ind}, then we have

$$S_{Ricci\,\mathrm{eff}} = 2\int_0^{\tilde{y}} e^{-A(y)} dy \int d^4x \sqrt{\tilde{g}} \tilde{R}^{(4)} = 2e^{A(\tilde{y})} \int_0^{\tilde{y}} e^{-A(y)} dy \int d^4x \sqrt{g^{ind}} R^{(4)}. \tag{2.20}$$

However, this is not the full story. In the GRS model (and the two strip model in general) the radion is relevant in the infrared, and induces additional operators in the effective brane action.

3. The Radion in the GRS Model

In this section we study the induced radion kinetic term on the effective brane, following the work of Pilo, Rattazzi and Zaffaroni [48]. We start with the action

$$S = \int \sqrt{g}\,(M_*^3 R - \Lambda(y))\, d^5x - \sum_i \int \sqrt{g_i^{ind}}\, V_i\, d^4x, \tag{3.1}$$

where the bulk integral can be split into two regions $(0, y_0)$ and (y_0, ∞) (where y_0 is the position of the negative tension brane) with cosmological constants Λ_1 and Λ_2 respectively, and the brane integrals involve the brane tensions V_i and induced metrics g_i^{ind} on the two branes.

The form for $B(y)$ in the metric (2.19), in the region between the positive and negative tension branes is determined from the linearized Einstein equations by insisting that the 4D graviton fluctuations decouple from the radion. The four-dimensional components of the linearized Einstein equations in the bulk are proportional to,

$$e^{2ky}(2k\, B(y) - B'(y)) + 2k(-4k\, \epsilon'(y) + \epsilon''(y)) = 0, \tag{3.2}$$

which is solved by [48],

$$B(y) = 2\left[\alpha e^{2ky} + k\, e^{-2ky}\, \partial_y \epsilon(y)\right]. \tag{3.3}$$

The coefficient α is arbitrary, but in order for the radion coupling to matter [42,50] to be normalized the same as the longitudinal graviton we choose it to be $\alpha = 1$. In the region beyond the negative tension brane, $y > y_0$, $B(y)$ takes the form,

$$B(y) = k'\, e^{-2k'y}\, \partial_y \epsilon(y), \tag{3.4}$$

where for the GRS scenario we take the limit $k' \to 0$. In this region (for any k') the radion dependent part of the metric is pure gauge and the equations of motion are satisfied for any $f(x)$ [47, 48]. The matching conditions at the positive and negative tension branes determine [48]

$$B(0) = 2, \quad B(y_0) = 2e^{2ky_0} \frac{k'}{(k' - k)}. \tag{3.5}$$

Note that $B(y_0) \to 0$ for the GRS model.

The pure gauge part of the action does not contribute to the effective theory on the brane, so with the above gauge choice for the metric the radion effective action is given completely by integrating over the region between the positive and negative tension brane.

As described in the previous section the fluctuation independent part of the action is made to vanish by the usual fine tuning of cosmological constants and brane tension, which for GRS is, $\Lambda_1 = -12\,k^2 M_*^3$ between the branes, $\Lambda_2 = 0$ beyond the negative tension brane; $V_1 = 12\,k\,M_*^3$ and $V_2 = -6\,k\,M_*^3$ are the brane tensions of the positive and negative tension brane, respectively.

We focus on the radion kinetic terms in the action. The calculation is simplified as in [48] by expanding about the background metric, so that,

$$S_{\rm radion} = -\int_{-y_0}^{y_0} dy \int d^4x\,\sqrt{g}\;\delta g^{\mu\nu}\left[M_*^3 R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\left(M_*^3 R - \Lambda - \sum_i V_i\,\delta\left(\sqrt{g_{yy}}(y-y_i)\right)\right)\right], \tag{3.6}$$

where $\delta g^{\mu\nu}$ is the $\mathcal{O}\left(f(x)\right)$ term in $g^{\mu\nu}$, and the equation of motion is expanded to linear order in $f(x)$. Given the ansatz (3.3) for $B(y)$, the equations of motion identically vanish to linear order except for the (yy) component. This follows from the requirement that the equations of motion for $h_{\mu\nu}$ and f are decoupled. The delta function contributions to the equations of motion identically vanish given the ansatz (2.19), which leaves a single surface term for the radion kinetic part of the action:

$$S_{\rm radion} = \frac{M_*^3}{k}\int_0^{y_0} dy \int d^4x\sqrt{\tilde{g}}\,3B'(y)\,f\Box f. \tag{3.7}$$

Separating the integral over the strip $(0,\tilde{y})$ from the remainder of the bulk, the full action for the reduced bulk and renormalized brane is:

$$\begin{aligned} S[\tilde{y}] = & \int_{|y|>\tilde{y}} \sqrt{g}\,(M_*^3 R - \Lambda(y))\,d^5x - \sum_i \int \sqrt{g_i^{ind}}\,V_i\,d^4x \\ & + \int d^4x\sqrt{\tilde{g}}\left[\frac{M_*^3}{k}(1-e^{-2k\tilde{y}})R^{(4)} + \frac{3M_*^3}{k}\left(B(\tilde{y}) - B(0)\right) f\Box f\right], \end{aligned} \tag{3.8}$$

where in the bulk integral the region $(0,\tilde{y})$ has been removed and the brane tension of the positive tension Planck brane V_1 is replaced by its renormalized value $V_1(\tilde{y})$ (which is, however, constant in the GRS scenario for $\tilde{y} < y_0$). When $\tilde{y} \geq y_0$ there is a single tensionless brane in the effective theory, which is (aside from the ghostlike radion contribution) the DGP model [49]. This is a new type of holographic duality between the GRS and DGP models, meaning that the DGP model is in fact (modulo the radion) nothing else but the low-energy effective theory of the GRS model.

4. The Graviton Propagator from Holographic RG in the GRS Model

Before we calculate the renormalized graviton Green's function in the GRS model, we pause to consider the rationale behind the holographic renormalization group in more detail. Performing a renormalization group transformation by moving branes through a five dimensional space may at first seem bizarre to those not familiar with recent developments in string theory. However we can arrive at a simple understanding of why this procedure is reasonable by recalling some simple classical mechanics. In order to calculate the potential due to some mass we can find an equipotential Gaussian surface and require that the integral of the gradient of the potential over this surface is equal to the mass up to appropriate factors (like Newton's constant). In a 5D setting we see that an equipotential surface of a gravitational source on the brane penetrates some distance into the bulk around the brane. The ratio of this distance to the corresponding distance on the brane depends in detail on the 5D metric. However as we look at longer distances on the brane, we are probing further into the bulk. Furthermore different energy/matter distributions (including distributions that penetrate into the bulk) can have the same equipotential surface. Thus we are performing an averaging over the short-distance propagation of the 5D gravitons. In GRS type geometries, at large enough distances the equipotential surface will include part of the 5D Minkowski space. At sufficiently large distances most of the interior of the equipotential surface will be 5D Minkowski and the gravitational potential will be that of 5D Minkowski space (ignoring the radion for now), falling like the inverse of the distance squared. Another way of saying this is that at distances much larger than the proper distance at which the geometry becomes flat, $\sim k^{-1}e^{ky_0}$, we would not be able to resolve the small slice of AdS_5 and we would effectively see a tensionless brane in 5D Minkowski space. However, this does not automatically imply that the theory on the brane will be 5D Minkowski gravity. This is because there are additional operators in the effective action on the tensionless brane, which can have important consequences. For example, the scalar anti-gravity behavior noticed in [27] is entirely due to the radion field which is localized on the tensionless brane as envisioned in [48], and does not probe the bulk.

We now discuss the long distance behavior of the GRS model from the point of view of the holographic picture explained above. We have seen that at the length scale $k^{-1}e^{ky_0}$ the holographic effective theory (corresponding to integrating out $\tilde{y}$ to that scale) is described by a tensionless brane in a 5D Minkowski space, with the action

$$\int d^5x M_*^3 e^{-2ky_0}\sqrt{g}R + \int d^4x \left(M_*^3 2b\sqrt{\tilde{g}}R^{(4)} - cf\Box f\right), \tag{4.1}$$

where

$$b = \frac{1-e^{-2ky_b}}{2k}, \quad c = \frac{6M_*^3}{k}. \tag{4.2}$$

In the first term of (4.1) the factor e^{-2ky_0} has been scaled out of the metric so as to restore the canonical flat 5D background metric $g_{\mu\nu} = \eta_{\mu\nu}$. This effective theory is nothing other than the model recently proposed by Dvali, Gabadadze and Porrati [49], in which there is a tensionless brane in 5D Minkowski space with an additional induced 4D curvature term, but with the ghost-like radion field necessary to cancel the effects of the extra graviton polarization automatically present on the brane. In terms of their notation the effective 5D Planck scale is given by

$$\tilde{M}^3 = e^{-2ky_0} M_*^3, \tag{4.3}$$

and the effective 4D Planck scale is given by

$$M_{Pl}^2 = \frac{M_*^3}{k}(1 - e^{-2ky_0}). \tag{4.4}$$

In [49] it was shown that the graviton propagator of this theory (without the radion) is approximately that of an almost massless 4D graviton up to distance scales of order $\tilde{r}_0 = \frac{M_{Pl}^2}{\tilde{M}^3}$, and after that the theory becomes 5D gravity. With our parameters, $\tilde{r}_0 = k^{-1}e^{2ky_0}$, but since this is already the effective theory, this scale $\tilde{r}_0$ is related to the original scales on the brane by $r_0 = \tilde{r}_0 e^{ky_0}$, therefore we recover the results of [26, 27, 48]: the GRS model reproduces ordinary 4D Einstein gravity up to the scale $r_0 = k^{-1}e^{3ky_0}$, and after that scale the radion will dominate and give 4D scalar anti-gravity (and the graviton will contribute as in ordinary 5D Minkowski gravity). This holographic approach thus sheds light on the appearance of the somewhat mysterious scale $k^{-1}e^{3ky_0}$.

We now show the detailed calculation of the renormalized propagator in the GRS model. Consider renormalizing from the Planck brane to a renormalized effective brane at the position of the negative tension brane y_0. The induced brane action for the graviton on the renormalized brane is given by the second term in Eq. (4.1) To find the Green's function in the effective theory we follow the method of Giddings, Katz, and Randall [15]. The equation for the graviton Green's function in transverse-traceless gauge is (after Fourier transforming from brane coordinates to the brane momentum p and taking $p^2 = -q^2$, and setting $M_* = 1$ for simplicity):

$$(\partial_y^2 - 2A'\partial_y + e^A q^2 + 2e^{2A} b q^2 \delta(y - y_0))\Delta(q, y, y') = e^{2A}\delta(y - y') \tag{4.5}$$

This can be put in self-adjoint form by rescaling Δ by metric factors:

$$\Delta(q, y, y') = e^{A(y)+A(y')}\hat{\Delta}(q, y, y') \ . \tag{4.6}$$

We then have

$$(\partial_y^2 + A'' - (A')^2 + e^A q^2 + 2e^{2A} b q^2 \delta(y - y_0))\hat{\Delta} = \delta(y - y') \tag{4.7}$$

For $y, y' \geq y_0$ the Green's function can be constructed by patching together the solutions of the corresponding homogeneous equation with $y < y'$ and $y > y'$, which we refer to as $\Delta_<$ and $\Delta_>$ respectively:

$$\hat{\Delta} = \theta(y - y')\Delta_> + \theta(y' - y)\Delta_< \tag{4.8}$$

Plugging the patched solution into Eq. (4.7) for $y' \neq y_0$ yields:

$$\Delta_<|_{y=y'} = \Delta_>|_{y=y'} \tag{4.9}$$
$$\partial_y(\Delta_> - \Delta_<)|_{y=y'} = 1 \tag{4.10}$$

Since we are interested in the graviton Green's function and not just the scalar Green's function, we will impose the boundary condition implied by the Israel jump condition which relates the jump in the derivative of the metric to the brane tension. For the linearized fluctuation h_{ij} around the background (in the absence of a Ricci scalar term in the brane action) we have

$$\left[\frac{\partial}{\partial y} h_{ij}\right]|_{y=y_0} = -2A'(y_0)h_{ij} \tag{4.11}$$

Taking account of the induced Ricci scalar term in the effective brane action we find that the correct boundary condition is:

$$\partial_y \Delta_<|_{y=y_0} = -(A' + e^{2A}b)\Delta_<|_{y=y_0} \ . \tag{4.12}$$

In that case the implied discontinuity in the derivative of the Green's function at $y = y_0$ is just what is required by Eq. (4.7) due to the δ function piece of A'' and the induced Ricci scalar.

Setting $y' = y_0$ in Eqs. (4.9) and (4.10) and combining with Eq. (4.12) yields

$$\partial_y \Delta_>|_{y=y_0} + (A' + e^{2A}bq^2)\Delta_>|_{y=\tilde{y}_0} = 1 \tag{4.13}$$

as the boundary condition at the renormalized brane, which we can use to determine the normalization of the homogeneous solution. The solution of the homogeneous equation in the Minkowski region are just plane waves. The choice that satisfies the outgoing wave boundary condition at infinity is

$$\Delta_> = N e^{iqye^{ky_0}} \ , \tag{4.14}$$

where N is a normalization factor.

Using the normalization condition (4.13) we can determine N and find that the Green's function on the renormalized brane ($y = y' = y_0$) is given by:

$$\hat{\Delta}(q, y_0, y_0) = \frac{1}{iqe^{ky_0} + bq^2 e^{4ky_0}} \tag{4.15}$$

Rescaling back to the physical Green's function via (4.6) we find

$$\Delta(q, y_0, y_0) = \frac{e^{4ky_0}}{iqe^{ky_0} + bq^2 e^{4ky_0}} \ , \tag{4.16}$$

which agrees for $q < ke^{-ky_0}$ with the full (and tedious) Green's function calculation in the two-strip model with $k' \to 0$. For $k \gg q \gg ke^{-3ky_0}$, using (4.2) this is approximately,

$$\Delta(q, y_0, y_0) \approx \frac{2k}{(1 - e^{-2ky_0})q^2} \ . \tag{4.17}$$

which is the expected 4D propagator for intermediate scales. Including the 5D Planck scale couplings we can read off the effective Newton's constant at intermediate scales to be:

$$G_N = \frac{2k}{(1 - e^{-2ky_0})M_*^2} \tag{4.18}$$

Alternatively, for small $q \ll ke^{-3ky_0}$, (4.16) reduces to

$$\Delta(q, y_0, y_0) \approx -\frac{ie^{3ky_0}}{q} + \frac{1}{2k}(e^{6ky_0} - e^{4ky_0}) \ . \tag{4.19}$$

Thus we see that (neglecting the effects of the radion) at long distances the graviton Green's function goes over to 5D behavior, with an associated gravitational potential that falls off like $1/r^2$.

5. Cosmology of Theories with Quasi-localized Gravity

We have seen in the previous section that, as predicted in [27, 48], the GRS model (and theories with quasi-localized gravity in general) interpolates between ordinary 4D Einstein gravity and (due to the presence of the radion field with negative kinetic term) 4D scalar anti-gravity. Since the cosmology of brane models has recently attracted a lot of attention [37–45], we will briefly sketch how the cosmology of the GRS model would work, even though due the radion instability this model is probably not a realistic model of the Universe. First we consider the case of pure radiation on the positive tension brane in the GRS model. Since the energy-momentum tensor for radiation is traceless, the radion does not couple to this type of source. Thus for pure radiation, the effect of the radion should be negligible at the classical level. Therefore, for a radiation dominated universe, the GRS model interpolates between the RS model at high (but below M_{Pl}) energies and a tensionless brane in 5D Minkowski space at very low energies (that is at very large distances). We expect the expansion of the Universe to be dominated by the largest scales, since the gravitational energy itself is dominated by the largest distances. This is so because the gravitational energy in a sphere grows as the square of the radius in the case of 4D gravity (and as the radius for 5D gravity). The reason is that even though the potential is decreasing with the distance, there is much more matter close to the edge of the sphere than in the middle. Therefore we will assume that the expansion of the Universe should be described by the effective holographic brane model with the cut-off given by the size of the Universe (that is, the region in causal contact since the Big Bang). This immediately indicates the type of expansion one expects here: as long as the size of the

Universe is smaller than the distance where Einstein gravity turns into 4D anti-gravity, we expect the expansion to be given by that of a brane in AdS space, that is the ordinary Friedmann equation. Once the size of the Universe reaches the critical distance, it will be given by the cosmology of a tensionless brane in 5D Minkowski space. Binétruy, Deffayet and Langlois (BDL) showed that the cosmology of a tensionless brane in Minkowski space does not reproduce the ordinary Friedmann equations; instead it predicts a Hubble law of the form $H^2 \propto \rho^2$, where ρ is the energy energy density of matter on the brane [37]. Thus, once the critical distance is reached, the expansion will change to that of BDL. Since in this case $H^2 \propto \rho^2$ instead of the ordinary $H^2 \propto \rho$, the expansion will change once the 5D phase is reached. The transition will not be as abrupt as in the largest-scale-dominance approximation, however, away from the transition region largest-scale-dominance should be a good approximation. The expansion equation for this case has been investigated in detail in [39]. The Friedmann equation is given by

$$\frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0}\right)^2 = -\frac{\kappa^4}{36}\rho(\rho + 3p), \tag{5.1}$$

where $a_0(t)$ is the scale factor at the positive tension brane, and ρ, p are the radiation energy and pressure densities introduced on the brane as

$$T^{\text{brane}\ \mu}_{\ \ \ \ \ \ \ \nu} = b^{-1}\delta(y)\ \text{diag}(-\rho, p, p, p)\ , \tag{5.2}$$

and the five dimensional metric (which includes both the background and the expansion due to the matter sources on the brane) is given by

$$ds^2 = b^2(y,t)dy^2 + a^2(y,t)d\vec{x}^2 - n^2(y,t)dt^2\ . \tag{5.3}$$

In [39] it was shown, that there are two types of solutions for the above equation (5.1). One is the solution obtained in [37], for which $a_0(t) \sim t^{\frac{1}{4}}$, while for the other solution

$$a_0(t) \sim t^{\frac{1}{2}}\left(1 - \frac{\kappa^4}{36}\frac{\rho^2(t_i)a_0(t_i)^8}{t^2} + \dots\right). \tag{5.4}$$

Thus for the case of pure radiation, there are two known solutions to the expansion equations. In one solution the expansion of the Universe would slow down to $t^{\frac{1}{4}}$, while for the other solution (if the densities are small compared to the expansion rate at the critical distance) the solution remains an essentially unchanged $t^{\frac{1}{2}}$ expansion law, up to small corrections. Which solution is actually realized depends on initial conditions but we expect that for generic initial conditions (where the radiation dominated universe expands normally in the intermediate 4D regime) that the second solution should hold to good approximation.

The situation will be very different for other types of matter, for example consider an ordinary matter dominated Universe. In this case, the expansion will again start out as an ordinary 4D expansion, given by the power law $a_0(t) \sim t^{\frac{1}{3}}$. However, for distances larger than the critical distance, the radion will start dominating the expansion. We will approximate the expansion equations for this case by assuming that we have a tensionless brane in 5D Minkowski space, plus the radion localized on in. At distances where the 5D nature of gravity becomes apparent, the induced 4D curvature term can be neglected compared to the 5D curvature term. In this case, one can derive the general expansion equations, which will now also involve the radion field φ. To find the expansion equations, we start with an effective action at very large distances of the form

$$\int d^5x \frac{1}{2\kappa^2}\sqrt{g}R + \int d^4x \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi, \tag{5.5}$$

where $\varphi = \sqrt{2c}f$ represents the radion field with the wrong-signed kinetic term, and κ here stands for the Newton's constant in the effective holographic theory, which is related by the warp factor to the fundamental parameter of the the model. Similarly to [37], one can derive the expansion equation for this action. The difference compared to [37] will be that the scalar is localized on the effective brane as well, therefore they will appear as additional delta-function like sources, which will modify the "jump-equations" to

$$\frac{[a']}{a_0 b_0} = -\frac{\kappa^2}{3}\rho + \frac{\kappa^2}{6n^2}\dot{\varphi}^2, \quad \frac{[n']}{n_0 b_0} = \frac{\kappa^2}{3}(3p+2\rho) + \frac{\kappa^2}{6n^2}\dot{\varphi}^2, \tag{5.6}$$

where $[a']$ denotes the jump of the y derivative of the scale factor at the position of the brane. Then similarly to [37] one obtains the expansion equation by substituting these jumps into the 55 component of Einstein's equation. The result is given by

$$\frac{\ddot{a}_0}{a_0} + \left(\frac{\dot{a}_0}{a_0}\right)^2 = -\frac{\kappa^4}{36}\rho(\rho+3p) + \frac{\kappa^4}{72}\dot{\varphi}^4 + \frac{\kappa^4}{72}\dot{\varphi}^2(\rho+3p), \tag{5.7}$$

where we have rescaled time such that $n_0 = n(0) = 1$. However, this is not the whole story, since ρ and p are also the sources for φ, which is given by the scalar equation of motion on the brane [26, 27]. The linear coupling to matter on the brane (neglecting the dimension 6 derivative couplings) is given by

$$\varphi\frac{\delta S}{\delta\varphi}|_{\phi=0} = \varphi\frac{\delta S}{\delta(g^{ind})^{\mu\nu}}\frac{\delta(g^{ind})^{\mu\nu}}{\delta\varphi} = -\sqrt{g^{ind}}\tilde{T}_{\mu\nu}e^{-A(0)}\tilde{g}^{\mu\nu}\frac{B(0)}{2}\frac{\varphi(x)}{\sqrt{2c}} \ . \tag{5.8}$$

Note, that the matter is assumed to be at the original Planck brane, therefore there will be no additional warp factor appearing in the coupling. Thus the radion equation of motion is

$$\Box\varphi = -\frac{1}{\sqrt{2c}}(3p-\rho), \tag{5.9}$$

which, assuming that there is only time dependence, would give the equation

$$\ddot{\varphi} = \frac{1}{\sqrt{2c}}(3p-\rho). \tag{5.10}$$

Thus for general matter the expansion is determined by the coupled equations (5.7) and (5.10). Due to the antigravitational nature of the interaction mediated by the radion we expect that (like in [39]) the Universe would reach a maximal size and then recollapses. Since the ghostlike radion is likely to make the model unstable anyway, we will not pursue the solutions of the expansion equation sketched above.

6. Conclusions

We have examined a class of 5D metrics with embedded 3-branes. Along the extra dimension these models are asymptotically Minkowski, and gravitons are quasi-localized on a brane. One would expect that at sufficiently large distances the details near the brane are irrelevant and effectively there is a tensionless brane in 5D Minkowski space (a brane with non-zero tension would curve the space around it). We find that a holographic renormalization group analysis confirms this intuitive picture for the graviton. The renormalization group analysis also shows how the radion effectively gives rise to scalar "anti-gravity" at long distances by an induced radion coupling on the brane with negative kinetic term. In fact, at intermediate and large distances the holographic effective theory is equivalent to the recently proposed model of Dvali et al., where the tensionless brane in 5D Minkowski space also has an induced 4D curvature term on the brane. Thus the behavior of quasi-localized gravity in GRS-type models at different length scales is as follows: at very short distances the theory is five dimensional (both scalar potential and tensor structure); at intermediate scales it is given by ordinary 4D gravity with corrections that can be arbitrarily small; and at ultra-large distances the graviton is again five dimensional and the 4D radion dominates. Thus these models do not seem to be internally consistent; however, if a generalized model could eliminate the radion from the light degrees of freedom (as can happen in RS models) they might produce viable cosmologies which decelerate after a late epoch. To be consistent with current observations this epoch must be later than the current epoch.

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